ON THE CONSTRUCTION OF MEMBERSHIP FUNCTIONS FOR FUZZY SETS ASSOCIATED WITH RISKS

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Abstract

The paper analyzes the methods for constructing membership functions on the example of fuzzy sets associated with risks using expert information. Methods of approximation and interpolation are used. The influence of noise in expert data on the accuracy of the obtained membership functions is analyzed.

Keywords: fuzzy sets, membership function, method of penalty function, approximation

I. Introduction

It is known that both at the design stage and in the management of complex processes, decision-making problems occupy an important place. These problems, as a rule, are associated with an assessment of the risks that the decisions taken lead to. Of great importance is the choice of specific values of the parameters that determine the chosen decision, in which the consequences of the decision lead to fewer risks, and even better - to their absence.

Many real problems cannot be formalized mathematically because of their complexity. This is due to the impossibility of constructing appropriate adequate mathematical models, objective functions and functions that describe constraints.

Given the above, to solve complex decision-making problems that are difficult to formalize and can be solved in the framework of classical mathematics, relatively new approaches based on the construction of intelligent systems are used. Examples of such approaches are theories of expert systems, fuzzy sets, theories of neural and neurophase networks. This work is devoted to an approach based on the theory of fuzzy sets, which is currently widely used in mathematical modeling of the functioning of complex technological processes and technical objects. The main characteristic of fuzzy sets is their membership functions. In practice, such functions as triangular, trapezoidal, Gaussian, bell-shaped and some other types of functions. Each of these classes of functions is characterized by a number of parameters that can be used to construct the membership function (FC) of a specific fuzzy set. The number of parameters of the known classes listed above is in many cases small to construct a sufficient adequate FP for specifically given fuzzy sets [1]-[5].

In this paper, the analysis of methods for constructing an FP from various classes of known functions is carried out. Thus, the analysis of functions from the class of piecewise continuous functions proposed in this paper is carried out. To construct functions, expert information about the degree of belonging to a fuzzy set of some given elements of the universal set is used.

To construct membership functions, namely, to determine the values of their parameters, mathematical methods such as various interpolation methods, approximation methods based on conditional and unconditional optimization methods are used.

The paper presents the results of numerical experiments using various approaches to constructing the FP, and analyzes and compares the results obtained.

II. Statement of the problem

Let there be expert information about the degree of membership of the elements $\bar{x}_i \in X, i = 1, \dots, n,$ to the fuzzy set A, here X is a universal set. Thus, $\bar{x}_i$ and $\bar{\mu}_i$are given, such that it is desirable to fulfill the conditions:

$$\bar{\mu}_i = \mu_A(\bar{x}_i; P), \quad i = 1 \dots, n. \tag{1}$$

Here the $\mu_A(\bar{x}_i; P) -$ is a membership function of the fuzzy set A; $P -$ is a vector of parameters of this membership function.

It is required, firstly, to determine the type of MF $\mu_A(x, p)$, i.e. to determine what class it is from (we call this problem the problem of structural identification). Secondly, it is required to determine the values of the r-dimensional vector of parameters $P \in R^r$, participating as coefficients in the function $\mu_A(x, p), r -$ is the dimension of the vector of parameters (we call this problem the problem of parametric identification).

There are no mathematical methods to solve the problem of structural identification, i.e. to choose the form of the function $\mu_A(x_i, p)$ when choosing a class of functions, it is necessary to consider:

1) the specifics of the fuzzy set itself;

2) $r -$the number is the dimension of the vector of parameters of functions participating in the membership functions of a given class;

3) $n -$ the number of elements, which have expert information about the membership degree to the fuzzy set [6]-[9].

To carry out parametric identification, such mathematical methods can be used, such as, for example, the Lagrange interpolation method when the condition $r = n$ is satisfied. In the case, if $n < r$ condition of uniqueness of the constructed membership function is not satisfied. The use of interpolation methods is also incorrect for $n > r$, due to the impossibility of constructing (non-existence) of such a membership function.

The application of approximation methods using methods, for example, minimizing the standard deviation, leads the original problem to the following:

$$S(P) = \frac{1}{n}\sum_{i=1}^{n}[\mu_A(\bar{x}_i; P) - \bar{\mu}_i]^2 + \varepsilon\left\|P - \tilde{P}\right\|_R^2 \to \min_{P \in R^r} \tag{2}$$

Here it is assumed that the condition $r < n$ is satisfied. This is necessary for the uniqueness of the solution of the problem (2). The second term in (2) is introduced to regularize the optimization problem; $\varepsilon > 0, p \in R^r -$ regularization parameters [10]-[12].

The problem (2) belongs to finite-dimensional optimization problems. To solve it, you can use well-known effective optimization methods such as conjugate gradient methods, variable metrics, and others. To solve it, there are ready-made packages of applied programs.

As a rule, membership functions are required to fulfill the following condition:

$$\mu_A(x; P) \leq 1, \qquad x \in X, \tag{3}$$

$$\mu_A(x; P) \geq 0, \qquad x \in X, \tag{4}$$

The condition (3), called the normalization condition, is optional in some problems. However, after solving the problem of parametric identification and determining the parameters P, the resulting function can be normalized by the formula:

$$\bar{\mu}_A(x) = \frac{\mu_A(x;P)}{\max_{y\in X} \mu_A(y;P)}, \quad x \in X. \tag{5}$$

It is easy to check that the (5) function $\bar{\mu}_A(x)$ reduced to satisfies the (3) condition. If the obtained function violates condition (4), then it can be transformed as follows:

$$\bar{\bar{\mu}}_A(x) = \frac{\mu_A(x;P) - \min_{y\in X} \mu_A(y;P)}{\max_{y\in X} \mu_A(y;P) - \min_{y\in X} \mu_A(y;P)}, \quad x \in X. \tag{6}$$

It is easy to check that the (6) function $\bar{\bar{\mu}}_A(x)$reduced to satisfies the conditions(3), (4).

You can do it in another way. Namely, to solve the problem of minimizing a function (2) under restrictions (3), (4). For this, for example, methods of penalty functions can be used. But in this case, constraints (3), (4) on the parameters P must be discretized with respect to $x \in X$. Namely, enter, for example, a grid with a given step h: $x_j = x_0 + jh$, $j = 1, \dots, M$, and require the fulfillment of the conditions (3), (4) at all points of the grid:

$$\mu_A(x_j;P) \le 1, \qquad j = 1, \dots, M, \tag{7}$$

$$\mu_A(x_j;P) \ge 0, \qquad j = 1, \dots, M. \tag{8}$$

To determine the parameter vector P minimizing the function (2), it is necessary to add $2M$ constraints (7), (8).

III. Results of numerical experiments

Let us present some of the results of computer experiments carried out on examples of hypothetical expert data.

Tables 1, 2, 3 show exact expert data on the degree of belonging of elements to some fuzzy sets. The fourth rows of these tables contain data on the degree of membership of the same elements with noises $\xi = 10\%$, i.e. $\bar{\mu}_i^{\xi} = (1 + \xi)\bar{\mu}_i$. Noises are random variables that have a uniform distribution on the interval$[-1; 1]$.

Table 1: *Expert data for triangular membership function*

No.	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
$\bar{\mathbf{x}}_{\mathbf{i}}$	0.1	0.21	0.3	0.38	0.48	0.65	0.71	0.83	0.88	0.9
$\bar{\boldsymbol{\mu}}_{\mathbf{i}}$	0	0.2749	0.4999	0.7	0.95	0.625	0.475	0.175	0.05	0
$\bar{\boldsymbol{\mu}}_i^{\xi}$	0	0.2954	0.5372	0.7522	1	0.6716	0.5104	0.188	0.0537	0

Table 2: *Expert data for the trapezoidal membership function*

No.	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
$\bar{\mathbf{x}}_{\mathbf{i}}$	0.1	0.21	0.3	0.38	0.48	0.65	0.71	0.83	0.88	0.9
$\bar{\boldsymbol{\mu}}_{\mathbf{i}}$	0	0.36	0.66	0.93	1	1	0.95	0.35	0.1	0
$\bar{\boldsymbol{\mu}}_i^{\xi}$	0	0.39	0.71	1	1	1	1	0.37	0.1	0

Table 3: *Expert data for the Gaussian membership function*

No.	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
$\bar{\mathbf{x}}_{\mathbf{i}}$	0.1	0.21	0.3	0.38	0.48	0.65	0.71	0.83	0.88	0.9
$\bar{\boldsymbol{\mu}}_{\mathbf{i}}$	0	0.01	0.14	0.49	0.98	0.32	0.11	0	0	0
$\bar{\boldsymbol{\mu}}_i^{\xi}$	0	0	0.1	0.5	1	0.3	0.1	0	0	0

Figure 1 shows graphs of membership functions from a triangular class of functions, the parameters of which were obtained using the above methods.

Figure 2 shows similar results for the trapezoidal membership function.

Figure 3 shows similar results for the Gaussian membership function.

In these figures, solid graphs are obtained for the exact values of expert data, dotted graphs correspond to expert data with noises given in the fourth rows of tables 1-3.

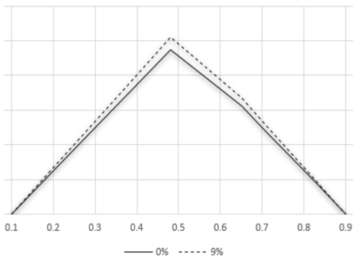

Fig.1: *Results for triangular function accessories*

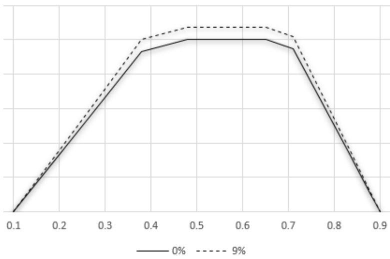

Fig.2: *Results for the trapezoidal function accessories*

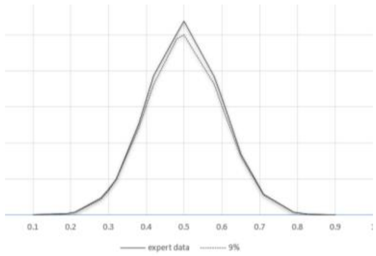

Fig.3: *Results for the Gaussian function accessories*

As can be seen from the figures, the quality of the approximation of expert data significantly depends on the chosen class of the membership function. Noises in expert data have a comparatively small effect, since they are smoothed out in the process of approximation.

IV. Conclusion

In recent years, much attention of researchers has been paid to the application of the theory of fuzzy sets in solving problems that are difficult to formalize. The main characteristic of fuzzy sets is the corresponding membership functions.

In this work, on the basis of expert data on the degree of risk of the decision being made for various values of the parameters on which the decision depends, an approach to the construction of membership functions of fuzzy sets is investigated. The approach is based on approximate methods of approximation.

The use of a class of triangular-like, trapezoidal, functions and Gaussian functions for the approximation of membership functions has been carried out. To determine the parameters of membership functions, approximation methods were used, leading to the problem of unconditional optimization of the mean square criterion.

An analysis of the stability of the constructed membership functions to the noise of expert data assignment is carried out.

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