# SEISMIC OSCILLATIONS OF CRUSTAL LAYER OF THE EARTH

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**Abstract**

*Seismic push causes low-frequency oscillations of structures. Since the structures have a large mass, inertia forces occur during oscillations, resulting in high mechanical stresses (compression-tension and shear)at different places of the structures, which can exceed the strength of the material at one place or another and lead to damage or even collapse of the entire structure. Under seismic influences, as a result of inertial forces, low-frequency vibrations of structures occur in structures, mechanical stresses are generated in various places of structures that exceed the strength of the material, and can lead to damage or to the collapse of the entire structure. For this reason, buildings with anti-seismic reinforcement of structural elements are being erected in seismic areas. A structure, as a free body in space, has six degrees of freedom and the corresponding vibration modes: three translational displacements (vertical and two horizontal) and three rotational displacements: pendulum oscillations, oscillations around the longitudinal axis; vibrations around the transverse axis; vibrations around the vertical axis. The vibrations of an arbitrary structure with a foundation are the result of the superposition of different vibration modes with free vibration frequencies. In this work for the first time the crustal layer oscillation is described, frequency of oscillations from wavelength is determined.*

**Keywords:** seismic effect, oscillation period, angular frequency, formation base, vibrations, rotational displacements

## I. Introduction

Obviously, the vibrations of the structure are also influenced by the soils on which it stands. Seismic impact is determined in three parameters: the level of amplitudes, period and duration of oscillations. These parameters are critical for the stability of structures, and even a short-term load with very high acceleration may not be dangerous for many of them.

The longest period of the Earth's oscillations is about 1.5 hours. The periods of vibrations of the Earth's strata during earthquakes are of the order of a fraction of a second. Therefore, it can be assumed that earthquakes are independent of the Earth's vibrations.

In this research paper, earth layer is considered independently of the vibrations of the Earth.

Modernity, complexity and diversity of phenomena of non-stationary interaction of waves with obstacles in the form of solid and deformable bodies will assist in providing the processes of interaction of bodies of different physical nature. For major changes in design, construction and evaluation to be accepted, it is necessary that innovative structures be monitored for their interacting with the medium and studied for their deformation. The practice of modern construction industries requires the calculation of elements of structures and structures on the action of shock waves propagating in the medium or in the medium, filling it. To assist in achieving this goal in the given work is developing two-dimensional problem of propagation of seismic waves in an elastic medium. The two-dimensional problems of wave propagation in an elastic medium are interesting not only from a theoretical, but also from a practical point of view. In particular, seismic waves, given their rapid attenuation in depth, should be considered as two-dimensional. To receive the analytical and numerical solution of a task with the parameters, which are instantly increasing on border or in, the form of pushes with the subsequent attenuation attracts great theoretical and practical interest.

## II. Methods

A layer of earth with a thickness of $y_0$ is considered, there are no stresses on the surface, $\sigma_{xy} = 0$ and и $\sigma_{yy} = 0$; at the base $v = 0$ and $\sigma_{xy} = 0$, i.e. there is no vertical displacement and the ground slides freely in the horizontal direction (Fig. 1)

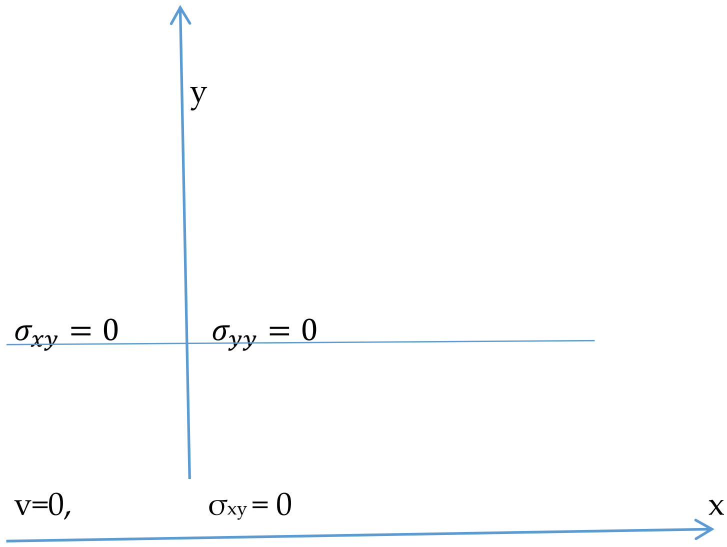


**Fig. 1:** *Horizontal ground slip*

Solutions of wave equations in polar coordinates $r$, θ are found

$$a^2\Delta\varphi - \frac{\partial^2\varphi}{\partial t^2} = 0$$
$$b^2\Delta\psi - \frac{\partial^2\psi}{\partial t^2} = 0 \qquad (1)$$

under boundary conditions on the surface of cylindrical inclusion

$$U(t) = H(t)\,V_0 \qquad (2)$$

in the form of

$$u_t = \frac{2r_0V_0\sqrt{ab}}{\pi}\left(\frac{1}{ab}\left(A_1(a,b) - \frac{a+b}{r_0\mu}\int_{\frac{r-r_0}{a}}^{t} A_1(a,b)\;\mu\;d\tau\right) +\right.$$

$$+\frac{1}{br\mu}\int_{\frac{r-r_0}{a}}^{t} A_2(a,b)\ \mu\ d\tau)+\frac{2}{ar_0\mu}\cdot\int_{\frac{r-r_0}{a}}^{t} A_3(a,b)\ \mu\ d\tau+\frac{2}{r_0\ r}(A_4(a,b)-$$

$$-\frac{a+b}{r_0\mu}\int_{\frac{r-r_0}{a}}^{t} A_4(a,b)\ \mu\ d\tau)\ -\ \frac{1}{ar\mu}\int_{\frac{r-r_0}{b}}^{t} A_2(b,a)\ \mu\ d\tau\ -\ \frac{2}{r_0\ r}(A_4(b,a)-$$

$$-\ \frac{a+b}{r_0\mu}\int_{\frac{r-r_0}{b}}^{t} A_4(b,a)\ \mu\ d\tau)) \tag{3}$$

where: φ and ψ - potential functions, described waves the transferring volume expansion and rotation;

magnitudes $a=\sqrt{\frac{\lambda+2\mu}{\rho}}$ and $b=\sqrt{\frac{\mu}{\rho}}$ are determine the velocity of propagation of waves of expansion and waves of rotation;

λ и μ - constants of Lama; ρ - density of medium; Δ - Laplacian; $V_0$ – constant speed of cylindrical inclusion; $H(t)$- Hevisayd's unit function, determined by a formula

$$H(t)=\begin{cases}1,\ t>0\\0,\ t<0\end{cases} \tag{4}$$

## III. Results

For simplicity, a two-dimensional problem for an elastic soil is considered. The equations of motion are

$$\rho\frac{\partial^2 u}{\partial t^2}=(\lambda+2\mu)\nabla^2 u \tag{5}$$

$$\rho\frac{\partial^2 v}{\partial t^2}=(\lambda+2\mu)\nabla^2 v \tag{6}$$

where $\rho$ – soil density, $\lambda,\mu$ – Lame's constants, $t$ - time, $u,v$ – displacements in horizontal and vertical directions. $\nabla^2=\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}$, $\lambda\ and\ \mu$ are considered permanent.

Assuming that there is an oscillatory motion with an angular frequency and a standing wave length $l$, we have

$$u=Usin\omega tcos\frac{2\pi}{l}x \tag{7}$$

and

$$v=Vsin\omega tsin\frac{2\pi}{l}x \tag{8}$$

where $u$ $and$ $v$ – fuction of $y$.

Substituting (7) and (8) in (5) and (6), we obtain

$$-\rho\omega^2 U = (\lambda + 2\mu)\left[-\left(\frac{2\pi}{l}\right)^2 U + U^{\shortparallel}\right] \quad (9)$$

$$-\rho\omega^2 V = (\lambda + 2\mu)\left[-\left(\frac{2\pi}{l}\right)^2 V + V^{\shortparallel}\right] \quad (10)$$

Solving equations (9) and (10) relatively to $u$ and $v$, we obtain

$$u = c_1 sin\Omega y + c_2 cos\Omega y \quad (11)$$

$$V = D_1 sin\Omega y + D_2 cos\Omega y \quad (12)$$

where

$$\Omega\sqrt{\frac{\rho\omega^2}{\lambda+2\mu} - \left(\frac{2\pi}{l}\right)^2} \quad (13)$$

Shear stress

$$\sigma_{xy} = \mu\varepsilon_{xy} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) = \mu\left(\frac{2\pi}{l}V + u'\right) sin\omega t cos\frac{2\pi}{l}x \quad (14)$$

Satisfying the conditions at the lower bound: $y = 0, v = 0, \sigma_{xy} = 0$,

we get

$$D_2 = 0; \quad c_1 = 0 \quad (15)$$

On the upper border $y = y_0$, $\sigma_{xy} = 0, \sigma_{yy} = 0$, we get

$$\sigma_{xy}\Big|_{y=y_0} = \mu(D_1 \sin\Omega y_0 - \Omega c_2 sin\Omega y_0) sin\omega t cos\frac{2\pi}{l}x = 0$$

or

$$\frac{2\pi}{l}D_1 - \Omega C_2 = 0 \quad (16)$$

From (11) and (15) expressions

$$U = c_2 cos\Omega y_0$$
$$V = D_1 sin\Omega y_0 \quad (17)$$

Further

$$\sigma_{yy} = \lambda\Delta + 2\mu\varepsilon_{yy} = \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + 2\mu\frac{\partial v}{\partial y} =$$
$$= \left[\lambda\left(-\frac{2\pi}{l}U + V'\right) + 2\mu V'\right] sin\omega t cos\frac{2\pi}{e}x =$$
$$= \left[-\frac{2\pi}{l}\lambda c_2 cos\Omega y_0 + (\lambda + 2\mu)\Omega D_1 cos\Omega y_0\right] sin\omega t cos\frac{2\pi}{e}x - \frac{2\pi}{l}\lambda c_2 + (\lambda + 2\mu)\Omega D_1 = 0 \quad (18)$$

From (16) and (18) expressions

$$\omega^2 = \frac{2(\lambda + \mu)}{\rho} \quad (19)$$

where $\nu$ – frequency

$$\nu = \frac{\omega}{2\pi}$$

One example of the carried-out calculation, considering bulkiness of the received results, corresponds to velocity of propagation of waves in rocky and semi-rocky breeds

$a$=2000m/sec; $b$=1400m/sec;

$r_0$ = 10 m; $r$ = 100 m; 1000 m; 10000 m

$r_0$ - distances of inclusion

In considered medium the non-stationary elastic wave interacts with inclusion and generates the reflected waves moves. The time changes in the range of 0 sec., $0 \le r \le 10$ sec, $\Delta t$ = 0,01 sec.

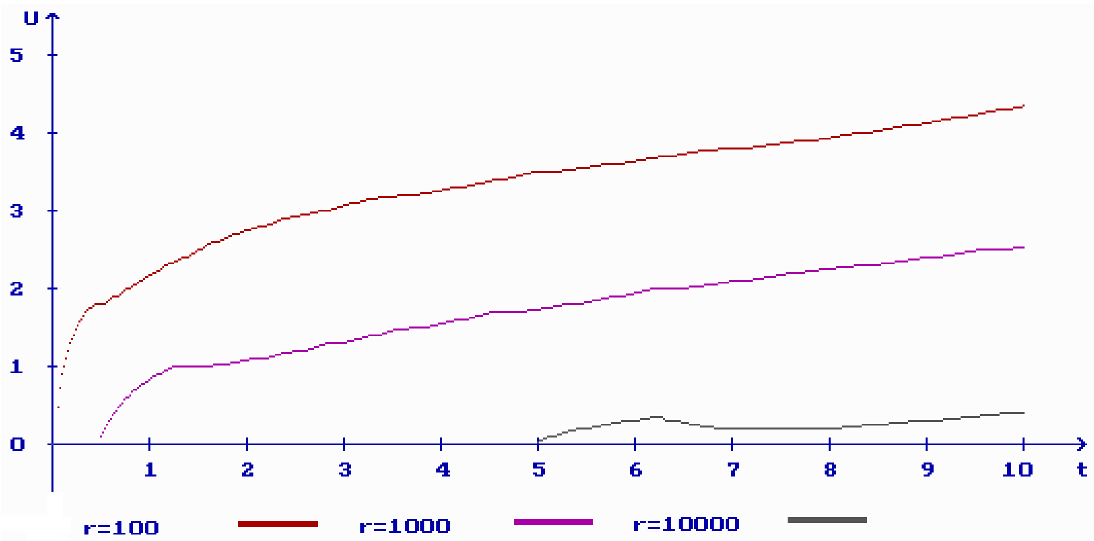


**Fig. 2:** *Attenuation of waves of displacements*

The top curve corresponds to dependence of $u_t(t)$ on $t$ at $r$=100m; the average curve corresponds to dependence of $u_t(t)$ on $t$ at $r$=1000m; the lower curve corresponds to dependence of $u_t(t)$ on t at r=10000m. As inclusion farther from the center, then more is observed attenuation of waves of displacements.

## IV. Discussion

The frequency of the earthquake can be used to estimate the length of the standing wave of soil vibrations.

Here are materials on possible phenomena during earthquakes. In particular, assuming the Lame constants for the soil

$$\lambda = 1 \cdot 10^9 Pa, \qquad \mu = 0{,}8 \cdot 10^9 Pa,$$

We have according to (19)

$$\omega = 2\pi\nu; \;\; E = 2 \cdot 10^9; \;\; \rho = 2 \cdot 10^3 \text{ кг/м}^3; \;\; \nu - \text{frequency}$$

1) $l = 100\text{m}; \;\; \nu = 27{,}6 \;\; 1/sec$
2) $l = 300m; \;\; \nu = 9{,}2 \;\; 1/sec$

In further consideration, obviously, consider the Lamé constants $(\lambda = 1 \cdot 10^9 Pa)$ $and$ $\mu$ $(\mu = 0{,}8 \cdot 10^9 Pa)$, depending on depth and, if possible, take into account friction at the bottom of the formation.

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