Inventory Model with Truncated Weibull Decay Under Permissible Delay in Payments and Inflation Having Selling Price Dependent Demand

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Abstract

For optimal utilization of resources, the inventory models are required in several places such as market yards, production processes, warehouses, oil exploration industries and food vegetable markets. Huge work has been produced by several researchers in inventory models for obtaining optimal ordering quantity and pricing policies. This paper addresses an EOQ model for deteriorating items having Weibull decay under inflation and permissible delay in payments. It is considered that the demand of items is a function of selling price. It is further assumed that the decay of items starts after certain period of time which can be well characterized by truncated Weibull probability model for the life time of the commodity. The optimal ordering and pricing policies of this system are derived and analyzed in the light of the input parameters and costs. Through sensitivity analysis it is demonstrated that the delay in the payments and rate of inflation have significant effect on the optimal policies. This model is very useful in the analyzing market yards where sea foods, vegetables, fruits, edible oils are stored and distributed.

Keywords: EOQ model, selling price depended demand, truncated decay.

1. Introduction

Decay is the major consideration for planning inventory and scheduling orders. The decay is in general random due to various factors such as environmental conditions, type of commodity, storage facility and natural life time. Considering the life time of commodity as random several authors developed various inventory model for deteriorating items with various plausible assumptions. The review on inventory models with deteriorating items is given by [1], [2], [3], [4]. Recently [5], [6], [7], [8], [9], and [10] have developed several inventory models with the assumption that the life time of a commodity is random and follows a specified distribution depending on the nature of commodity. In all these papers they assumed that the decay starts immediately after the procurement. But in many practical situations the deterioration of items in the stock starts only after certain period of time. This type of delay in decay can be characterized by truncated Weibull life time distribution which is often known as three parameter Weibull distribution.

Another basic assumption made by all these authors is that the payments must be made to the supplier immediately after receiving the items. However, it is a common phenomenon that the supplier allows a certain fixed period for finalizing the accounts and does not charge any interest during that period from the retailer. In [11] studied an EOQ model with assumption of permissible delay in payments. His work was extended to deteriorating items by [12]. Later [13], [14], [15] and others have developed EOQ models with permissible delay in payments.

In today's business transaction, the supplier will offer a cash discount to encourage the retailer in addition to allowing a fixed period for settlement of account. In addition to this there is a change in money value over time. Ignoring inflation may leads falsification in the model. Recently [16] has studied Inventory Model with Generalized Pareto life time under permissible delay in payments while deriving the optimal pricing and ordering policies. Considering the inflation several authors have studied various inventory model with permissible delay in payments. However, they assumed the decay is constant or independent of time, but in many practical situations the deteriorating rate is time dependent. An EOQ model with time quadratic demand by [17]. They considered the inflation while determining the optimal policies.

Little work has been reported regarding EOQ models under permissible delay in payments having inflation and selling price dependent demand, which are very useful for analyzing many practical situations arising at market yards, warehouse etc. Hence in this paper we develop and analyze the Economic Order Quantity model with truncated Weibull decay under permissible delay in payments and inflation having selling price dependent demand.

Section (2) of this paper deals with the assumptions of the model and notation. Section (3) is to develop the instantaneous inventory level at any given time t. The optimal ordering and pricing policies of the model are derived in Section (4). Section (5) considers Numerical illustration of the model. The sensitivity analysis is presented in Section (6). Section (7) deals with conclusions.

2. Assumptions

For developing the Economic Order Quantity model, the following assumptions are made

- Deterioration start time is γ.
- Weibull distribution is the life time distribution of the commodity. Its p.d.f is

$$f(t) = \alpha\beta(t-\gamma)^{\beta-1}e^{-\alpha(t-\gamma)^{\beta}}$$

Where α is the scale parameter, β is the shape parameter and γ is the location parameter

The instantaneous deterioration rate is

$$h(t) = \alpha(t-\gamma)^{\beta}, \qquad t \geq \gamma$$

- Demand function is

$$R(p(t)) = a - bp(t) = a - bpe^{rt}$$

Which is selling price dependent demand. Where, a is the fixed demand, $a > 0$, b is the demand parameter, $b > 0$, and $a > b$, $p(t)$ is the selling price of an item at time t and p is the selling price of the item at time $t = 0$.
- Rate of inflation is r, $0 < r < 1$
- Shortages are not allowed.
- Zero lead time.
- During the permissible delay period (M), the account is not settled, the generated sales revenue is deposited in an interest-bearing account. At the end of the trade credit period, the customer pays off for all the units ordered.
- There is no repair or replacement of the deteriorated units during the cycle time.

Notation

H : Finite horizon length.

$R(p(t))$: Demand per unit time as a function of selling price.

h : Holding cost of inventory per unit time after excluding interest.

r : Rate of inflation.

$p(t) = pe^{rt}$: Per unit selling price.

$g(t) = ge^{rt}$: Purchase cost of a unit at time t.

$A(t) = Ae^{rt}$: Per order cost at time t.

I_C : Interest charged per Rs. INR in stock per a year by the supplier.

I_e : Interest earned in Rs. INR per a year.

M : Permissible delay period which is allowed in settling the account.

Q : Order quantity per a cycle.

T : Cycle length

$I(t)$: On-hand inventory at time $t, 0 \le t \le T$.

$TC(p,T)$: Total cost over $(0, H)$.

$NP(p,T)$: Net profit rate function over planning period.

3. Inventory Model

Let Q be the inventory level of the system at time $t = 0$. During $(0,\gamma)$ inventory will decrease due to demand and during (γ, T) inventory will decrease due to demand and deterioration. Since no shortages are allowed, at time T the inventory level reaches zero, the stock is replenished instantaneously. The schematic diagram representing the inventory level is shown in Figure-3.1.

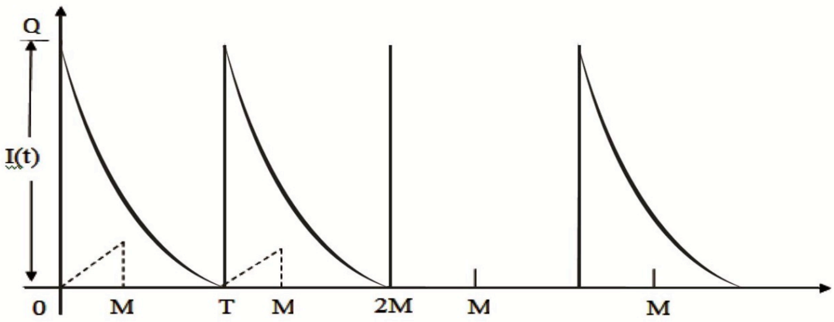


Figure -1: *Schematic diagram representing the inventory level of selling price dependent demand model*

Let $I(t)$ be the on-hand inventory at time t. The differential equations governing the on-hand inventory at time t are

$$\frac{d}{dt}I(t) = -R\big(p(t)\big) \qquad 0 \le t \le \gamma \qquad (1)$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -R\big(p(t)\big) \qquad \gamma \le t \le T \qquad (2)$$

where $h(t) = \alpha\beta(t-\gamma)^{\beta-1}$ $\quad \gamma \le t \le T$

and $R(p(t)) = a - bp(t) = a - bpe^{rt}$

with initial conditions $I(0) = Q \; and \; I(T) = 0$.

Solving equation (1) and using the initial condition $I(0) = Q$, we get

$$I(t) = Q - at + \frac{bp}{r}(e^{rt} - 1) \qquad 0 \le t \le \gamma \qquad (3)$$

Solving equation (2) and using the initial condition $I(T) = 0$, we get

$$I(t) = e^{-\alpha(t-\gamma)^{\beta}} \left[a \int_{t}^{T} e^{\alpha(u-\gamma)^{\beta}} du - bp \int_{t}^{T} e^{ru+\alpha(u-\gamma)^{\beta}} du \right] \quad \gamma \le t \le T \tag{4}$$

Equating equations (3) and (4) when $t = \gamma$, we get

$$Q = a\gamma - \frac{bp}{r}(e^{r\gamma} - 1) + a \int_{\gamma}^{T} e^{\alpha(u-\gamma)^{\beta}} du - bp \int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^{\beta}} du \tag{5}$$

Substituting Q in equation (3), we get

$$I(t) = a(\gamma - t) + \frac{bp}{r}(e^{rt} - e^{r\gamma}) + a \int_{\gamma}^{T} e^{\alpha(u-\gamma)^{\beta}} du - bp \int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^{\beta}} du \quad 0 \le t \le \gamma \tag{6}$$

Since the length of time intervals are all the same, we have

$$I(jT + t) = \begin{cases} a(\gamma - t) + \frac{bp}{r}(e^{rt} - e^{r\gamma}) + a \int_{\gamma}^{T} e^{\alpha(u-\gamma)^{\beta}} du - bp \int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^{\beta}} du & 0 \le t \le \gamma \\ e^{-\alpha(t-\gamma)^{\beta}} \left[a \int_{t}^{T} e^{\alpha(u-\gamma)^{\beta}} du - bp \int_{t}^{T} e^{ru+\alpha(u-\gamma)^{\beta}} du \right] & \gamma \le t \le T \end{cases} \tag{7}$$

4. The Optimal Ordering and Pricing Policies

Total cost function is the sum of Ordering Cost (OC), Cost Deterioration (CD), Inventory Carrying Cost (ICC), Interest Charged (IC_1) and Interest Earned (IE_1).
Each cost component is computed as follows:
Ordering Cost, OC is

$$OC = A(0) + A(T) + A(2T) + \ldots + A(n-1)T = A\left[\frac{e^{rH} - 1}{e^{rT} - 1}\right] \tag{8}$$

Cost Deterioration, CD is

$$CD = \sum_{j=0}^{n-1} g e^{rjT} \left[Q - \int_{0}^{T} (a - bpe^{rt}) dt \right]$$

where, Q is as given in equation (5). On simplification, we get

$$CD = g\left[a\gamma - aT - \frac{bp}{r}(e^{r\gamma} - e^{rT}) + a \int_{\gamma}^{T} e^{\alpha(u-\gamma)^{\beta}} du - bp \int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^{\beta}} du \right] \left[\frac{e^{rH} - 1}{e^{rT} - 1}\right] \tag{9}$$

Inventory Carrying Cost, ICC is

$$ICC = h \sum_{j=0}^{n-1} g(jT) \left[\int_{0}^{T} I(jT + t) dt \right]$$

$$= hg\left[\frac{a\gamma^2}{2} + \frac{bp}{r^2}[e^{r\gamma}(1 - r\gamma) - 1] + \gamma \left[a \int_{\gamma}^{T} e^{\alpha(u-\gamma)^{\beta}} du - bp \int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^{\beta}} du \right] + \int_{\gamma}^{T} e^{-\alpha(t-\gamma)^{\beta}} \left[a \int_{t}^{T} e^{\alpha(u-\gamma)^{\beta}} du - bp \int_{t}^{T} e^{ru+\alpha(u-\gamma)^{\beta}} du \right] dt \right] \left[\frac{e^{rH} - 1}{e^{rT} - 1}\right] \tag{10}$$

For computing interest charged and earned, there are two possibilities based on the customer's choice. Interest Charges (IC) for unsold items at the initial time or after the permissible delay period M and interest Earned (IE) from the sales revenue during the permissible delay period.
Case (i): Optimum cycle length T is larger than or equal to M i.e., $T \ge M$
Interest Charged in $(0, H)$, IC_1 is

$$IC_1 = I_c \sum_{j=0}^{n-1} g(jT)\left[\int_M^T I(jT+t)dt\right]$$

$$= I_c g\left[\frac{a}{2}(\gamma^2 + M^2 - 2M\gamma) + \frac{bp}{r^2}\left[e^{r\gamma}\big(1 - r(\gamma - M)\big) - e^{rM}\right] + (\gamma - M)\left[a\int_\gamma^T e^{\alpha(u-\gamma)^\beta}du - bp\int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta}du\right]\right.$$

$$\left. + \int_\gamma^T e^{-\alpha(t-\gamma)^\beta}\left[a\int_t^T e^{\alpha(u-\gamma)^\beta}du - bp\int_t^T e^{ru+\alpha(u-\gamma)^\beta}du\right]dt\right]\left[\frac{e^{rH}-1}{e^{rT}-1}\right] \tag{11}$$

Interest Earned in $(0, H)$, IE_1 is

$$IE_1 = I_e \sum_{j=0}^{n-1} p(jT)\left[\int_0^M (a - bpe^{rt})t\,dt\right]$$

$$= I_e p\left[\frac{aM^2}{2} - \frac{bp}{r^2}\left[e^{rM}(rM-1)+1\right]\right]\left[\frac{e^{rH}-1}{e^{rT}-1}\right] \tag{12}$$

The total cost over $(0, H)$ is $TC(p,T)$ and is given by

$$TC(p,T) = OC + CD + ICC + IC_1 - IE_1 \tag{13}$$

Substituting equations (8), (9), (10), (11) and (12) in (13), we get

$TC(p,T) =$

$$\left[A + g\left[a\gamma - aT - \frac{bp}{r}(e^{r\gamma} - e^{rT}) + a\int_\gamma^T e^{\alpha(u-\gamma)^\beta}du - bp\int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta}du\right]\right.$$

$$+hg\left[\frac{a\gamma^2}{2} + \frac{bp}{r^2}\left[e^{r\gamma}(1 - r\gamma) - 1\right] + \gamma\left[a\int_\gamma^T e^{\alpha(u-\gamma)^\beta}du - bp\int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta}du\right]\right.$$

$$\left. + \int_\gamma^T e^{-\alpha(t-\gamma)^\beta}\left[a\int_t^T e^{\alpha(u-\gamma)^\beta}du - bp\int_t^T e^{ru+\alpha(u-\gamma)^\beta}du\right]dt\right]$$

$$+I_c g\left[\frac{a}{2}(\gamma^2 + M^2 - 2M\gamma) + \frac{bp}{r^2}\left[e^{r\gamma}\big(1 - r(\gamma - M)\big) - e^{rM}\right] + (\gamma - M)\left[a\int_\gamma^T e^{\alpha(u-\gamma)^\beta}du - bp\int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta}du\right]\right.$$

$$\left. + \int_\gamma^T e^{-\alpha(t-\gamma)^\beta}\left[a\int_t^T e^{\alpha(u-\gamma)^\beta}du - bp\int_t^T e^{ru+\alpha(u-\gamma)^\beta}du\right]dt\right]$$

$$\left. - I_e p\left[\frac{aM^2}{2} - \frac{bp}{r^2}\left[e^{rM}(rM-1)+1\right]\right]\right]\left[\frac{e^{rH}-1}{e^{rT}-1}\right] \tag{14}$$

The net profit is the difference of gross revenue and total cost.

The gross revenue is $(pe^{rT} - ge^{rT})(a - bpe^{rT})$

Hence, the net profit is $NP(p,T) = (pe^{rT} - ge^{rT})(a - bpe^{rT}) - TC(p,T)$ (15)

where, $TC(p,T)$ is as given in (14)

For obtaining the optimal policies of the model, maximize $NP(p,T)$ with respect to T and p. The conditions for obtaining optimality are

$$\frac{\partial NP(p,T)}{\partial T}=0,\frac{\partial NP(p,T)}{\partial p}=0 \quad and \quad D=\begin{vmatrix}\frac{\partial^2 NP(p,T)}{\partial p^2} & \frac{\partial^2 NP(p,T)}{\partial T\partial p}\\ \frac{\partial^2 NP(p,T)}{\partial T\partial p} & \frac{\partial^2 NP(p,T)}{\partial T^2}\end{vmatrix}<0$$

where D is the determinant of Hessian matrix

$\frac{\partial NP(p,T)}{\partial T}=0$ implies,

$$(p-g)[are^{rT}-2pbre^{2rT}]$$

$$\left\{\left[\frac{e^{rH}-1}{e^{rT}-1}\right]\left[g\left[-a+bpe^{rT}+ae^{\alpha(T-\gamma)^\beta}-bpe^{rT+\alpha(T-\gamma)^\beta}\right]\right.\right.$$

$$+hg\left[\gamma\left[ae^{\alpha(T-\gamma)^\beta}-bpe^{rT+\alpha(T-\gamma)^\beta}\right]+\int_\gamma^T e^{-\alpha(t-\gamma)^\beta}\left[ae^{\alpha(T-\gamma)^\beta}-bpe^{rT+\alpha(T-\gamma)^\beta}\right]dt\right]$$

$$+I_cg\left[(\gamma-M)\left[ae^{\alpha(T-\gamma)^\beta}-bpe^{rT+\alpha(T-\gamma)^\beta}\right]+\int_\gamma^T e^{-\alpha(t-\gamma)^\beta}\left[ae^{\alpha(T-\gamma)^\beta}-bpe^{rT+\alpha(T-\gamma)^\beta}\right]d\,t\right]$$

$$+\left[A+g\left[a\gamma-aT-\frac{bp}{r}(e^{r\gamma}-e^{rT})+a\int_\gamma^T e^{\alpha(u-\gamma)^\beta}du-bp\int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta}du\right]\right.$$

$$+hg\left[\frac{a\gamma^2}{2}+\frac{bp}{r^2}[e^{r\gamma}(1-r\gamma)-1]+\gamma\left[a\int_\gamma^T e^{\alpha(u-\gamma)^\beta}du-bp\int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta}du\right]\right.$$

$$\left.+\int_\gamma^T e^{-\alpha(t-\gamma)^\beta}\left[a\int_t^T e^{\alpha(u-\gamma)^\beta}du-bp\int_t^T e^{ru+\alpha(u-\gamma)^\beta}du\right]dt\right]$$

$$+I_cg\left[\frac{a}{2}(\gamma^2+M^2-2M\gamma)+\frac{bp}{r^2}\left[e^{r\gamma}\big(1-r(\gamma-M)\big)-e^{rM}\right]+(\gamma-M)\left[a\int_\gamma^T e^{\alpha(u-\gamma)^\beta}du-bp\int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta}du\right]\right.$$

$$\left.+\int_\gamma^T e^{-\alpha(t-\gamma)^\beta}\left[a\int_t^T e^{\alpha(u-\gamma)^\beta}du-bp\int_t^T e^{ru+\alpha(u-\gamma)^\beta}du\right]dt\right]$$

$$\left.\left.-I_ep\left[\frac{aM^2}{2}-\frac{bp}{r^2}[e^{rM}(rM-1)+1]\right]\right]\left[\frac{e^{rH}-1}{(e^{rT}-1)^2}\right]re^{rT}\right\}=0 \qquad (16)$$

$\frac{\partial NP(p,T)}{\partial p}=0$ implies,

$$e^{rT}(a+bge^{rT}-2pbe^{rT})$$

$$-\left\{g\left[-\frac{b}{r}(e^{r\gamma}-e^{rT})-b\int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta}du\right]+hg\left[\frac{b}{r^2}[e^{r\gamma}(1-r\gamma)-1]-b\gamma\left[\int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta}du\right]\right.\right.$$

$$\left.-b\int_\gamma^T e^{-\alpha(t-\gamma)^\beta}\left[\int_t^T e^{ru+\alpha(u-\gamma)^\beta}du\right]dt\right]+I_cg\left[\frac{b}{r^2}\left[e^{r\gamma}\big(1-r(\gamma-M)\big)-e^{rM}\right]\right.$$

$$-b(\gamma-M)\left[\int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]-b\int_{\gamma}^{T} e^{-\alpha(t-\gamma)^{\beta}}\left[\int_{t}^{T} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]dt\Bigg]$$

$$-I_e\left[\frac{aM^2}{2}-\frac{2bp}{r^2}\left[e^{rM}(rM-1)+1\right]\right]\Bigg\}\left[\frac{e^{rH}-1}{e^{rT}-1}\right]=0 \tag{17}$$

For given values of the parameters and costs, equations (16) and (17) are solved using MATHCAD to get the optimal cycle length $T=T_1$ and selling price $p=p_1$. Substituting the optimal values T_1 and p_1 in equation (14) we get the minimum total cost. Substituting this minimum total cost, T_1 and p_1 in equation (15), we get the maximum profit as

$$NP^{*}(p_1,T_1)=(p_1e^{rT_1}-ge^{rT_1})(a-bp_1e^{rT_1})$$

$$-\left[A+g\left[a\gamma-aT_1-\frac{bp_1}{r}(e^{r\gamma}-e^{rT_1})+a\int_{\gamma}^{T_1} e^{\alpha(u-\gamma)^{\beta}}du-bp_1\int_{\gamma}^{T_1} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]\right.$$

$$+hg\left[\frac{a\gamma^2}{2}+\frac{bp_1}{r^2}\left[e^{r\gamma}(1-r\gamma)-1\right]+\gamma\left[a\int_{\gamma}^{T_1} e^{\alpha(u-\gamma)^{\beta}}du-bp_1\int_{\gamma}^{T_1} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]\right.$$

$$\left.+\int_{\gamma}^{T_1} e^{-\alpha(t-\gamma)^{\beta}}\left[a\int_{t}^{T_1} e^{\alpha(u-\gamma)^{\beta}}du-bp_1\int_{t}^{T_1} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]dt\right]$$

$$+I_cg\left[\frac{a}{2}(\gamma^2+M^2-2M\gamma)+\frac{bp_1}{r^2}\left[e^{r\gamma}\left(1-r(\gamma-M)\right)-e^{rM}\right]+(\gamma-M)\left[a\int_{\gamma}^{T_1} e^{\alpha(u-\gamma)^{\beta}}du-bp_1\int_{\gamma}^{T_1} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]\right.$$

$$\left.\left.+\int_{\gamma}^{T_1} e^{-\alpha(t-\gamma)^{\beta}}\left[a\int_{t}^{T_1} e^{\alpha(u-\gamma)^{\beta}}du-bp_1\int_{t}^{T_1} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]dt\right]-I_ep_1\left[\frac{aM^2}{2}-\frac{bp_1}{r^2}\left[e^{rM}(rM-1)+1\right]\right]\right]\left[\frac{e^{rH}-1}{e^{rT_1}-1}\right] \tag{18}$$

Case (ii): Cycle Length T is smaller than M i.e., $T<M$

Interest Earned, IE_2 is

$$IE_2=I_e\sum_{j=0}^{n-1}p(j,T)\left\{\int_0^T R(p(t))t\,dt+R(p(T))[T(M-T)]\right\}$$

$$=pI_e\left\{\int_0^T (a-bpe^{rt})t\,dt+(a-bpe^{rT})\,[T(M-T)]\right\}\left[\frac{e^{rH}-1}{e^{rT}-1}\right]$$

$$=pI_e\left[\frac{aT^2}{2}-\frac{bp}{r^2}\left[e^{rT}(rT-1)-1\right]+(a-bpe^{rT})[T(M-T)]\right]\left[\frac{e^{rH}-1}{e^{rT}-1}\right] \tag{19}$$

Thus, the total cost over $(0,H)$ is $TC(p,T)$

$$TC(p,T)=OC+CD+ICC-IE_2 \tag{20}$$

Substituting equations (8), (9), (10) and (19) in (20), we get

$$TC(p,T)=\left[A+g\left[a\gamma-aT-\frac{bp}{r}(e^{r\gamma}-e^{rT})+a\int_{\gamma}^{T} e^{\alpha(u-\gamma)^{\beta}}du-bp\int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]\right.$$

$$+hg\left[\frac{a\gamma^2}{2}+\frac{bp}{r^2}[e^{r\gamma}(1-r\gamma)-1]+\gamma\left[a\int_{\gamma}^{T}e^{\alpha(u-\gamma)^\beta}du-bp\int_{\gamma}^{T}e^{ru+\alpha(u-\gamma)^\beta}du\right]\right.$$

$$\left.+\int_{\gamma}^{T}e^{-\alpha(t-\gamma)^\beta}\left[a\int_{t}^{T}e^{\alpha(u-\gamma)^\beta}du-bp\int_{t}^{T}e^{ru+\alpha(u-\gamma)^\beta}du\right]dt\right]$$

$$\left.-pI_e\left[\frac{aT^2}{2}-\frac{bp}{r^2}[e^{rT}(rT-1)-1]+(a-bpe^{rT})[T(M-T)]\right]\right]\left[\frac{e^{rH}-1}{e^{rT}-1}\right] \tag{21}$$

The net profit is the difference of gross revenue and total cost.

The gross revenue is $(pe^{rT}-ge^{rT})(a-bpe^{rT})$

Hence, the net profit is $NP(p,T)=(pe^{rT}-ge^{rT})(a-bpe^{rT})-TC(p,T)$ (22)

where, $TC(p,T)$ is as given in equation (21)

For obtaining the optimal policies of the model we maximize $NP(p,T)$ with respect to T and p. The conditions for obtaining optimality are

$$\frac{\partial NP(p,T)}{\partial T}=0,\frac{\partial NP(p,T)}{\partial p}=0 \quad and \quad D=\begin{vmatrix}\frac{\partial^2 NP(p,T)}{\partial p^2} & \frac{\partial^2 NP(p,T)}{\partial T\partial p}\\ \frac{\partial^2 NP(p,T)}{\partial T\partial p} & \frac{\partial^2 NP(p,T)}{\partial T^2}\end{vmatrix}<0$$

where D is the determinant of Hessian matrix

$\frac{\partial NP(p,T)}{\partial T}=0$ implies,

$$(p-g)[are^{rT}-2brpe^{2rT}]-\left\{\left[\frac{e^{rH}-1}{e^{rT}-1}\right]\left[g\left[-a+bpe^{rT}+ae^{\alpha(T-\gamma)^\beta}-bpe^{rT+\alpha(T-\gamma)^\beta}\right]\right.\right.$$

$$+hg\left[\gamma\left[ae^{\alpha(T-\gamma)^\beta}-bpe^{rT+\alpha(T-\gamma)^\beta}\right]+\int_{\gamma}^{T}e^{-\alpha(t-\gamma)^\beta}\left[ae^{\alpha(T-\gamma)^\beta}-bpe^{rT+\alpha(T-\gamma)^\beta}\right]dt\right]$$

$$\left.-I_ep[aT-bpTe^{rT}+(a-bpe^{rT})(M-2T)+(MT-T^2)(-bpre^{rT})]\right]$$

$$+\left[A+g\left[a\gamma-aT-\frac{bp}{r}(e^{r\gamma}-e^{rT})+a\int_{\gamma}^{T}e^{\alpha(u-\gamma)^\beta}du-bp\int_{\gamma}^{T}e^{ru+\alpha(u-\gamma)^\beta}du\right]\right.$$

$$+hg\left[\frac{a\gamma^2}{2}+\frac{bp}{r^2}[e^{r\gamma}(1-r\gamma)-1]+\gamma\left[a\int_{\gamma}^{T}e^{\alpha(u-\gamma)^\beta}du-bp\int_{\gamma}^{T}e^{ru+\alpha(u-\gamma)^\beta}du\right]\right.$$

$$\left.+\int_{\gamma}^{T}e^{-\alpha(t-\gamma)^\beta}\left[a\int_{t}^{T}e^{\alpha(u-\gamma)^\beta}du-bp\int_{t}^{T}e^{ru+\alpha(u-\gamma)^\beta}du\right]dt\right]$$

$$\left.\left.-I_ep\left[\frac{aT^2}{2}-\frac{2bp}{r^2}[e^{rT}(rT-1)]+(a-bpe^{rT})[T(M-T)]\right]\right]\left[\frac{e^{rH}-1}{(e^{rT}-1)^2}\right]re^{rT}\right\}=0 \tag{23}$$

$\frac{\partial NP(p,T)}{\partial p}=0$ implies,

$$e^{rT}(a+bge^{rT}-2pbe^{rT})-\left[g\left[-\frac{b}{r}(e^{r\gamma}-e^{rT})-b\int_{\gamma}^{T}e^{ru+\alpha(u-\gamma)^\beta}du\right]\right.$$

$$+hg\left[\frac{b}{r^2}\left[e^{r\gamma}(1-r\gamma)-1\right]-b\gamma\int_{\gamma}^{T}e^{ru+\alpha(u-\gamma)^{\beta}}du-b\int_{\gamma}^{T}e^{-\alpha(t-\gamma)^{\beta}}\left[\int_{t}^{T}e^{ru+\alpha(u-\gamma)^{\beta}}du\right]dt\right]$$

$$-I_e\left[\frac{aT^2}{2}-\frac{2pb}{r^2}\left[e^{rT}(rT-1)\right]+(a-2pbe^{rT})[T(M-T)]\right]\Bigg]\left[\frac{e^{rH}-1}{e^{rT}-1}\right]=0 \quad (24)$$

For given values of the parameters and costs, equations (23) and (24) are solved using MATHCAD to get the optimal cycle length $T = T_2$ and selling price $p = p_2$. Substituting the optimal values of T_2 and p_2 in equation (21), we get the minimum total cost. Substituting this minimum total cost, T_2 and p_2 in equation (22), we get the maximum profit as

$$NP^*(p_2,T_2) = (p_2e^{rT_2}-ge^{rT_2})(a-bp_2e^{rT_2})$$

$$-\Bigg[A+g\left[a\gamma-aT_2-\frac{bp_2}{r}(e^{r\gamma}-e^{rT_2})+a\int_{\gamma}^{T_2}e^{\alpha(u-\gamma)^{\beta}}du-bp_2\int_{\gamma}^{T_2}e^{ru+\alpha(u-\gamma)^{\beta}}du\right]$$

$$+hg\Bigg[\frac{a\gamma^2}{2}+\frac{bp_2}{r^2}\left[e^{r\gamma}(1-r\gamma)-1\right]+\gamma\left[a\int_{\gamma}^{T_2}e^{\alpha(u-\gamma)^{\beta}}du-bp_2\int_{\gamma}^{T_2}e^{ru+\alpha(u-\gamma)^{\beta}}du\right]$$

$$+\int_{\gamma}^{T_2}e^{-\alpha(t-\gamma)^{\beta}}\left[a\int_{t}^{T_2}e^{\alpha(u-\gamma)^{\beta}}du-bp_2\int_{t}^{T_2}e^{ru+\alpha(u-\gamma)^{\beta}}du\right]dt\Bigg]$$

$$-p_2I_e\left[\frac{aT_2{}^2}{2}-\frac{bp_2}{r^2}\left[e^{rT_2}(rT_2-1)-1\right]+(a-bp_2e^{rT_2})[T_2(M-T_2)]\right]\Bigg]\left[\frac{e^{rH}-1}{e^{rT_2}-1}\right] \quad (25)$$

5. Numerical Illustration

The optimal values of selling price (p) and cycle length (T) are obtained by using the equation (16) and (17) or (23) and (24). The optimal values of T are taken as $T = T_1$ if $T_1 \geq M$ and $T = T_2$ if $T_2 < M$.

To illustrate the developed model of Case (i) i.e, if $T_1 \geq M$, a numerical example with the following parameter values is considered. The deteriorating parameters α, β and γ vary from 0.020 to 0.024, 0.06 to 0.72 and 0.06 to 0.72 respectively. The values of the other parameters and costs are considered as follows: $a = 1000$ $to 1200$, $b = 0.010$ to 0.012 units, $\mathrm{A} = \mathrm{Rs.}\,250.0$ to 300.0, $g = Rs.\,0.20$ to $0.24 = \mathrm{Rs.}\,0.100$ to 0.120 $I_c = Rs.\,0.150$ to 0.180, $I_e = Rs.\,0.120$ to 0.144, $M = 15$ $days = \frac{15}{30} = 0.500$ to 0.600, $r = 0.010$ to 0.012, $H = 12.0$ to 14.4 months.

By substituting the above values in equations (16) and (17) and solving, the optimal values of cycle length T and selling price p are obtained. Substituting the optimal values of cycle length T and selling price p in equations (5) and (15), the optimal values of Order quantity Q and net profit NP are obtained and presented in Table-1.

From Table-1, it is observed that when the parameter 'a' is increasing from 1000 to 1200 units, the optimal ordering quantity 'Q', the cycle length 'T' and the net profit 'NP' are increasing from 1250.845 to 1585.738 units, 1.245 to 1.314 and Rs.1977.152 to Rs.2050.474 respectively and the unit selling price 'p' is decreasing from Rs. 4.275 to Rs. 3.625, when other parameters and costs are fixed.

When the parameter 'b' is increasing from 0.010 to 0.012 units, the optimal ordering quantity 'Q' increasing from 1250.845 to 1250.849, cycle length 'T', selling price 'p' are remains constant at 1.245, Rs.4.275 and the net profit 'NP' is decreasing from Rs.1977.151 to Rs.1977.150 respectively, when other parameters and costs are fixed.

As the deterioration parameter α is increasing from 0.020 to 0.024, the optimal ordering quantity 'Q' and the cycle length 'T' are increasing from 1250.845 to 1375.107 units, 1.245 to 1.365 respectively and the unit selling price 'p' and the net profit 'NP' are decreasing from Rs. Rs.4.275 to Rs. 4.140 and Rs.1977.152 to Rs.1953.603 respectively, when other parameters and costs are fixed.

Table-1: *Optimal values of Q, NP, T and p for different values of parameters and costs*

For h=0.1, I_c=0.15, I_e=0.12, M=0.5, r=0.01, H=12

a	b	α	β	γ	A	g	Q	T	p	NP
1000	0.01	0.02	0.6	0.6	250	0.2	1250.845	1.245	4.275	1977.152
1050							1328.542	1.259	4.090	1994.236
1100							1410.353	1.275	3.921	2012.050
1150							1496.130	1.294	3.766	2030.755
1200							1585.738	1.314	3.625	2050.474
	0.0105						1250.847	1.245	4.275	1977.151
	0.0110						1250.847	1.245	4.275	1977.151
	0.0115						1250.849	1.245	4.275	1977.150
	0.0120						1250.849	1.245	4.275	1977.150
		0.021					1281.746	1.275	4.239	1971.233
		0.022					1312.768	1.305	4.204	1965.332
		0.023					1343.894	1.335	4.171	1959.454
		0.024					1375.107	1.365	4.140	1953.603
			0.63				1290.509	1.284	4.227	1970.017
			0.66				1332.079	1.325	4.181	1962.661
			0.69				1375.594	1.368	4.135	1955.097
			0.72				1421.082	1.413	4.090	1947.340
				0.63			1252.355	1.247	4.272	1976.975
				0.66			1253.806	1.248	4.269	1976.811
				0.69			1255.199	1.250	4.266	1976.658
				0.72			1256.534	1.252	4.263	1976.517
					262.5		1170.346	1.165	4.489	1984.103
					275.0		1167.304	1.162	4.498	1984.386
					287.5		1161.260	1.156	4.516	1984.953
					300.0		1160.959	1.156	4.517	1984.982
						0.21	1255.235	1.249	4.277	1961.926
						0.22	1259.979	1.254	4.279	1946.543
						0.23	1265.073	1.259	4.280	1930.992
						0.24	1270.516	1.264	4.281	1915.263

For a=1000, b=0.01, α=0.02, β=0.6, γ=0.6, A=250, g=0.2

h	I_c	I_e	M	r	H	Q	T	p	NP
0.105						1252.897	1.247	4.275	1971.381
0.110						1255.002	1.249	4.274	1965.584
0.115						1257.161	1.251	4.274	1959.762
0.120						1259.374	1.253	4.273	1953.913
	0.1575					1251.530	1.245	4.272	1970.434
	0.1650					1252.233	1.246	4.270	1964.532
	0.1725					1253.034	1.247	4.267	1958.606
	0.1800					1254.068	1.248	4.264	1951.804
		0.126				1268.103	1.262	4.228	1975.626
		0.132				1285.617	1.279	4.183	1974.133

h	I_c	I_e	M	r	H	Q	T	p	NP
		0.138				1303.390	1.296	4.139	1972.676
		0.144				1321.422	1.314	4.097	1971.258
			0.525			1282.482	1.276	4.182	1976.588
			0.550			1316.396	1.309	4.091	1976.187
			0.575			1352.685	1.345	4.002	1976.004
			0.600			1391.447	1.383	3.916	1976.099
				0.0105		1242.297	1.236	4.292	1979.547
				0.0110		1242.297	1.236	4.292	1979.547
				0.0115		1233.796	1.228	4.309	1981.935
				0.0120		1233.796	1.228	4.309	1981.935
					12.6	1228.266	1.222	4.330	1974.585
					13.2	1206.405	1.201	4.386	1972.158
					13.8	1185.239	1.180	4.443	1969.864
					14.4	1164.742	1.160	4.501	1967.694

When the parameter β is increasing from 0.60 to 0.72 the optimal ordering quantity 'Q' and the cycle length 'T' are increasing from 1250.845 to 1421.082 units, 1.245 to 1.413 respectively and the unit selling price 'p' and the net profit 'NP' are decreasing from Rs. 4.275 to Rs. 4.090 and Rs.1977.152 to Rs.1947.340 respectively, when other parameters and costs are fixed.

As the deterioration parameter γ is increasing from 0.60 to 0.72, the optimal ordering quantity 'Q' and the cycle length 'T' are increasing from 1250.845 to 1256.534 units, 1.245 to 1.252 respectively and the unit selling price 'p' and the net profit 'NP' are decreasing from Rs. 4.275 to Rs. 4.263 and Rs.1977.152 to Rs.1976.517 respectively, when other parameters and costs are fixed.

If the ordering cost 'A' increases from Rs.250 to 300, the optimal ordering quantity 'Q' and the cycle length 'T' are decreasing from 1250.845 to 1160.959 units, 1.245 to 1.156 respectively and the unit selling price 'p' and the net profit 'NP' are increasing from Rs. 4.275 to Rs. 4.517 and Rs.1977.152 to Rs.1984.982 respectively, when other parameters and costs are fixed.

When the unit cost 'g' is increasing from Rs.0.20 to 0.24, the optimal ordering quantity 'Q', the cycle length 'T' and the unit selling price 'p' are increasing from 1250.845 to 1270.516 units, 1.245 to 1.264 and Rs. 4.275 to Rs. 4.281 respectively and the net profit 'NP' is decreasing from Rs.1977.152 to Rs.1915.263 respectively, when other parameters and costs are fixed.

When holding cost 'h' is increasing from Rs.0.100 to 0.120, the optimal ordering quantity 'Q' and the cycle length 'T' are increasing from 1250.845 to 1259.374 units, 1.245 to 1.253 respectively and the unit selling price 'p' and the net profit 'NP' are decreasing from Rs. 4.275 to Rs. 4.273 and Rs.1977.152 to Rs.1953.913 respectively, when other parameters and costs are fixed.

When interest charged 'I_C' increases from Rs.0.150 to 0.180, the optimal ordering quantity 'Q' and the cycle length 'T' are increasing from 1250.845 to 1254.068 units, 1.245 to 1.248 respectively and the unit selling price 'p' and the net profit 'NP' are decreasing from Rs. 4.275 to Rs. 4.264 and Rs.1977.152 to Rs.1951.804 respectively, when other parameters and costs are fixed.

If interest charged 'I_e' increases from Rs.0.120 to 0.144, the optimal ordering quantity 'Q' the the cycle length 'T' are increasing from 1250.845 to 1321.422 units, 1.245 to 1.314 respectively and the unit selling price 'p' and the net profit 'NP' are decreasing from Rs. 4.275 to Rs. 4.097 and Rs.1977.152 to Rs.1971.258 respectively, when other parameters and costs are fixed.

If the permissible delay period 'M' increases from 0.5 months to 0.6 months, the optimal ordering quantity 'Q' and the cycle length 'T' are increasing from 1250.845 to 1391.447 units, 1.245 to 1.383 respectively and the unit selling price 'p' and the net profit 'NP' are decreasing from Rs. 4.275 to Rs. 3.916 and Rs.1977.152 to Rs.1976.099 respectively, when other parameters and costs are fixed.

The inflation rate 'r' increases from 0.010 to 0.0120 the optimal ordering quantity 'Q' and the cycle length 'T' are decreasing from 1250.845 to 1233.796 units, 1.245 to 1.228 respectively and the unit selling price 'p' and the net profit 'NP' are decreasing from Rs. 4.275 to Rs. 4.309 and Rs.1977.152 to Rs.1981.935 respectively, when other parameters and costs are fixed.

When the time horizon 'H' increases from 12 months to 13.8 then the optimal ordering quantity 'Q', the cycle length 'T' and the net profit 'NP' are decreasing from 1250.845 to 1164.742 units, 1.245 to 1.16 and Rs.1977.152 to Rs.1967.694 respectively and the unit selling price 'p' is increasing from Rs. 4.275 to Rs. 4.501, when other parameters and costs are fixed.

6. Sensitivity Analysis

To study the effect of changes in the model parameters and costs on the optimal values of the order quantity, cycle length, selling price and net profit, the sensitivity analysis is carried by considering $a = 1000$, $b = 0.01$ units, $\alpha = 0.02$, $\beta = 0.60$, $\gamma = 0.60$, $A = Rs. 250$, $g = Rs. 0.20$, $h = Rs. 0.100$, $I_c = Rs. 0.150, I_e = Rs. 0.120$, $M = 0.500, r = 0.01$, $H = 12$ months. Table-2 summarizes these results for variations of -15%, -10%, -5%, 0, 5%, 10%, 15% of the parameters and costs.

As the parameter a increases from -15% to +15%, the optimal order quantity Q is increases from 1044.252 to 1496.13, cycle length 'T' increases from 1.223 to 1.294, selling price 'p' decreases from Rs.4.940 to Rs.3.766 and the net profit increases from Rs.1927.777 to Rs.2030.755.

When the total demand during the cycle period b increases from -15% to +15%, the optimal order quantity 'Q' increases from 1250.843 to 1250.849, cycle length 'T' and selling price 'p' remains constant 1.245 and Rs.4.275 and the net profit 'NP' decreases from Rs.1977.153 to Rs.1977.150.

As the deterioration parameter α increases from -15% to +15%, the optimal order quantity 'Q' increases from 1159.039 to 1343.894, cycle length 'T' increases from 1.155 to 1.335, selling price 'p' decreases from Rs.4.394 to Rs.4.171 and the net profit 'NP' decreases from Rs.1994.984 to Rs.1959.454.

If the parameter β increases from -15% to +15%, the optimal order quantity 'Q' increases from 1165.449 to 1375.594, cycle length 'T' increases from 1.160 to 1.368, selling price 'p' decreases from Rs.4.388 to Rs.4.135 and the net profit 'NP' decreases from Rs.1992.891 to Rs.1955.097

When the deterioration parameter γ increases from -15% to +15%, the optimal order quantity 'Q' increases from 1245.965 to 1255.199, cycle length 'T' increases from 1.238 to 1.250, selling price 'p' decreases from Rs.4.285 to Rs.4.266 and the net profit 'NP' decreases from Rs.1977.753 to Rs.1976.658.

When the ordering cost A increases from -15% to +15%, the optimal order quantity 'Q' decreases from 1636.158 to 1161.260, cycle length 'T' decreases from 1.623 to 1.156, selling price 'p' increases from Rs.3.693 to Rs.4.516 and the net profit 'NP' increases from Rs.1950.119 to Rs.1984.953.

As the unit cost g increases from -15% to +15%, the optimal order quantity 'Q' increases from 1239.817 to 1265.073, cycle length 'T' increases from 1.234 to 1.259, selling price 'p' increases from Rs.4.267 to Rs.4.280 and the net profit 'NP' decreases from Rs.2021.981 to Rs.1930.992.

As the holding cost h increases from -15% to +15%, the optimal order quantity 'Q' increases from 1245.014 to 1257.161, cycle length 'T' increases from 1.239 to 1.251, selling price 'p' decreases from Rs.4.276 to Rs.4.274 and the net profit decreases from Rs.1994.320 to Rs.1959.762.

When the interest charged I_c increases from -15% to +15%, the optimal order quantity 'Q' increases from 1249.630 to 1253.034, cycle length 'T' increases from 1.243 to 1.247, selling price 'p' decreases from Rs.4.281 to Rs.4.267 and the net profit decreases from Rs.1995.489 to Rs.1958.606.

If the interest earned I_e increases from -15% to +15%, the optimal order quantity 'Q' increases from 1200.594 to 1303.390, cycle length 'T' increases from 1.195 to 1.296, selling price 'p' decreases from Rs.4.425 to Rs.4.139 and the net profit 'NP' decreases from Rs.1981.910 to Rs.1972.676.

When the permissible delay period M increases from -15% to +15%, the optimal order quantity 'Q' increases from 1168.658 to 1352.685, cycle length 'T' increases from 1.164 to 1.345, selling price 'p' decreases from Rs.4.562 to Rs.4.002 and the net profit 'NP' decreases from Rs.1979.374 to Rs.1976.004.

If the inflation rate r increases from -15% to +15%, the optimal order quantity 'Q' decreases from 1259.440 to 1233.796, cycle length 'T' decreases from 1.253 to 1.228, selling price 'p' increases from

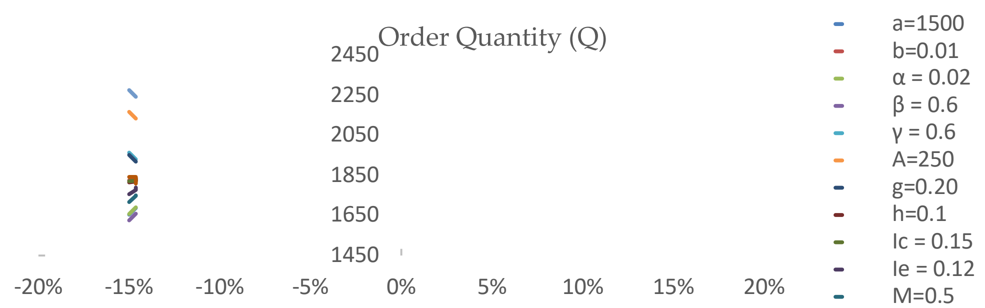
Rs.4.258 to Rs.4.309 and the net profit 'NP' increases from Rs.1974.749 to Rs.1981.935.

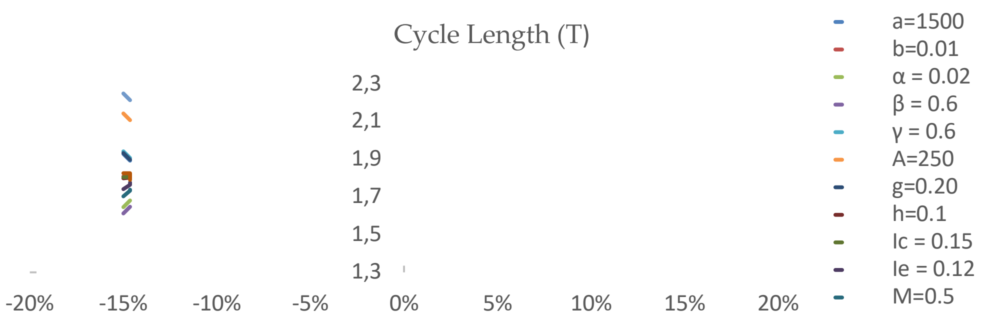
When the time horizon H increases from -15% to +15%, the optimal order quantity 'Q' decreases from 1490.623 to 1185.239, cycle length 'T' decreases from 1.480 to 1.180, selling price 'p' increases from Rs.3.820 to Rs.4.443 and the net profit 'NP' decreases from Rs.2008.083 to Rs.1969.864.

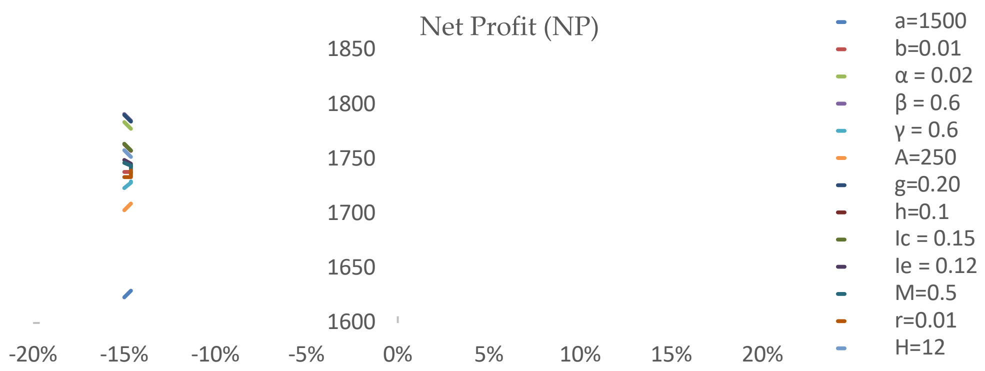
Table-2: *Efect on Optimal Values with Respect to Parameters Variation*

Variation Parameters		Percentage change in parameter						
		-15	-10	-5	0	5	10	15
a	Q	1044.252	1108.494	1177.432	1250.845	1328.542	1410.353	1496.130
	T	1.223	1.226	1.233	1.245	1.259	1.275	1.294
	p	4.940	4.699	4.477	4.275	4.090	3.921	3.766
	NP	1927.777	1944.253	1960.585	1977.152	1994.236	2012.050	2030.755
b	Q	1250.843	1250.843	1250.845	1250.845	1250.847	1250.847	1250.849
	T	1.245	1.245	1.245	1.245	1.245	1.245	1.245
	p	4.275	4.275	4.275	4.275	4.275	4.275	4.275
	NP	1977.153	1977.153	1977.152	1977.152	1977.151	1977.151	1977.150
α	Q	1159.039	1189.474	1220.082	1250.845	1281.746	1312.768	1343.894
	T	1.155	1.185	1.215	4.275	1.275	1.305	1.335
	p	4.394	4.353	4.313	1977.152	4.239	4.204	4.171
	NP	1994.984	1989.032	1983.086	1.245	1971.233	1965.332	1959.454
β	Q	1165.449	1177.053	1213.044	1250.845	1290.509	1332.079	1375.594
	T	1.160	1.172	1.207	1.245	1.284	1.325	1.368
	p	4.388	4.372	4.323	4.275	4.227	4.181	4.135
	NP	1992.891	1990.722	1984.056	1977.152	1970.017	1962.661	1955.097
γ	Q	1245.965	1247.650	1249.277	1250.845	1252.355	1253.806	1255.199
	T	1.238	1.241	1.243	1.245	1.247	1.248	1.250
	p	4.285	4.281	4.278	4.275	4.272	4.269	4.266
	NP	1977.753	1977.540	1977.340	1977.152	1976.975	1976.811	1976.658
A	Q	1636.158	1440.839	1340.631	1250.845	1170.346	1167.304	1161.260
	T	1.623	1.432	1.333	1.245	1.165	1.162	1.156
	p	3.693	3.899	4.078	4.275	4.489	4.498	4.516
	NP	1950.119	1964.813	1970.615	1977.152	1984.103	1984.386	1984.953
g	Q	1239.817	1243.134	1246.810	1250.845	1255.235	1259.979	1265.073
	T	1.234	1.237	1.241	1.245	1.249	1.254	1.259
	p	4.267	4.270	4.273	4.275	4.277	4.279	4.280
	NP	2021.981	2007.169	1992.229	1977.152	1961.926	1946.543	1930.992
h	Q	1245.014	1246.903	1248.847	1250.845	1252.897	1255.002	1257.161
	T	1.239	1.241	1.243	1.245	1.247	1.249	1.251
	p	4.276	4.275	4.275	4.275	4.275	4.274	4.274
	NP	1994.320	1988.621	1982.898	1977.152	1971.381	1965.584	1959.762
I_c	Q	1249.630	1249.910	1250.290	1250.845	1251.530	1252.233	1253.034
	T	1.243	1.244	1.244	1.245	1.245	1.246	1.247
	p	4.281	4.279	4.277	4.275	4.272	4.270	4.267
	NP	1995.489	1989.675	1983.842	1977.152	1970.434	1964.532	1958.606
I_e	Q	1200.594	1217.093	1233.842	1250.845	1268.103	1285.617	1303.390
	T	1.195	1.211	1.228	1.245	1.262	1.279	1.296
	p	4.425	4.373	4.323	4.275	4.228	4.183	4.139
	NP	1981.910	1980.296	1978.709	1977.152	1975.626	1974.133	1972.676

Variation Parameters		Percentage change in parameter						
		-15	-10	-5	0	5	10	15
M	Q	1168.658	1194.025	1221.391	1250.845	1282.482	1316.396	1352.685
	T	1.164	1.189	1.216	1.245	1.276	1.309	1.345
	p	4.562	4.466	4.370	4.275	4.182	4.091	4.002
	NP	1979.374	1978.584	1977.831	1977.152	1976.588	1976.187	1976.004
r	Q	1259.440	1259.440	1250.845	1250.845	1242.297	1242.297	1233.796
	T	1.253	1.253	1.245	1.245	1.236	1.236	1.228
	p	4.258	4.258	4.275	4.275	4.292	4.292	4.309
	NP	1974.749	1974.749	1977.152	1977.152	1979.547	1979.547	1981.935
H	Q	1490.623	1402.823	1323.141	1250.845	1228.266	1206.405	1185.239
	T	1.480	1.394	1.316	1.245	1.222	1.201	1.180
	p	3.820	3.963	4.114	4.275	4.330	4.386	4.443
	NP	2008.083	1995.997	1985.781	1977.152	1974.585	1972.158	1969.864







7. Conclusion

In this paper an EOQ model for deteriorating items with permissible delay in payments having truncated Weibull distribution with inflation is proposed and analyzed. In inventory control, permissible delay in payments has significance influence in obtaining the optimal pricing and ordering policies. The truncated Weibull distribution is one of the most significant life time distributions for items such as food and vegetables markets, market yards and chemical industries, etc., where the deterioration is skewed and having long upper tail. The truncated Weibull distribution includes exponential distribution as a particular case. The sensitivity analysis of the model revealed that the pricing and ordering are highly influenced by the parameters and costs. The model with constraints on warehouse capacity and budget can also be developed with permissible delay in payment and truncated Weibull decay which will be published elsewhere.

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