ON PRINCIPLES OF RISK ANALYSIS WITH A PRACTICAL EXAMPLE

Vladimir Rykov[1,3,4], Nika Ivanova[1,2], Mais Farhadov[2]

[1]Peoples' Friendship University of Russia (RUDN University), Moscow, Russia
nm_ivanova@bk.ru
[2]V.A. Trapeznikov Institute of Control Sciences of RAS, Moscow, Russia
mais.farhadov@gmail.com
[3]Gubkin Oil & Gas Russian State University, Moscow, Russia
vladimir_rykov@mail.ru
[4]Kharkevich Institute for Information Transmission Problems of RAS, Moscow, Russia

Abstract

As a scientific term "risk'' originally appeared in insurance mathematics and meant the "ruin probability'' in a "collective risk'' model of an insurance company. Recently, this term has become widespread in different spheres of human activity and has been applied to various models of "individual risk". At the same time, despite its increasing popularity, its strict definition still does not exist, and various sources interpret it differently. The report discusses various interpretations of this notion and it reasons. The authors adhere to the concept of risk as a random phenomenon and from this point of view, consider its characteristics and their measuring methods. The risk tree is used for the probabilistic space of the risk phenomenon construction. In the talk, the proposed approach will be demonstrated with the help of one real-world example.

Keywords: individual risk: notion, measurement, risk tree analysis.

I. Introduction and Motivation

The term "risk" has become very popular recently and used in various fields of human activity. However, an exact definition of this notion still does not exist, and various sources interpret it differently (see, for example, the analysis of sources in [1-5]). Such a variety in the interpretation of this notion relates to the fact that different authors are based on different concepts of "uncertainty", which has recently received several directions of formalization: (a) randomness, which is investigated by probabilistic methods, (b) subjective uncertainty, studied within the framework of subjective probabilities, (c) fuzzy uncertainty, and (c) expert uncertainty (other methods of studying uncertainties may exist or are acceptable). From another side, the variety of characteristics and indicators of risk leads to the fact that different authors use to define it.

From our point of view, to study the risk phenomena and construct its mathematical models, it is necessary, from the very beginning, to define within what type of uncertainties it will be done. In this paper, we will study the risk as a random phenomenon in terms of "objective probability". In the interpretation of A.N. Kolmogorov, a random phenomenon should have two main features:

- the possibility of multiple observation of this phenomenon and
- in homogeneous (identical) conditions.

Then randomness is measured by probability, which is estimated using frequency.

Other concepts of risk considerations are also possible, but we leave them for other authors and other discussions.

This talk deals with the notion of individual risk, its main characteristics, and their measurements based on randomness conception, and uses probabilistic terminology.

II. Risk: notion and measurements

Analysis of numerous examples of phenomena, in which the term "risk" appears, carried out in [6] and [7], allowed us to conclude that the risk is associated with the occurrence of some random event A, which we will call risk event, from a possible family $\mathscr{F}$ of events describing the considered risk situation. These events are usually distributed in time and accompanied by certain damages (material or other) generally speaking, also random in its magnitude. Thus, the risk is characterized by two values: T time of the risk event nonoccurrence and the value of X the damage connected with them. It gives the right to state the following

Definition 1. Risk is a *random phenomenon*, modeled by a probabilistic space (Ω, $\mathscr{F}$, P), on which two-dimensional random variable (r.v.) (T, X) is determined, where T is the time to the risk event A ∈ $\mathscr{F}$ occurs, and the other one X shows a *damage*, connected with it[1].

Thus, as for any r.v. its main characteristic is its cumulative distribution function (c.d.f.)

$$F(t,x) = \mathrm{P}\{\mathrm{T} \le t,\, \mathrm{X} \le x\}, \quad t \ge 0,\, x \ge 0 \tag{1}$$

In the most real cases, the information about the joint distribution of these r.v.'s does not available and one should be limited oneself with one-dimensional distributions of times to risk event and the value of the damage

$$F_{\mathrm{T}}(t) = \mathrm{P}\{\mathrm{T} \le t\}, \quad F_{\mathrm{X}}(t) = \mathrm{P}\{\mathrm{X} \le x\}. \tag{2}$$

All other risk indexes can be found from this one mail characteristics. For example, for risks during fixed time interval instead of time T to risk events one should consider its indicator function $1_{\{\mathrm{A}\}}$, and measured damage by its conditional distribution $G(x;\mathrm{A}) = \mathrm{P}\{\mathrm{X} \le x \mid \mathrm{A}\}$. At that unconditional distribution of the damage value is the joint distribution of the risk event A occurrence and conditional damage value distribution that has a jump in zero, since the damage equal zero under the absence of risk event:

$$F_{\mathrm{X}}(x) = 1 - \mathrm{P}(\mathrm{A})(1 - G(x;\mathrm{A}))$$

where $\mathrm{P}(\mathrm{A})$ – is a probability of event A.

In general, it is natural to measure a risk by the distribution of the time moment of risk event occurrence $F_{\mathrm{T}}(t) = F(t) = F(t,\infty)$ and the conditional distribution of a damage $G(x;t) = \mathrm{P}\{X \le x \mid T = t\}$. At that its joint distribution is

$$F(t,x) = \int_0^t G(x;u)\, f_{\mathrm{T}}(u)\, du\,. \tag{3}$$

The simplest assumption consists in independent of these values

$$G(x) = F_{\mathrm{X}}(t) = F(t,\infty); \quad F(x,t) = G(x)F(t)\,.$$

In the most real situations, this assumption is quite admissible with the only remark that the value of future damage in the recent time evaluated as to its *present value*

$$\hat{\mathrm{X}} = e^{-s\mathrm{T}}\mathrm{X}, \tag{4}$$

where s is a discount factor.

Further, it is supposed that the r.v.'s T, and X are independent and the following notations are used:

[1] The time T can depend on the time t_0 of the risk situation beginning. Moreover, the risk situation can change during its development. In this case, one should model the risk situation by the two-dimensional absorbing *risk process* (Z(t), X(t)) with absorption as a risk event. But we leave this construction for another work.

$$F_{\mathrm{T}}(t) = F(t,\infty) = F(t); \quad F_{\mathrm{X}}(t) = F(\infty, x) = G(x).$$

In practice, for concrete calculations, more simple characteristics of risks are used, such as expectations of the time to risk event $\mu_{\mathrm{T}} = \mathrm{M[T]}$ and mean value of damages caused by it $\mu_{\mathrm{X}} = \mathrm{M[X]}$ and their variances

$$\sigma_T^2 = D\mathrm{T} = \mathrm{M}(\mathrm{T} - \mu_{\mathrm{T}})^2; \quad \sigma_{\mathrm{X}}^2 = D\mathrm{X} = \mathrm{M}(\mathrm{X} - \mu_{\mathrm{X}})^2,$$

as well as a probability q of a risk event A в occurrence during the fixed time t_0,

$$q = \mathrm{P}\{\mathrm{T} \le t_0\} = \mathrm{P(A)} = \mathrm{M}1_{\{\mathrm{A}\}},$$

and so on.

III. Risk tree

For investigation of complex phenomena, such as development of failures in complex systems, the development of risk situations in technique, business, medicine, etc., it is convenient to use an *event tree* notion. Originally this notion has been used under the name of a *failure tree* in the very beginning of the 60-th last century by H.A. Watson from Bell Lab for analysis of systems' reliability. At that as risk events the units, subsystems, and the whole system failures are considered. Further, this idea has been used in other applications: for study business, medicine, biological and other processes. We will use it under the name *risk tree*.

Definition 2. A *risk tree* is a labeled graph, the root of which is the main risk event, and the branches associate it with initial (leaf) events, which are taken as elementary.

In practice, it is more convenient to depict the risk tree upside down.

For a description of the events hierarchy in the risk tree, it is convenient to index them with the vector indexes $\boldsymbol{i}_r = (i_1, i_2, \dots, i_r)$, whose components denote the consequence of risk events numbers, beginning from the main up to the elementary one. Thus denote

- i_0 is the number of the main risk event in the considered risk situation;
- i_1 is the number of the first level event that can be one of the reasons the main event occurrence;
- i_2 is the number of the second level event that can be one of the reasons the event with the number i_1 of the previous level occurrence; etc.
- i_k is the number of the k-th level event that can be one of the reasons the event with the number i_{k-1} of the previous level occurrence; thus
- the vector $\boldsymbol{i}_r = (i_1, i_2, \dots, i_r)$ denotes the number one of the leave (elementary) events; where
- r is a hierarchy level of this event (its *rank*), at that different elementary events can have different ranks;
- by truncated vector $\boldsymbol{i}_k = (i_1, i_2, \dots, i_k)$ the number of the according event of k-th level is denoted; and
- the j-th event, which can be the cause of the event $\boldsymbol{i}_k$, is denoted by $j(\boldsymbol{i}_k)$;
- at that, the different events are indexed by appropriate indexes $A_{\boldsymbol{i}_k}$

Thus, all events, starting from the main and intermediate ones up to elementary ones, the considered risk situation are completely determined by their numbers.

Further, analogously to the failure tree in reliability, for the risk characteristics calculation, we will use structural variables and functions. Denote *structural variable* of an event $A_{\boldsymbol{i}_k}$ by

$$x_{\boldsymbol{i}_k} = \begin{cases} 1, \textit{if an event } A_{\boldsymbol{i}_k} \textit{ occurs}; \\ \quad 0, \quad \textit{otherwise}. \end{cases}$$

Corresponding *structural functions* of the main, intermediate, and elementary events are

$$\varphi_k\left(x_{(\boldsymbol{i}_k,1)}, \dots, x_{\left(\boldsymbol{i}_k, n(\boldsymbol{i}_{k,})\right)}\right) = x_{\boldsymbol{i}_{k-1}} \tag{4}$$

and they are calculated accordingly to the rules of Boolean algebra. For the risk tree construction, it is convenient to use special symbols of the events and gates that are taken from the book of Henley & Kumamoto [8] and can be found in [6]-[7].

In the talk, this approach will be demonstrated with the help of the example of the risks, connected with the system of underwater pipeline system monitoring.

IV. Conclusion and the further work

In the talk, the risk notion, its measurements, and its model construction in the framework of Kolmogorov's probabilistic approach are proposed. This approach is demonstrated with the help of some example. The development of a risk situation is considered in an unchanging external environment. More wide risk model investigation is also possible. It must be based on the model of risk process modeling and considering the risk event as an adsorbing of this process. This approach can be the subject of further research.

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