RELIABILITY ANALYSIS FOR GDC SYSTEM USING REPAIR AND REPLACEMENT FACILITY IN PISTON FOUNDRY PLANT

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Abstract

The system in industries is greatly impacted by failure. Eliminating these defects is therefore essential for enhancing system performance. This study aims to assess the range of repair/replacement facilities in the GDC (Gravity Die Casting) system at the Piston Foundry Plant. Two sub-units are connected to one main unit, which makes up the GDC system. Any component failure results in system failure. In this situation, the system will first attempt to be repaired, and if that is unsuccessful, it will be replaced. To operate effectively, the primary unit needs to be built of aluminium alloy (Al). Lack of raw materials is what leads to a system failing.Using semi-Markov processes and the regenerating point method, the aforementioned measurements were computed numerically and graphically. The results of this study are unusual since no prior research has concentrated on the GDC system repair/replacement facilities at piston foundries. The conclusions, according to the discussion, are very helpful for businesses who manufacture pistons and utilise the GDC system.

Keywords: GDC, repair, replacement, semi-Markov process, regenerating point technique.

1. Introduction

A system is made up of a variety of parts that function in concert to create a whole. Finally, the system's operation is influenced by how well each component performs. A component-based system can be in both an operational and a failing state. Failures have an impact on the system's use and dependability. As a result, many systems, such as those that control hydraulic, computer, and electric power supplies, nuclear power plants, aviation engines, vehicle engines, and so on, now demand reliability as a component. For systems that function in the dependability domain, researchers have significantly contributed to the creation of reliability models. There have been several research on backup systems, including: Srinivasan [10] gave an examination of warm standby system dependability for a repair facility. The stochastic standby system behaviour with repair time was handled by Kumar et al. [4]. Sharma and Kaur [8] conducted a cost-benefit analysis of a compressor standby system. A power plant system's cold standby unit was stochastically modelled by Sharma and Sharma[9].

Some authors provided an overview of the different reliability modelling methodologies used in die casting systems such as: High Pressure Grain structure and segregation in die casting of magnesium and aluminium alloys Characteristics mentioned by Laukli [5]. High pressure die cast AlSi9Cu3 (Fe) alloys are provided by Timelli [11] using constitutive and stochastic models to anticipate the impact of casting flaws on the mechanical properties. Die Casting Process Modeling and Optimization for ZAMAK Alloy given by Sharma [7]. Existing epistemic uncertainty in die-casting is modelled for reliability and optimised by Yourui et al.[12]. Sensitivity study for the casting method provided by Kumar [3]. An Early Investigation of a Lightweight provided by Muller et al. [6] Die Casting Die Using a Modular Design Approach. High pressure die casting machine reliability analysis of two unit standby system offered by Bhatia and Sharma [1]. The Casting Process Optimization Case Study: A Review of the Reliability Techniques used by Chaudhari and Vasudevan [2]. According to the discussion above, every researcher has addressed reliability analysis of the die casting method used in piston foundries. Research findings pertaining to the GDC system in piston foundries have not been discovered. There are a variety of systems in piston foundry operations that must be analysed using real data at various rates and costs. Our efforts are closing this gap by gathering genuine data from a company called Federal-Mogul Powertrain, India Limited, which is based in Bahadurgarh, Punjab, near Patiala. Federal-Mogul is the world's leading maker of world-class pistons, piston rings and cylinder linears, with products for two-and three -wheelers, vehicles and tractors, among other applications.

We create a Reliability model for the Gravity Die Casting (GDC) system at the piston foundry using the ideas presented above as our inspiration. This model includes the ability for repair and replacement. The goal of this study is to evaluate the range of repair and replacement capabilities offered by the GDC (Gravity Die Casting) system at the Piston Foundry Plant. One primary unit and two supporting units make up the GDC system. The system as a whole fails if even one of the constituent components fails. In this instance, the system will be fixed, and if repair is not possible, it will be replaced. The primary component needs to be made of aluminum alloy for it to work effectively (Al). Lack of raw materials might cause a system to fail.

There are a few assumptions that must be made for the model:

- The system works initially at state S0.
- All failures/repairs /replacement times are exponential distribution.
- In the states, the system is restored to working order after each repair/replacement.
- The unit is brought online as quickly as possible.
- Visit of repairman is immediate upon failure.

2. Methods

The following tools and procedures were utilised to accomplish this study:

In order to overcome the difficulties, semi-Markov processes and regenerating point techniques are used. System availability, mean time to system failure, busy period for repairs/replacements, and expected number of repairs/replacements are only a few of the data that have been collected about system efficiency. Also made are the profits. For a particular case, graphical assessments are produced using the programming languages C++, Python, and MS Excel.

3. Notations and States for the Model

$\lambda \rightarrow$ Failure rate of the main unit i.e. DC.

$\lambda_1 \rightarrow$ Failure rate of the sub-unit one.

$\lambda_2 \rightarrow$ Failure rate of the sub-unit two.

$p \rightarrow$ Probability that raw material is Available.

$q \rightarrow$ Probability that raw material is Non-Available.

$a \rightarrow$ Probability that repair is feasible.

$b \rightarrow$ Probability that replacement is feasible.

$\beta \rightarrow$ Rate of metal treatment.

$O \rightarrow$ Operative unit.

$DC \rightarrow$ Main unit of the system i.e. DC.

$$\begin{aligned}
O(DC) &\rightarrow \text{Main unit of the system is in operating state.}\\
SU_1, SU_2 &\rightarrow \text{Sub-unit one and sub-unit two of the system.}\\
O(SU_1) &\rightarrow \text{Sub unit one is in operating state.}\\
O(SU_2) &\rightarrow \text{Sub unit two is in operating state.}\\
Al &\rightarrow \text{Aluminum alloy.}\\
Av(Al) &\rightarrow \text{Aluminum alloy is Available.}\\
NA(Al) &\rightarrow \text{Aluminum alloy is Non- Available .}\\
CS(DC) &\rightarrow \text{Main unit is in cold standby state.}\\
CS(SU_1) &\rightarrow \text{Sub unit one is in cold standby state.}\\
CS(SU_2) &\rightarrow \text{ Sub unit two is in cold standby state.}\\
F(t), f(t) &\rightarrow \text{c.d.f. and p.d.f. of availablility of the raw material.}\\
G(t), g(t) &\rightarrow \text{c.d.f. and p.d.f of time to repair/replacement of the main unit.}\\
G_1(t), g_1(t) &\rightarrow \text{c.d.f. and p.d.f of time to repair/replacement of the sub-unit one.}\\
G_2(t), g_2(t) &\rightarrow \text{c.d.f. and p.d.f of time to repair/replacement of the sub-unit two.}\\
Fr(DC) &\rightarrow \text{Main unit is under repair.}\\
Fr(SU_1), Fr(SU_2) &\rightarrow \text{Sub-unit one and Sub-unit two are under repair.}\\
Frp(DC) &\rightarrow \text{Main unit is under replacement.}\\
Frp(SU_1), Frp(SU_2) &\rightarrow \text{ Sub-unit one and Sub-unit two are under replacement.}
\end{aligned} \tag{1}$$

4. The System's Reliability Measures

4.1. Transition Probabilities

The transition diagram in **Fig.1** depicts the system's many states.

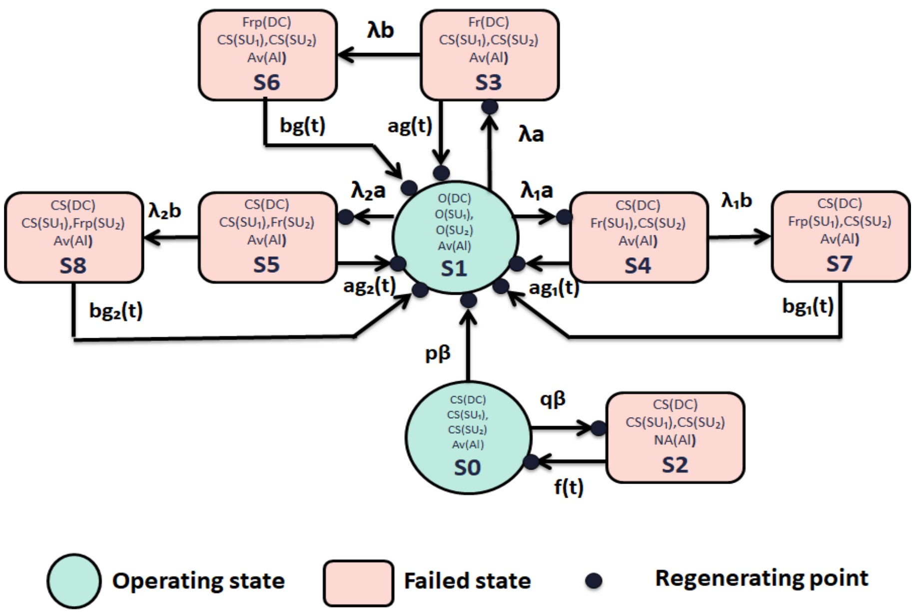

Figure 1: *State Transition Diagram*

The epochs of entry into states S0, S1, S2, S3, S4, and S5 are regenerative states; the remaining states are non-regenerative stages. States S0 and S1 are operating states, while states S2, S3, S4, S5, S6, S7, and S8 are failed states. The following sources provide the transition probabilities:

$$\begin{aligned}
&dQ_{01}(t) = p\beta e^{-\beta t}dt && dQ_{02}(t) = q\beta e^{-\beta t}dt \\
&dQ_{13}(t) = \lambda a e^{-(\lambda+\lambda_1+\lambda_2)at}dt && dQ_{14}(t) = \lambda_1 a e^{-(\lambda+\lambda_1+\lambda_2)at}dt \\
&dQ_{15}(t) = \lambda_2 a e^{-(\lambda+\lambda_1+\lambda_2)at}dt && dQ_{36}(t) = \lambda b e^{-\lambda bt}\bar{G}(t)dt \\
&dQ_{31}^{(6)}(t) = [\lambda b e^{-\lambda bt}©1]g(t)dt && dQ_{31}(t) = g(t)e^{-\lambda bt}dt \\
&dQ_{41}(t) = g_1(t)e^{-\lambda_1 bt}dt && dQ_{47}(t) = \lambda_1 b e^{-\lambda_1 bt}\bar{G_1}(t)dt \\
&dQ_{41}^{(7)}(t) = [\lambda_1 b e^{-\lambda_1 bt}©1]g_1(t)dt && dQ_{51}(t) = g_2(t)e^{-\lambda_2 bt}dt \\
&dQ_{58}(t) = \lambda_2 b e^{-\lambda_2 bt}\bar{G_2}(t)dt && dQ_{51}^{(8)}(t) = [\lambda_2 b e^{-\lambda_2 bt}©1]g_2(t)dt \\
&dQ_{20}(t) = f(t)dt && dQ_{61}(t) = g(t)dt \\
&dQ_{71}(t) = g_1(t)dt && dQ_{81}(t) = g_2(t)dt
\end{aligned} \quad (2)$$

The non-zero elements p_{ij} can be represented as below:

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}dt \quad (3)$$

As we get

$$\begin{aligned}
&p_{01} = p && p_{02} = q \\
&p_{13} = \frac{\lambda a}{(\lambda+\lambda_1+\lambda_2)a} && p_{14} = \frac{\lambda_1 a}{(\lambda+\lambda_1+\lambda_2)a} \\
&p_{15} = \frac{\lambda_2 a}{(\lambda+\lambda_1+\lambda_2)a} && p_{31} = g^*(\lambda b) \\
&p_{36} = p_{31}^{(6)} = \frac{\lambda b[1-g^*(\lambda b)]}{\lambda b} && p_{41} = g_1^*(\lambda_1 b) \\
&p_{47} = p_{41}^{(7)} = \frac{\lambda_1 b[1-g_1^*(\lambda_1 b)]}{\lambda_1 b} && p_{51} = g_2^*(\lambda_2 b) \\
&p_{58} = p_{51}^{(8)} = \frac{\lambda_2 b[1-g_2^*(\lambda_2 b)]}{\lambda_2 b} && p_{20} = f^*(0) \\
&p_{61} = g^*(0) && p_{71} = g_1^*(0) \\
&p_{81} = g_2^*(0)
\end{aligned} \quad (4)$$

It is also verified that:

$$\begin{aligned}
&p_{01} + p_{02} = 1 && p_{13} + p_{14} + p_{15} = 1 \\
&p_{31} + p_{36} = 1 && p_{31} + p_{31}^{(6)} = 1 \\
&p_{41} + p_{47} = 1 && p_{41} + p_{41}^{(7)} = 1 \\
&p_{51} + p_{58} = 1 && p_{51} + p_{51}^{(8)} = 1 \\
&p_{20} = p_{61} = p_{71} = p_{81} = 1
\end{aligned} \quad (5)$$

The unconditional mean time taken by the system to transit for any regenerative state 'j'when it (time) is counted from the epoch of entrance into state 'i'is mathematically state as:

$$m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}^*(0) \quad (6)$$

it is also verified that

$$\begin{aligned}
&m_{01} + m_{02} = \mu_0 && m_{13} + m_{14} + m_{15} = \mu_1 \\
&m_{31} + m_{36} = \mu_3 && m_{31} + m_{31}^{(6)} = K \\
&m_{41} + m_{47} = \mu_4 && m_{41} + m_{41}^{(7)} = K_1 \\
&m_{51} + m_{58} = \mu_5 && m_{51} + m_{51}^{(8)} = K_2 \\
&m_{20} = \mu_2 && m_{61} = \mu_6 \\
&m_{71} = \mu_7 && m_{81} = \mu_8
\end{aligned} \tag{7}$$

where

$$\begin{aligned}
m_{01} &= \int_0^\infty tp\beta e^{-(\beta)t} dt & m_{02} &= \int_0^\infty tq\beta e^{-(\beta)t} dt \\
m_{13} &= \int_0^\infty t\lambda a e^{-(\lambda+\lambda_1+\lambda_2)at} dt & m_{14} &= \int_0^\infty \lambda_1 a t e^{-(\lambda+\lambda_1+\lambda_2)at} dt \\
m_{15} &= \int_0^\infty \lambda_2 a t e^{-(\lambda+\lambda_1+\lambda_2)at} dt & m_{31} &= \int_0^\infty g(t) t e^{-(\lambda b)t} dt \\
m_{36} &= \int_0^\infty \lambda b t e^{-(\lambda b)t} \bar{G(t)} dt & m_{31}^{(6)} &= \int_0^\infty t[\lambda b e^{-(\lambda b)t} ©1] g(t) dt \\
m_{41} &= \int_0^\infty g_1(t) t e^{-(\lambda_1 b)t} dt & m_{47} &= \int_0^\infty \lambda_1 b t e^{-(\lambda_1 b)t} \bar{G_1(t)} dt \\
m_{41}^{(7)} &= \int_0^\infty t[\lambda_1 b e^{-(\lambda_1 b)t} ©1] g_1(t) dt & m_{51} &= \int_0^\infty g_2(t) t e^{-(\lambda_2 b)t} dt \\
m_{58} &= \int_0^\infty \lambda_2 b t e^{-(\lambda_2 b)t} \bar{G_2(t)} dt & m_{51}^{(8)} &= \int_0^\infty t[\lambda_2 b e^{-(\lambda_2 b)t} ©1] g_2(t) dt \\
m_{20} &= \int_0^\infty t f(t) dt & m_{61} &= \int_0^\infty t g(t) dt \\
m_{71} &= \int_0^\infty t g_1(t) dt & m_{81} &= \int_0^\infty t g_2(t) dt \\
K &= \int_0^\infty \bar{G(t)} dt & K_1 &= \int_0^\infty \bar{G_1(t)} dt \\
K_2 &= \int_0^\infty \bar{G_2(t)} dt
\end{aligned} \tag{8}$$

The mean sojourn time (μ_i) in the regenerative state 'i'is defined as time of stay in that state before transition to any other state :

$$\begin{aligned}
\mu_0 &= \frac{1}{\beta} & \mu_1 &= \frac{1}{\lambda + \lambda_1 + \lambda_2} \\
\mu_3 &= \frac{1 - g^*(\lambda b)}{\lambda b} & \mu_4 &= \frac{1 - g_1^*(\lambda_1 b)}{\lambda_1 b} \\
\mu_5 &= \frac{1 - g_2^*(\lambda_2 b)}{\lambda_2 b} & \mu_2 &= -f^*(0) \\
\mu_6 &= -g^*(0) & \mu_7 &= -g_1^*(0) \\
\mu_8 &= -g_2^*(0)
\end{aligned} \tag{9}$$

5. Reliability Analysis

5.1. Mean Time To System Failure

To determine the mean time to system failure (MTSF) of the system, we regard the failed states of system absorbing. By probabilities arguments; we obtain the following recursive relation for

$\phi_i(t)$:

$$\phi_0(t) = Q_{01}(t) Ⓢ \phi_1(t) + Q_{02}(t)$$
$$\phi_1(t) = Q_{13}(t) + Q_{14}(t) + Q_{15}(t) \tag{10}$$

Taking Laplace Stieltje Transforms(L.S.T) of these relations in equations(10) and solving for $\phi_o^{**}(s)$ we obtain

$$\phi_o^{**}(s) = \frac{N(s)}{D(s)} \tag{11}$$

where

$$N(s) = Q_{01}^{**}(s)[Q_{13}^{**}(s) + Q_{14}^{**}(s) + Q_{15}^{**}(s)] + Q_{02}^{**}(s)$$
$$D(s) = 1 \tag{12}$$

Now the mean time to system failure (MTSF) , when the system started at the beginning of state S0 is

$$T = \lim_{s \longrightarrow 0} \frac{1 - \phi_o^{**}(s)}{s} \tag{13}$$

Using L' Hospital rule and putting the value of $\phi_o^{**}(s)$ from equation(13), we have

$$T_0 = \frac{N}{D} \tag{14}$$

where

$$N = \mu_0 + \mu_1[p_{01}]$$
$$D = 1 \tag{15}$$

5.2. Availability Analysis

Let $A_i(t)$ be the probability that the system is in up state at instant t given that the system entered regenerative state i at t=0. The availability $A_i(t)$ is to satisfy the following recursive relations:

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©A_2(t)$$
$$A_1(t) = M_1(t) + q_{13}(t)©A_3(t) + q_{14}(t)©A_4(t) + q_{15}(t)©A_5(t)$$
$$A_2(t) = q_{20}(t)©A_0(t)$$
$$A_3(t) = q_{31}(t)©A_1(t) + q_{31}^{(6)}(t)©A_1(t)$$
$$A_4(t) = q_{41}(t)©A_1(t) + q_{41}^{(7)}(t)©A_1(t)$$
$$A_5(t) = q_{51}(t)©A_1(t) + q_{51}^{(8)}(t)©A_1(t) \tag{16}$$

where

$$M_0(t) = e^{-\beta t} \qquad M_1(t) = e^{-(\lambda+\lambda_1+\lambda_2)t} \tag{17}$$

Taking Laplace Transformation of the above equation(17) and letting s$\longrightarrow$ 0, we get

$$M_0^*(0) = \mu_0 \qquad M_1^*(0) = \mu_1 \tag{18}$$

Taking Laplace transform of the above equations(16) and solving them for

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)} \tag{19}$$

where

$$N_1(s) = M_0^*(s)[1 - q_{13}^*(s)(q_{31}^*(s) + q_{31}^{(6)*}(s)) - q_{14}^*(s)(q_{41}^*(s) + q_{41}^{(7)*}(s)) - q_{15}^*(s)$$
$$(q_{51}^*(s) + q_{51}^{(8)*}(s))] + M_1^*(s)q_{01}^*(s) \quad (20)$$

$$D_1(s) = 1 - q_{13}^*(s)(q_{31}^*(s) + q_{31}^{(6)*}(s)) - q_{14}^*(s)(q_{41}^*(s) + q_{41}^{(7)*}(s)) - q_{15}^*(s)(q_{51}^*(s) + q_{51}^{(8)*}(s))$$
$$- q_{02}^*(s)q_{20}^*(s)[1 - q_{13}^*(s)(q_{31}^*(s) + q_{31}^{(6)*}(s)) - q_{14}^*(s)(q_{41}^*(s) + q_{41}^{(7)*}(s))$$
$$- q_{15}^*(s)(q_{51}^*(s) + q_{51}^{(8)*}(s))] \quad (21)$$

In steady state, system availability is given as

$$A_0 = \lim_{s \longrightarrow 0} sA_0^*(s) = \frac{N_1}{D_1} \quad (22)$$

where

$$N_1 = \mu_1[p_{01}] \quad (23)$$

$$D_1 = \mu_1[p_{01}] + K[p_{01}p_{13}] + K_1[p_{01}p_{14}] + K_2[p_{01}p_{15}] \quad (24)$$

5.3. Busy Period Analysis for the Repairman

Let $BR_i(t)$ be the probability that the repairman is busy at time t given that the system entered regenerative state i at i=0. The recursive relation for $BR_i(t)$ are as follows:

$$BR_0(t) = q_{01}(t)©BR_1(t) + q_{02}(t)©BR_2(t)$$
$$BR_1(t) = q_{13}(t)©BR_3(t) + q_{14}(t)©BR_4(t) + q_{15}(t)©BR_5(t)$$
$$BR_2(t) = W_2(t) + q_{20}(t)©BR_0(t)$$
$$BR_3(t) = W_3(t) + q_{31}(t)©BR_1(t)$$
$$BR_4(t) = W_4(t) + q_{41}(t)©BR_1(t)$$
$$BR_5(t) = W_5(t) + q_{51}(t)©BR_1(t) \quad (25)$$

where

$$W_2(t) = F\bar{(t)} \qquad W_3(t) = G\bar{(t)}$$
$$W_4(t) = G_1\bar{(t)} \qquad W_5(t) = G_2\bar{(t)} \quad (26)$$

Taking Laplace Transformation of the above equation(26) and letting s$\longrightarrow$ 0, we get

$$W_2^*(0) = \mu_2 \qquad W_3^*(0) = K \qquad W_4^*(0) = K_1$$
$$W_5^*(0) = K_2$$

Taking Laplace transform of the above equations(25) and solving them for

$$BR_0^*(s) = \frac{N_2(s)}{D_1(s)}$$

where

$$N_2(s) = W_3^*(s)q_{01}^*(s)q_{13}^*(s) + W_4^*(s)q_{01}^*(s)q_{14}^*(s) + W_5^*(s)q_{01}^*(s)q_{15}^*(s) + W_2^*(s)q_{02}^*(s)$$
$$[1 - q_{13}^*(s)q_{31}^*(s) - q_{14}^*(s)q_{41}^*(s) - q_{15}^*(s)q_{51}^*(s)]$$

The value of $D_1(s)$ is already defined in equation(21).

System total fraction of the time when it is under repair in steady state is given by

$$BR_0 = \lim_{s \longrightarrow 0} sBR_0^*(s) = \frac{N_2}{D_1} \tag{27}$$

where

$$N_2 = \mu_2 p_{02}[1 - p_{13}p_{31} - p_{14}p_{41} - p_{15}p_{51}] + K[p_{01}p_{13}] + K_1[p_{01}p_{14}] + K_2[p_{01}p_{15}]$$

The value of D_1 is already defined in equation(24).

5.4. Busy Period Analysis for the Replacement

Let $BRP_i(t)$ be the probability that the repairman is busy at time t given that the system entered regenerative state i at i=0. The recursive relation for $BRP_i(t)$ are as follows:

$$\begin{aligned}
BRP_0(t) &= q_{01}(t)©BRP_1(t) + q_{02}(t)©BRP_2(t)\\
BRP_1(t) &= q_{13}(t)©BRP_3(t) + q_{14}(t)©BRP_4(t) + q_{15}(t)©BRP_5(t)\\
BRP_2(t) &= q_{20}(t)©BRP_0(t)\\
BRP_3(t) &= W_3(t) + q_{31}^{(6)}(t)©BRP_1(t)\\
BRP_4(t) &= W_4(t) + q_{41}^{(7)}(t)©BRP_1(t)\\
BRP_5(t) &= W_5(t) + q_{51}^{(8)}(t)©BRP_1(t)
\end{aligned} \tag{28}$$

where

$$W_3(t) = \bar{G}(t) \qquad W_4(t) = \bar{G_1}(t) \qquad W_5(t) = \bar{G_2}(t) \tag{29}$$

Taking Laplace Transformation of the above equation(29) and letting s$\longrightarrow$ 0, we get

$$W_3^*(0) = K \qquad W_4^*(0) = K_1 \qquad W_5^*(0) = K_2 \tag{30}$$

Taking Laplace transform of the above equations(28) and solving them for

$$BRP_0^*(s) = \frac{N_3(s)}{D_1(s)} \tag{31}$$

where

$$\begin{aligned}
N_3(s) = {} & W_3^*(s)q_{01}^*(s)q_{13}^*(s) + W_4^*(s)q_{01}^*(s)q_{14}^*(s) + W_5^*(s)q_{01}^*(s)q_{15}^*(s) + q_{02}^*(s)\\
& [1 - q_{13}^*(s)q_{31}^{(6)*}(s) - q_{14}^*(s)q_{41}^{(7)*}(s) - q_{15}^*(s)q_{51}^{(8)*}(s)]
\end{aligned} \tag{32}$$

The value of $D_1(s)$ is already defined in equation(21).

System total fraction of the time when it is under repair in steady state is given by

$$BRP_0 = \lim_{s \longrightarrow 0} sBRP_0^*(s) = \frac{N_3}{D_1} \tag{33}$$

where

$$N_3 = p_{02}[1 - p_{13}p_{31}^{(6)} - p_{14}p_{41}^{(7)} - p_{15}p_{51}^{(8)}] + K[p_{01}p_{13}] + K_1[p_{01}p_{14}] + K_2[p_{01}p_{15}] \tag{34}$$

The value of D_1 is already defined in equation(24).

5.5. Expected Number of Repairs

Let $ER_i(t)$ be the expected no. of repairs in (0,t] given that the system entered regenerative state i at i=0. The recursive relations for $ER_i(t)$ are as follows:

$$\begin{aligned}
ER_0(t) &= Q_{01}(t)Ⓢ ER_1(t) + Q_{02}(t)Ⓢ ER_2(t)\\
ER_1(t) &= Q_{13}(t)Ⓢ ER_3(t) + Q_{14}(t)Ⓢ ER_4(t) + Q_{15}(t)Ⓢ ER_5(t)\\
ER_2(t) &= Q_{20}(t)Ⓢ[1 + ER_0(t)]\\
ER_3(t) &= Q_{31}(t)Ⓢ[1 + ER_1(t)]\\
ER_4(t) &= Q_{41}(t)Ⓢ[1 + ER_1(t)]\\
ER_5(t) &= Q_{51}(t)Ⓢ[1 + ER_1(t)]
\end{aligned} \tag{35}$$

Taking L.S.T.of above relations and obtain the value of $ER_0^{**}(s)$, we get

$$ER_0^{**}(s) = \frac{N_4(s)}{D_1(s)}$$

where

$$N_4(s) = Q_{01}^{**}(s)[Q_{13}^{**}(s)Q_{31}^{**}(s) + Q_{14}^{**}(s)Q_{41}^{**}(s) + Q_{15}^{**}(s)Q_{51}^{**}(s)] + Q_{02}^{**}(s)Q_{20}^{**}(s)[1 - Q_{13}^{**}(s)Q_{31}^{**}(s) - Q_{14}^{**}(s)Q_{41}^{**}(s) - Q_{15}^{**}(s)Q_{51}^{**}(s)]$$

The value of $D_1(s)$ is already defined in equation(21).

For system steady state, the number of repairs per unit time is given by

$$ER_0 = \lim_{s \longrightarrow 0} sER_0^{**}(s) = \frac{N_4}{D_1} \tag{36}$$

where

$$N_4 = p_{01}[p_{13}p_{31} + p_[14]p_{41} + p_{15}p_{51}] + p_{02}[1 - p_{13}p_{31} - p_{14}p_{41} - p_{15}p_{51}]$$

The value of D_1 is already defined in equation(24).

5.6. Expected Number of Replacements

Let $ERP_i(t)$ be the expected no. of replacements in (0,t] given that the system entered regenerative state i at i=0. The recursive relations for $ERP_i(t)$ are as follows:

$$\begin{aligned}
ERP_0(t) &= Q_{01}(t)Ⓢ ERP_1(t) + Q_{02}(t)Ⓢ ERP_2(t)\\
ERP_1(t) &= Q_{13}(t)Ⓢ ERP_3(t) + Q_{14}(t)Ⓢ ERP_4(t) + Q_{15}(t)Ⓢ ERP_5(t)\\
ERP_2(t) &= Q_{20}(t)Ⓢ ERP_0(t)\\
ERP_3(t) &= Q_{31}^{(6)}(t)Ⓢ[1 + ERP_1(t)]\\
ERP_4(t) &= Q_{41}^{(7)}(t)Ⓢ[1 + ERP_1(t)]\\
ERP_5(t) &= Q_{51}^{(8)}(t)Ⓢ[1 + ERP_1(t)]
\end{aligned} \tag{37}$$

Taking L.S.T.of above relations and obtain the value of $ERP_0^{**}(s)$, we get

$$ERP_0^{**}(s) = \frac{N_5(s)}{D_1(s)}$$

where

$$N_5(s) = Q_{01}^{**}(s)[Q_{13}^{**}(s)Q_{31}^{(6)**}(s) + Q_{14}^{**}(s)Q_{41}^{(7)**}(s) + Q_{15}^{**}(s)Q_{51}^{(8)**}(s)]$$

The value of $D_1(s)$ is already defined in equation(21).

For system steady state, the number of replacements per unit time is given by

$$ERP_0 = \lim_{s \longrightarrow 0} sERP_0^{**}(s) = \frac{N_5}{D_1} \quad (38)$$

where

$$N_5 = p_{01}[p_{13}p_{31}^{(6)} + p_{14}p_{41}^{(7)} + p_{15}p_{51}^{(8)}]$$

The value of D_1 is already defined in equation(24).

6. Profit Analysis

Profit incurred to the system model in steady state is given by

$$P = Z_0A_0 - Z_1BR_0 - Z_2BRP_0 - Z_3ER_0 - Z_4ERP_0 \quad (39)$$

where

P = Profit Analysis.

Z_0 = Revenue per unit up time.

Z_1 = Cost per unit up time for which the repairman is busy for repair.

Z_2 = Cost per unit up time for which the repairman is busy for replacement.

Z_3 = Cost per repair.

Z_4 = Cost per replacement.

7. Particular Cases

For the particular case, the failure rates and repair rates are exponentially distributed as follows:

$$f(t) = \gamma e^{-\gamma t} \qquad g(t) = \alpha e^{-\alpha t}$$
$$g_1(t) = \alpha_1 e^{-\alpha_1 t} \qquad g_2(t) = \alpha_2 e^{-\alpha_2 t}$$

As we get,

$$p_{01} = p \qquad p_{02} = q$$
$$p_{13} = \frac{\lambda}{(\lambda + \lambda_1 + \lambda_2)} \qquad p_{14} = \frac{\lambda_1}{(\lambda + \lambda_1 + \lambda_2)}$$
$$p_{15} = \frac{\lambda_2}{(\lambda + \lambda_1 + \lambda_2)} \qquad p_{31} = \frac{\alpha}{\lambda + \alpha}$$
$$p_{36} = p_{31}^{(6)} = \frac{\lambda}{\lambda + \alpha} \qquad p_{41} = \frac{\alpha_1}{\lambda_1 + \alpha_1}$$
$$p_{47} = p_{41}^{(7)} = \frac{\lambda_1}{\lambda_1 + \alpha_1} \qquad p_{51} = \frac{\alpha_2}{\lambda_2 + \alpha_2}$$
$$p_{58} = p_{51}^{(8)} = \frac{\lambda_2}{\lambda_2 + \alpha_2} \qquad p_{20} = p_{61} = p_{71} = p_{81} = 1$$
$$\mu_0 = \frac{1}{\beta} \qquad \mu_1 = \frac{1}{(\lambda + \lambda_1 + \lambda_2)}$$
$$\mu_2 = \frac{1}{\gamma} \qquad \mu_3 = \frac{1}{\lambda + \alpha}$$

$$\mu_4 = \frac{1}{\lambda_1 + \alpha_1} \qquad \mu_5 = \frac{1}{\lambda_2 + \alpha_5}$$
$$\mu_6 = K = \frac{1}{\alpha} \qquad \mu_7 = K_1 = \frac{1}{\alpha_1}$$
$$\mu_8 = K_2 = \frac{1}{\alpha_2} \tag{40}$$

To study the reliability and profit analysis of the GDC systems, We have visited the piston foundry of a firm named **Federal-Mogul Powertrain** and contacted the concerned persons and obtain the information regarding the failures/repairs and replacements. Based on the facts received i.e.,

Table 1: *Information Gathered*

Description	Notation	Rate(/hr)
Failure Rate of Main unit	λ	$0.001391281/hr$
Failure Rate of Sub-unit one	λ_1	$0.001390573/hr$
Failure Rate of Sub-unit two	λ_2	$0.001420852/hr$
Repair/Replacement Rate of Main unit	α	$0.193686798/hr$
Repair/Replacement Rate of Sub-unit one	α_1	$0.206397204/hr$
Repair/Replacement Rate of Sub-unit two	α_2	$0.201849607/hr$

The remaining values are assumed and are listed in Table 2:

Table 2: *Assumed Values*

Description	Notation	Rate(/hr)
Rate of Metal Treatment	β	$1.1762493/hr$
Rate of raw material is available	γ	$0.1158713/hr$
Probability that raw material is available	p	0.75
Probability that raw material is non-available	q	0.25
Probability that repair is feasible	a	0.75
Probability that replacement is feasible	b	0.25
Revenue per unit uptime of the system(per month)	Z_0	$Rs.8,85,000$
Cost per unit uptime, when repairman is busy for repair(per month)	Z_1	$Rs.12,466$
Cost per unit uptime, when repairman is busy for replacement(per month)	Z_2	$Rs.19,480$
Cost per repair(per month)	Z_3	$Rs.18,350$
Cost per replacement(per month)	Z_4	$Rs.25,650$

Various measures of system effectiveness are shown in Table 3:

Table 3: *Results*

Description	Notation	Rate(/hr)
Mean Time to System Failure	T_0	$179.0661/hrs$
Availability of the system	A_0	0.994787
Busy period of Repairman	BR_0	0.020706
Busy period of Replacement	BRP_0	0.002816
Expected no. of Repairs	ER_0	0.004181
Expected no. of Replacements	ERP_0	0.000056
Profit	P	$Rs.8,80,328.59$

8. Graphical Representation

Graphical study has been made for the MTSF, Profit with respect to failure rate of sub-unit one(λ_1), revenue per unit uptime of the system(Z_0) for different values of cost of repairman for busy in doing repair(Z_1).

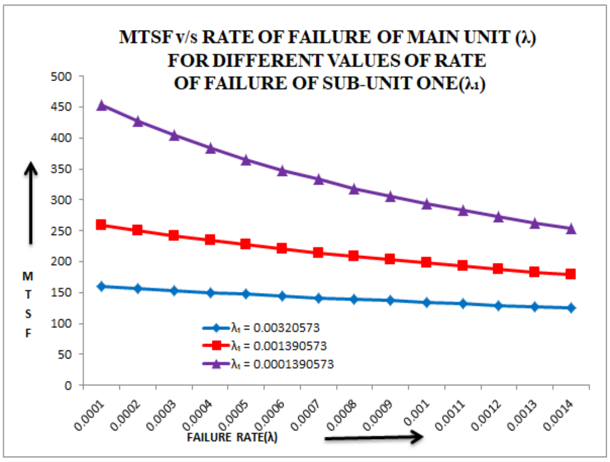

Figure 2: *MTSF v/s Failure Rate*

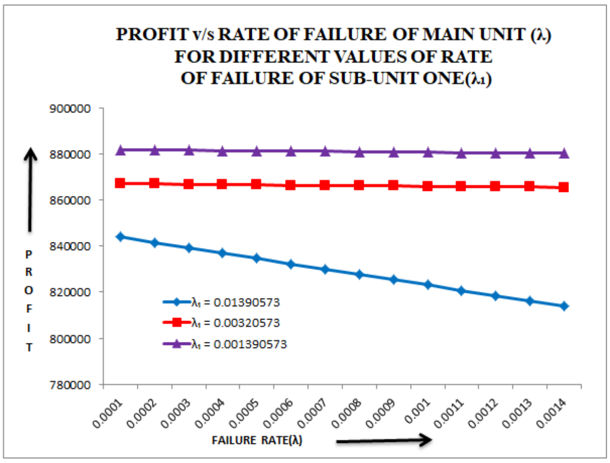

Figure 3: *Profit v/s Failure Rate*

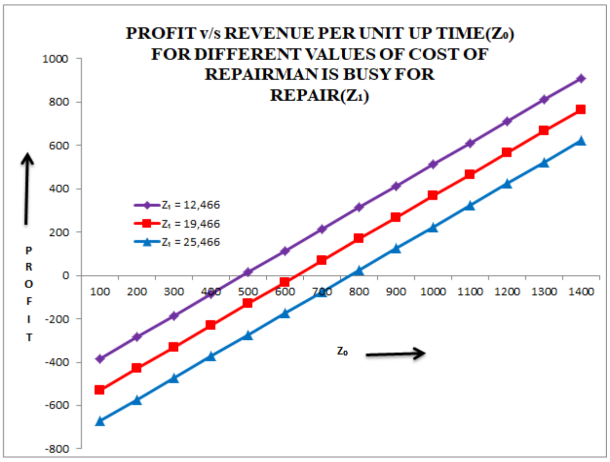

Figure 4: *Profit v/s Revenue*

9. Discussion

Discussion for the FAILURE RATE v/s MTSF and PROFIT v/s FAILURE RATE in the **(Table 4)**

Table 4: *Results*

Variation	Effect
λ/λ_1 increasing ($\uparrow$)	MTSF decreases ($\downarrow$)
λ/λ_1 increasing ($\uparrow$)	Profit decreases ($\downarrow$)

As shown in above table, the behaviour of MTSF and Profit w.r.t. rate of failure of Main unit for the different values of the rate of failure of sub-unit one. It clear from the table that MTSF and Profit gets decreased with increase in values of rate of failure of Main unit i.e. λ. Also MTSF and Profit decreases as failure rate of sub-unit one i.e. λ_1 increases.

Discussion for the PROFIT v/s REVENUE in the **(Table 5)** as below:

Table 5: *Results*

Variation	Effect
Z_0 increasing ($\uparrow$)	Profit increases ($\uparrow$)
$Z_1 = 12,466$; Profit $>=<$ according as z_0	when Z_0 is $>=<$ INR 428
$Z_1 = 19,466$; Profit $>=<$ according as z_0	when it Z_0 $>=<$ INR 582
$Z_1 = 25,466$; Profit $>=<$ according as z_0	when it Z_0 $>=<$ INR 722

Above table depicts the behaviour of the profit w.r.t. revenue per unit uptime of the system (Z_0) for different values of cost of repairman is busy under repair (Z_1). The graph exhibits that there is inclination in the trend of profit increases with increases in the values of Z_0. Also, following conclusion can be drawn from the discussion for Profit v/s Revenue :

For $Z_1 = 12,466$, the profit is positive or zero or negative according as Z_0 is $>=<$ INR 428. Hence, for this case the revenue per unit up time should be fixed, equal or greater than INR 428.

Similarly, discussion for other values of Z_1.

10. Conclusion

It's vital to use the outcomes of the mathematical metrics to enhance the reliability model (Table 3). To better comprehend the significant genuine influencing factors, these results must be employed. This study's findings are ground-breaking since no prior research has emphasised the critical function that GDC system repair and replacement facilities play at piston plants. The analysis's conclusions are very intriguing, and employing the GDC system by piston manufacturing companies is advantageous, according to the argument. Similar to how it is used in other domains, system designers can apply the proposed strategy in their own. Utilize the acquired equations to assess the applicability of various mechanism-type systems.

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