The Seasonal Effect of Working Conditions of an Ice-cream Plant

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Abstract

An ice-cream plant's workings are analyzed in the summer and winter seasons of the paper. The ice-cream unit along with the other three units i.e., flavoring, freezing and combined flavouring and freezing units are always operational in summers, due to the high demand, while in winters the combined flavoring and freezing unit is kept in cold standby as a backup in case there is a demand for ice-cream. In this work, the semi-Markov process and the regenerative point technique have been used to analyze the system. Numerical analysis has been conducted using MATLAB. A variety of measures have been developed to evaluate the effectiveness of a system. The Code Blocks have been used in interpreting the graph in the specific case presented. All evaluation is based on the milk production data collected by the plant. Improvements to the system performance will lead to increased profits. Similar techniques can be applied to other systems.

Keywords: Seasonal functioning, semi-Markov process, Regenerative point technique, profit.

1. Introduction

The primary goal of any industry is to upgrade production through technological interventions and so, is the goal of the dairy industry i.e to improve their production operations to attain competitiveness. Physical models for predicting ice cream's thermal properties were developed by [1]. Analysis of reliability modeling for 2-out-of-3 redundant system was done by [2]. Profit analysis of a two unit standby oil delivering system where priority is given to partially failed unit over the completely failed unit for repair was analyzed by [3]. Reliability models for the fertilzer industry were pioneered by [4], [5]. Availability optimization of ice cream making unit of milk plant was discussed by [6]. Optimized scheduling, production planning and RAM study of an ice-cream plant was given by [7], [12] respectively. Reliability and profit where stand by units functions to accommodate the required demand was analyzed by [8] for system evaluation. Availability analysis of a skim milk powder and profit analysis of a butter-oil production in dairy industry was discussed by [9] respectively. Profit analysis of a system where operation is affected by temperature was discussed by [10]. Description of the four subsystems of the butter-oil production process- the melting vats, the boilers, the clarifier and the settling tanks was given by [11]. The probability of a three-unit induced draft fan system with one standby unit in a working condition was given by [13]. Modeling of two-Unit cold standby system was discussed by [14]. On the basis of progressively censored first-failure data, a problem of estimating parameters for an exponential distribution class and hazard rate functions is studied by [15].
But, none of them have discussed the working of an ice-cream plant. So, in this paper functioning of an ice-cream plant is discussed. The production of functional ice-cream plant consists of one main unit and three units grouped in parallel but in series with the unit 1. The main unit (unit 1) consists of heating, emulsifying, pasteurization, homogenization and ageing. Unit 2 is flavoring unit, unit 3 is freezing unit and unit 4 is combined flavoring and freezing. In summers, due to

high demand the whole system is operative whereas in winters, the system goes to cold standby state and undergoes maintenance. It only operates when the demand occurs. In that case the units 2, 3 along with the unit 1 operates and the unit 4 is in cold standby state and operates on the failure of either of the units 2, 3 or on the failure of both. The system goes to a failed state a) on the failure of the unit 1 or b) on the failure of unit 2 with unit 4 or c) on the failure of unit 3 with unit 4.
Semi-Markov process and regenerative point technique is used to obtain measures of system effectiveness in steady state that include MTSF, availability of the system, busy period of repairman for repair and maintenance, expected number of repair and maintenances, profit of the system.

2. Methods

The stages of methodlogy carried out is given below:

1. In the beginning, industry data on failure rates and maintenance was collected over a five-year period.
2. A comprehensive understanding of how the unit operates is the second step. Through that, reliability models are generated.
3. MATLAB is used to obtain reliability measures using semi-Markov processes and regenerative point techniques that include:
   - Transition probabilities and mean sojourn time in steady state
   - MTSF of the system.
   - Long term availability for the system.
   - Bus period analysis of the repairman.
   - Expected number of repairs.
   - Additionally, the system's profit potential is analyzed graphically.
4. In the following step, graphical analysis is performed by using excel and code blocks on a particular example of exponential distribution.
5. Furthermore, reliability can be improved by identifying key machines and faults, making better decisions, and formulating better strategies.

3. Annotations and Symbols

Table 1

Notations of the model	
Notations	Descriptions
λ	Failure rate of unit 1.
λ_1	Failure rate of unit 2.
λ_2	Failure rate of unit 3.
λ_3	Failure rate of unit 4.
λ_4	Maintenance rate of the unit.
α, β	Rate of going to winters and summers respectively.
θ	Repair rate of unit 1.
θ_1	Repair rate of unit 2.
θ_2	Repair rate of unit 3.
θ_3	Repair rate of unit 4.

Notations of the model

Notations	Descriptions
θ_4	Repair rate after maintenance.
$G(t), g(t)$	c.d.f and p.d.f of repair time of unit 1.
$G_1(t), g_1(t)$	c.d.f and p.d.f of the repair time of unit 2.
$G_2(t), g_2(t)$	c.d.f and p.d.f of the repair time of unit 3.
$G_3(t), g_3(t)$	c.d.f and p.d.f of the repair time of unit .
$G_4(t), g_4(t)$	c.d.f and p.d.f of the maintenance time.
$\star$	Symbol for convolution.
Ⓢ	Laplace Stieltjes Treansform.
$M_l(t)$	Probability that the system is in up state at time t.
$W_l(t)$	Probability that the system is busy for repair at time t.
$A_l(t)$	Availability of the system at time t.
$B_l(t)$	Busy period for repair/maintenance the system at time t.
$V_l(t)$	Expected number of repairs/maintenances.

Descrpition

States	Description and symbols of the model
S_j	These are the states when the system is operative, j=0,15,17,18.
S_4, S_5, S_6	These are the states when the system works in a reduced capacity.
S_2	This is the cold stanby state.
S_i	These are the failed states, i=3,7,8,9,10,11,12,13,14,16,19,20,21,22.
S, W	Symbols for summer and winter respectively.
o(u1,2,3,4)	The units are in operating state.
o(u1,4)	Units 1 and 4 are in operating state.
o(u1,2,3)	Units 1,2,3 are in operating state.
csu4	Unit 4 is in cold standby state.
fric	Ice-cream unit is under repair due to failure in unit 1.
umic	Ice-cream unit is under repair.
fRu4	Unit 4 is under continuous repair.
fru2,fru3,fru4	Units 2,3,4 are under repair respectively.
fwru2,fwru3	Units 2,3 are waiting for repair.

4. Model Descriptions and Assumptions

Reliability analysis is done of the working of an ice-crream plant w.r.t. seasons. At an initial stage the system is operative, in summers since the demand is high all the units are operative whereas in winters the system is in cold standby state and only operates when there is some demand.The system consists of four units, unit 1 is the main unit of the system where the process starts after that it moves to unit 2 i.e., the flavouring unit and after that unit 3 i.e., the freezing unit. Unit 4 is combined freezing and flavouring unit. In summers, the system operates at reduced capacity when any of the units 2,3,4 fails. The system goes to failed state on the failure of unit 1; unit 2, 4 and unit 3, 4.
Following are the assumptions of the system:

- The system is operating initially.
- A distribution of exponential failure times is assumed for all failure times.
- Unit 1 and unit 4 receive the most priority for repair.

- States always restores the system to its original functionality after every repair.

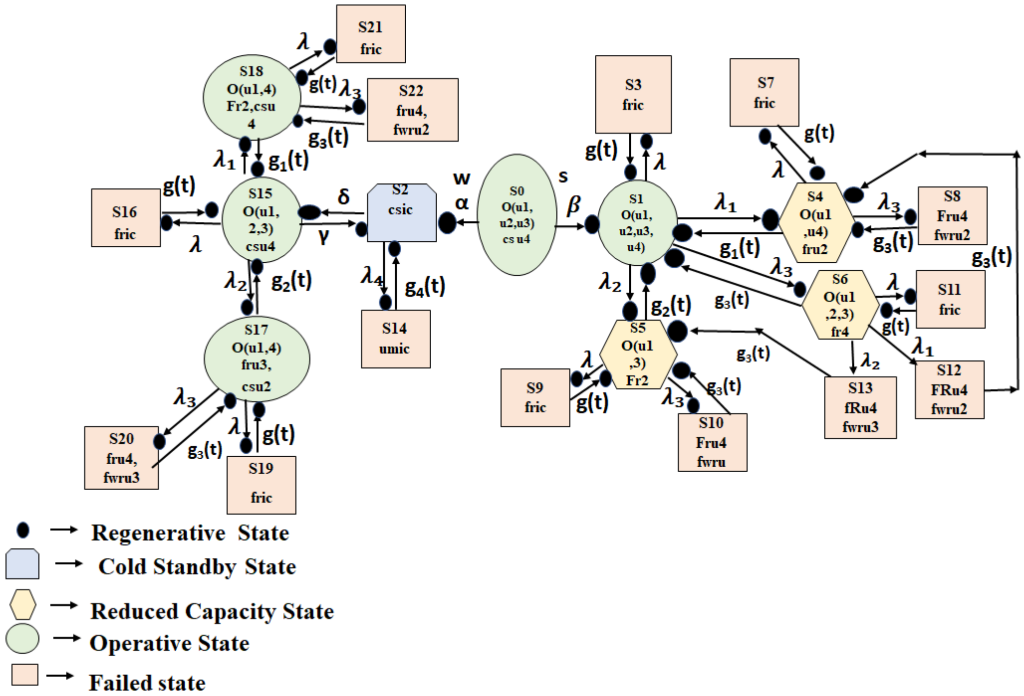

Figure 1: *State Transition Diagram*

5. System Effectiveness Measures

In this model the states $S_0, S_1, S_{15}, S_{17}, s_{18}$ are the operating states. States S_4, S_5, S_6 are the states operating in a reduced capacity. S_2 is a cold standby state, rest are the failed states.

5.1. Mean time to system failure (MTSF)

System effectiveness measures have been achieved using semi-Markov processes and regenerative point techniques. A mean time to failure (MTSF) is determined for the system when considering the failed state as an absorbent state. In terms of probabilistic arguments, we can get the following recursive relation for $\phi_l(t)$:

$$\phi_l(t) = \sum_n Q_{ln}(t) \circledS \phi_n(t) + \sum_e Q_{le}(t)$$

where S_n indicates an un-failed regenerative state into which the given regenerative state $S_l$can transit and Se indicates a failed state into which the state S_l can transit directly. By applying the Laplace-Stieltjes Transform (L.S.T.) to the relationships given by the above equation and solving them for $\phi_0^{**}(t)$, we are able to calculate:

$$\phi_0^{**}(t) = \frac{N(s)}{D(s)}$$

The mean time to system failure (MTSF), when the system started at the beginning of state S_0 is: MTSF=$\int_0^\infty R(t)dt = \lim_{s\to 0} R^*(s)$ Using L' Hospital rule and putting the value of $\phi_0^{**}(s)$ we get

$$\text{MTSF} = T_0 = \lim_{s\to 0} = \frac{1-\phi_0^{**}(s)}{s} = \frac{N}{D}$$

where

N=$\mu_0(p_{13} + p_{14}p_{47} + p_{14}p_{48} + p_{15}p_{59} + p_{15}p_{5,10} + p_{16}p_{6,11} + p_{16}p_{6,12} + p_{16}p_{6,13} - p_{13}p_{15,2}p_{2,15} - p_{13}p_{15,17}p_{17,15} - p_{13}p_{15,18}p_{18,15} - p_{14}p_{47}p_{15,2}p_{2,15} - p_{14}p_{48}p_{15,2}p_{2,15} - p_{15}p_{59}p_{15,2}p_{2,15} - p_{15}p_{15,2}p_{2,15}p_{5,10} - p_{16}p_{15,2}p_{2,15}p_{6,11} - p_{16}p_{15,2}p_{2,15}p_{6,12} - p_{16}p_{15,2}p_{2,15}p_{6,13} - p_{14}p_{47}p_{15,17}p_{17,15} -$

$p_{14}p_{48}p_{15,17}p_{17,15} - p_{15}p_{59}p_{15,17}p_{17,15} - p_{14}p_{47}p_{15,18}p_{18,15} - p_{14}p_{48}p_{15,18}p_{18,15} - p_{15}p_{59}p_{15,18}p_{18,15} - p_{15}p_{5,10}p_{15,17}p_{17,15} - p_{15}p_{5,10}p_{15,18}p_{18,15} - p_{16}p_{6,11}p_{15,17}p_{17,15} - p_{16}p_{6,12}p_{15,17}p_{17,15} - p_{16}p_{6,13}p_{15,17}p_{17,15} - p_{16}p_{6,11}p_{15,18}p_{18,15} - p_{16}p_{6,12}p_{15,18}p_{18,15} - p_{16}p_{6,13}p_{15,18}p_{18,15}) + \mu_1(p_{01} - p_{01}p_{15,2}p_{2,15} - p_{01}p_{15,17}p_{17,15} - p_{01}p_{15,18}p_{18,15}) + \mu_2(p_{02} - p_{02}p_{14}p_{41} - p_{02}p_{15}p_{51} - p_{02}p_{16}p_{61} - p_{02}p_{15,17}p_{17,15} - p_{02}p_{15,18}p_{18,15} + p_{02}p_{14}p_{41}p_{15,17}p_{17,15} + p_{02}p_{15}p_{51}p_{15,17}p_{17,15} + p_{02}p_{16}p_{61}p_{15,17}p_{17,15} + p_{02}p_{14}p_{41}p_{15,18}p_{18,15} + p_{02}p_{15}p_{51}p_{15,18}p_{18,15} + p_{02}p_{16}p_{61}p_{15,18}p_{18,15}) + \mu_4(p_{01}p_{14} - p_{01}p_{14}p_{15,2}p_{2,15} - p_{01}p_{14}p_{15,17}p_{17,15} - p_{01}p_{14}p_{15,18}p_{18,15}) + \mu_5(p_{01}p_{15} - p_{01}p_{15}p_{15,2}p_{2,15} - p_{01}p_{15}p_{15,17}p_{17,15} - p_{01}p_{15}p_{15,18}p_{18,15}) + \mu_6(p_{01}p_{16} - p_{01}p_{16}p_{15,2}p_{2,15} - p_{01}p_{16}p_{15,17}p_{17,15} - p_{01}p_{16}p_{15,18}p_{18,15}) + \mu_{15}(p_{02}p_{2,15} - p_{02}p_{14}p_{41}p_{2,15} - p_{02}p_{15}p_{51}p_{2,15} - p_{02}p_{16}p_{61}p_{2,15}) + \mu_{17}(p_{02}p_{2,15}p_{15,17} - p_{02}p_{14}p_{41}p_{2,15}p_{15,17} - p_{02}p_{15}p_{51}p_{2,15}p_{15,17} - p_{02}p_{16}p_{61}p_{2,15}p_{15,17}) + \mu_{18}(p_{02}p_{2,15}p_{15,18} - p_{02}p_{14}p_{41}p_{2,15}p_{15,18} - p_{02}p_{15}p_{51}p_{2,15}p_{15,18} - p_{02}p_{16}p_{61}p_{2,15}p_{15,18})$

D=$p_{14}p_{41}p_{15,2}p_{2,15} - p_{16}p_{51} - p_{16}p_{61} - p_{15,2}p_{2,15} - p_{15,17}p_{17,15} - p_{15,18}p_{18,15} - p_{14}p_{41} + p_{16}p_{51}p_{15,2}p_{2,15} + p_{16}p_{61}p_{15,2}p_{2,15} + p_{14}p_{41}p_{15,17}p_{17,15} + p_{16}p_{51}p_{15,17}p_{17,15} + p_{16}p_{61}p_{15,17}p_{17,15} + p_{14}p_{41}p_{15,18}p_{18,15} + p_{16}p_{51}p_{15,18}p_{18,15} + p_{16}p_{61}p_{15,18}p_{18,15} + 1$

6. Cost Measures

6.1. Long Term Availability of the System in Summers at Full Capacity

Using the theory of regeneration process, using $A_l(t)$ where l = 0, 1 as the probability that the system will be in upstate at instant t given that it is in state i at t=0, we can find that this value will satisfy the following recursive relations:

$$A_l(t) = M_l(t) + \sum_n q_{ln}(t) \star A_n(t)$$

In this case, Sn can represent any state to which S_l can transit. $M_l(t)$ is the probability that the system will be accessible at time t before visiting any other state.

$M_0 = e^{-(\alpha+\beta)t}, M_1 = e^{-(\lambda+\lambda_1+\lambda_2)t}$

Taking Laplace transform of equations and solving for A_0 we obtain:

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)}$$

Steady state availability is given by:

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_1}{D_1}$$

where

$N_1 =$

$\mu_0 + \mu_1 p_{01} - \mu_0 p_{13} - \mu_0 p_{47} - \mu_0 p_{48} - \mu_0 p_{59} - \mu_0 p_{5,10} - \mu_0 p_{6,11} - \mu_1 p_{01} p_{47} - \mu_1 p_{01} p_{48} - \mu_0 p_{14} p_{41} + \mu_0 p_{13} p_{47} + \mu_0 p_{13} p_{48} - \mu_1 p_{01} p_{59} - \mu_0 p_{15} p_{51} + \mu_0 p_{13} p_{59} - \mu_0 p_{16} p_{61} + \mu_0 p_{47} p_{59} + \mu_0 p_{48} p_{59} - \mu_1 p_{01} p_{5,10} + \mu_0 p_{13} p_{5,10} + \mu_0 p_{47} p_{5,10} + \mu_0 p_{48} p_{5,10} - \mu_1 p_{01} p_{6,11} + \mu_0 p_{13} p_{6,11} + \mu_0 p_{47} p_{6,11} + \mu_0 p_{48} p_{6,11} + \mu_0 p_{59} p_{6,11} + \mu_0 p_{5,10} p_{6,11} + \mu_1 p_{01} p_{47} p_{59} + \mu_1 p_{01} p_{48} p_{59} + \mu_0 p_{15} p_{47} p_{51} + \mu_0 p_{14} p_{41} p_{59} + \mu_0 p_{15} p_{48} p_{51} - \mu_0 p_{13} p_{47} p_{59} - \mu_0 p_{13} p_{48} p_{59} - \mu_0 p_{16} p_{41} p_{64} + \mu_0 p_{16} p_{47} p_{61} + \mu_0 p_{16} p_{48} p_{61} - \mu_0 p_{16} p_{51} p_{65} + \mu_0 p_{16} p_{59} p_{61} + \mu_1 p_{01} p_{47} p_{5,10} + \mu_1 p_{01} p_{48} p_{5,10} + \mu_0 p_{14} p_{41} p_{5,10} - \mu_0 p_{13} p_{47} p_{5,10} - \mu_0 p_{13} p_{48} p_{5,10} + \mu_0 p_{16} p_{61} p_{5,10} + \mu_1 p_{01} p_{47} p_{6,11} + \mu_1 p_{01} p_{48} p_{6,11} + \mu_0 p_{14} p_{41} p_{6,11} - \mu_0 p_{13} p_{47} p_{6,11} - \mu_0 p_{13} p_{48} p_{6,11} + \mu_1 p_{01} p_{59} p_{6,11} + \mu_0 p_{15} p_{51} p_{6,11} - \mu_0 p_{13} p_{59} p_{6,11} - \mu_0 p_{47} p_{59} p_{6,11} - \mu_0 p_{48} p_{59} p_{6,11} + \mu_1 p_{01} p_{5,10} p_{6,11} - \mu_0 p_{13} p_{5,10} p_{6,11} - \mu_0 p_{47} p_{5,10} p_{6,11} - \mu_0 p_{48} p_{5,10} p_{6,11} + \mu_0 p_{16} p_{47} p_{51} p_{65} + \mu_0 p_{16} p_{41} p_{59} p_{64} + \mu_0 p_{16} p_{48} p_{51} p_{65} - \mu_0 p_{16} p_{47} p_{59} p_{61} - \mu_0 p_{16} p_{48} p_{59} p_{61} + \mu_0 p_{16} p_{41} p_{64} p_{5,10} - \mu_0 p_{16} p_{47} p_{61} p_{5,10} - \mu_0 p_{16} p_{48} p_{61} p_{5,10} - \mu_1 p_{01} p_{47} p_{59} p_{6,11} - \mu_1 p_{01} p_{48} p_{59} p_{6,11} - \mu_0 p_{15} p_{47} p_{51} p_{6,11} - \mu_0 p_{14} p_{41} p_{59} p_{6,11} - \mu_0 p_{15} p_{48} p_{51} p_{6,11} + \mu_0 p_{13} p_{47} p_{59} p_{6,11} + \mu_0 p_{13} p_{48} p_{59} p_{6,11} - \mu_1 p_{01} p_{47} p_{5,10} p_{6,11} - \mu_1 p_{01} p_{48} p_{5,10} p_{6,11} - \mu_0 p_{14} p_{41} p_{5,10} p_{6,11} + \mu_0 p_{13} p_{47} p_{5,10} p_{6,11} + \mu_0 p_{13} p_{48} p_{5,10} p_{6,11}$

$D_1 = (\mu_3 p_{13} + \mu_1)(p_{51} - p_{47}p_{51} - p_{48}p_{51} - p_{51}p_{6,11} + p_{47}p_{51}p_{6,11} + p_{48}p_{51}p_{6,11}) + (\mu_4 + \mu_7 p_{47} + \mu_8 p_{48})(p_{14} + p_{16}p_{64} - p_{14}p_{59} - p_{14}p_{5,10} - p_{14}p_{6,11} - p_{16}p_{59}p_{64} - p_{16}p_{64}p_{5,10} + p_{14}p_{59}p_{6,11} + p_{14}p_{5,10}p_{6,11}) + (\mu_5 + \mu_9 p_{59} + \mu_{10} p_{5,10})(p_{15} + p_{16}p65 - p_{15}p_{47} - p_{15}p_{48} - p_{15}p_{6,11} - p_{16}p_{47}p65 - p_{16}p_{48}p65 + p_{15}p_{47}p_{6,11} + p_{15}p_{48}p_{6,11}) + (k + \mu_{11} p_{6,11})(p_{16} - p_{16}p_{47} - p_{16}p_{48} - p_{16}p_{59} - p_{16}p_{5,10} + p_{16}p_{47}p_{59} + p_{16}p_{48}p_{59} + p_{16}p_{47}p_{5,10} + p_{16}p_{48}p_{5,10})$......(1)

6.2. Long Term Availability of the System in Summers at Half Capacity

Using the theory of regeneration process, using $A_l(t)$ where l = 4,5,6 as the probability that the system will be in upstate at instant t given that it is in state i at t=0, we can find that this value will satisfy the following recursive relations:

$$A_l(t) = M_l(t) + \sum_n q_{ln}(t) \star A_n(t)$$

In this case, Sn can represent any state to which S_l can transit. $M_l(t)$ is the probability that the system will be accessible at time t before visiting any other state.
$M_4 = e^{-(\lambda+\lambda_3)t}\bar{G_1}(t), M_5 = e^{-(\lambda+\lambda_3)t}\bar{G_2}(t), M_6 = e^{-(\lambda+\lambda_1+\lambda_2)t}\bar{G_3}(t)$
Taking Laplace transform of equations and solving for A_0 we obtain:

$$A_0^*(s) = \frac{N_2(s)}{D_1(s)}$$

Steady state availability is given by:

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_1}$$

where
$N_2 = -p_{01}(\mu_5 p_{15} p_{47} - \mu_5 p_{15} - kp_{16} - \mu_4 p_{14} + \mu_5 p_{15} p_{48} + kp_{16} p_{47} + kp_{16} p_{48} + \mu_4 p_{14} p_{59} + kp_{16} p_{59} - \mu_4 p_{16} p_{64} - \mu_5 p_{16} p_{65} + \mu_4 p_{14} p_{5,10} + kp_{16} p_{5,10} + \mu_4 p_{14} p_{6,11} + \mu_5 p_{15} p_{6,11} - kp_{16} p_{47} p_{59} - kp_{16} p_{48} p_{59} + \mu_5 p_{16} p_{47} p_{65} + \mu_5 p_{16} p_{48} p_{65} + \mu_4 p_{16} p_{59} p_{64} - kp_{16} p_{47} p_{5,10} - kp_{16} p_{48} p_{5,10} + \mu_4 p_{16} p_{64} p_{5,10} - \mu_5 p_{15} p_{47} p_{6,11} - \mu_5 p_{15} p_{48} p_{6,11} - \mu_4 p_{14} p_{59} p_{6,11} - \mu_4 p_{14} p_{5,10} p_{6,11})$
D_1 is already defined in equation (1).

6.3. Busy Period Analysis for Repair in Summers

Using the theory of regeneration process, using $B_l(t)$ where l = 4,5,6,7,8,9,10,11 as the probability that the system is under repair at an instant t given that it is in state i at t=0, we can find that this value will satisfy the following recursive relations:

$$B_l(t) = W_l(t) + \sum_n q_{ln}(t) \star B_n(t)$$

In this case, Sn can represent any state to which S_l can transit. $W_l(t)$ is the probability that the system will be busy for repair at time t before visiting any other state.
$W_3 = W_7 = W_9 = W_{11} = \bar{G}(t), W_4 = e^{-(\lambda+\lambda_3)}\bar{G_1}(t), W_5 = e^{-(\lambda+\lambda_3)}\bar{G_2}(t), W_6 = e^{-(\lambda+\lambda_1+\lambda_2)}\bar{G_3}(t) + (\lambda_1 e^{-(\lambda+\lambda_1+\lambda_2)} \star 1)\bar{G_3}(t) + (\lambda_2 e^{-(\lambda+\lambda_1+\lambda_2)} \star 1)\bar{G_3}(t), W_8 = W_{10} = W_{12} = W_{13} = \bar{G_3}(t)$
Taking Laplace transform of equations and solving for B_0 we obtain:

$$B_0^*(s) = \frac{N_3(s)}{D_1(s)}$$

Steady state availability is given by:

$$B_0 = \lim_{s\to 0} sB_0^*(s) = \frac{N_3}{D_1}$$

where
$N_3 = -p_{01}(p_{13} p_{47} \mu_3 - p_{14} \mu_4 - p_{15} \mu_5 - p_{16} k - p_{14} p_{47} \mu_7 - p_{14} p_{48} \mu_8 - p_{15} p_{59} \mu_9 - p_{16} p_{64} \mu_4 - p_{16} p_{65} \mu_5 - p_{15} p_{5,10} \mu_{10} - p_{16} p_{6,11} \mu_{11} - p_{16} p_{47} p_{64} \mu_7 - p_{16} p_{48} p_{64} \mu_8 - p_{13} \mu_3 + p_{15} p_{47} \mu_5 + p_{16} p_{47} k + p_{13} p_{48} \mu_3 - p_{16} p_{59} p_{65} \mu_9 + p_{15} p_{48} \mu_5 + p_{16} p_{48} k + p_{13} p_{59} \mu_3 + p_{14} p_{59} \mu_4 + p_{16} p_{59} k - p_{16} p_{65} p_{5,10} \mu_{10} + p_{13} p_{5,10} \mu_3 + p_{14} p_{5,10} \mu_4 + p_{16} p_{5,10} k + p_{13} p_{6,11} \mu_3 + p_{14} p_{6,11} \mu_4 + p_{15} p_{6,11} \mu_5 + p_{15} p_{47} p_{59} \mu_9 + p_{16} p_{47} p_{65} \mu_5 + p_{15} p_{48} p_{59} \mu_9 + p_{16} p_{48} p_{65} \mu_5 + p_{14} p_{47} p_{59} \mu_7 + p_{14} p_{48} p_{59} \mu_8 + p_{16} p_{59} p_{64} \mu_4 + p_{15} p_{47} p_{5,10} \mu_{10} + p_{15} p_{48} p_{5,10} \mu_{10} + p_{14} p_{47} p_{5,10} \mu_7 + p_{14} p_{48} p_{5,10} \mu_8 + p_{16} p_{64} p_{5,10} \mu_4 + p_{16} p_{47} p_{6,11} \mu_{11} + p_{16} p_{48} p_{6,11} \mu_{11} + p_{16} p_{59} p_{6,11} \mu_{11} + p_{14} p_{47} p_{6,11} \mu_7 + p_{14} p_{48} p_{6,11} \mu_8 + p_{15} p_{59} p_{6,11} \mu_9 + p_{16} p_{5,10} p_{6,11} \mu_{11} + p_{15} p_{5,10} p_{6,11} \mu_{10} + p_{16} p_{47} p_{59} p_{65} \mu_9 + p_{16} p_{48} p_{59} p_{65} \mu_9 + p_{16} p_{47} p_{59} p_{64} \mu_7 + p_{16} p_{48} p_{59} p_{64} \mu_8 - p_{13} p_{47} p_{59} \mu_3 - p_{16} p_{47} p_{59} k - p_{13} p_{48} p_{59} \mu_3 - p_{16} p_{48} p_{59} k + p_{16} p_{47} p_{65} p_{5,10} \mu_{10} + p_{16} p_{48} p_{65} p_{5,10} \mu_{10} + p_{16} p_{47} p_{64} p_{5,10} \mu_7 + p_{16} p_{48} p_{64} p_{5,10} \mu_8 - p_{13} p_{47} p_{5,10} \mu_3 - p_{16} p_{47} p_{5,10} k - p_{13} p_{48} p_{5,10} \mu_3 - p_{16} p_{48} p_{5,10} k - p_{13} p_{47} p_{6,11} \mu_3 - p_{15} p_{47} p_{6,11} \mu_5 - p_{13} p_{48} p_{6,11} \mu_3 - p_{15} p_{48} p_{6,11} \mu_5 - p_{13} p_{59} p_{6,11} \mu_3 - p_{14} p_{59} p_{6,11} \mu_4 - p_{13} p_{5,10} p_{6,11} \mu_3 - p_{14} p_{5,10} p_{6,11} \mu_4 - p_{16} p_{47} p_{59} p_{6,11} \mu_{11} - p_{16} p_{48} p_{59} p_{6,11} \mu_{11} - p_{15} p_{47} p_{59} p_{6,11} \mu_9 - p_{15} p_{48} p_{59} p_{6,11} \mu_9 - p_{14} p_{47} p_{59} p_{6,11} \mu_7 - p_{14} p_{48} p_{59} p_{6,11} \mu_8 - p_{16} p_{47} p_{5,10} p_{6,11} \mu_{11} - p_{15} p_{47} p_{5,10} p_{6,11} \mu_{10} - p_{16} p_{48} p_{5,10} p_{6,11} \mu_{11} - p_{15} p_{48} p_{5,10} p_{6,11} \mu_{10} - p_{14} p_{47} p_{5,10} p_{6,11} \mu_7 - p_{14} p_{48} p_{5,10} p_{6,11} \mu_8 + p_{13} p_{47} p_{59} p_{6,11} \mu_3 + p_{13} p_{48} p_{59} p_{6,11} \mu_3 + p_{13} p_{47} p_{5,10} p_{6,11} \mu_3 + p_{13} p_{48} p_{5,10} p_{6,11} \mu_3)$
D_1 is already defined in equation (1).

6.4. Expected Number of Repairs in Summers

Leting $V_l(t)$ be the expected number of repairs in 0 < l $\leq$ t such that it is given the system entered the state S_l at t=0, we get

$$V_l(t) = \sum_n Q_{ln}(t)[h_l + v_l(t)];\ l=4,5,6,7,8,9,10,11$$

$$h_l = \begin{cases} 1, \text{when state } S_l \text{ is the regenerative state} \\ 0, \text{otherwise} \end{cases}$$

Taking LST of equations, we get:

$$V_0^{**}(s) = \frac{N_4}{D_1}$$

The equation describing the number of repairs per unit time in steady state

$$V_0 = \lim_{s\to 0} s v_0^{**}(s) = \frac{N_4}{D_1}$$

where

$N_4 = -p_{01}(p_{13}p_{47} - p_{14}p_{41} - p_{15}p_{51} - p_{16}p_{61} - p_{14}p_{47} - p_{14}p_{48} - p_{15}p_{59} - p_{16}p_{64}p_{41} - p_{16}p_{65}p_{51} - p_{15}p_{5,10} - p_{16}p_{6,11} - p_{16}p_{47}p_{64} - p_{16}p_{48}p_{64} - p_{13} + p_{15}p_{47}p_{51} + p_{16}p_{47}p_{61} + p_{13}p_{48} - p_{16}p_{59}p_{65} + p_{15}p_{48}p_{51} + p_{16}p_{48}p_{61} + p_{13}p_{59} + p_{14}p_{59}p_{41} + p_{16}p_{59}p_{61} - p_{16}p_{65}p_{5,10} + p_{13}p_{5,10} + p_{14}p_{5,10}p_{41} + p_{16}p_{5,10}p_{61} + p_{13}p_{6,11} + p_{14}p_{6,11}p_{41} + p_{15}p_{6,11}p_{51} + p_{15}p_{47}p_{59} + p_{16}p_{47}p_{65}p_{51} + p_{15}p_{48}p_{59} + p_{16}p_{48}p_{65}p_{51} + p_{14}p_{47}p_{59} + p_{14}p_{48}p_{59} + p_{16}p_{59}p_{64}p_{41} + p_{15}p_{47}p_{5,10} + p_{15}p_{48}p_{5,10} + p_{14}p_{47}p_{5,10} + p_{14}p_{48}p_{5,10} + p_{16}p_{64}p_{5,10}p_{41} + p_{16}p_{47}p_{6,11} + p_{16}p_{48}p_{6,11} + p_{16}p_{59}p_{6,11} + p_{14}p_{47}p_{6,11} + p_{14}p_{48}p_{6,11} + p_{15}p_{59}p_{6,11} + p_{16}p_{5,10}p_{6,11} + p_{15}p_{5,10}p_{6,11} + p_{16}p_{47}p_{59}p_{65} + p_{16}p_{48}p_{59}p_{65} + p_{16}p_{47}p_{59}p_{64} + p_{16}p_{48}p_{59}p_{64} - p_{13}p_{47}p_{59} - p_{16}p_{47}p_{59}p_{61} - p_{13}p_{48}p_{59} - p_{16}p_{48}p_{59}p_{61} + p_{16}p_{47}p_{65}p_{5,10} + p_{16}p_{48}p_{65}p_{5,10} + p_{16}p_{47}p_{64}p_{5,10} + p_{16}p_{48}p_{64}p_{5,10} - p_{13}p_{47}p_{5,10} - p_{16}p_{47}p_{5,10}p_{61} - p_{13}p_{48}p_{5,10} - p_{16}p_{48}p_{5,10}p_{61} - p_{13}p_{47}p_{6,11} - p_{15}p_{47}p_{6,11}p_{51} - p_{13}p_{48}p_{6,11} - p_{15}p_{48}p_{6,11}p_{51} - p_{13}p_{59}p_{6,11} - p_{14}p_{59}p_{6,11}p_{41} - p_{13}p_{5,10}p_{6,11} - p_{14}p_{5,10}p_{6,11}p_{41} - p_{16}p_{47}p_{59}p_{6,11} - p_{16}p_{48}p_{59}p_{6,11} - p_{15}p_{47}p_{59}p_{6,11} - p_{15}p_{48}p_{59}p_{6,11} - p_{14}p_{47}p_{59}p_{6,11} - p_{14}p_{48}p_{59}p_{6,11} - p_{16}p_{47}p_{5,10}p_{6,11} - p_{15}p_{47}p_{5,10}p_{6,11} - p_{16}p_{48}p_{5,10}p_{6,11} - p_{15}p_{48}p_{5,10}p_{6,11} - p_{14}p_{47}p_{5,10}p_{6,11} - p_{14}p_{48}p_{5,10}p_{6,11} + p_{13}p_{47}p_{59}p_{6,11} + p_{13}p_{48}p_{59}p_{6,11} + p_{13}p_{47}p_{5,10}p_{6,11} + p_{13}p_{48}p_{5,10}p_{6,11})$

D_1 is already defined in equation (1).

6.5. Availability of the System in Winters

The availability A_0^w of the system in winters is:

$$A_0^w = \lim_{s\to 0}(sA_0^{*w}) = \frac{N_5}{D_2}$$

where

$M_0 = e^{-(\alpha+\beta)}, M_{15} = e^{-(\gamma+\lambda+\lambda_1+\lambda_2)}t, M_{17} = e^{-(\lambda+\lambda_3)t}\bar{G_2}(t), M_{18} = e^{-(\lambda+\lambda_3)t}\bar{G_3}(t)$

$N_5 = \mu_0 + \mu_{15}p_{02}p_{2,15} - \mu_0p_{2,14} - \mu_0p_{15,2}p_{2,15} - \mu_0p_{15,16} - \mu_0p_{15,17}p_{17,15} - \mu_0p_{15,18}p_{18,15} - \mu_0p_{17,19} - \mu_0p_{17,20} - \mu_0p_{18,21} - \mu_0p_{18,22} + \mu_{17}p_{02}p_{2,15}p_{15,17} + \mu_{18}p_{02}p_{2,15}p_{15,18} + \mu_0p_{2,14}p_{15,16} + \mu_0p_{2,14}p_{15,17}p_{17,15} + \mu_0p_{2,14}p_{15,18}p_{18,15} - \mu_{15}p_{02}p_{2,15}p_{17,19} - \mu_{15}p_{02}p_{2,15}p_{17,20} + \mu_0p_{2,14}p_{17,19} + \mu_0p_{15,2}p_{2,15}p_{17,19} + \mu_0p_{2,14}p_{17,20} + \mu_0p_{15,2}p_{2,15}p_{17,20} - \mu_{15}p_{02}p_{2,15}p_{18,21} - \mu_{15}p_{02}p_{2,15}p_{18,22} + \mu_0p_{2,14}p_{18,21} + \mu_0p_{15,2}p_{2,15}p_{18,21} + \mu_0p_{2,14}p_{18,22} + \mu_0p_{15,2}p_{2,15}p_{18,22} + \mu_0p_{15,16}p_{17,19} + \mu_0p_{15,16}p_{17,20} + \mu_0p_{15,18}p_{17,19}p_{18,15} + \mu_0p_{15,16}p_{18,21} + \mu_0p_{15,18}p_{17,20}p_{18,15} + \mu_0p_{15,16}p_{18,22} + \mu_0p_{15,17}p_{17,15}p_{18,21} + \mu_0p_{15,17}p_{17,15}p_{18,22} + \mu_0p_{17,19}p_{18,21} + \mu_0p_{17,19}p_{18,22} + \mu_0p_{17,20}p_{18,21} + \mu_0p_{17,20}p_{18,22} - \mu_{18}p_{02}p_{2,15}p_{15,18}p_{17,19} - \mu_{18}p_{02}p_{2,15}p_{15,18}p_{17,20} - \mu_{17}p_{02}p_{2,15}p_{15,17}p_{18,21} - \mu_{17}p_{02}p_{2,15}p_{15,17}p_{18,22} - \mu_0p_{2,14}p_{15,16}p_{17,19} - \mu_0p_{2,14}p_{15,16}p_{17,20} - \mu_0p_{2,14}p_{15,18}p_{17,19}p_{18,15} - \mu_0p_{2,14}p_{15,16}p_{18,21} - \mu_0p_{2,14}p_{15,18}p_{17,20}p_{18,15} - \mu_0p_{2,14}p_{15,16}p_{18,22} - \mu_0p_{2,14}p_{15,17}p_{17,15}p_{18,21} - \mu_0p_{2,14}p_{15,17}p_{17,15}p_{18,22} + \mu_{15}p_{02}p_{2,15}p_{17,19}p_{18,21} + \mu_{15}p_{02}p_{2,15}p_{17,19}p_{18,22} + \mu_{15}p_{02}p_{2,15}p_{17,20}p_{18,21} - \mu_0p_{2,14}p_{17,19}p_{18,21} - \mu_0p_{15,2}p_{2,15}p_{17,19}p_{18,21} + \mu_{15}p_{02}p_{2,15}p_{17,20}p_{18,22} - \mu_0p_{2,14}p_{17,19}p_{18,22} - \mu_0p_{2,14}p_{17,20}p_{18,21} - \mu_0p_{15,2}p_{2,15}p_{17,19}p_{18,22} - \mu_0p_{15,2}p_{2,15}p_{17,20}p_{18,21} - \mu_0p_{2,14}p_{17,20}p_{18,22} - \mu_0p_{15,2}p_{2,15}p_{17,20}p_{18,22} - \mu_0p_{15,16}p_{17,19}p_{18,21} - \mu_0p_{15,16}p_{17,19}p_{18,22} - \mu_0p_{15,16}p_{17,20}p_{18,21} - \mu_0p_{15,16}p_{17,20}p_{18,22} + \mu_0p_{2,14}p_{15,16}p_{17,19}p_{18,21} + \mu_0p_{2,14}p_{15,16}p_{17,19}p_{18,22} + \mu_0p_{2,14}p_{15,16}p_{17,20}p_{18,21} + \mu_0p_{2,14}p_{15,16}p_{17,20}p_{18,22}$

$D_2 = (\mu_{14}p_{2,14} + \mu_2)(p_{15,2} - p_{15,2}p_{17,19} - p_{15,2}p_{17,20} - p_{15,2}p_{18,21} - p_{15,2}p_{18,22} + p_{15,2}p_{17,19}p_{18,21} + p_{15,2}p_{17,19}p_{18,22} + p_{15,2}p_{17,20}p_{18,21} + p_{15,2}p_{17,20}p_{18,22}) + (\mu_{15} + \mu_{16}p_{15,16})(p_{17,15} - p_{2,14}p_{17,15} -$

$p_{17,15}p_{18,21} - p_{17,15}p_{18,22} + p_{2,14}p_{17,15}p_{18,21} + p_{2,14}p_{17,15}p_{18,22}) + (\mu_{17} + \mu_{19}p_{17,19} + \mu_{20}p_{17,20})(p_{15,17} - p_{2,14}p_{15,17} - p_{15,17}p_{18,21} - p_{15,17}p_{18,22} + p_{2,14}p_{15,17}p_{18,21} + p_{2,14}p_{15,17}p_{18,22}) + (\mu_{18} + \mu_{21}p_{18,21} + \mu_{22}p_{18,22})(p_{15,18} - p_{2,14}p_{15,18} - p_{15,18}p_{17,19} - p_{15,18}p_{17,20} + p_{2,14}p_{15,18}p_{17,19} + p_{2,14}p_{15,18}p_{17,20})$.......(2).

6.6. Busy Period for Repair in Winters

The busy period for repair B_0^{wR} of the system in winters is:

$$B_0^{wR} = \lim_{s\to 0}(sB_0^{*wR}) = \frac{N_6}{D_2}$$

where

$W_{17} = e^{-(\lambda+\lambda_3)}\bar{G_2}(t), W_{18} = e^{-(\lambda+\lambda_3)}\bar{G_3}(t), W_{19} = W_{21} = \bar{G}(t), W_{20} = W_{22} = \bar{G_3}(t)$

$N_6 = -p_{02}(p_{2,15}p_{15,16}p_{17,19}\mu_{16} - p_{2,15}p_{15,17}\mu_{17} - p_{2,15}p_{15,18}\mu_{18} - p_{2,15}p_{15,17}p_{17,19}\mu_{19} - p_{2,15}p_{15,17}p_{17,20}\mu_{20} - p_{2,15}p_{15,18}p_{18,21}\mu_{21} - p_{2,15}p_{15,18}p_{18,22}\mu_{22} - p_{2,15}p_{15,16}\mu_{16} + p_{2,15}p_{15,18}p_{17,19}\mu_{18} + p_{2,15}p_{15,16}p_{17,20}\mu_{16} + p_{2,15}p_{15,18}p_{17,20}\mu_{18} + p_{2,15}p_{15,16}p_{18,21}\mu_{16} + p_{2,15}p_{15,17}p_{18,21}\mu_{17} + p_{2,15}p_{15,16}p_{18,22}\mu_{16} + p_{2,15}p_{15,17}p_{18,22}\mu_{17} + p_{2,15}p_{15,18}p_{17,19}p_{18,21}\mu_{21} + p_{2,15}p_{15,18}p_{17,19}p_{18,22}\mu_{22} + p_{2,15}p_{15,18}p_{17,20}p_{18,21}\mu_{21} + p_{2,15}p_{15,18}p_{17,20}p_{18,22}\mu_{22} + p_{2,15}p_{15,17}p_{17,19}p_{18,21}\mu_{19} + p_{2,15}p_{15,17}p_{17,20}p_{18,21}\mu_{20} + p_{2,15}p_{15,17}p_{17,19}p_{18,22}\mu_{19} + p_{2,15}p_{15,17}p_{17,20}p_{18,22}\mu_{20} - p_{2,15}p_{15,16}p_{17,19}p_{18,21}\mu_{16} - p_{2,15}p_{15,16}p_{17,19}p_{18,22}\mu_{16} - p_{2,15}p_{15,16}p_{17,20}p_{18,21}\mu_{16} - p_{2,15}p_{15,16}p_{17,20}p_{18,22}\mu_{16})$

D_2 is already defined in equation (2).

6.7. Busy Period for Maintenance in Winters

The busy period for maintenance B_0^{wM} of the system in winters is:

$$B_0^{wM} = \lim_{s\to 0}(sB_0^{*wM}) = \frac{N_7}{D_2}$$

where

$W_{14} = \bar{G_4}(t)$

$N_7 = p_{02}p_{2,14}\mu_{14}(p_{15,16}p_{17,19} - p_{15,17}p_{17,15} - p_{15,18}p_{18,15} - p_{17,19} - p_{17,20} - p_{18,21} - p_{18,22} - p_{15,16} + p_{15,16}p_{17,20} + p_{15,18}p_{17,19}p_{18,15} + p_{15,16}p_{18,21} + p_{15,18}p_{17,20}p_{18,15} + p_{15,16}p_{18,22} + p_{15,17}p_{17,15}p_{18,21} + p_{15,17}p_{17,15}p_{18,22} + p_{17,19}p_{18,21} + p_{17,19}p_{18,22} + p_{17,20}p_{18,21} + p_{17,20}p_{18,22} - p_{15,16}p_{17,19}p_{18,21} - p_{15,16}p_{17,19}p_{18,22} - p_{15,16}p_{17,20}p_{18,21} - p_{15,16}p_{17,20}p_{18,22} + 1)$

D_2 is already defined in equation (2).

6.7.1. Expected Number of Repairs in Winters

The expected number of repair V_0^{wR} of the system in winters is:

$$V_0^{wR} = \lim_{s\to 0}(sV_0^{*wR}) = \frac{N_8}{D_2}$$

where

$N_8 = -p_{02}(p_{2,15}p_{15,16}p_{17,19} - p_{2,15}p_{15,17}p_{17,15} - p_{2,15}p_{15,18}p_{18,15} - p_{2,15}p_{15,17}p_{17,19} - p_{2,15}p_{15,17}p_{17,20} - p_{2,15}p_{15,18}p_{18,21} - p_{2,15}p_{15,18}p_{18,22} - p_{2,15}p_{15,16} + p_{2,15}p_{15,18}p_{17,19}p_{18,15} + p_{2,15}p_{15,16}p_{17,20} + p_{2,15}p_{15,18}p_{17,20}p_{18,15} + p_{2,15}p_{15,16}p_{18,21} + p_{2,15}p_{15,17}p_{18,21}p_{17,15} + p_{2,15}p_{15,16}p_{18,22} + p_{2,15}p_{15,17}p_{18,22}p_{17,15} + p_{2,15}p_{15,18}p_{17,19}p_{18,21} + p_{2,15}p_{15,18}p_{17,19}p_{18,22} + p_{2,15}p_{15,18}p_{17,20}p_{18,21} + p_{2,15}p_{15,18}p_{17,20}p_{18,22} + p_{2,15}p_{15,17}p_{17,19}p_{18,21} + p_{2,15}p_{15,17}p_{17,20}p_{18,21} + p_{2,15}p_{15,17}p_{17,19}p_{18,22} + p_{2,15}p_{15,17}p_{17,20}p_{18,22} - p_{2,15}p_{15,16}p_{17,19}p_{18,21} - p_{2,15}p_{15,16}p_{17,19}p_{18,22} - p_{2,15}p_{15,16}p_{17,20}p_{18,21} - p_{2,15}p_{15,16}p_{17,20}p_{18,22})$

D_2 is already defined in equation (2).

6.7.2. Expected Number of Maintenances in Winters

The expected number of maintenances V_0^{wM} of the system in winters is:

$$V_0^{wM} = \lim_{s\to 0}(sV_0^{*wM}) = \frac{N_9}{D_2}$$

where

$N_9 = p_{02}p_{2,14}(p_{15,16}p_{17,19} - p_{15,17}p_{17,15} - p_{15,18}p_{18,15} - p_{17,19} - p_{17,20} - p_{18,21} - p_{18,22} - p_{15,16} + p_{15,16}p_{17,20} + p_{15,18}p_{17,19}p_{18,15} + p_{15,16}p_{18,21} + p_{15,18}p_{17,20}p_{18,15} + p_{15,16}p_{18,22} + p_{15,17}p_{17,15}p_{18,21} + p_{15,17}p_{17,15}p_{18,22} + p_{17,19}p_{18,21} + p_{17,19}p_{18,22} + p_{17,20}p_{18,21} + p_{17,20}p_{18,22} - p_{15,16}p_{17,19}p_{18,21} - p_{15,16}p_{17,19}p_{18,22} - p_{15,16}p_{17,20}p_{18,21} - p_{15,16}p_{17,20}p_{18,22} + 1)$
D_2 is already defined in equation (2).

7. Transition Probabilities and Mean Sojourn Time

$p_{01} = \frac{\beta}{\alpha+\beta}, p_{02} = \frac{\alpha}{\alpha+\beta}, p_{13} = \frac{\lambda}{\lambda+\lambda_1+\lambda_2+\lambda_3}, p_{14=\frac{\lambda_1}{\lambda+\lambda_1+\lambda_2+\lambda_3}}, p_{15} = \frac{\lambda_2}{\lambda+\lambda_1+\lambda_2+\lambda_3}, p_{16} = \frac{\lambda_3}{\lambda+\lambda_1+\lambda_2+\lambda_3}, p_{2,14} = \frac{\lambda_4}{\lambda_4+\gamma}, p_{2,15} = \frac{\gamma}{\lambda_4+\gamma}, p_{41} = g_1^*(\lambda+\lambda_3), p_{47} = \frac{\lambda}{\lambda+\lambda_3}(1-g_1^*(\lambda+\lambda_3)), p_{48} = \frac{\lambda_3}{\lambda+\lambda_3}(1-g_1^*(\lambda+\lambda_3)), p_{51} = g_2^*(\lambda+\lambda_3), p_{59} = \frac{\lambda}{\lambda+\lambda_3}(1-g_2^*(\lambda+\lambda_3)), p_{5,10} = \frac{\lambda_3}{\lambda+\lambda_3}(1-g_2^*(\lambda+\lambda_3)), p_{61} = g_3^*(\lambda+\lambda_1+\lambda_2), p_{6,11} = \frac{\lambda}{\lambda+\lambda_1+\lambda_2}(1-g_3^*(\lambda+\lambda_1+\lambda_2)), p_{6,12} = p_{64}^{(12)} = \frac{\lambda_1}{\lambda+\lambda_1+\lambda_2}(1-g_3^*(\lambda+\lambda_1+\lambda_2)), p_{6,13} = p_{65}^{(13)} = \frac{\lambda_2}{\lambda+\lambda_1+\lambda_2}(1-g_3^*(\lambda+\lambda_1+\lambda_2)), p_{15,2} = \frac{\delta}{\delta+\lambda+\lambda_1+\lambda_2}, p_{15,17} = \frac{\lambda_2}{\delta+\lambda+\lambda_1+\lambda_2}, p_{15,18} = \frac{\lambda_1}{\delta+\lambda+\lambda_1+\lambda_2}, p_{17,15} = g_2^*(\lambda+\lambda_3), p_{17,19} = \frac{\lambda}{\lambda+\lambda_3}(1-g_2^*(\lambda+\lambda_3)), p_{17,20} = \frac{\lambda_3}{\lambda+\lambda_3}(1-g_2^*(\lambda+\lambda_3)), p_{18,15} = g_1^*(\lambda+\lambda_3), p_{18,21} = \frac{\lambda}{\lambda+\lambda_3}(1-g_1^*(\lambda+\lambda_3)), p_{18,22} = \frac{\lambda_3}{\lambda+\lambda_3}(1-g_1^*(\lambda+\lambda_3))$
mean sojourn times are as follows:
$\mu_0 = \frac{1}{\alpha+\beta}, \mu_1 = \frac{1}{\lambda+\lambda_1+\lambda_2+\lambda_3}, \mu_2 = \frac{1}{\delta+\lambda_4}, \mu_4 = \frac{1}{\lambda+\lambda_3}(1-g_1^*(\lambda+\lambda_3)), \mu_5 = \frac{1}{\lambda+\lambda_3}(1-g_2^*(\lambda+\lambda_3)), \mu_6 = \frac{1}{\lambda+\lambda_1+\lambda_2}(1-g_3^*(\lambda+\lambda_1+\lambda_2)), \mu_{15} = \frac{1}{\delta+\lambda+\lambda_1+\lambda_2}, \mu_{17} = \frac{1}{\lambda+\lambda_3}(1-g_2^*(\lambda+\lambda_3)), \mu_{18} = \frac{1}{\lambda+\lambda_3}(1-g_1^*(\lambda+\lambda_3))$
If time is measured from the epoch of entry into state S_n, the unconditional mean transit time of the system from any state S_l is:

$$m_{ln}=\int_0^\infty t dQ_{ln}(t) = -q_{ln}^{*}{}'(0)$$

where,
$m_{01} + m_{02} = \mu_0, m_{13} + m_{14} + m_{15} = \mu_1, m_{2,14} + m_{2,15} = \mu_2, m_{41} + m_{47} + m_{48} = \mu_4, m_{51} + m_{59} + m_{5,10} = \mu_5, m_{61} + m_{6,11} + m_{6,12} + m_{6,13} = \mu_6, m_{61} + m_{6,11} + m_{64}^{(12)} + m_{65}^{(13)} = K, m_{15,2} + m_{15,16} + m_{15,17} + m_{15,18} = \mu_{15}, m_{17,15} + m_{17,19} + m_{17,20} = \mu_{17}, m_{18,15} + m_{18,21} + m_{18,22} = \mu_{18}$
$K = \frac{\lambda}{(\lambda+\lambda_1+\lambda_2)^2} + \frac{\lambda_1+\lambda_2}{\lambda+\lambda_1+\lambda_2}\int_0^\infty t g_3(t)dt + \frac{1}{\lambda+\lambda_1+\lambda_2}\int_0^\infty e^{-(\lambda+\lambda_1+\lambda_2)}g_3(t)dt - \frac{\lambda_1+\lambda_2}{(\lambda+\lambda_1+\lambda_2)^2}\int_0^\infty e^{-(\lambda+\lambda_1+\lambda_2)}g_3(t)dt$
From the above transition probabilites it is verified that:
$p_{01} + p_{02} = 1, p_{13} + p_{14} + p_{15} = 1, p_{2,14} + p_{2,15} = 1, p_{41} + p_{47} + p_{48} = 1, p_{51} + p_{59} + p_{5,10} = 1, p_{61} + p_{6,11} + p_{6,12} + p_{6,13} = 1, p_{61} + p_{6,11} + p_{64}^{(12)} + p_{65}^{(13)} = 1, p_{15,2} + p_{15,16} + p_{15,17} + p_{15,18} = 1, p_{17,15} + p_{17,19} + p_{17,20} = 1, p_{18,15} + p_{18,21} + p_{18,22} = 1$

8. Profit analysis

The profit incurred to the system is:
$P = A_0C_0 + A_0^1C_1 + A_0^wC_2 - (B_0C_3 + B_0^{WR}C_4 + B_0^{WM}C_5 + V_0C_6 + V_0^{WR}C_7 + V_0^{WM}C_8)$
Where
C_0, C_1 are the revenues generated in summers when the system operates at full and half capacity respectively. C_2 is the revenue generated in winters.
C_3, C_4 is the cost per unit time when the repairman in busy for reapir in summers and winters respectively.C_5 is the cost per unit time when the repairman is busy for maintenance.
C_6, C_7 cost per repair in summers and winters respectively.C_8 cost per maintenance.

Rates of the system	Associated Values (per hr)
Failure of unit 1	0.000778752
Failure rate of unit 2	0.0007787652
Failure rate of unit 3	0.00077528
Failure rate of unit 4	0.00053279
Maintenance rate of the unit	0.000594484
Repair rate of unit 1	0.065642055
Repair rate of unit 2	0.0432694
Repair rate of unit 3	0.014205127
Repair rate of unit 4	0.02134
Rate of maintenance	0.00279431
Rate of going to summers	0.00019841
Rate of going to winters	0.0002777
Rate of going to operating state	.0033
Rate of going to standby state	.00027

Figure 2: *Values computed from the data collected*

Costs of the system	Associated Values (Rs.)
Revenue per up time in summers when system works at full capacity	256000
Revenue per up time in summers when system works at half capacity	125000
Revenue per up time in winters	21000
Cost per unit time when repairman is busy for repair in summers	10000
Cost per unit time when repairman is busy for repair in winters	5000
Cost per unit time when repairman is busy for maintenance in winters	12500
Cost per repair in summers	18000
Cost per repair in winters	9000
Cost per maintenance	22000

Figure 3: *Values computed from the data collected*

9. Graphical Analysis and Conclusion using Particular Case

Let us assume an exponential distribution for all the repair rates such that
$g(t) = \theta e^{-\theta(t)}, g_1(t) = \theta_1 e^{-\theta_1(t)}, g_2(t) = \theta_2 e^{-\theta_2(t)}, g_3(t) = \theta_3 e^{-\theta_3(t)}, g_4(t) = \theta_4 e^{-\theta_4(t)}$
$p_{01} = \frac{\beta}{\alpha+\beta}, p_{02} = \frac{\alpha}{\alpha+\beta}, p_{13} = \frac{\lambda}{\lambda+\lambda_1+\lambda_2+\lambda_3}, p_{14} = \frac{\lambda_1}{\lambda+\lambda_1+\lambda_2+\lambda_3}, p_{15} = \frac{\lambda_2}{\lambda+\lambda_1+\lambda_2+\lambda_3}, p_{16} = \frac{\lambda_3}{\lambda+\lambda_1+\lambda_2+\lambda_3}, p_{2,14} = \frac{\lambda_4}{\lambda_4+\gamma}, p_{2,15} = \frac{\gamma}{\lambda_4+\gamma}, p_{41} = \frac{\theta_1}{\theta_1+\lambda+\lambda_3}, p_{47} = \frac{\lambda}{\theta_1+\lambda+\lambda_3}, p_{48} = \frac{\lambda_3}{\theta_1+\lambda+\lambda_3}, p_{51} = \frac{\theta_2}{\theta_2+\lambda+\lambda_3}, p_{59} = \frac{\lambda}{\theta_2+\lambda+\lambda_3}, p_{5,10} = \frac{\lambda_3}{\theta_2+\lambda+\lambda_3}, p_{61} = \frac{\theta_3}{\theta_3+\lambda+\lambda_1+\lambda_2}, p_{6,11} = \frac{\lambda}{\theta_3+\lambda+\lambda_1+\lambda_2}, p_{6,12} = p_{64}^{(12)} = \frac{\lambda_1}{\theta_3+\lambda+\lambda_1+\lambda_2}, p_{6,13} = p_{65}^{(13)} = \frac{\lambda_2}{\theta_3+\lambda+\lambda_1+\lambda_2}, p_{15,2} = \frac{\delta}{\delta+\lambda+\lambda_1+\lambda_2}, p_{15,17} = \frac{\lambda_2}{\delta+\lambda+\lambda_1+\lambda_2}, p_{15,18} = \frac{\lambda_1}{\delta+\lambda+\lambda_1+\lambda_2}, p_{17,15} = \frac{\theta_2}{\theta_2+\lambda+\lambda_3}, p_{17,19} = \frac{\lambda}{\theta_2+\lambda+\lambda_3}, p_{17,20} = \frac{\lambda_3}{\theta_2+\lambda+\lambda_3}, p_{18,15} = \frac{\theta_1}{\theta_1+\lambda+\lambda_3}, p_{18,21} = \frac{\lambda}{\theta_2+\lambda+\lambda_3}, p_{18,22} = \frac{\lambda_3}{\theta_2+\lambda+\lambda_3}, \mu_0 = \frac{1}{\alpha+\beta}, \mu_1 = \frac{1}{\lambda+\lambda_1+\lambda_2+\lambda_3}, \mu_2 = \frac{1}{\delta+\lambda_4}, \mu_4 = \frac{1}{\lambda+\lambda_3}(1 - g_1^*(\lambda + \lambda_3)), \mu_5 = \frac{1}{\lambda+\lambda_3+\theta_2}, \mu_6 = \frac{1}{\lambda+\lambda_1+\lambda_2+\theta_3} \mu_{15} = \frac{1}{\delta+\lambda+\lambda_1+\lambda_2}, \mu_{17} = \frac{1}{\lambda+\lambda_3+\theta_2}, \mu_{18} = \frac{1}{\lambda+\lambda_3+\theta_1}$

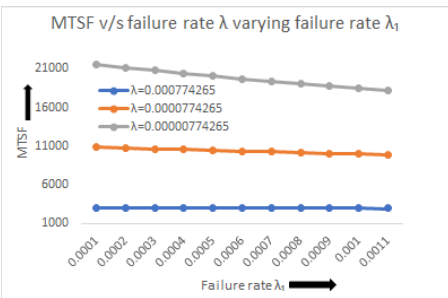

Figure 4: *MTSF v/s Failure rate*

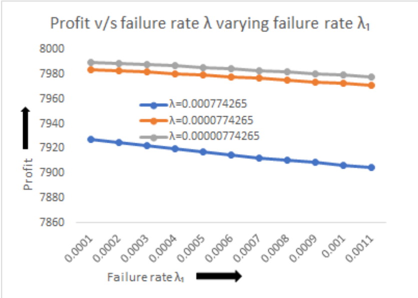

Figure 5: *Profit v/s Failure rate*

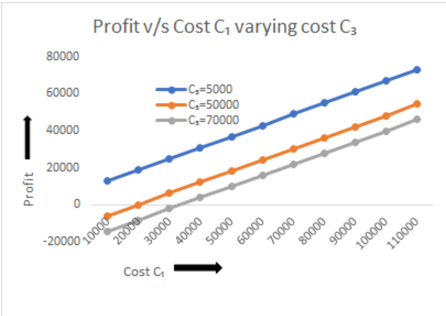

Figure 6: *Profit v/s Cost*

Figures 4, 5 are the MTSF and profit graphs showing a similar trend against the failure rate λ_1 varying failure rate λ.
It shows that as the failure rate λ or λ_1 increases the MTSF and profit decreases.
cut off points for figure 6 are as follows:

Table 2: *Cut-off Points*

Cost C_3Rs.	Revenue per up timeRs.
50000	19736.6955
70000	33564.2752

In table 2 the cut-off points have been shown and from figure 6 it is also clear that with increase in the cost C_1 the profit of the system increases. A number of research papers have been reviewed in this review that have contributed greatly to the field of reliability engineering over the years. The authors have conducted a substantial literature review with an aim of providing reliability engineers and industry leaders with recommendations on improving system reliability. As a final point, we see a wide range of potential uses for the new methods, techniques, and models. The system analysis was performed using a semi-Markov process and regenerative point technique. Code Blocks and Excel are used for graphical analysis, while MATLAB is used for calculation.In conclusion, researchers should consider cost factors as well as reliability factors in order to attain maximum reliability at a minimal cost in the future.

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