Comparison of $FM/FD/1$ Queuing Performance Using Fuzzy Queuing Model and Intuitionistic Fuzzy Queuing Model with Infinite capacity

S. Aarthi, M. Shanmugasundari

•

Department of Mathematics & Statistics, College of Science and Humanities, SRM Institute of Science and Technology, Kattankulathur, Tamil Nadu, India.
as6238@srmist.edu.in
shanmugm@srmist.edu.in

Abstract

Under assorted fuzzy numbers, we portray an FM/FD/1 queuing model with an unrestrained limit. The foremost target of this paper is to compare the efficacy of an FM/FD/1 queuing model based on fuzzy queuing theory and intuitionistic fuzzy queuing theory. Birth (arrival) and death (service) rates are thought to be triangular and triangular intuitionistic fuzzy numbers. The fuzzy consequence of unpredictability modeling is a fuzzy random variable because arbitrary events can only be recognized in an undefined manner. As a consequence, it is essential to interpret the direct correlation between volatility and vagueness. The lining miniature's prosecution dimensions are fuzzified and then examined using arithmetic and logical operations. The evaluation metrics for the fuzzy queuing theory model are furnished as a range of outcomes, meanwhile, the intuitionistic fuzzy queuing theory model has plenty of virtues. An approach is conducted to ascertain quality measures using a methodological approach in which fuzzy values are preserved without being incorporated into crisp values, allowing us to draw scientific conclusions in an uncertain environment. The arithmetical precepts are defined in dealing with various fuzzy numbers to test the model's technical feasibility. A comparison illustration is constituted for each fuzzy number.

Keywords: queuing theory, triangular fuzzy number, triangular intuitionistic fuzzy number, infinite capacity.

1. Introduction

A queue is made up of at least one queue or one or more remodeled offices that carry a system of regulations. To initiate propagation in the queuing hypothesis, the specifications birth rate (birth) and death rate (service) are required. Kaufmann [1] featured an introduction to fuzzy subset theory in 1975. John F Shortle et al [2] envisioned several basic queuing suppositions in 1985. In 1986, Yager [3] proposed a different interpretation of the fuzzy set extension principle. In 1989, Lie et al [4] proposed a fuzzy queuing model. In 1992, Negi et al [5] made an overview of queuing systems. Recently, Lofti A Zadeh [6] depicted fuzzy sets and logic in 1995. Chen [7, 8] posited a parameterized nonlinear optimization strategy to fuzzy queues with the widespread regime in 2005, and an arithmetic programming approach to dealing with equipment interruption with fuzzy parameters in 2006. He evolved FM/FM/1/1/FCFS is a fuzzified exponential time dependent on queuing hypothesis. In 2007, K. Gupta et al [9] published a book that was knowledgeable regarding queuing models. Fuzzy logic with engineering disciplines was proposed by Timothy J. Rose [10] in 2010. In 2012, S. Barak et al [11] published a paper on the cost analysis of fuzzy queuing systems. Srinivasan [12] proposed a fuzzy queuing model based on the DSW algorithm in 2014. Shanmugasundaram et al [13] postulated a DSW computation version on fuzzy multi-server queuing in 2015. Using the DSW algorithm, Mohd Zaki et al [16] likened the queuing model and the fuzzy queuing model. The basic framework is focused on Atanassov's extension principle and α - cut method, and Narayana Moorthy et al [20] used intuitionistic fuzzy numbers as hyperparameters. Arpita Kabiraj et al [21] used intuitionistic notions in a linear programming problem to solve fuzzy linear programming problems. In this analysis, G. Chen et al [22] glanced at optimized and enthalpies techniques in fuzzy M/M/1 queues, using all fuzzy numbers as a covariate. In their paper, S. Hanumantha Rao et al [23] proposed a single semi-Markov queueing system with constraints, encouraging or discouraging arrivals, and a rejigged customer reneging policy. S. Hanumantha Rao et al [24] proposed the membership function of the fuzzy cost function to procure optimistic prognostications for certain key metrics of a customizable 2 different service dedicated server markovian limiting queues with server starts and breakdown over N-policy. The prediction generating function was used by S.S. Sanga et al [25] to generate the stable flow mathematical formulation for predictive distributions and systems assessment processes. R. Sethi et al. [26] used an iterative method for extracting steady queue distributions, making multiple performance indices, and wandering numerical experiments to typify the attitude of the system coefficients as numerous system parameters are updated. F. Ferdowsi [28] envisioned an intuitionistic fuzzy measure to negotiate with uncertainty, in which he used a credibility indicator to integrate a fuzzy model into a crisp model. To investigate the performance of a system, B. R. Kumar et al [29] used estimation theory and defuzzification. A. Tamilarasi [27] researched the intuitionistic fuzzy and queuing model utilizing trapezoidal intuitionistic fuzzy numbers. The fuzzy queues are assessed by transferring fuzzy values into crisp values, as shown in the aforementioned overview. As a consequence, we've proposed a technique for tackling deterministic queues in both fuzzy and intuitionistic fuzzy environments without shifting their nature in this paper. In comparison to previous strategies, this initiative is favorable in that it is eloquent, resilient, and noteworthy. According to the results of the analysis, the fuzzy queuing model's performance measurements are within the spectrum of the intuitionistic fuzzy queuing model's computed performance measures. In the queuing theory, both the specifications, that is, the birth (arrival) times and death (service) times, are geared towards achieving predefined appropriations in the customary lining model. The birth rate and death rate are commonly characterized using terminological terms, such as large, small, incredibly low, and modest, which are better reflected by fuzzy and intuitionistic fuzzy sets. Probably the easiest queue with deterministic service time is the FM/FD/1 queue, which has a multitude of uses in performance measurement, network technologies, and other areas. The main concept is to obtain exact fuzzy values, that is, without attempting to convert them to crisp values, and then apply the queuing performance formulas to two types of participatory cognitive abilities, namely triangular and intuitionistic fuzzy enrolment capacities.

A deterministic queueing model is perhaps the most basic type of queuing problem, as it does not necessitate the use of probability to describe the arrival and service pattern. The set of inputs entering at particular times and the processing times are both fixed. The arrival rate is Poisson in this model, but the service rate is deterministic, i.e., it is consistent. If we can make the service deterministic, that is, suspend evolutionary divergence from the service, we can substantially reduce the virtues of the number in the system as well as the waiting time if we want to optimize the queuing parameters. One solution is to add further servers, and besides, if we can also completely eradicate chance variations, whether, through digitalization or any other means, things will get better remarkably.

Section 2 introduces some underlying concepts and definitions. The insinuations and nomenclature are discussed in Section 3. The suggested queuing model is presented in Section 4. Two numerical precepts are solved in Section 5. The discussion of findings is presented in Section 6. The prototype is checked in Section 7.

2. Preliminaries

The motive of this division is to give some basic definitions, annotations, and outcomes that are used in our subsequent calculations.

Definition 2.1: [14] A fuzzy set $\tilde{A}$ is defined on R, the set of real numbers is called a **fuzzy number** if its membership function $\mu_{\tilde{A}} : R \to [0,1]$ has the following conditions:

(a) $\tilde{A}$ is convex, which means that there exists $x_1, x_2 \in R$ and $\lambda \in [0,1]$, such that $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$

(b) $\tilde{A}$ is normal, which means that there exists an $x \in R$ such that $\mu_{\tilde{A}}(x) = 1$

(c) $\tilde{A}$ is piecewise continuous.

Definition 2.2: [14] A fuzzy number $\tilde{A}$ is defined on R, the set of real numbers is said to be a **triangular fuzzy number (TFN)** if its membership function $\mu_{\tilde{A}} : R \to [0,1]$ which satisfies the following conditions:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-\tilde{a}_1}{\tilde{a}_2-\tilde{a}_1} \text{ for } \tilde{a}_1 \leq x \leq \tilde{a}_2 \\ 1 \text{ for } x = \tilde{a}_2 \\ \frac{\tilde{a}_3-x}{\tilde{a}_3-\tilde{a}_2} \text{ for } \tilde{a}_2 \leq x \leq \tilde{a}_3 \\ 0 \text{ otherwise} \end{cases}$$

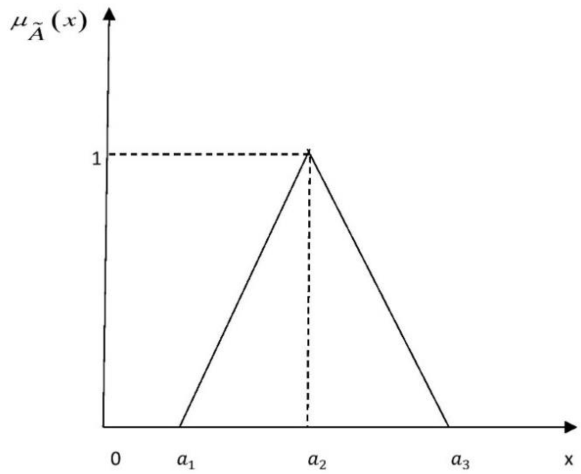

Figure 1: *Triangular fuzzy number*

The triangular fuzzy number is illustrated in Fig 1.

Definition 2.3:[14] Let the two triangular fuzzy numbers be $\tilde{P} \approx (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3)$ and $\tilde{Q} \approx (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3)$ and then the **arithmetic operations on TFN** be given as follows:

(A)Addition

$$\tilde{P}+\tilde{Q}\approx\left(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3\right)+\left(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3\right)$$
$$\tilde{P}+\tilde{Q}\approx\left(\tilde{m}_1,\tilde{\alpha}_1,\tilde{\beta}_1\right)+\left(\tilde{m}_2,\tilde{\alpha}_2,\tilde{\beta}_2\right)$$
$$\tilde{P}+\tilde{Q}\approx\left(\tilde{m}_1+\tilde{m}_2,\max\left\{\tilde{\alpha}_1,\tilde{\alpha}_2\right\},\max\left\{\tilde{\beta}_1,\tilde{\beta}_2\right\}\right) \quad (1)$$

(B)Subtraction

$$\tilde{P}-\tilde{Q}\approx\left(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3\right)-\left(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3\right)$$
$$\tilde{P}-\tilde{Q}\approx\left(\tilde{m}_1,\tilde{\alpha}_1,\tilde{\beta}_1\right)-\left(\tilde{m}_2,\tilde{\alpha}_2,\tilde{\beta}_2\right)$$
$$\tilde{P}-\tilde{Q}\approx\left(\tilde{m}_1-\tilde{m}_2,\max\left\{\tilde{\alpha}_1,\tilde{\alpha}_2\right\},\max\left\{\tilde{\beta}_1,\tilde{\beta}_2\right\}\right) \quad (2)$$

(C) Multiplication

$$\tilde{P}.\tilde{Q}\approx\left(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3\right).\left(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3\right)$$
$$\tilde{P}.\tilde{Q}\approx\left(\tilde{m}_1,\tilde{\alpha}_1,\tilde{\beta}_1\right).\left(\tilde{m}_2,\tilde{\alpha}_2,\tilde{\beta}_2\right)$$
$$\tilde{P}.\tilde{Q}\approx\left(\tilde{m}_1.\tilde{m}_2,\max\left\{\tilde{\alpha}_1,\tilde{\alpha}_2\right\},\max\left\{\tilde{\beta}_1,\tilde{\beta}_2\right\}\right) \quad (3)$$

(D) Division

$$\frac{\tilde{P}}{\tilde{Q}}\approx\frac{\left(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3\right)}{\left(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3\right)}$$
$$\frac{\tilde{P}}{\tilde{Q}}\approx\frac{\left(\tilde{m}_1,\tilde{\alpha}_1,\tilde{\beta}_1\right)}{\left(\tilde{m}_2,\tilde{\alpha}_2,\tilde{\beta}_2\right)}$$
$$\frac{\tilde{P}}{\tilde{Q}}\approx\left(\frac{\tilde{m}_1}{\tilde{m}_2},\max\left\{\tilde{\alpha}_1,\tilde{\alpha}_2\right\},\max\left\{\tilde{\beta}_1,\tilde{\beta}_2\right\}\right) \quad (4)$$

Definition 2.4: For every triangular fuzzy number $\tilde{P}\approx\left(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3\right)\in F(R)$ **ranking function** $\Re:F(R)\to R$ is defined by graded mean as

$$\Re\left(\tilde{P}\right)=\frac{\left(\tilde{a}_1+4\tilde{a}_2+\tilde{a}_3\right)}{6}$$

For any two TFN $\tilde{P}\approx\left(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3\right)$ and $\tilde{Q}\approx\left(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3\right)$ We have the following comparisons,

$(a)\tilde{P}\succ\tilde{Q}\Leftrightarrow\Re\left(\tilde{P}\right)>\Re\left(\tilde{Q}\right)$

$(b)\tilde{P}\prec\tilde{Q}\Leftrightarrow\Re\left(\tilde{P}\right)<\Re\left(\tilde{Q}\right)$

$(c)\tilde{P}\approx\tilde{Q}\Leftrightarrow\Re\left(\tilde{P}\right)=\Re\left(\tilde{Q}\right)$

$(d)\tilde{P}-\tilde{Q}\approx 0\Leftrightarrow\Re\left(\tilde{P}\right)-\Re\left(\tilde{Q}\right)=0$

A triangular fuzzy number $\tilde{P}\approx\left(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3\right)\in F(R)$ is known to be **positive** if $\Re\left(\tilde{P}\right)>0$ and defined by $\tilde{P}\succ 0$.

Definition 2.5:[15] Let a non–empty set be X. An **Intuitionistic fuzzy set (IFS)** $\tilde{A}'$ is defined as $\tilde{A}'=\left\{\left(x,\mu_{\tilde{A}'}(x),\gamma_{\tilde{A}'}(x)/x\in X\right)\right\}$, where $\mu_{\tilde{A}'}:X\to[0,1]$ and $\gamma_{\tilde{A}'}:X\to[0,1]$ denotes the degree of membership and degree of non– membership functions respectively where $x\in X$, for every $x\in X, 0\le\mu_{\tilde{A}'}(x)+\gamma_{\tilde{A}'}(x)\le 1$.

Definition 2.6:[15] An intuitionistic fuzzy set $\tilde{A}'$ described on R, the real numbers are said to be an **Intuitionistic fuzzy number (IFN)** if its membership function $\mu_{\tilde{A}'}:\mathrm{R}\to[0,1]$ and its non –

membership function $\gamma_{\tilde{A}'} : R \rightarrow [0,1]$ should agreeable to the following conditions:

i) $\tilde{A}'$ is normal, which means that there exists an $x \in R$, such that $\mu_{\tilde{A}'}(x) = 1, \gamma_{\tilde{A}'}(x) = 0$

ii) $\tilde{A}'$ is convex for the membership function $\mu_{\tilde{A}'}$, which means that there exists $x_1, x_2 \in R$ and $\lambda \in [0,1]$ such that $\mu_{\tilde{A}'}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}'}(x_1), \mu_{\tilde{A}'}(x_2)\}$.

iii) $\tilde{A}'$ is concave for the non – membership function $\gamma_{\tilde{A}'}$, which means that there exists $x_1, x_2 \in R$ and $\lambda \in [0,1]$ such that $\gamma_{\tilde{A}'}(\lambda x_1 + (1-\lambda)x_2) \leq \max\{\gamma_{\tilde{A}'}(x_1), \gamma_{\tilde{A}'}(x_2)\}$.

Definition 2.7:[15] A fuzzy number $\tilde{A}'$ on R is said to be a **triangular intuitionistic fuzzy number (TIFN)** if its membership function $\mu_{\tilde{A}'} : R \rightarrow [0,1]$ and non – membership function $\gamma_{\tilde{A}'} : R \rightarrow [0,1]$ has the following conditions:

$$\mu_{\tilde{A}'}(x) = \begin{cases} \frac{x-\tilde{a}_1}{\tilde{a}_2-\tilde{a}_1} \text{ for } \tilde{a}_1 \leq x \leq \tilde{a}_2 \\ 1 \text{ for } x = \tilde{a}_2 \\ \frac{\tilde{a}_3-x}{\tilde{a}_3-\tilde{a}_2} \text{ for } \tilde{a}_2 \leq x \leq \tilde{a}_3 \\ 0 \text{ otherwise} \end{cases}$$

and

$$\gamma_{\tilde{A}'}(x) = \begin{cases} 1 \text{ for } x < \tilde{a}'_1, x > \tilde{a}'_3 \\ \frac{\tilde{a}_2-x}{\tilde{a}_2-\tilde{a}'_1} \text{ for } \tilde{a}'_1 \leq x \leq \tilde{a}_2 \\ 0 \text{ for } x = \tilde{a}_2 \\ \frac{x-\tilde{a}_2}{\tilde{a}_3-\tilde{a}_2} \text{ for } \tilde{a}_2 \leq x \leq \tilde{a}'_3 \end{cases}$$

and is given by $\tilde{A}' = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3; \tilde{a}'_1, \tilde{a}_2, \tilde{a}'_3)$ where $\tilde{a}'_1 \leq \tilde{a}_1 \leq \tilde{a}_2 \leq \tilde{a}_3 \leq \tilde{a}'_3$.

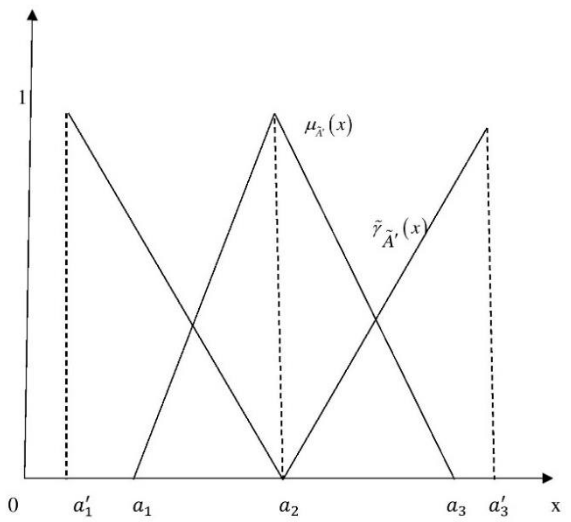

Figure 2: *Intuitionistic triangular fuzzy number*

The intuitionistic triangular fuzzy number is illustrated in Fig 2.

Cases: Let $\tilde{A}' = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3; \tilde{a}'_1, \tilde{a}_2, \tilde{a}'_3)$ be a TIFN then the following cases arises.

Case:1 If $\tilde{a}'_1 = \tilde{a}_1, \tilde{a}'_3 = \tilde{a}_3$ then $\tilde{A}'$ represent a TFN.

Case:2 If $\tilde{a}_1' = \tilde{a}_1 = \tilde{a}_2 = \tilde{a}_3' = \tilde{a}_3 = \tilde{m}$ then $\tilde{A}'$ represent a real number $\tilde{m}$. The parametric form of TIFN $\tilde{A}'$ is defined as $\tilde{A}' = (\tilde{\alpha}, \tilde{m}, \tilde{\beta}; \tilde{\alpha}', \tilde{m}, \tilde{\beta}')$ where $\tilde{\alpha}, \tilde{\alpha}'$ & $\tilde{\beta}, \tilde{\beta}'$ represents the left spread and right spread of membership functions and non – membership functions respectively.

Definition 2.8:[15] TIFN $\tilde{A}' \in F(R)$, (where $F(R)$ is the set of all TIFN) can also be represented as a pair $\tilde{A}' = (\tilde{a}, \tilde{\bar{a}}; \tilde{a}', \tilde{\bar{a}}')$ of functions $\tilde{a}(\tilde{r}'), \tilde{\bar{a}}(\tilde{r}'), \tilde{a}'(\tilde{r}')$ & $\tilde{\bar{a}}'(\tilde{r}')$ for $0 \le \tilde{r}' \le 1$ which satisfies the following requirements:

i) $\tilde{a}(\tilde{r}')$ & $\tilde{\bar{a}}'(\tilde{r}')$ is a bounded monotonic increasing left continuous function for membership and non–membership functions respectively.

ii) $\tilde{\bar{a}}(\tilde{r}')$ & $\tilde{a}'(\tilde{r}')$ is a bounded monotonic decreasing left continuous function for membership and non–membership functions respectively.

iii) $\tilde{a}(\tilde{r}') \le \tilde{\bar{a}}(\tilde{r}'), 0 \le \tilde{r}' \le 1.$

iv) $\tilde{a}'(\tilde{r}') \le \tilde{\bar{a}}'(\tilde{r}'), 0 \le \tilde{r}' \le 1.$

Definition 2.9: The extension of fuzzy arithmetic operations of Ming Ma et al [14] to the set of TIFN based upon both location indices and functions of fuzziness indices. The location indices number is taken in the regular arithmetic while the functions of fuzziness indices are assumed to follow the lattice rule which is the least upper bound in the lattice $\tilde{I}'$. For any two arbitrary TIFN $\tilde{P}' \approx (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1')$ and $\tilde{Q}' \approx (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2', \tilde{\beta}_2')$ and $* \in \{+, -, \times, \div\}$, then the **arithmetic operations on TIFN** are defined by $\tilde{P}' * \tilde{Q}' = (\tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}_1 \vee \tilde{\alpha}_2, \tilde{\beta}_1 \vee \tilde{\beta}_2; \tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}_1' \vee \tilde{\alpha}_2', \tilde{\beta}_1' \vee \tilde{\beta}_2')$.

In particular, for any two TIFNs $\tilde{P}' \approx (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1')$ and $\tilde{Q}' \approx (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2', \tilde{\beta}_2')$ the arithmetic operations are defined as

$$\tilde{P}' * \tilde{Q}' = (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1') * (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2', \tilde{\beta}_2')$$

$$\tilde{P}' * \tilde{Q}' = (\tilde{m}_1 * \tilde{m}_2, \max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, \max\{\tilde{\beta}_1, \tilde{\beta}_2\}; \tilde{m}_1 * \tilde{m}_2, \max\{\tilde{\alpha}_1', \tilde{\alpha}_2'\}, \max\{\tilde{\beta}_1', \tilde{\beta}_2'\})$$

$$\tilde{P}' * \tilde{Q}' = (\tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}_1 \vee \tilde{\alpha}_2, \tilde{\beta}_1 \vee \tilde{\beta}_2; \tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}_1' \vee \tilde{\alpha}_2', \tilde{\beta}_1' \vee \tilde{\beta}_2')$$

In particular, for any two TIFNs $\tilde{P}' \approx (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3; \tilde{a}_1', \tilde{a}_2', \tilde{a}_3') \approx (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1')$, $\tilde{Q}' \approx (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3; \tilde{b}_1', \tilde{b}_2', \tilde{b}_3') \approx (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2', \tilde{\beta}_2')$ we define:

Addition

$$\tilde{P}' + \tilde{Q}' = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3; \tilde{a}_1', \tilde{a}_2, \tilde{a}_3') + (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3; \tilde{b}_1', \tilde{b}_2, \tilde{b}_3')$$

$$\tilde{P}' + \tilde{Q}' = (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1') + (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2', \tilde{\beta}_2')$$

$$\tilde{P}' + \tilde{Q}' = (\tilde{m}_1 + \tilde{m}_2, \max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, \max\{\tilde{\beta}_1, \tilde{\beta}_2\}; \tilde{m}_1 + \tilde{m}_2, \max\{\tilde{\alpha}_1', \tilde{\alpha}_2'\}, \max\{\tilde{\beta}_1', \tilde{\beta}_2'\}) \quad (5)$$

Subtraction

$$\tilde{P}' - \tilde{Q}' = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3; \tilde{a}_1', \tilde{a}_2, \tilde{a}_3') - (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3; \tilde{b}_1', \tilde{b}_2, \tilde{b}_3')$$

$$\tilde{P}' - \tilde{Q}' = (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1') - (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2', \tilde{\beta}_2')$$

$$\tilde{P}' - \tilde{Q}' = (\tilde{m}_1 - \tilde{m}_2, \max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, \max\{\tilde{\beta}_1, \tilde{\beta}_2\}; \tilde{m}_1 - \tilde{m}_2, \max\{\tilde{\alpha}_1', \tilde{\alpha}_2'\}, \max\{\tilde{\beta}_1', \tilde{\beta}_2'\}) \quad (6)$$

Multiplication

$$\tilde{P}' \times \tilde{Q}' = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3; \tilde{a}_1', \tilde{a}_2, \tilde{a}_3') \times (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3; \tilde{b}_1', \tilde{b}_2, \tilde{b}_3')$$

$$\tilde{P}' \times \tilde{Q}' = (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1') \times (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2', \tilde{\beta}_2')$$

$$\tilde{P}'\times\tilde{Q}'=\left(\tilde{m}_1\times\tilde{m}_2,\max\left\{\tilde{\alpha}_1,\tilde{\alpha}_2\right\},\max\left\{\tilde{\beta}_1,\tilde{\beta}_2\right\};\tilde{m}_1\times\tilde{m}_2,\max\left\{\tilde{\alpha}'_1,\tilde{\alpha}'_2\right\},\max\left\{\tilde{\beta}'_1,\tilde{\beta}'_2\right\}\right) \quad (7)$$

Division

$$\tilde{P}'\div\tilde{Q}'=\left(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3;\tilde{a}'_1,\tilde{a}_2,\tilde{a}'_3\right)\div\left(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3;\tilde{b}'_1,\tilde{b}_2,\tilde{b}'_3\right)$$

$$\tilde{P}'\div\tilde{Q}'=\left(\tilde{m}_1,\tilde{\alpha}_1,\tilde{\beta}_1;\tilde{m}_1,\tilde{\alpha}'_1,\tilde{\beta}'_1\right)\div\left(\tilde{m}_2,\tilde{\alpha}_2,\tilde{\beta}_2;\tilde{m}_2,\tilde{\alpha}'_2,\tilde{\beta}'_2\right)$$

$$\tilde{P}'\div\tilde{Q}'=\left(\tilde{m}_1\div\tilde{m}_2,\max\left\{\tilde{\alpha}_1,\tilde{\alpha}_2\right\},\max\left\{\tilde{\beta}_1,\tilde{\beta}_2\right\};\tilde{m}_1\div\tilde{m}_2,\max\left\{\tilde{\alpha}'_1,\tilde{\alpha}'_2\right\},\max\left\{\tilde{\beta}'_1,\tilde{\beta}'_2\right\}\right) \quad (8)$$

Definition 2.10: Consider an arbitrary TIFN $\tilde{A}'=\left(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3;\tilde{a}'_1,\tilde{a}_2,\tilde{a}'_3\right)=\left(\tilde{m},\tilde{\alpha},\tilde{\beta};\tilde{m},\tilde{\alpha}',\tilde{\beta}'\right)$ and the magnitude of TIFN $\tilde{A}'$ is given by

$$mag\left(\tilde{A}'\right)=\frac{1}{2}\int\left(\tilde{a}+\tilde{\tilde{a}}+2\tilde{m}+\tilde{a}'+\tilde{\tilde{a}}'\right)\tilde{f}\left(\tilde{r}'\right)d\tilde{r}'$$

$$mag\left(\tilde{A}'\right)=\frac{1}{2}\int\left(\tilde{\beta}+\tilde{\beta}'+6\tilde{m}-\tilde{\alpha}-\tilde{\alpha}'\right)\tilde{f}\left(r'\right)d\tilde{r}'$$

In real life scenario, decision-makers select the value of $\tilde{f}\left(\tilde{r}'\right)$ based on their circumstances. Here for our ease, we choose $\tilde{f}\left(\tilde{r}'\right)=\tilde{r}'^2$

$$mag\left(\tilde{A}'\right)=\frac{\left(\tilde{\beta}+\tilde{\beta}'+6\tilde{m}-\tilde{\alpha}-\tilde{\alpha}'\right)}{6}$$

$$mag\left(\tilde{A}'\right)=\frac{\left(\tilde{a}+\tilde{\tilde{a}}+2\tilde{m}+\tilde{a}'+\tilde{\tilde{a}}'\right)}{6}$$

For any two TIFN $\tilde{P}'\approx\left(\tilde{m}_1,\tilde{\alpha}_1,\tilde{\beta}_1;\tilde{m}_1,\tilde{\alpha}'_1,\tilde{\beta}'_1\right)$, $\tilde{Q}'\approx\left(\tilde{m}_2,\tilde{\alpha}_2,\tilde{\beta}_2;\tilde{m}_2,\tilde{\alpha}'_2,\tilde{\beta}'_2\right)$ in $F\left(R\right)$, we define

$(a)\tilde{P}'\geq\tilde{Q}'\Leftrightarrow mag\left(\tilde{P}'\right)\geq mag\left(\tilde{Q}'\right)$

$(b)\tilde{P}'\leq\tilde{Q}'\Leftrightarrow mag\left(\tilde{P}'\right)\leq mag\left(\tilde{Q}'\right)$

$(c)\tilde{P}'\approx\tilde{Q}'\Leftrightarrow mag\left(\tilde{P}'\right)=mag\left(\tilde{Q}'\right)$

3. Suppositions and Diacritical marks

3.1. Suppositions

The following are the accompanying presumptions used in the current model:

i) With only one server, the $\left(FM\,/\,FD\,/\,1\right):\left(\infty\,/\,FCFS\right)$ queuing model has no bounds.
ii) Service discipline First-Come-First-Served (FCFS)
iii) Arrival times that are widely disseminated.
iv) Fixed deterministic service fetishization.
v) The birth(arrival) rate and death(service) rate are both ambiguous figures.

3.2. Diacritical marks

Here we are using the following notations:

$\tilde{\lambda},\tilde{\lambda}'\rightarrow$ The mean no. of consumers who arrive in a predetermined period of time.

$\tilde{\mu},\tilde{\mu}'\rightarrow$ The mean no. of consumers being serviced per unit of time.

$\tilde{\rho}\rightarrow$ Traffic intensity

$\tilde{N}_q,\tilde{N}'_q\rightarrow$ The mean no. of consumers in the line.

$\tilde{N}_s,\tilde{N}'_s\rightarrow$ The mean no. of consumers in the system.

$\tilde{T}_q, \tilde{T}'_q \rightarrow$ The mean sojourn time of the consumers in the queue.

$\tilde{T}_S, \tilde{T}'_S \rightarrow$ The mean sojourn time of the consumers in the system.

$FM \rightarrow$ Fuzzified exponential distribution.

$FD \rightarrow$ Fuzzified Regular service distribution.

$\tilde{P}, \tilde{P}' \rightarrow$ Interarrival rate.

$\tilde{Q}, \tilde{Q}' \rightarrow$ Service rate.

4. Single server deterministic fuzzy queuing model with infinite capacity

We envisage a solo server queuing model governed by the First Come, First Served (FCFS) principle. It's composed as $(FM / FD / 1):(\infty / FCFS)$ in Kendall's notation. Fuzzified exponential distribution with arrival rate is denoted by FM, and fuzzified stable (consistent) dispersion with service rate is denoted by FD. This is a stochastic process, and the state vector is the collection $\{0, 1, 2, ...\}$ in which the value implies the number of customers in the system, which encompasses any enterprise currently in the establishment. Because it is unbounded in size, there is no limit to the number of customers it can hold. Let $\tilde{\lambda}$ and $\tilde{\lambda}'$ be the fuzzy and intuitionistic fuzzy arrival rates respectively. Let $\tilde{\mu}$ and $\tilde{\mu}'$ be the fuzzy and intuitionistic fuzzy service rates respectively. At the steady-state, the FCFS discipline is upheld but the capacity is unlimited.
The following are the fabrication characteristics of the above model:

i) The anticipated number of customers in the system is given as

$$\tilde{N}_S = \tilde{\rho} + \frac{\tilde{\rho}^2}{2(1-\tilde{\rho})}, \tilde{\rho} = \frac{\tilde{\lambda}}{\tilde{\mu}} \tag{9}$$

ii) The anticipated number of customers in the queue is given as

$$\tilde{N}_q = \frac{\tilde{\rho}^2}{2(1-\tilde{\rho})} \tag{10}$$

iii) The anticipated waiting time of customers in the queue is given as

$$\tilde{T}_q = \frac{\tilde{\rho}}{2(1-\tilde{\rho})\tilde{\mu}} \tag{11}$$

iv) The anticipated waiting time of customers in the system is given as

$$\tilde{T}_S = \frac{1}{\tilde{\mu}} + \frac{\tilde{\rho}}{2(1-\tilde{\rho})\tilde{\mu}} \tag{12}$$

5. Solo server deterministic fuzzy queuing model with unlimited capability

The plaza has a ginormous market and over 10 different manufacturers. Contemplate a shopping centre with a parking slot facility on one floor, with 12 access points and 12 exit ramps for vehicles that are free of charge. Envision two vehicles arriving at the parking spot every minute. Under this scenario, we enumerate the examples and solve them. Interpret the entry rate and the departure rate as both TFNs and TIFNs symbolized by $\tilde{\lambda}$, $\tilde{\lambda}'$ and $\tilde{\mu}$, $\tilde{\mu}'$ respectively. We postulate the system's limit, is infinity.

5.1. Single server deterministic fuzzy queuing model with infinite capacity

Let $\tilde{\lambda} = (1, 2, 3)$ is the arrival rate and $\tilde{\mu} = (11, 12, 13)$ is the service rate of the queuing model. Determine the TFN in the form of $(\tilde{m}, \tilde{\alpha}, \tilde{\beta})$ as $\tilde{\lambda} = (2, 1, 1)$ and $\tilde{\mu} = (12, 1, 1)$. To determine the values of a number of customers and their sojourn time in the queue as well as a system using suitable formulas among (9), (10), (11), & (12). It is necessary to use the appropriate arithmetic operations described in (1), (2), (3), and (4) for add, sub, multiply, and divide, respectively.

The metrics of performance are calculated and tabulated in Table 1.

Table 1: *Performance measures using TFN*

Quantifiable metrics using TFN	
$\tilde{N}_q$	$(-0.9834, 0.0166, 1.0166)$
$\tilde{N}_S$	$(-0.8168, 0.1832, 1.1832)$
$\tilde{T}_q$	$(-0.9917, 0.0083, 1.0083)$
$\tilde{T}_S$	$(-0.9084, 0.0916, 1.0916)$

5.2. Single server deterministic intuitionistic fuzzy queuing model with infinite capacity

Let $\tilde{\lambda}' = (1.5, 2, 2.5; 1, 2, 3)$ is the arrival rate and $\tilde{\mu}' = (11.5, 12, 12.5; 11, 12, 13)$ is the service rate of the queuing model. Determine the TIFN in the form of $(\tilde{m}, \tilde{\alpha}, \tilde{\beta}; \tilde{m}, \tilde{\alpha}', \tilde{\beta}')$ as $\tilde{\lambda}' = (2, 0.5, 0.5; 2, 1, 1)$ and $\tilde{\mu}' = (12, 0.5, 0.5; 12, 1, 1)$. To determine the values of a number of customers and their sojourn time in the queue as well as a system using suitable formulas among (9), (10), (11), & (12). It is necessary to use the appropriate arithmetic operations described in (5), (6), (7), and (8) for add, sub, multiply, and divide, respectively.

The metrics of performance are calculated and tabulated in Table 2.

Table 2: *Performance measures using TIFN*

Quantifiable metrics using TIFN	
$\tilde{N}'_q$	$(-0.4834, 0.0166, 0.5166; -0.9834, 0.0166, 1.0166)$
$\tilde{N}'_S$	$(-0.3168, 0.1832, 0.6832; -0.8168, 0.1832, 1.1832)$
$\tilde{T}'_q$	$(-0.4917, 0.0083, 0.5083; -0.9917, 0.0083, 1.0083)$
$\tilde{T}'_S$	$(-0.4084, 0.0916, 0.5916; -0.9084, 0.0916, 1.0916)$

The following figures depict the visualizations of Tables 1 and 2.

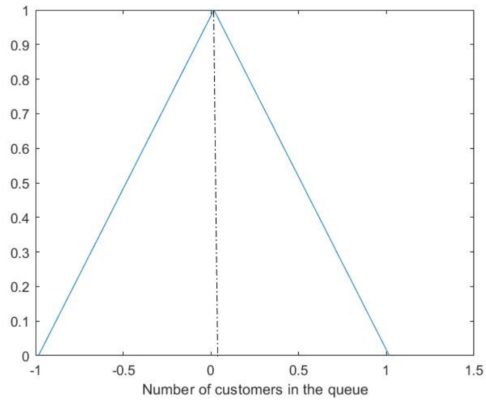

Figure 3: *The number of customers in the queue* $\tilde{N}_q$

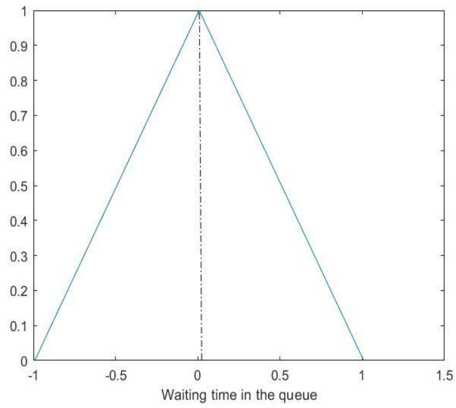

Figure 4: *The waiting time of the customers in the queue* $\tilde{T}_q$

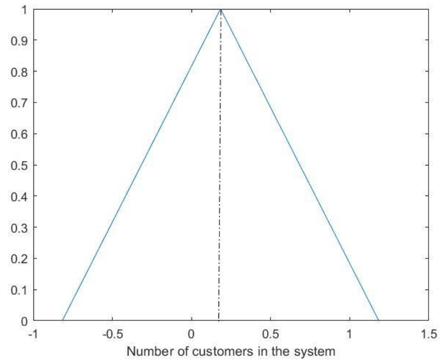

Figure 5: *The number of customers in the system* $\tilde{N}_s$

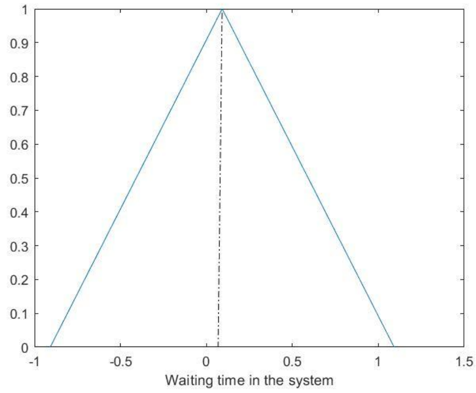

Figure 6: *The waiting time of customers in the system* $\tilde{T}_s$

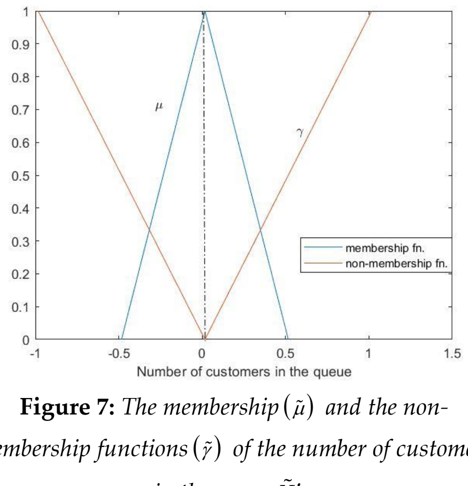

Figure 7: *The membership* $(\tilde{\mu})$ *and the non-membership functions* $(\tilde{\gamma})$ *of the number of customers in the queue* $\tilde{N}'_q$

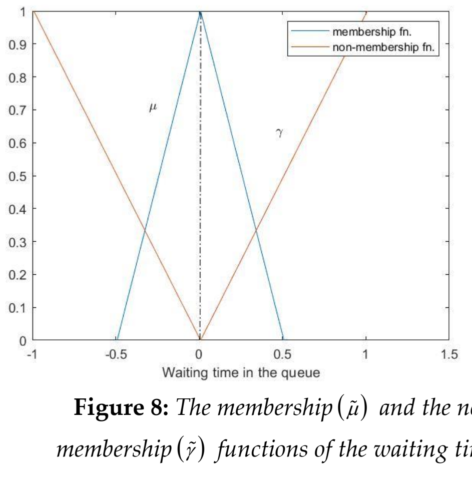

Figure 8: *The membership* $(\tilde{\mu})$ *and the non-membership* $(\tilde{\gamma})$ *functions of the waiting time of consumers in the queue* $\tilde{T}'_q$

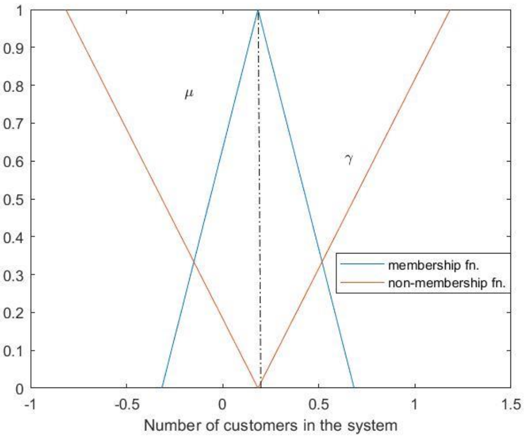

Figure 9: *The membership* $(\tilde{\mu})$ *and the non-membership* $(\tilde{\gamma})$ *functions of the number of consumers in the system* $\tilde{N}'_S$

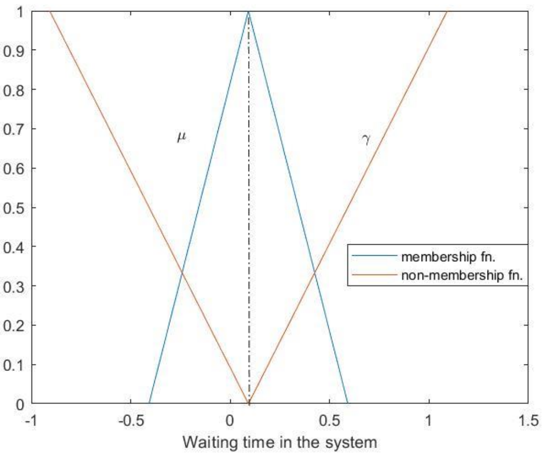

Figure 10: *The membership* $(\tilde{\mu})$ *and the non-membership* $(\tilde{\gamma})$ *functions of the waiting time of consumers in the system* $\tilde{T}'_s$

6. Results and Discussions

Tables 1-2 provide the results, which show different assessments for a multitude of membership functions (TFN and TIFN).

i) The mean value of $\tilde{N}_q = 0.0166$ and the left and right stretched values are -0.9834 and 1.0166 respectively emphasizing that the queue length of consumers is closely between -0.9834 and 1.0166. Its most assured value is 0.0166.

ii) The mean value of $\tilde{N}_S = 0.1832$ and the left and right stretched values are -0.8168 and 1.1832 respectively emphasizing that the system length of consumers is closely between -0.8168 and 1.1832. Its most assured value is 0.1832.

iii) The mean value of $\tilde{T}_q = 0.0083$ and the left and right stretched values are -0.9917 and 1.0083 respectively emphasizing that the sojourn time of consumers in the queue is closely between -0.9917 and 1.0083. Its most assured value is 0.0083.

iv) The mean value of $\tilde{T}_S = 0.0916$ and the left and right stretched values are -0.9084 and 1.0916 respectively emphasizing that the sojourn time of consumers in the system is closely between -0.9084 and 1.0916. Its most assured value is 0.0916.

v) The mean value of $\tilde{N}'_q = 0.0166$ and the left and right fuzziness of membership $(\tilde{\mu})$ functions are -0.4834 and 0.5166 respectively and the left and right fuzziness of non-membership $(\tilde{\gamma})$ functions are -0.9834 and 1.0166 respectively. Its most assured value is 0.0166.

vi) The mean value of $\tilde{N}'_S = 0.1832$ and the left and right fuzziness of membership $(\tilde{\mu})$ functions are -0.3168 and 0.6832 respectively and the left and right fuzziness of non-membership $(\tilde{\gamma})$ functions are -0.8168 and 1.1832 respectively. Its most assured value is 0.1832.

vii) The mean value of $\tilde{T}'_q = 0.0083$ and the left and right fuzziness of membership $(\tilde{\mu})$ functions are -0.4917 and 0.5083 respectively and the left and right fuzziness of non-membership $(\tilde{\gamma})$ functions are -0.9917 and 1.0083 respectively. Its most assured value is 0.0083.

viii) The mean value of $\tilde{T}'_S = 0.0916$ and the left and right fuzziness of membership $(\tilde{\mu})$ functions are -0.4084 and 0.5916 respectively and the left and right fuzziness of non-membership $(\tilde{\gamma})$ functions are -0.9084 and 1.0916 respectively. Its most assured value is 0.0916.

The findings demonstrate that the exhibition measures $\tilde{N}_q, \tilde{N}_S, \tilde{T}_q, \tilde{T}_S, \tilde{N}'_q, \tilde{N}'_S, \tilde{T}'_q$ & $\tilde{T}'_S$ for both the fuzzy queuing theory and intuitionistic fuzzy queuing models were metabolized and tested in this study. Because the intuitionistic fuzzy set supposition is more efficaciously customizable, the intuitionistic fuzzy queuing model is significantly more effective and efficient in measuring the exhibition of the deterministic FM/FD/1 queuing model framework. The intuitionistic fuzzy queuing model produces more comprehensive data, which is immensely beneficial when characterizing a model framework. As a result, this study concludes that intuitionistic fuzzy lining is one of the options for registering exposition parameters because the data obtained from the implementation is much better to recognize and perceive.

7. Conclusion

In this manuscript, IFS is shown as a quite crucial asset to fuzzy set theory when interacting with ostensible implementation in single server deterministic queuing models with infinite capacity. We validated the system using comparison sorting rubrics such as the extrapolated length of the customer line and the system for both classifications of arrivals without changing the composition of the queues from fuzzy to crisp. In addition, fuzzy values and intuitionistic fuzzy values are used to ascertain the assumptive sojourn time of customers in the line and throughout the system. Another cause to use the suggested technique indicator is that it delivers viable paths to beliefs in the queuing utilizing multiple forms of membership functions (TFN and TIFN) while retaining exactness within the shuttered crisp interval.

The birth rate and death rate are fuzzy allegiances, so we make a comparison between the fuzzy and intuitionistic fuzzy set hypothesis. System length, queue length, system sojourn time, queue sojourn time, and other execution proportions are interestingly ambiguous. The proposed method's effectiveness is supported by mathematical precepts. It should be acknowledged that by raising the number of variables, the accomplishment of the queuing model can be enhanced. Entrepreneurs, retail outlets, and dealerships can use the proposed model to precisely determine the best queueing system execution.

The prediction model is used to reach scientific claims, and the fuzzy and intuitionistic fuzzy queue with infinite capacity is explained in greater detail. The envisaged queuing system's authenticity and conciseness are appraised using the TFN and TIFN mathematical manifestations. A numerical model demonstrates the design flexibility of the preferred methodology. The intuitionistic fuzzy queuing model is certainly better and more favorable in appraising dimensions of queuing models because the intuitionistic fuzzy theory is more configurable. As an outcome, intuitionistic fuzzy queuing is one of the healthiest modes of computing performance standards, according to this study, because the evidence found from the application is easier to spot and explore. The paper can be extended from various angles. One is to consider birth and death rates as random variables, or fuzzy random variables. Consider neutrosophic fuzzy numbers as an additional aspect to protrude this paper.

References

[1] Kaufmann, A. (1975). Introduction to the theory of fuzzy subsets, Vol. I. Academic Press, New York.

[2] Shortle, John. F., Thompson, James. M., Gross, Donald. and Harris, Carl. M. (2018). Fundamentals of queuing theory, fifth edition, (Wiley Series in Probability and Statistics). New York: John Wiley & Sons, Inc. Doi:10.1002/9781119453765

[3] Yager, R. R. (1986). A characterization of the extension principle. *Fuzzy Sets and Systems*, 18(3):205 - 217. Doi: 10.1016/0165-0114(86)90002-3

[4] Li, R. J. and Lee, E. S. (1989). Analysis of fuzzy queues. *Computers and Mathematics with Applications*, 17(7):1143-1147. Doi: 10.1016/0898-1221(89)90044-8

[5] Negi, D. S. and Lee, E. S. (1992). Analysis and simulation of fuzzy queues. *Fuzzy Sets and Systems*, 46(3):321-330. Doi: 10.1016/0165-0114(92)90370-j

[6] Lofti Zadeh, A. (1996). Fuzzy sets, fuzzy logic and fuzzy systems. *Advances in Fuzzy Systems-Applications and Theory*, Vol.6. Doi: 10.1142/2895

[7] Chen, S. P. (2005). Parametric nonlinear programming approach to fuzzy queues with bulk service. *European Journal of Operational Research*, 163(2):434-444. Doi: 10.1016/j.ejor.2003.10.041

[8] Chen, S. P. (2006). A mathematical programming approach to the machine interference problem with fuzzy parameters. *Applied Mathematics and Computation*, 174(1):374-387. Doi: 10.1016/j.amc.2005.05.012

[9] Gupta, P. K. and Hira, D.S. (2014). Operations Research, Seventh revised edition. S. Chand & company Pvt. Ltd.

[10] Timothy, J. Ross, (2010). Fuzzy Logic with Engineering Applications (3rd Ed). John Wiley and Sons, Ltd. Doi: 10.1002/9781119994374.

[11] Barak, S. and Fallahnezhad, M. S. (2012). Cost analysis of fuzzy queuing systems. *International Journal of Applied Operational Research*, 2(2):25-36. http://ijorlu.liau.ac.ir/article-1-138-en.pdf.

[12] Srinivasan, R. (2014). Fuzzy queuing model using DSW algorithm. *International Journal of Advanced Research in Mathematics and Applications*, 1(1):57-62. http://www.isrjournals.org/journal-view/fuzzy-queueing-model-using-dsw-algorithm.

[13] Shanmugasundaram, S. and Venkatesh, B. (2015). Study on fuzzy multi server queueing model through DSW algorithm. *International Journal of Latest Trends in Engineering and Technology*, 5(3):452-457. https://www.ijltet.org/journal_details.php?id=884&j_id=2012.

[14] Ming Ma, Menahem Friedman, and Abraham Kandel. (1999). A new fuzzy arithmetic. *Fuzzy sets and systems*, 108(1):83-90.

[15] Shaw, A. K. and Roy, T. K. (2012). Some Arithmetic Operations on Triangular Intuitionistic Fuzzy Number and its Application on reliability evaluation. *International Journal of Fuzzy Mathematics and Systems*, 2(4):363-382.

[16] Noor Hidayah Mohd Zaki, Aqilah Nadirah Salliman, Nur Atikah Abdullah, Nur Su Ain Abu Hussain, and Norani Amit. (2019). Comparison of queuing performance using queuing theory model and fuzzy queuing model at check-in counter in airport. *Mathematics and Statistics*, 7(4A):17-23.

[17] Hassan, H.E., Nagib. G. and Ibrahiem, K. H. (2021). A novel task scheduling approach for dependent non-preemptive tasks using fuzzy logic. *IET Computers and Digital Techniques*, 15(3):214-222.

[18] Usha Prameela, K., Richard Wurmbrand, R. and Jayakar, R. P. S. (2021). An Interpretation of non-preemptive priority fuzzy queuing model with asymmetrical service rates. *Pakistan Journal of Statistics and Operation Research*, 17(4):791-797.

[19] Kim, B., Kim, J. and Bueker, O. (2021). Non-preemptive priority M/M/m queue with servers' vacations, *Computers and Industrial Engineering*, 160(107390).

[20] Narayanamoorthy, S., Anuja, A., Murugesan, V. and Kang, D. (2020). A distinctive analyzation of intuitionistic fuzzy queueing system using Erlang service model. *International Conference on Advances in Applicable Mathematics, ICAAM, Coimbatore, AIP Conference Proceedings*, 2261(030040).

[21] Arpita Kabiraj, Prasun Kumar Nayak, and Swapan Raha. (2019). Solving intuitionistic fuzzy linear programming problem. *International Journal of Intelligence Science*. 9(1):44-58.

[22] Chen, G., Liu, Z. and Zhang, J. (2020). Analysis of strategic customer behavior in fuzzy queueing systems. *Journal of Industrial and Management Optimization*, 16(1):371-386.

[23] Hanumantha Rao, S., Vasanta Kumar, V. and Sathish Kumar, K. (2020). Encouraged or discouraged arrivals of an m/m/1/n queuing system with modified reneging. *Advances in Mathematics: Scientific Journal*, 9(9):6641-6647.

[24] Hanumantha Rao, Sama., Vasanta Kumar, Vemuri. and Venkata Siva Nageswara Hari Prasad Boppana. (2021). Optimal control policy for a two-phase M/M/1 unreliable gated queue under N-policy with a fuzzy environment. *Ingénierie des Systèmes d'Information*, 26(4):357-364.

[25] Sanga, S.S. and Jain, M. (2019). FM/FM/1 double orbit retrial queue with customers' joining strategy: A parametric nonlinear programming approach. *Applied Mathematics and Computation*, 362(124542).

[26] Sethi, R., Jain, M., Meena, R. K. and Garg, D. (2020). Cost optimization and ANFIS computing of an unreliable M/M/1 queuing system with customers' impatience under N-policy. *International Journal of Applied and Computational Mathematics*. 6(2), No. 51:1-14.

[27] Tamilarasi, A. (2019). A study on intuitionistic fuzzy model for classical queuing system. *International Journal of Pharmacy and Technology*, 11(4):31824-31846.

[28] Ferdowsi, F., Maleki, H. R. and Niroomand, S. (2019). Refueling problem of alternative fuel vehicles under intuitionistic fuzzy refueling waiting times: A fuzzy approach. *Iranian Journal of Fuzzy Systems*, 16(3):47-62.

[29] Kumar, B. R., Narayansamy, P. and Bhuvaneswari, S. (2021). Optimal control analyze the bulk arrival queueing system using Estimation theory. *2nd International Conference on Mathematical Modeling and Computational Methods in Science and Engineering, ICMMCMSE, Karaikudi, Sivagangai, Journal of Physics: Conference Series*, 1850(1), No. 012072.

[30] Vijaya Laxmi, P. and Girija Bhavani, E. (2021). Multi-server markovian queue with successive optional services. *Reliability: Theory and Applications*, 16, No.4(65):32-43.

[31] Su Wai, Khin Su., Htoon, Ei Chaw. and Myint Thein, Nwe New. (2020). Performance evaluation of M/D/1 queuing model on hyperledger fabric. *Proceedings of the 4th International Conference on Advanced Information Technologies, ICAIT 2020*, 9261808:36-41.

[32] Mounir Mahmoud Moghazy Abdel-Aal. (2020). Survey-based calibration of a parking entry as a single-server mathematical queuing model: A case study. *Alexandria Engineering Journal*, 59:829-838.