

ON ONE APPROACH TO NON-DESTRUCTIVE CONTROL OVER THE STATE OF PIPELINE SYSTEMS

Reshad Ismibayli¹, Yegana Ashrafova²

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¹Azerbaijan University of Architecture and Construction

²Baku State University, Azerbaijan

reshadismibayli@gmail.com ashrafova.yegana@gmail.com

Abstract

In the process of long-term operation of pipeline systems for the transport of hydrocarbons (oil, gas) and other liquids, leaks occur in some of its sections over time. An important role of timely detection of leaks is occupied by methods of indestructible control over the state of pipeline systems that do not require the production of any technical actions. Non-destructive testing methods are based, as a rule, on mathematical methods and modern computer technologies. In this paper, using the example of the problem of determining the locations of fluid leaks, an approach is proposed based on solving an inverse problem with respect to a system of differential equations with partial derivatives that describes the process of fluid flow.

Keywords: oil-gas pipelines, non-destructive control methods, leaks, environmental risks, inverse problem.

I. Introduction

In the process of long-term operation of pipeline systems for the transport of hydrocarbons (oil, gas) and other liquids, leaks occur in some of its sections over time. Leaks occur due to pipeline breaks, which can have different causes and sizes.

The emerging leaks lead to great risks, which have both environmental consequences, but also causing great economic damage to mining and transport organizations.

Known and used are various methods of timely detection of leaks of raw materials. An important role is occupied by methods of indestructible control over the state of pipeline systems that do not require the production of any technical actions. In particular, in the case of underground pipeline networks, such methods do not require, for example, any earthworks.

Non-destructive testing methods are based, as a rule, on mathematical methods and modern computer technologies. In this paper, using the example of the problem of determining the locations of fluid leaks, an approach is proposed based on solving an inverse problem with respect to a system of differential equations with partial derivatives that describes the process of fluid flow.

An inverse problem for a pipeline network of complex loopback structure is solved numerically. The problem is to determine the locations and amounts of leaks from unsteady flow characteristics measured at some pipeline points. The features of the problem include impulse functions involved in a system of hyperbolic differential equations, the absence of classical initial conditions, and boundary conditions specified as nonseparated relations between the states at the endpoints of adjacent pipeline segments. The problem is reduced to a parametric optimal control problem without initial conditions,

but with nonseparated boundary conditions. The latter problem is solved by applying first-order optimization methods [1]. Results of numerical experiments are presented.

This paper differs from many other studies [2] in which leak locations and amounts were determined either in a steady flow regime in a pipeline of complex structure or in a transient flow regime in a pipeline consisting of a single linear segment. In this study, we numerically solve an inverse problem [3] of determining leak locations and amounts in an unsteady flow in a pipeline network of complex (loopback) structure. The problem is described by a system made up of numerous subsystems of two hyperbolic partial differential equations with impulse actions specified at possible leakage points on pipeline network segments.

Another feature of the problem is the assumption that, due to the long duration of the process under study, exact information on its initial state is not available at the time of monitoring and that the states of the process (which is distributed in space) cannot be quickly measured at all points. Instead, there is information on a variety of possible initial states of the process and some state (regime) characteristics are measured at certain pipeline points starting at this time. One more feature of the problem is that its boundary conditions are specified as nonseparated relations (determined by physical laws) between the states at the endpoints of adjacent pipeline segments.

II. Problem statement

To simplify presentation of numerical schemes and to be specific, let us consider the pipe network, containing 8 segments as shown in figure 1. Numbers in brackets identify the nodes (or junctions). The set of nodes we denote by $I : I = \{k_1, \dots, k_N\}$; where $k_i, i = \overline{1, N}$ are the nodes; $N = |I|$ is the numbers of nodes in the network. Two numbers in parentheses identify two-index numbers of segments. The flow in these segments goes from the first index to the second (for example, the flow in the segment (1,2) is obviously from the node 1 to node 2).

Let $J : J = \{(k_i, k_j) : k_i, k_j \in I\}$ is the set of segments and $M = |J|$ is its quantity; $l_{k_i k_j}$, $d_{k_i k_j}, k_i, k_j \in I$ is a length and diameter of the segment (k_i, k_j) respectively; I_k^+ is the set of nodes connected with node k by segments where flow goes into the node, I_k^- is the set of nodes connected with node k by segments where flow goes out of the node; $I_k = I_k^+ \cup I_k^-$ is the set of total nodes connected with node k and $N_k = |I_k|, N_{k^+} = |I_k^+|, N_{k^-} = |I_k^-|, N_k = N_{k^-} + N_{k^+}$.

Beside of inflows and outflows in the segments of the network there can be external inflows (sources) and outflows (sinks) with the rate $\tilde{q}_i(t)$ at some nodes $i \in I$ of the network. Positive and negative values of $\tilde{q}_i(t)$ indicate the existence of external inflow or outflow at the node i . However, in general case, assuming that the case $\tilde{q}_i(t) \equiv 0$ for the sources is admissible one can consider all nodes of the network as the nodes with external inflows or outflows. Let $I^f \subset I$ denote the set of nodes $i \in I$, where i is such that the set $I_i^+ \cup I_i^-$ consists of only one segment. It means that the node i is a node of external inflow or outflow for the whole pipe network (for example $I^f = \{1, 4, 5, 8\}$ in fig.1). Let $N_f = |I^f|$ is the number of such nodes, it is obviously that $N_f \leq N$; I^{int} is the set of nodes not belonging to I^f , so $N_{\text{int}} = |I^{\text{int}}|$, i.e., $I^{\text{int}} = I / I^f$, $N_{\text{int}} = N - N_f$. In actual conditions, the pumping stations are placed, the measuring equipment is installed and the quantitative accounting is conducted at the nodes from the set I^f .

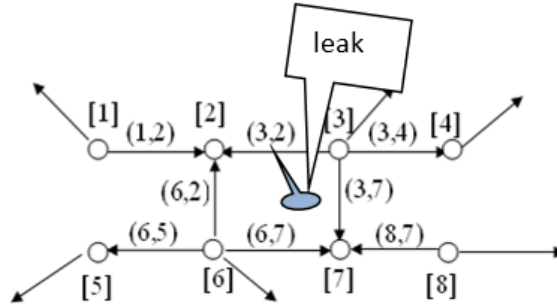


Fig.1: The scheme of pipe network with 8 nodes

We assume that at some instants of time $t \geq t_0$ at some points $\xi_{ks} \in (0, l)$, of any (ks) -th section of the pipeline network, fluid leakage with the flow rates $q_{ks}^{loss}(t)$ began. Using the generalized Dirac function $\delta(x)$, we can describe the motion of the liquid by the following linearized system of differential equations for unsteady flow of dripping liquid with constant density ρ in a linear pipe (k, s) of length l_{ks} and diameter d_{ks} of oil pipeline network can be written in the following form [4-8]:

$$\begin{cases} -\frac{\partial P^{ks}(x, t)}{\partial x} = \frac{\rho}{S^{ks}} \frac{\partial Q^{ks}(x, t)}{\partial t} + 2a^{ks} \frac{\rho}{S^{ks}} Q^{ks}(x, t), & x \in (0, l^{ks}), \\ -\frac{\partial P^{ks}(x, t)}{\partial t} = c^2 \frac{\rho}{S^{ks}} \frac{\partial Q^{ks}(x, t)}{\partial x} + c^2 \frac{\rho}{S^{ks}} q_{ks}^{loss}(t) \delta(x - \xi_{ks}), & t \in (0, T], \end{cases} \quad s \in I_k^+, k \in I. \quad (1)$$

here c is the sound velocity in the fluid; S^{ks} is the area of an internal cross-section of the segment (k, s) ; a^{ks} is the coefficient of dissipation (we may consider that the kinematic coefficient of viscosity γ is independent of pressure and the condition $2a^{ks} = \frac{32\gamma}{(d^{ks})^2} = const$ is quite accurate for a laminar flow). $Q^{k_i, k_j}(x, t)$, $P^{k_i, k_j}(x, t)$ are the flow rate and pressure of flow, respectively, at the time instance t in the point $x \in (0, l^{k_i, k_j})$ of the segment (k_i, k_j) of the pipe network. $P^k(t)$, $Q^k(t)$ are the pressure and flow rate at the node $k \in I$, respectively.

The conditions of Kirchhoff's first law (total flow into the node must be equal to total flow out of the node) are satisfied at the nodes of the network at $t \in [0, T]$:

$$\sum_{s \in I_k^+} Q^{ks}(l^{ks}, t) - \sum_{s \in I_k^-} Q^{ks}(0, t) = \tilde{q}^k(t), \quad k \in I. \quad (2)$$

Also, the following conditions of flow continuity for the nodes of the net (the equality of the values of pressures on all adjacent ends of the segments of the network) hold:

$$P^k(t) = P^{k_i, k}(l^{k_i, k}, t) = P^{k, k_j}(0, t), \quad k_i \in I_k^+, k_j \in I_k^-, k \in I, \quad (3)$$

where $\tilde{q}^k(t)$ is the external inflow ($\tilde{q}^k(t) > 0$) or outflow ($\tilde{q}^k(t) < 0$) for the node k , $P^k(t)$ is the value of the pressure in the node k . We must note that they have significant specific features, consisting in the fact that the conditions (2) and (3) are non-separated (nonlocal) boundary conditions unlike classical cases of boundary conditions for partial differential equations.

The total number of conditions for all nodes from I^f is N_f . So, the total number of conditions in (2) and (3) is $[N_f + N_{int}] + [(2M - N_f) - N_{int}] = 2M$. As it was noted above the

number of conditions in (2) is N , but in view of the condition of material balance ($\sum_{k \in I} \tilde{q}^k(t) = 0$) for the whole pipeline network, we conclude that the number of linearly independent conditions is $N-1$. So, it is necessary to add any one independent condition. As a rule the value of pressure at one of the nodes $s \in I^f$ is given for this purpose, in place of the flow rate $q^s(t)$:

$$P^s(t) = \tilde{P}^s(t). \quad (4)$$

In the case of unknown points of leakages and their rates $\xi_{ks}, q_{ks}^{loss}(t)$ we will assume that at the ends of the pipeline sections a constant and rather long observation on pressure is made, i.e., the values of $P_{mes}^n(t), n \in I_p^f$ or $Q_{mes}^m(t), m \in I_q^f$ are known. It is quite natural to suppose that the sought leak spots do not coincide with the points of observation of regimes. In more general case, for every node from $I^f = I_q^f \cup I_p^f$, it is necessary to give the values of pressure ($I_p^f \subset I^f$ denotes the set of such nodes) or the values of flow rate (the set $I_q^f \subset I^f$) and I_p^f must not be an empty: $I_p^f \neq \emptyset$. So, we will add the following conditions to the condition (3):

$$\begin{cases} P^n(t) = P^{ns}(0, t) = P_{mes}^n(t), & s \in I_n^+, \text{ if } I_n^- = \emptyset, \\ P^n(t) = P^{sn}(l^{sn}, t) = P_{mes}^n(t), & s \in I_n^-, \text{ if } I_n^+ = \emptyset, \end{cases} \quad n \in I_p^f, \quad (5)$$

$$\begin{cases} Q^m(t) = Q^{ms}(0, t) = Q_{mes}^m(t), & s \in I_m^+, \text{ если } I_m^- = \emptyset, \\ Q^m(t) = Q^{sm}(l^{sm}, t) = Q_{mes}^m(t), & s \in I_m^-, \text{ если } I_m^+ = \emptyset, \end{cases} \quad m \in I_q^f, \quad (6)$$

When the spots of oil leakages from a pipeline and the rates of these leakages are known $\xi_{ks}, q_{ks}^{loss}(t), (k, s) \in J$, it is sufficient to use one of the boundary-value conditions (5) or (6) to calculate the regime of liquid motion in the pipeline from (1) on the time interval $[t_0, T]$. One of them we will use in the functional, the form which will be given below.

The problem consists in the detection of the points of leakage $\xi = \{\xi_{ks}, (k, s) \in J\}$ and corresponding losses of raw material $q^{loss}(t) = \{q_{ks}^{loss}(t), (ks) \in J\}$ at $t \in [t_0, T]$ with the use of the given mathematical model and obtained information.

It is important to note that if process (1) is rather long, then, due to the presence of friction typical of any real physical system, the influence of the initial state of the pipeline on the regimes of oil motion in it becomes weaker with time. Therefore, when the process is observed for a long time, i.e., within a large time interval $[t_0, T]$, the influence of the initial regime of oil flow in a pipeline (at $t = t_0$) on the current state of the process decreases, and there exists such τ ($\tau < T$) that at $t > \tau$ the regime of oil motion experiences only the influence of the boundary-value conditions on the time interval $[t_0, T]$, where the quantity τ is determined by the parameters of the process and the characteristics of the pipeline [9]. Thus, we arrive at the problem without initial conditions.

III. Approach to the solution of the problem

In order to solve the problem posed, we will consider the functional that determines the derivation of regimes of oil flow at the given points of the oil pipeline section from those predicted:

$$\mathfrak{A}(\xi, q^{loss}) = \int_{\mathcal{D}} [\Phi(\xi, q^{loss}; \gamma) + \mathfrak{R}(\xi, q)] \mu_{\mathcal{D}}(\gamma) d\gamma \rightarrow \min, \quad (7)$$

$$\Phi(\xi, q^{loss}; \gamma) = \sum_{m \in \tilde{I}_q^f} \int_{\tau}^T [Q^m(t; \xi, q(t), \gamma) - Q_{mes}^m(t)]^2 dt,$$

$$\mathfrak{R}(\xi, q) = \varepsilon_1 \|q(t) - \hat{q}\|_{L_2^2[t_0, T]}^2 + \varepsilon_2 \|\xi - \hat{\xi}\|_{R^Z}^2,$$

where $Q^m(t; \xi, q(t), \gamma), m \in \tilde{I}_q^f$ – is the solution of the problem (1)–(5) at the given values of $(\xi, q^{loss}(t))$, $[\tau, T]$ is the time interval of monitoring the process whose regimes already do not depend on the initial conditions; $\tilde{\xi}, \tilde{q} \in R^m, \varepsilon_1, \varepsilon_2$ – are the regularization parameters. Since the initial conditions at time t_0 do not influence the process in the interval $[\tau, T]$, exact knowledge of the initial value of t_0 is not of primary importance.

Proceeding from the meaning of the problem considered, technological conditions, and technical requirements, we will assume that are restrictions on the identified functions and parameters:

$$0 < \xi_{ks} \leq l^{ks}, \quad \underline{q} \leq q^{loss}(t) \leq \bar{q}, \quad t \in [t_0, T],$$

where \underline{q}, \bar{q} are the given quantities.

As is seen, as to the determination of the points and rates of leakages the posed problem is the problem of parametric optimal control of an object described by a hyperbolic system. For its solution we use numerical methods (projections of the conjugated gradient) based on iteration procedures of first order optimization. To carry out this procedure, we obtain formulas for the gradient of functional (7). If as a result of the solution of posed problem we obtain that $|q^{loss}(t)| \leq \varepsilon, t \in [\tau, T]$, this will mean that in this section of the pipeline network there is no leakage of raw material.

IV. Results of numerical experiments

We consider the following specially constructed test problem for oil pipeline network consisting of 5 nodes, as shown in figure 2. Here $N = 6, M = 5, I^f = \{1, 3, 4, 6\}, N_f = 4, N_{int} = 2$. There are no external inflows and outflows inside the network. We assume that in the course of 30 min we observe the process (mode of operation of pumping plants at the ends of the sections) of oil transportation with the kinematic viscosity $\nu = 1.5 \cdot 10^{-4} (m^2 / s)$ and density $\rho = 920 (kg / m^3)$ ($2a = 0.017$ for case being considered; the sound velocity in oil is $1200 (m / s)$) in the sections of pipeline of diameter 530 (mm), of the lengths of the segments:

$$l^{(1,2)} = 100 \text{ (km)}, l^{(5,2)} = 30 \text{ (km)}, l^{(3,2)} = 70 \text{ (km)}, l^{(5,4)} = 100 \text{ (km)}, l^{(5,6)} = 60 \text{ (km)}.$$

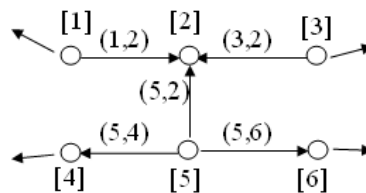


Fig.2: The scheme of oil pipeline network with 5 nodes

Let there was regime in the pipes at initial time instance $t = 0$ with the following values of pressure and flow rate in the pipes:

$$\begin{aligned} \hat{P}^{1,2}(x) &= 23\,00000 - 5.8955x \text{ (Pa)}, \hat{P}^{5,2}(x) = 1745669 - 1.17393x \text{ (Pa)}, \\ \hat{P}^{3,2}(x) &= 1827844 - 1.677043x \text{ (Pa)}, \hat{P}^{5,4}(x) = 1827844 - 2.35786x \text{ (Pa)}, \hat{P}^{5,6}(x) = 1827844 - 0.94415x \text{ (Pa)}, \\ \hat{Q}^{1,2}(x) &= 300 \text{ (m}^3/\text{hour)}, \hat{Q}^{5,2}(x) = 200 \text{ (m}^3/\text{hour)}, \hat{Q}^{3,2}(x) = 100 \text{ (m}^3/\text{hour)}, \\ \hat{Q}^{5,4}(x) &= 120 \text{ (m}^3/\text{hour)}, \hat{Q}^{5,6}(x) = 80 \text{ (m}^3/\text{hour)}, \end{aligned}$$

Let the oil flow rate at the ends of this pipeline section be defined by the functions:

$$\begin{aligned} \tilde{P}_0^1(t) &= 2000000 + 300000 e^{-0.0003t} \text{ (Pa)}, \tilde{P}_0^3(t) = 1900000 - 72156 e^{-0.0004t} \text{ (Pa)}, \\ \tilde{P}_l^4(t) &= 1800000 - 66571 e^{-0.0007t} \text{ (Pa)}, \tilde{P}_l^6(t) = 1600000 + 86372 e^{-0.0002t} \text{ (Pa)}. \end{aligned}$$

On the assumption that the point of leakage is located at the point $\xi = 30 \text{ (km)}$ of the first section of pipeline network and the rate of leakage is determined by the function $q^{loss}(t) = 50 - 10e^{-0.0003t} \text{ (m}^3/h)$, we solved the boundary-value problem (1)-(5) numerically and determined the numerical values of pressure at the ends of the section $P^n(t), n \in I_p^f$. Thereafter, with the aid of the probe of uniformly distributed random numbers these values were changed within 2% (to simulate the error of measurements) and used as the observed regimes of the process. The point and rate of leakage $\xi, q^{loss}(t)$ "forgotten" in this case.

To determine $\xi, q^{loss}(t)$, we used the method of the projection of conjugate gradients. The numerical solution of the boundary-value problem (1)-(5) was made using the scheme of the sweep method introduced in [10], on the grids with the steps $h_x = 10m$ and $h_t = 100(\text{sec})$.

Table 1 presents the obtained results of the minimization of functional (7) for different initial values of the identified parameters $(\xi, q^{loss}(t))^0$, as well as the required number of iterations (one-dimensional minimizations) of the method of projection of conjugate gradients. Here are the results of solving the problem under the conditions that the observed values of the flow rate at the ends of the network have measurement errors.

For this experiment, to generate observations for the inverse problem, by using a random number generator we add noises $\eta\chi_i Q^m(t_i)$ to the values $Q^m(t_i) = Q^m(t_i; \xi, q)$, $t_i = ih_t$, $m \in \tilde{I}_q^f$, $i = 1, \dots, N_t$ obtained by solving the direct problem, where χ_i - random variable, uniformly distributed on a segment $[-1, 1]$, $i = 1, \dots, N_t$, η takes values equal to 0.0, 0.005 and 0.01, which corresponds to the noise level when measuring flow rates in vertices \tilde{I}_q^f respectively in 0% (without noise), 0.5% and 1% from the measured value.

We use the following designations in table 1: $\tilde{\xi}^{(1,2)}$ – obtained leak location value, \mathfrak{S}_0 – initial value of the functional, $\tilde{\mathfrak{S}}$ – the resulting optimal value of the functional, N_{iter} – the number of iterations (one-dimensional minimizations) required by the conjugate gradient

projection method, $\delta\xi^{\xi(1,2)} = |\xi^{(1,2)*} - \tilde{\xi}^{(1,2)}| / \xi^{(1,2)}$, $\delta q^{(1,2)} = \max_{t \in [t_0, T]} |q^{(1,2)*}(t) - \tilde{q}^{(1,2)}(t)| / |q^{(1,2)}(t)|$ – relative error values, respectively, at the location of leakage and its volume.

As can be seen from Table 1, an increase in the accuracy of measurements most significantly affects the accuracy of determining the volume of leaks, but in general, the order of error of the obtained values of the identified parameters is the same as the order of measurement error.

Table 1: The results of numerical experiments

η	$\xi_0^{(1,2)}$	60	20	90	10	45,685
	$q_0^{(1,2)}(t)$	$90 - 10e^{-0.0003t}$	$20 - 10e^{-0.0003t}$	$30 - 10e^{-0.0003t}$	$66 + 20e^{-0.0003t}$	$66 + 20e^{-0.0003t}$
0%	\mathfrak{I}_0	76.104	16.704	11.664	42.48	57.636
	$\tilde{\mathfrak{I}}$	$5.73 \cdot 10^{-7}$	$1.26 \cdot 10^{-7}$	$3.19 \cdot 10^{-6}$	$1.85 \cdot 10^{-6}$	$7.43 \cdot 10^{-7}$
	$\tilde{\xi}^{(1,2)}$	30.003	29.998	30.008	29.994	29.998
	N_{iter}	6	5	16	14	8
	$\delta\xi^{\xi(1,2)}$	0.00009	0.00006	0.0003	0.0002	0.00006
	$\delta q^{(1,2)}$	0.0003	0.0006	0.0003	0.0002	0.0003
0.5%	\mathfrak{I}_0	76.352	16.837	11.683	42.148	57.732
	$\tilde{\mathfrak{I}}$	0.023	0.014	0.024	0.017	0.020
	$\tilde{\xi}^{(1,2)}$	29.841	30.332	30.068	29.654	29.796
	N_{iter}	6	5	14	12	7
	$\delta\xi^{\xi(1,2)}$	-0.005	0.011	0.002	-0.011	-0.006
	$\delta q^{(1,2)}$	0.043	0.030	0.049	0.041	0.042
1%	\mathfrak{I}_0	77.119	16.924	11.832	43.744	57.413
	$\tilde{\mathfrak{I}}$	0.067	0.071	0.073	0.062	0.065
	$\tilde{\xi}^{(1,2)}$	28.527	29.392	29.923	30.597	29.839
	N_{iter}	6	5	14	12	7
	$\delta\xi^{\xi(1,2)}$	-0.049	-0.020	-0.002	0.020	-0.005
	$\delta q^{(1,2)}$	0.109	0.092	0.107	0.094	0.093

Figure 3 shows the graphs of the exact leak function and the resulting loss functions when solving problem assuming the presence of a leakage.

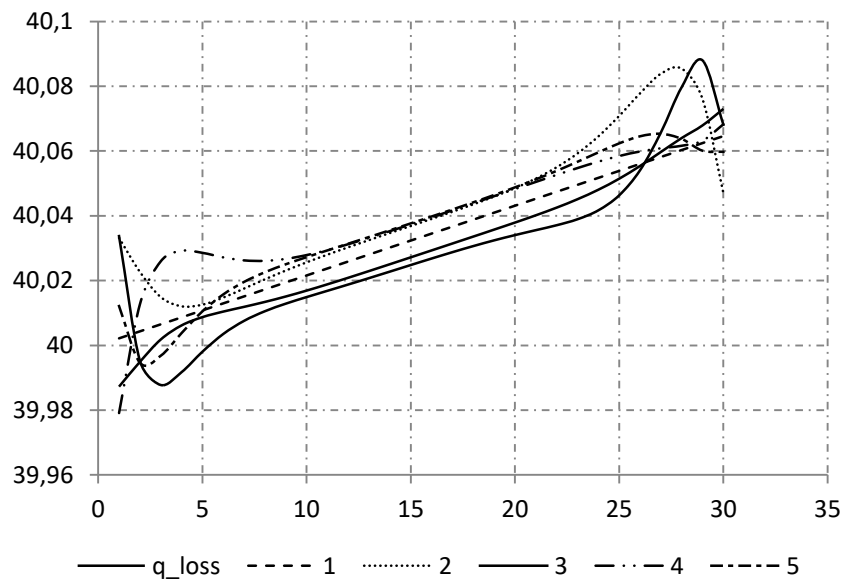


Fig.3: The exact and experimental time dependences of raw material leakage.

V. Conclusion

We propose a numerical solution to an inverse problem in the hydraulic network of complex loopback structure in the paper. The problem consists in determining the places and volume of leakage in the presence at some points of the pipeline the results of additional observations on non-stationary regimes of fluid flow. We reduce the stated problem to the parametric optimal control problem with unknown initial and non-separated boundary conditions, and use numerical methods of first-order optimization to solve the problem. The results of numerical experiments are given.

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