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A DIFFERENT INITIATIVE TO FIND AN OPTIMAL SOLUTION TO THE TRIANGULAR FUZZY TRANSPORTATION PROBLEM BY IMPLEMENTING THE ROW-COLUMN MAXIMA METHOD

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Abstract

In this paper, we discussed an issue in fuzzy transportation problem, which involves fuzzy costs, fuzzy supply, and fuzzy product needs. The goal of this article is to convey the item from point of origin to point of destination at the least possible cost. For fuzzy transportation problems with balance and unbalance types, the proposed technique provides a superior optimal. Transportation costs, supply, and demand are represented by generalized triangular fuzzy numbers using this proposed named Row - Column Maxima Method (RCMM). A numerical example of a fuzzy transportation problem is illustrated and the solution is compared with the outcomes of other approaches. This method reduces iterations and which help to understand and implement easily in real life applications.

Keywords: Fuzzy set, Fuzzy Number, Triangular fuzzy number, Fuzzy Transportation problem, RCM- Method, Fuzzy optimal solution.

1. Introduction

In 1941, Hitchcock had his initial idea regarding the transportation problem. In 1965, L.A. Zadeh [17] created fuzzy set theory and successfully applied it to a variety of fields. There is a need to send products from various origins (Factories) to various destinations in a variety of real-world situations (warehouses). The decision maker's goal is to figure out how much product to order. Many distribution challenges in today's actual world, such as in business or industrial settings are imprecise in nature due to parameter variances. However, due to some unavoidable circumstances, all of these elements of the transportation problem may not be precisely understood in real time. In 1978, the fuzzy decision-making method was introduced. Zimmermann developed a variety of fuzzy optimization algorithms for TP and FTP [18]. Hitchcock [5] was the first to come up with the basic transportation problem. To handle the totally fuzzy transportation problem, Dhanaseker et al. [4] presented the Hungarian-Modi technique. Muthuperumal et al. [9] offered an algorithmic solution to the problem of unbalanced triangular fuzzy transportation. Senthil Kumar et al. [13] suggested the Harmonic Mean Way as a new method for solving the Generalized Fuzzy Transportation problem. A new strategy for finding an optimal solution to Generalized Fuzzy Transportation Problems was proposed by Srinivasaro Thota and Raja [16]. Fuzzy Transportation Problem By Using Triangular Fuzzy Numbers With Ranking Using Area Of Trapezium, Rectangle, And Centroid At Different Levels Of -Cut was discussed by Ambadas Deshmukh et al[1]. Balasubramanian et al. [2], [3] explored utilizing a ranking function to solve the Fuzzy Transportation Problem. Srinivasan et al. [14] established a method for handling fully fuzzy transportation problems in which the materials are transformed, and this method is straightforward to evaluate and can rank many forms of triangular fuzzy numbers. This study by Ladji Kane et al. [8] addressed a Simplified approach for Solving Transportation Problems with Triangular Fuzzy Numbers in Fuzzy Environments. Purushoth kumar et al. [10] proposed employing the diagonal optimum method to address fully fuzzy transportation problems. Indira Singuluri et al. [6] proposed their strategies to address a novel transportation approach to solving type-2 triangular intuitionistic fuzzy transportation problems.

In this study, we offer a new method for solving the fuzzy transportation problem called the RCM method, which assumes supply, demand, and unit transportation cost as triangular fuzzy integers. It provides a minimal value when compared to other approaches such as the NWCM [North-West Corner Method], LCM [Least Cost Method], VAM [Vogel's Approximation Method], and RMM [Row Minima Method]. Finally, an example is provided to aid in the comprehension of the method.

The remainder of this work is arranged in the following manner. Present the fundamental definitions and mathematical constructions of transportation problems in section 2. Present a new algorithm to handle the fully fuzzy transportation problem in section 3. The proposed approach is illustrated numerically in Section 4. The conclusion and future study is presented in section 5.

2. Preliminaries

Definition 2.1[17]

Let U is a collection of elements indicated by u then a fuzzy set $\mathcal{P}$ is a set of ordered pairs in U: $\mathcal{P} = \{(u,\ \mu_{\mathcal{P}}(u)) |\ u \in \mathrm{U}\}$, where the membership function or grade of membership of u in $\mathcal{P}$ is $\mu_{\mathcal{P}}(u)$:U $\longrightarrow$ [0,1].

Definition 2.2[4]

P is a fuzzy set of real numbers that is defined on the universal set of real numbers. If R's membership function satisfies the following properties, R is said to be a fuzzy number.

1. $\mu_{\mathcal{P}}(u)$ is a piecewise continuous
2. $\mathcal{P}$ is convex. $\mu_{\mathcal{P}}(\delta u_1 + (1-\delta)u_2) \geq \min(\mu_{\mathcal{P}}(u_1),\ \mu_{\mathcal{P}}(u_2)), \forall u_1,\ u_2 \in \mathcal{R}\ \&\ \forall \delta \in [0,\ 1]$.
3. $\mathcal{P}$ is Normal.

Definition 2.3[4]

If the membership function $\mathcal{P}: \mathcal{R} \rightarrow [0,1]$ of a fuzzy number P on R satisfies the following characteristics, it is said to be a triangular fuzzy number (TFN) or linear fuzzy number.

$$\mathcal{P}(u) = \begin{cases} \dfrac{u-p_1}{p_2-p_1}, \text{for } p_1 \le u \le p_2 \\ 1 \quad , \quad u = p_2 \\ \dfrac{p_3-u}{p_3-p_2}, \text{for } p_2 \le u \le p_3 \\ 0, \text{ elsewhere} \end{cases}$$

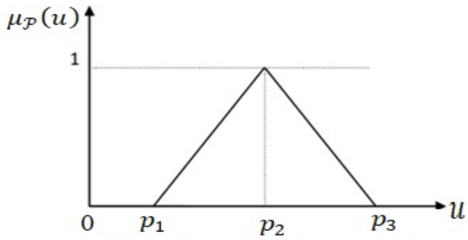

Fig 1.Triangular fuzzy number

2.4 Arithmetic Operation on Fuzzy Numbers [4]:

The operations that can be performed on triangular fuzzy numbers are as follows: Then, if $\mathcal{P} = (p_1, p_2, p_3)$ and $\mathcal{Q} = (q_1, q_2, q_3)$.

(i) Addition: $\mathcal{P} + \mathcal{Q} = (p_1 + q_1, p_2 + q_2, p_3 + q_3)$.

(ii) Subtraction: $\mathcal{P} - \mathcal{Q} = (p_1 - q_3, p_2 - q_2, p_3 - q_1)$.

(iii) Multiplication: $\mathcal{P} \times \mathcal{Q} = (p_1 q_1, p_2, p_3 q_3)$.

2.5 MATHEMATICAL CONSTRUCTION [9]:

A fuzzy transportation problem can be expressed mathematically as follows:

Minimize (Total cost) $\mathcal{Z} = \sum_{i=1}^{m} c_{ij} \sum_{j=1}^{n} x_{ij}$

Subject to the constraints

$\sum_{j=1}^{n} x_{ij} = s_i, \quad i = 1,2, \dots \dots \dots, m$(Fuzzy Supply constraints)

$\sum_{i=1}^{m} x_{ij} = d_j, \quad j = 1,2, \dots \dots \dots, n$(Fuzzy Demand constraints)

$x_{ij} \ge 0, \quad i = 1,2, \dots \dots \dots, m$ and $j = 1,2, \dots \dots \dots, n$

Where *m*: Total number of sources, *n*: Total number of destinations

Notations:

s_i: The product's fuzzy availability at i^{th} the source.
d_j: The product's fuzzy demand at j^{th} destination.
c_{ij}: The fuzzy transportation cost of transporting one unit of commodity from i^{th}source to j^{th} destinations.
x_{ij}: To minimize total fuzzy transportation, a fuzzy quantity is delivered from i^{th}source to j^{th}destination (or fuzzy decision variables).
$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$: The fuzzy cost of transporting one unit of the product from i^{th}source to the j^{th} destination.
$\sum_{i=1}^{m} s_i$: The product's total fuzzy availability
$\sum_{j=1}^{n} d_j$: The product's total fuzzy demand

	Destination 1	**Destination 2**	**…**	**Destination n**	**Supply**
Source 1	$c_{11}x_{11}$	$c_{12}x_{12}$	…	$c_{1n}x_{1n}$	s_1
Source 2	$c_{21}x_{21}$	$c_{22}x_{22}$	…	$c_{2n}x_{2n}$	s_2
⋮	⋮	⋮	⋮	⋮	⋮
Source m	$c_{m1}x_{m1}$	$c_{m2}x_{m2}$	…	$c_{mn}x_{mn}$	s_m
Demand	d_1	d_2	…	d_n	$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$

2.6 Balanced and unbalanced FTP [11]:

Balanced fuzzy transportation problem: The total fuzzy supply is equal to total fuzzy demand

$$\text{i.e. } \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

Unbalanced fuzzy transportation problem:

The total fuzzy supply is not equal to total fuzzy demand

$$\text{i.e. } \sum_{i=1}^{m} s_i \neq \sum_{j=1}^{n} d_j$$

2.7 To Modify Unbalanced FTP to Balanced FTP: An Unbalanced FTP may occur in two different forms: (i) Excess of availability, (ii) Shortage in availability.

We now discuss these two cases by considering the usual m- sources, n- destinations FTP with the condition that $\sum_{i=1}^{m} s_i \neq \sum_{j=1}^{n} d_j$

Case1: (Excess of Availability, i.e. $\sum s_i \geq \sum d_j$)

The general FTP may be stated as follows:

Minimize (Total cost) $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}c_{ij}$

Subject to the constraints

$\sum_{j=1}^{n} x_{ij} \leq s_i, \quad i = 1,2,\dots\dots\dots,m$(Fuzzy Supply constraints)

$\sum_{i=1}^{m} x_{ij} = d_j, \quad j = 1,2,\dots\dots\dots,n$(Fuzzy Demand constraints)

and $x_{ij} \geq 0, \quad i = 1,2,\dots\dots\dots,m$ and $j = 1,2,\dots\dots\dots,n$

The problem will possess a fuzzy feasible solution if $\sum s_i \geq \sum d_j$. In the first constraints, the introduction of slack variable $x_{i,n+1}(i = 1,2,\dots\dots\dots,m)$ gives

$$\Rightarrow \sum_{j=1}^{n} x_{ij} + x_{i,n+1} = s_i, \quad i = 1,2,\dots\dots\dots,m$$

$$\Rightarrow \sum_{i=1}^{m}\left(\sum_{j=1}^{n} x_{ij} + x_{i,n+1}\right) = \sum_{i=1}^{m} s_i$$

$$\Rightarrow \sum_{j=1}^{n}\left(\sum_{i=1}^{m} x_{ij}\right) + \sum_{i=1}^{m} x_{i,n+1} = \sum_{i=1}^{m} s_i$$

$$\Rightarrow \sum_{j=1}^{n} \boldsymbol{d_j} + \sum_{i=1}^{m} x_{i,n+1} = \sum_{i=1}^{m} s_i \quad (\because \sum_{i=1}^{m} x_{ij} = d_j$$

$$\Rightarrow \sum_{i=1}^{m} x_{i,n+1} = \sum_{i=1}^{m} s_i - \sum_{j=1}^{n} \boldsymbol{d_j} = \text{Excess of Availability}$$

If this excess availability is denoted by d_{n+1}, the modified FTP, can be reformulated as:

Minimize $Z = \sum_{i=1}^{m}\sum_{j=1}^{n+1} x_{ij}c_{ij}$,

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} + x_{i,n+1} = s_i, \quad i = 1,2,\dots\dots\dots,m$$

$\sum_{i=1}^{m} x_{ij} = d_j, \quad j = 1,2,\dots\dots\dots,n+1$ and $x_{ij} \geq 0$, for all i and j

and $c_{i,n+1} = 0$, for $i = 1,2,\dots\dots\dots,m$ and $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n+1} d_j$

This is clearly the balanced FTP and thus can be easily solved by fuzzy transportation algorithm.

Working Rule: If $\sum s_i \geq \sum d_j$, avoid using a fake row or column when converting to balance. Let see $\omega = \sum_{i=1}^{m} s_i - \sum_{j=1}^{n} d_j$. The difference ω added to the demand (d_1, d_2, d_3) minimum. Reconstruct the provided Fuzzy transportation table using $(d_1 + \omega_1, d_2 + \omega_2, d_3 + \omega_3)$.

Case2: (Shortage in Availability, i.e. $\sum s_i \le \sum d_j$)

In this case, the general FTP becomes:

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij}$

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = s_i, \quad i = 1,2, \dots \dots \dots, m$$

$$\sum_{i=1}^{m} x_{ij} \le d_j, \quad j = 1,2, \dots \dots \dots, n$$

and $x_{ij} \ge 0, \quad i = 1,2, \dots \dots \dots, m; j = 1,2, \dots \dots \dots, n$

Now, introducing the slack variable $x_{m+1,j} (j = 1,2, \dots \dots \dots, n)$ in the second constraint, we get

$$\Rightarrow \sum_{i=1}^{m} x_{ij} + x_{m+1,j} = d_j, \quad j = 1,2, \dots \dots \dots, n$$

$$\Rightarrow \sum_{j=1}^{n} \left(\sum_{i=1}^{m} x_{ij} + x_{m+1,j} \right) = \sum_{j=1}^{n} d_j$$

$$\Rightarrow \sum_{i=1}^{m} \left(\sum_{j=1}^{n} x_{ij} \right) + \sum_{j=1}^{n} x_{m+1,j} = \sum_{j=1}^{n} d_j$$

$$\Rightarrow \sum_{i=1}^{m} s_i + \sum_{j=1}^{n} x_{m+1,j} = \sum_{j=1}^{n} d_j \quad (\because \sum_{j=1}^{n} x_{ij} = s_i$$

$$\Rightarrow \sum_{j=1}^{n} x_{m+1,j} = \sum_{j=1}^{n} d_j - \sum_{i=1}^{m} s_i = \text{Shortage in availability } s_{m+1}, \text{ say}$$

Thus the modified FTP, in this case becomes:

Minimize $Z = \sum_{i=1}^{m+1} \sum_{j=1}^{n} x_{ij} c_{ij}$,

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = s_i, \quad i = 1,2, \dots \dots \dots, m+1$$

$\sum_{i=1}^{m} x_{ij} + x_{m+1,j} = d_j, \quad j = 1,2, \dots \dots \dots, n$ and $x_{ij} \ge 0$, for all i and j

where $c_{m+1,j} = 0$, for $j = 1,2, \dots \dots \dots, n$ and $\sum_{i=1}^{m+1} s_i = \sum_{j=1}^{n} d_j$

This is clearly the balanced FTP and thus can be easily solved by fuzzy transportation algorithm.

Working Rule: If $\sum_{i=1}^{m} s_i \le \sum_{j=1}^{n} d_j$, avoid using a fake row or column when converting to balance. Let see $\omega = \sum_{i=1}^{m} d_j - \sum_{j=1}^{n} s_i$. The difference ω added to the supply (s_1, s_2, s_3) minimum. Reconstruct the provided Fuzzy transportation table using $(s_1 + \omega_1, s_2 + \omega_2, s_3 + \omega_3)$.

2.8 Fuzzy Feasible Solution [9]:

A fuzzy feasible solution is any set of fuzzy non negative allocations $x_{ij} (x_{ij} \ge 0)$ that fulfills (in the sense equivalent) the row and column requirements.

2.9 Fuzzy Basic Feasible Solution [9]:

If the number of positive allocations is exactly equal to $(m + n - 1)$, a fuzzy feasible solution to a fuzzy transportation problem with m origins and n destinations is said to be fuzzy basic feasible solution.

2.10 Fuzzy Optimal Solution [9]:

If the entire fuzzy transportation cost is minimized, a fuzzy feasible solution is said to be fuzzy optimum.

Theorem 2.11 [11]: (Existence of Fuzzy feasible solution)

A necessary and sufficient condition for the existence of feasible solution of a fuzzy transportation problem is $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$ $(i = 1,2, \dots\dots\dots, m; j = 1,2, \dots\dots\dots, n)$.

Proof: The condition is necessary: Let there exist a feasible solution to the fuzzy transportation problem. Then,

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} s_i, \quad (1)$$

$$\sum_{j=1}^{n}\sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} d_j, \quad (2)$$

From equation (1) and (2), we get

$$\Leftrightarrow \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j.$$

The condition is sufficient: Let $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j = \mathcal{K}$ (say).

If $\mu_i \neq 0$ be any real number such that $x_{ij} = \mu_i d_j \forall\, i, j$, then μ_i is given by

$$\sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n} \mu_i d_j = \mu_i \sum_{j=1}^{n} d_j = \mathcal{K}\mu_i$$

$$\Rightarrow \mu_i = \frac{1}{\mathcal{K}} \sum_{j=1}^{n} x_{ij} = \frac{s_i}{\mathcal{K}} \left(\because \sum_{j=1}^{n} x_{ij} = s_i\right)$$

Thus, $x_{ij} = \mu_i d_j = \frac{s_i d_j}{\mathcal{K}} \geq 0$, since $s_i > 0, d_j > 0 \;\forall\, i, j$. Hence a Fuzzy feasible solution exists.

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3. Proposed algorithm

In this paper, we proposed Row-Column maxima method [RCMM] to find optimum solution and this result compared with NWCM, LCM, RMM, VAM methods.

Step 1: Check to see if the given FTP is balanced or not.

Case1: If $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$. then go to step 3.

Case2: If $\sum_{i=1}^{m} s_i \neq \sum_{j=1}^{n} d_j$, possible, avoid using a fake row or column when converting to balanced. Let see $(i)\omega = \sum_{i=1}^{m} s_i - \sum_{j=1}^{n} d_j \; if \sum_{j=1}^{n} d_j < \sum_{i=1}^{m} s_i$ or $(ii)\omega = \sum_{i=1}^{m} d_j - \sum_{j=1}^{n} s_i \; if \sum_{i=1}^{m} s_i < \sum_{j=1}^{n} d_j$.

Step 2: The difference ω will be divided into three parts $(\omega_1, \omega_2, \omega_3)$ such that $\omega = \sum_{i=1}^{3} \omega_i$ and added to the supply (s_1, s_2, s_3) or demand (d_1, d_2, d_3) minimum. Reconstruct the provided Fuzzy transportation table using $(s_1 + \omega_1, s_2 + \omega_2, s_3 + \omega_3)/(d_1 + \omega_1, d_2 + \omega_2, d_3 + \omega_3)$.

Step 3: For each row, find the difference between the first and second maximum values and use that value instead of the first maximum value.

Step 4: After completing step 3, calculate the difference between the 1st and 2nd maximum values and use that value to replace the 1st maximum value in each column.

Step 5: Choose the fuzzy cost's minimum value in either a row or a column. Then determine the minimum supply and demand value and assign it.

Step 6: After step 5, delete the row/column in which supply/demand has reached its limit.

Step 7: Steps 5 – 6 should be repeated until (m + n-1) cells have been allotted.

Step 8: Calculate the minimum Fuzzy Transportation Cost. That is,

Total Cost= $\sum_{i=1}^{m} c_{ij} \sum_{j=1}^{n} x_{ij}$.

4. Numerical Example

A manufacturing company produces diesel engines in 10 cities C_1,C_2, C_3,C_4, C_5, C_6, C_7, C_8, C_9, C_{10} and they are purchased by ten trucking companies T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9, T_{10}. The table below indicates how many engines are required by T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9, T_{10}. It also displays the cost of transportation per engine from origin to destination. The corporation wants to maintain the total transportation cost to a minimum.

Table 1: *Triangular Fuzzy Transportation Problem*

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	**Supply**
C_1	(3,5,7)	(4,6,8)	(3,6,9)	(11,12,13)	(2,3,4)	(7,8,9)	(5,6,7)	(12,14,16)	(1,4,7)	(7,8,9)	(15,20,25)
C_2	(2,5,8)	(5,7,9)	(4,5,6)	(7,8,9)	(14,16,18)	(12,13,14)	(5,7,9)	(0,1,2)	(2,4,6)	(6,7,8)	(5,10,15)
C_3	(5,6,7)	(4,6,8)	(11,13,15)	(3,6,9)	(14,15,16)	(2,3,4)	(8,9,10)	(4,8,16)	(9,10,11)	(14,16,18)	(25,30,35)
C_4	(9,10,11)	(2,5,7)	(2,3,4)	(3,5,7)	(10,15,20)	(5,6,7)	(7,8,9)	(1,3,5)	(3,6,9)	(10,11,12)	(40,45,50)
C_5	(8,9,10)	(2,4,6)	(8,10,12)	(6,8,10)	(3,6,7)	(10,12,14)	(1,4,7)	(14,16,18)	(1,2,3)	(8,9,10)	(90,95,100)
C_6	(6,7,8)	(12,13,14)	(14,16,18)	(1,2,3)	(1,3,5)	(3,5,7)	(6,8,10)	(2,4,6)	(7,8,9)	(13,15,17)	(70,75,80)
C_7	(5,6,7)	(12,14,16)	(13,15,17)	(5,6,7)	(0,1,2)	(11,13,15)	(14,16,18)	(2,4,6)	(7,9,11)	(3,5,7)	(50,55,60)
C_8	(16,18,20)	(1,3,5)	(7,8,9)	(8,10,12)	(3,6,9)	(4,5,6)	(10,11,12)	(3,6,9)	(14,15,16)	(4,6,8)	(65,70,75)
C_9	(4,6,8)	(1,2,3)	(2,4,6)	(11,12,13)	(1,2,3)	(2,4,6)	(3,5,7)	(5,6,7)	(8,9,10)	(4,5,6)	(85,90,95)
C_{10}	(7,8,9)	(5,7,9)	(6,8,10)	(9,11,13)	(4,6,8)	(14,15,16)	(11,12,13)	(14,16,18)	(0,2,4)	(2,3,4)	(55,60,65)
Demand	(15,20,25)	(40,45,50)	(25,30,35)	(5,10,15)	(50,55,60)	(70,75,80)	(90,95,100)	(85,90,95)	(65,70,75)	(55,60,65)	

Applying the proposed algorithm [RCMM]:

Step 1:

$\sum_{i=1}^{m} s_i = (500,550,600)$ and $\sum_{j=1}^{n} d_j = (500,550,600)$.

$\Rightarrow \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$ (Total supply = Total demand).

Since the given Fuzzy Transportation Problem is balanced. So go to step 3,

Step 3:

In first row, First maximum value = $(12,14,16)$

Second maximum value = $(11,12,13)$

The difference between 1st and 2nd maximum value

i.e., $(12,14,16) - (11,12,13) = (-1,2,5)$

Then replace the subtracted value instead of the first maximum value

i.e., $(12,14,16) = (-1,2,5)$

Similarly, apply step 3 other 2nd, 3rd up to 10th row, then we get table 2.

Table 2: *Row-wise Difference Table*

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	**Supply**
C_1	(3,5,7)	(4,6,8)	(3,6,9)	(11,12,13)	(2,3,4)	(7,8,9)	(5,6,7)	**(-1,2,5)**	(1,4,7)	(7,8,9)	(15,20,25)
C_2	(2,5,8)	(5,7,9)	(4,5,6)	(7,8,9)	**(0,3,6)**	(12,13,14)	(5,7,9)	(0,1,2)	(2,4,6)	(6,7,8)	(5,10,15)
C_3	(5,6,7)	(4,6,8)	(11,13,15)	(3,6,9)	(14,15,16)	(2,3,4)	(8,9,10)	(4,8,16)	(9,10,11)	**(-2,1,4)**	(25,30,35)
C_4	(9,10,11)	(2,5,7)	(2,3,4)	(3,5,7)	**(-2,4,10)**	(5,6,7)	(7,8,9)	(1,3,5)	(3,6,9)	(10,11,12)	(40,45,50)
C_5	(8,9,10)	(2,4,6)	(8,10,12)	(6,8,10)	(3,6,7)	(10,12,14)	(1,4,7)	**(0,4,8)**	(1,2,3)	(8,9,10)	(90,95,100)
C_6	(6,7,8)	(12,13,14)	**(-3,1,5)**	(1,2,3)	(1,3,5)	(3,5,7)	(6,8,10)	(2,4,6)	(7,8,9)	(13,15,17)	(70,75,80)
C_7	(5,6,7)	(12,14,16)	(13,15,17)	(5,6,7)	(0,1,2)	(11,13,15)	**(-3,1,5)**	(2,4,6)	(7,9,11)	(3,5,7)	(50,55,60)
C_8	**(0,3,6)**	(1,3,5)	(7,8,9)	(8,10,12)	(3,6,9)	(4,5,6)	(10,11,12)	(3,6,9)	(14,15,16)	(4,6,8)	(65,70,75)
C_9	(4,6,8)	(1,2,3)	(2,4,6)	**(1,3,5)**	(1,2,3)	(2,4,6)	(3,5,7)	(5,6,7)	(8,9,10)	(4,5,6)	(85,90,95)
C_{10}	(7,8,9)	(5,7,9)	(6,8,10)	(9,11,13)	(4,6,8)	(14,15,16)	(11,12,13)	**(-2,1,4)**	(0,2,4)	(2,3,4)	(55,60,65)
Demand	(15,20,25)	(40,45,50)	(25,30,35)	(5,10,15)	(50,55,60)	(70,75,80)	(90,95,100)	(85,90,95)	(65,70,75)	(55,60,65)	

Step 4: In table 2, apply the step 4 of proposed algorithm

In first column, First maximum value = $(9,10,11)$, Second maximum value = $(8,9,10)$

The difference between 1st and 2nd maximum value = $(9,10,11) - (8,9,10) = (-1,1,3)$

Then replace the subtracted value instead of the first maximum value. $[(9,10,11) = (-1,1,3)]$
Similarly, apply step 4 other 2nd, 3rd up to 10th column, then we get table 3.

Table 3: *Column-wise difference table*

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	**Supply**
C_1	(3,5,7)	(4,6,8)	(3,6,9)	**(-2,1,4)**	(2,3,4)	(7,8,9)	(5,6,7)	(-1,2,5)	(1,4,7)	(7,8,9)	(15,20,25) - (5,10,15) = (0,10,20)
C_2	(2,5,8)	(5,7,9)	(4,5,6)	(7,8,9)	(0,3,6)	(12,13,14)	(5,7,9)	(0,1,2)	(2,4,6)	(6,7,8)	(5,10,15)
C_3	(5,6,7)	(4,6,8)	(11,13,15)	(3,6,9)	**(6,9,12)**	(2,3,4)	(8,9,10)	**(-5,2,13)**	(9,10,11)	(-2,1,4)	(25,30,35)
C_4	**(-1,1,3)**	(2,5,7)	(2,3,4)	(3,5,7)	(-2,4,10)	(5,6,7)	(7,8,9)	(1,3,5)	(3,6,9)	(10,11,12)	(40,45,50)
C_5	(8,9,10)	(2,4,6)	(8,10,12)	(6,8,10)	(3,6,7)	(10,12,14)	(1,4,7)	(0,4,8)	(1,2,3)	(8,9,10)	(90,95,100)
C_6	(6,7,8)	(12,13,14)	(-3,1,5)	(1,2,3)	(1,3,5)	(3,5,7)	(6,8,10)	(2,4,6)	(7,8,9)	**(1,4,7)**	(70,75,80)
C_7	(5,6,7)	**(-2,1,4)**	**(-2,2,6)**	(5,6,7)	(0,1,2)	(11,13,15)	(-3,1,5)	(2,4,6)	(7,9,11)	(3,5,7)	(50,55,60)
C_8	(0,3,6)	(1,3,5)	(7,8,9)	(8,10,12)	(3,6,9)	(4,5,6)	(10,11,12)	(3,6,9)	**(3,5,7)**	(4,6,8)	(65,70,75)
C_9	(4,6,8)	(1,2,3)	(2,4,6)	(1,3,5)	(1,2,3)	(2,4,6)	(3,5,7)	(5,6,7)	(8,9,10)	(4,5,6)	(85,90,95)
C_{10}	(7,8,9)	(5,7,9)	(6,8,10)	(9,11,13)	(4,6,8)	**(-1,2,5)**	**(-1,1,3)**	(-2,1,4)	(0,2,4)	(2,3,4)	(55,60,65)
Demand	(15,20,25)	(40,45,50)	(25,30,35)	(5,10,15)	(50,55,60)	(70,75,80)	(90,95,100)	(85,90,95)	(65,70,75)	(55,60,65)	

Step 5: Follow step 5 of the outlined procedure in table 4 to assign the initial allocation.

Table 4: *First allocation table*

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	**Supply**
C_1	(3,5,7)	(4,6,8)	(3,6,9)	**(5,10,15)** (-2,1,4)	(2,3,4)	(7,8,9)	(5,6,7)	(-1,2,5)	(1,4,7)	(7,8,9)	(15,20,25) - (5,10,15) = (0,10,20)
C_2	(2,5,8)	(5,7,9)	(4,5,6)	(7,8,9)	(0,3,6)	(12,13,14)	(5,7,9)	(0,1,2)	(2,4,6)	(6,7,8)	(5,10,15)
C_3	(5,6,7)	(4,6,8)	(11,13,15)	(3,6,9)	(6,9,12)	(2,3,4)	(8,9,10)	(-5,2,13)	(9,10,11)	(-2,1,4)	(25,30,35)
C_4	(-1,1,3)	(2,5,7)	(2,3,4)	(3,5,7)	(-2,4,10)	(5,6,7)	(7,8,9)	(1,3,5)	(3,6,9)	(10,11,12)	(40,45,50)
C_5	(8,9,10)	(2,4,6)	(8,10,12)	(6,8,10)	(3,6,7)	(10,12,14)	(1,4,7)	(0,4,8)	(1,2,3)	(8,9,10)	(90,95,100)
C_6	(6,7,8)	(12,13,14)	(-3,1,5)	(1,2,3)	(1,3,5)	(3,5,7)	(6,8,10)	(2,4,6)	(7,8,9)	(1,4,7)	(70,75,80)
C_7	(5,6,7)	(-2,1,4)	(-2,2,6)	(5,6,7)	(0,1,2)	(11,13,15)	(-3,1,5)	(2,4,6)	(7,9,11)	(3,5,7)	(50,55,60)
C_8	(0,3,6)	(1,3,5)	(7,8,9)	(8,10,12)	(3,6,9)	(4,5,6)	(10,11,12)	(3,6,9)	(3,5,7)	(4,6,8)	(65,70,75)
C_9	(4,6,8)	(1,2,3)	(2,4,6)	(1,3,5)	(1,2,3)	(2,4,6)	(3,5,7)	(5,6,7)	(8,9,10)	(4,5,6)	(85,90,95)
C_{10}	(7,8,9)	(5,7,9)	(6,8,10)	(9,11,13)	(4,6,8)	(-1,2,5)	(-1,1,3)	(-2,1,4)	(0,2,4)	(2,3,4)	(55,60,65)
Demand	(15,20,25)	(40,45,50)	(25,30,35)	(5,10,15)	(50,55,60)	(70,75,80)	(90,95,100)	(85,90,95)	(65,70,75)	(55,60,65)	

Step 6: Using step 6 of the proposed method, remove T_4 from table 4, and then the new reduction indicated in table 5, and again execute steps 5 to 6 for the second allocation shown in table 6.

Table 5: *New Reduced Table*

	T_1	T_2	T_3	T_5	T_6	T_7	T_8	T_9	T_{10}	**Supply**
C_1	(3,5,7)	(4,6,8)	(3,6,9)	(2,3,4)	(7,8,9)	(5,6,7)	(-1,2,5)	(1,4,7)	(7,8,9)	(0,10,20)
C_2	(2,5,8)	(5,7,9)	(4,5,6)	(0,3,6)	(12,13,14)	(5,7,9)	(0,1,2)	(2,4,6)	(6,7,8)	(5,10,15)
C_3	(5,6,7)	(4,6,8)	(11,13,15)	(6,9,12)	(2,3,4)	(8,9,10)	(-5,2,13)	(9,10,11)	(-2,1,4)	(25,30,35)
C_4	(-1,1,3)	(2,5,7)	(2,3,4)	(-2,4,10)	(5,6,7)	(7,8,9)	(1,3,5)	(3,6,9)	(10,11,12)	(40,45,50)
C_5	(8,9,10)	(2,4,6)	(8,10,12)	(3,6,7)	(10,12,14)	(1,4,7)	(0,4,8)	(1,2,3)	(8,9,10)	(90,95,100)
C_6	(6,7,8)	(12,13,14)	(-3,1,5)	(1,3,5)	(3,5,7)	(6,8,10)	(2,4,6)	(7,8,9)	(1,4,7)	(70,75,80)
C_7	(5,6,7)	(-2,1,4)	(-2,2,6)	(0,1,2)	(11,13,15)	(-3,1,5)	(2,4,6)	(7,9,11)	(3,5,7)	(50,55,60)
C_8	(0,3,6)	(1,3,5)	(7,8,9)	(3,6,9)	(4,5,6)	(10,11,12)	(3,6,9)	(3,5,7)	(4,6,8)	(65,70,75)
C_9	(4,6,8)	(1,2,3)	(2,4,6)	(1,2,3)	(2,4,6)	(3,5,7)	(5,6,7)	(8,9,10)	(4,5,6)	(85,90,95)
C_{10}	(7,8,9)	(5,7,9)	(6,8,10)	(4,6,8)	(-1,2,5)	(-1,1,3)	(-2,1,4)	(0,2,4)	(2,3,4)	(55,60,65)
Demand	(15,20,25)	(40,45,50)	(25,30,35)	(50,55,60)	(70,75,80)	(90,95,100)	(85,90,95)	(65,70,75)	(55,60,65)	

Table 6: *Second allocation table*

	T_1	T_2	T_3	T_5	T_6	T_7	T_8	T_9	T_{10}	**Supply**
C_1	(3,5,7)	(4,6,8)	(3,6,9)	(2,3,4)	(7,8,9)	(5,6,7)	(-1,2,5)	(1,4,7)	(7,8,9)	(0,10,20)
C_2	(2,5,8)	(5,7,9)	(4,5,6)	(0,3,6)	(12,13,14)	(5,7,9)	**(5,10,15)** (0,1,2)	(2,4,6)	(6,7,8)	(5,10,15)
C_3	(5,6,7)	(4,6,8)	(11,13,15)	(6,9,12)	(2,3,4)	(8,9,10)	(-5,2,13)	(9,10,11)	(-2,1,4)	(25,30,35)
C_4	(-1,1,3)	(2,5,7)	(2,3,4)	(-2,4,10)	(5,6,7)	(7,8,9)	(1,3,5)	(3,6,9)	(10,11,12)	(40,45,50)
C_5	(8,9,10)	(2,4,6)	(8,10,12)	(3,6,7)	(10,12,14)	(1,4,7)	(0,4,8)	(1,2,3)	(8,9,10)	(90,95,100)
C_6	(6,7,8)	(12,13,14)	(-3,1,5)	(1,3,5)	(3,5,7)	(6,8,10)	(2,4,6)	(7,8,9)	(1,4,7)	(70,75,80)
C_7	(5,6,7)	(-2,1,4)	(-2,2,6)	(0,1,2)	(11,13,15)	(-3,1,5)	(2,4,6)	(7,9,11)	(3,5,7)	(50,55,60)
C_8	(0,3,6)	(1,3,5)	(7,8,9)	(3,6,9)	(4,5,6)	(10,11,12)	(3,6,9)	(3,5,7)	(4,6,8)	(65,70,75)
C_9	(4,6,8)	(1,2,3)	(2,4,6)	(1,2,3)	(2,4,6)	(3,5,7)	(5,6,7)	(8,9,10)	(4,5,6)	(85,90,95)
C_{10}	(7,8,9)	(5,7,9)	(6,8,10)	(4,6,8)	(-1,2,5)	(-1,1,3)	(-2,1,4)	(0,2,4)	(2,3,4)	(55,60,65)
Demand	(15,20,25)	(40,45,50)	(25,30,35)	(50,55,60)	(70,75,80)	(90,95,100)	(85,90,95) -(5,10,15) =(70,80,90)	(65,70,75)	(55,60,65)	

Step 7: Using Steps 5 to 6 of the proposed technique once again, all allocations are made as indicated in Table 7.

Table 7: *Final allocations of fuzzy transportation table*

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}
C_1	(3,5,7)	(4,6,8)	(3,6,9)	**(5,10,15)** (-2,1,4)	(2,3,4)	(7,8,9)	(5,6,7)	**(0,10,20)** (-1,2,5)	(1,4,7)	(7,8,9)
C_2	(2,5,8)	(5,7,9)	(4,5,6)	(7,8,9)	(0,3,6)	(12,13,14)	(5,7,9)	**(5,10,15)** (0,1,2)	(2,4,6)	(6,7,8)
C_3	(5,6,7)	(4,6,8)	(11,13,15)	(3,6,9)	(6,9,12)	(2,3,4)	(8,9,10)	(-5,2,13)	(9,10,11)	**(25,30,35)** (-2,1,4)
C_4	**(15,20,25)** (-1,1,3)	(2,5,7)	(2,3,4)	(3,5,7)	(-2,4,10)	(5,6,7)	(7,8,9)	**(15,25,35)** (1,3,5)	(3,6,9)	(10,11,12)
C_5	(8,9,10)	(2,4,6)	(8,10,12)	(6,8,10)	(3,6,7)	(10,12,14)	**(15,25,35)** (1,4,7)	(0,4,8)	**(65,70,75)** (1,2,3)	(8,9,10)
C_6	(6,7,8)	(12,13,14)	**(25,30,35)** (-3,1,5)	(1,2,3)	(1,3,5)	(3,5,7)	(6,8,10)	**(35,45,55)** (2,4,6)	(7,8,9)	(1,4,7)
C_7	(5,6,7)	**(40,45,50)** (-2,1,4)	(-2,2,6)	(5,6,7)	**(0,10,20)** (0,1,2)	(11,13,15)	(-3,1,5)	(2,4,6)	(7,9,11)	(3,5,7)
C_8	(0,3,6)	(1,3,5)	(7,8,9)	(8,10,12)	(3,6,9)	**(5,30,55)** (4,5,6)	**(-10,10,30)** (10,11,12)	**(-40,0,40)** (3,6,9)	(3,5,7)	**(20,30,40)** (4,6,8)
C_9	(4,6,8)	(1,2,3)	(2,4,6)	(1,3,5)	**(30,45,60)** (1,2,3)	**(25,45,65)** (2,4,6)	(3,5,7)	(5,6,7)	(8,9,10)	(4,5,6)
C_{10}	(7,8,9)	(5,7,9)	(6,8,10)	(9,11,13)	(4,6,8)	(-1,2,5)	**(55,60,65)** (-1,1,3)	(-2,1,4)	(0,2,4)	(2,3,4)

As a result, $(m + n - 1) = (10 + 10 - 1 = 19,$ cells are assigned and we have a feasible solution. Then find the minimum fuzzy transportation cost.

Step 8: Calculate the minimum Fuzzy Transportation Cost. Total cost $Z = \sum_{i=1}^{m} \mathcal{C}_{ij} \sum_{j=1}^{n} \mathcal{X}_{ij}$.

⇒Z = (5,10,15) (-2,1,4) + (0,10,20) (-1,2,5) + (5,10,15) (0,1,2) + (25,30,35) (-2,1,4) + (15,20,25) (-1,1,3) + (15,25,35) (1,3,5) + (15,25,35) (1,4,7) + (65,70,75) (1,2,3) + (25,30,35) (-3,1,5) + (35,45,55) (2,4,6) + (40,45,50) (-2,1,4) + (0,10,20) (0,1,2) + (5,30,55) (4,5,6) + (-10,10,30) (10,11,12) + (-40,0,40) (3,6,9) + (20,30,40) (4,6,8) + (30,45,60) (1,2,3) + (25,45,65) (2,4,6) + (55,60,65) (-1,1,3)

Z = (-160, 1440, 3930)

4.1 Result and discussion:

The fuzzy transportation cost Z of the given FTP is a TFN as given below:

Z = (-160, 1440, 3930).

The result can be explained (Refer to Fig. 2) as follows:

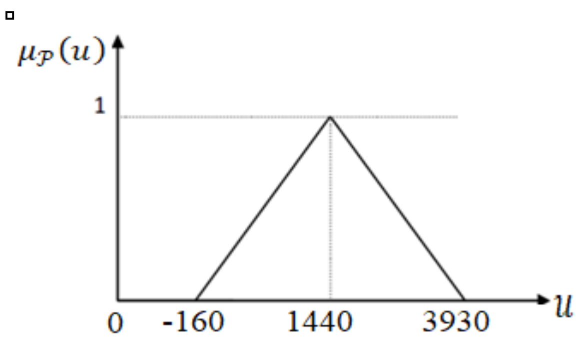

Fig. 2 fuzzy transportation cost

The least amount of the minimum total transportation cost is -160.

The most possible amount of the minimum total transportation cost is 1440.

The greatest amount of the minimum total transportation cost is 3930. i.e., the minimum total transportation cost will always be greater than -160 and less than 1440, and highest chances are that the minimum total transportation cost will be 3930.
The above result was verified by MATLAB.

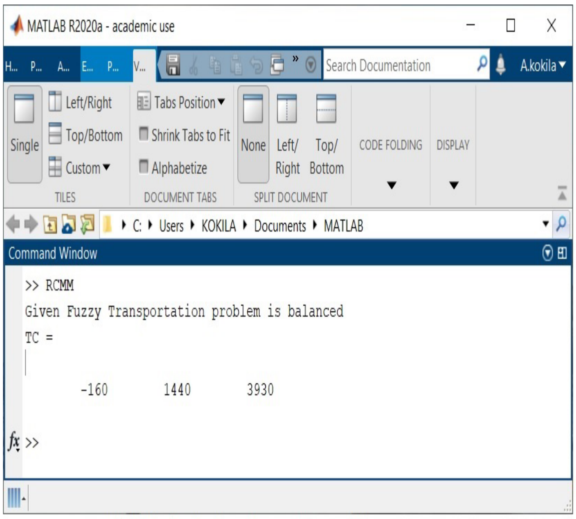

Fig 3. Result of the Fuzzy Transportation problem

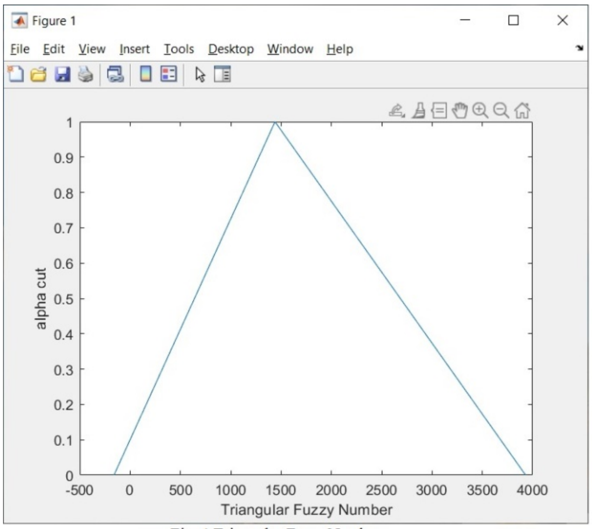

Fig. 4 Triangular Fuzzy Number

Table 8: *Comparative results of NWCM, LCM, RMM, VAM and proposed method (RCMM) for example 1*

Numerical example	NWCM	LCM	RMM	VAM	Proposed method (RCMM)
1	(-570,4050,11080)	(445,2160,4755)	(455,2110,4775)	(-70,1760,4260)	(-160,1440,3930)

The comparative results in table 8 are also depicted using bar graphs and the results are given in the Figure 5.

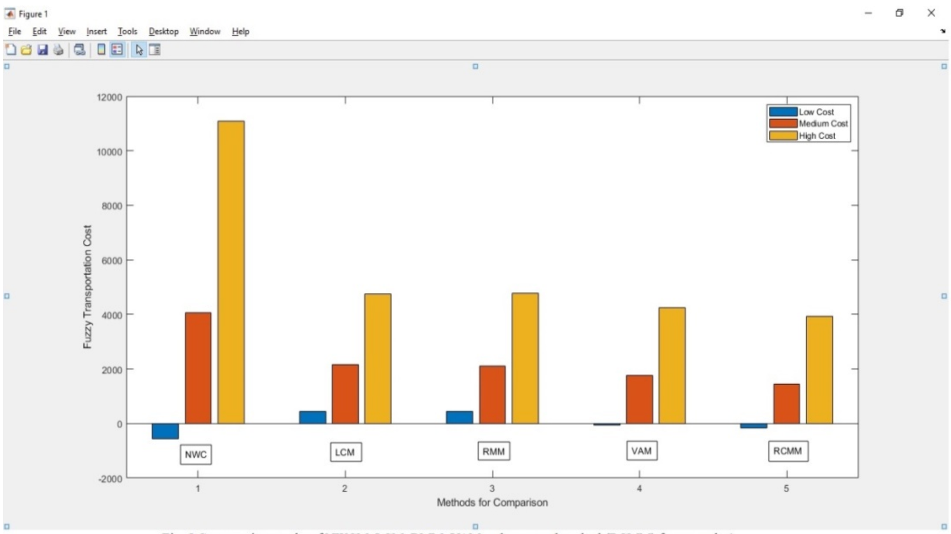

Fig. 5 Comparative results of NWCM, LCM, RMM, VAM and proposed method (RCMM) for example 1

4.2 Comparison of results: The numerical examples 2, 3, 4, 5, 6 are taken from the referred journals 1, 8, 9, 10, 14 respectively, and it is verified with our proposed method and the existing methods NWCM, LCM, RMM, VAM.

Table 9: *Comparative results of NWCM, LCM, RMM, VAM and proposed method (RCMM) for example 2 to 6*

Numerical examples	NWCM	LCM	RMM	VAM	Proposed method	RCMM
2[Ref:1]	(68,176,316)	(62,150,258)	(62,150,258)	(52,149,274)	(62, 150, 258)	(-2,83,208)
3[Ref:8]	(1850,6609,12882)	(1790,6609,12998)	(1900,8292,19080)	(1850,6609,12882)	(3532, 6609, 9852)	(-462,3249,8798)
4[Ref:9]	(40,1230.3560)	(-120,1210,3860)	(140,1250,3220)	(120,1210,3140)	(270,1210,2750)	(-1140,500,3760)
5[Ref:10]	(125,1000,2950)	(-175,850,2925)	(-275,950,3450)	(-25,850,2625)	(-75,850,2750)	(-175,350,2425)
6[Ref:14]	(-270,4285,10470)	(160,2455,5470)	(-330,2290,6500)	(-25,2220,5455)	(1825,2455,3085)	(-340,1025,4180)

5. Conclusion and future study

Our proposed method uses the comparison table to find the best initial feasible solution to the balanced and unbalanced fuzzy transportation problems. We compared our strategy to others and discovered that ours is the most effective. This technique considers the entire fuzzy cost of each origin and destination for allotment, allowing for a reduction in iterations to provide the best basic feasible solution to FTP. In addition, the proposed method is used to achieve the best solution for an unbalanced TP by converting it to a balanced TP without the need of a dummy source/destination, saving time and space. The proposed method is simple to implement and can be used to solve a variety of fuzzy transportation problems, including minimizing the total transportation costs. In the future, this technique might be expanded to fuzzy multiple objective transportation problems and used to solve real-world transportation problems using fuzzy numbers.

Conflict of interest: There are no conflicts of interest declared by the authors.

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