STUDY OF OPERATING MODES OF SMALL HPSS' HYDROELECTRIC UNITS EQUIPPED WITH SYNCHRONOUS GENERATOR

L.H. Hasanova[1], H.S. Aliyev[1], M.M. Musayev[2]

•

[1]Azerbaijan Technical University, Baku, Azerbaijan
AZ1073, 25 Hussein Javid pros.
e-mail: gasanovalg@mail.ru, hikmetaliyev@aztu.edu.az

[2]Azerbaijan Scientific-Research and Designed-Prospecting
Institute of Energetics Baku, Azerbaijan
AZ1012, Aven. H.Zardabi-94
e-mail: musahil@gmail.com

Abstract

A universal analytical expression is obtained for the power of hydraulic turbines of small hydropower plants such as Francis, Pelton and Kaplan with fixed blades as a function of water flow. This expression allows us to predict the generation of active power of each of the turbines, depending on the adjusted water flow rate, which varies in the seasonal, monthly and daily periods of the year. It also allows you to solve the inverse problem, when it is necessary, depending on the load schedule dictated by the electrical system to which these small hydroelectric power stations are connected, to regulate the amount of water flow by changing the opening angles of the guiding devices of the hydraulic turbines. Studies on the proposed mathematical model containing a synchronous generator with electromagnetic excitation coupled to a Francis hydraulic turbine with a 60% change in water flow from q=1 (r.u) to q=0.4 (r.u). The value of the required active power of a small hydroelectric power station, i.e. power output of the generator will jam (either by the dispatcher, or by the value of the load schedule), i.e. it is an input value, and as the output value of the controller is the opening angle of the guiding apparatus of the turbine.

Keywords: small hydropower plants, Francis turbine, Pelton turbine, axial turbines, synchronous generator with electromagnetic excitation, electrical grid.

1. Introduction

In recent years small HPSs (hydroelectric power station) are successfully adopted as the electric power producers at regional level in many world countries. One of the main advantages of this power type is its environmental purity, and a disadvantage is the demand for significant investments in their construction.

In small HPSs, whose power varies from several kW to 25–30 MW, in most cases as the hydraulic turbines the Francis and Pelton turbines are used, and a series of hydraulic units with axial flow turbines.

The purpose of this paper is the modeling of behavior of stated turbines in general analytic form and the solution of some issues of their combined action with electromechanical converter connected to electric power network of power system.

2. Materials and methods

To deduce the analytical relations, describing the processes in above turbines, as a basis the approach presented in [1] is assumed.

It is known, that the active power of hydraulic turbines is described by the expression of [2] form:

$$P = g \cdot \eta \cdot Q \cdot H \tag{1}$$

where P – powerin [kW]; η – efficiency, expressed in relative units; Q – water discharge [m^3/s]], H – head [m], g=9.81– acceleration of gravity [m/s^2].

It is clear that the rated power of hydraulic turbine will be:

$$P_{rat} = 9{,}81 \cdot \eta_{rat} \cdot Q_{rat} \cdot H_{rat} \tag{2}$$

If to write down the current power in relative units, taking the rated values as the basis, we will obtain:

$$p^* = \eta^* \cdot q^* \cdot h^* \tag{3}$$

where $p^* = \dfrac{P}{P_{rat}}$; $\eta^* = \dfrac{\eta}{\eta_{rat}}$; $q^* = \dfrac{Q}{Q_{rat}}$; $h^* = \dfrac{H}{H_{rat}}$.

The expression for efficiency of hydraulic turbines is presented in [1] in the form of:

$$\eta^* = a \cdot q^{*2} + b \cdot q^* + c \tag{4}$$

In (4) equation the a, b and c indexes depend on turbine's type. For example for the Francis hydraulic turbines they are equal to a=–0.537; b=1.047; c=0.49 with η_{rat}=0.9 for Pelton turbines they are equal to a=–0.224; b=0.483; c=0.741 also with; and finally for axial flow turbines a=–0.219; b=0.476; c=0.743 with η_{rat}=0.9 [1].

It is also known, that in general case the rated head is described by the expression of following form:

$$H_{rat} = H_0 \cdot (1 - \lambda) \tag{5}$$

where H_0 – head corresponded to theoretical fall, i.e. without taking into account the friction loss, λ – index, taking into account the water friction when passing from top level to hydraulic turbine.

It is natural that a value of this λ index depends on a value of water flow q^*, this dependence is imaged in the expression for current value of head, which is presented in the form of:

$$H = H_0(1 - \lambda \cdot q^{*2}) \tag{6}$$

Thus with taking into account (5) and (6) expressions the expression of current head in relative units has the form:

$$h^* = \frac{H}{H_{rat}} = \frac{1 - \lambda \cdot q^{*2}}{1 - \lambda} \tag{7}$$

Inserting the (4) and (7) expressions into (3) expression and making the simple conversions we finally obtain the expression for relative power of hydraulic turbine so small HPS sin the form of:

$$p^* = -A \cdot q^{*5} - B \cdot q^{*4} + C \cdot q^{*3} + D \cdot q^{*2} + E \cdot q^* \tag{8}$$

where $A = \dfrac{\lambda \cdot a}{1-\lambda}$; $B = \dfrac{\lambda \cdot b}{1-\lambda}$; $C = \dfrac{a - c \cdot \lambda}{1-\lambda}$; $D = \dfrac{b}{1-\lambda}$; $E = \dfrac{c}{1-\lambda}$.

Thus the expression (8) for power of hydraulic turbines is the universal analytic expression, linking a power value of three types of turbines –Francis, Pelton and axial flow with water discharge q, in this process only change the values of a, b, c indexes and friction factor λ.

Table 1. Calculatedrelationship $p^*=f(q^*)$ for Francis turbine

q^*	0.4	0.5	0.6	0.7	0.8	0.9	1	1.2	1.4
p^*	0.33	0.47	0.59	0.7	0.816	0.915	1	1.11	1.12

Table 2. Calculated relationship $p^*=f(q^*)$ for Pelton turbine

q^*	0.4	0.5	0.6	0.7	0.8	0.9	1	1.2	1.4
p^*	0.39	0.502	0.61	0.718	0.819	0.916	1	1.14	1.22

Table 3. Calculated relationship $p^*=f(q^*)$ for axial (Kaplan) turbine water rate

q^*	0.4	0.5	0.6	0.7	0.8	0.9	1	1.2	1.4
p^*	0.39	0.5	0.61	0.718	0.818	0.914	1	1.1	1.2

If to assume the friction factor λ is equal to λ=0.1, then the calculated dependences $p^*=f(q^*)$ for Francis turbines will be presented in Table 1, for Pelton turbines in Table 2 and for axial flow turbines in Table 3.

Analyzing the results, given in 1÷3 Tables, the single-valued conclusions can be drawn, that the power factors of Pelton turbine and axial flow turbine in turndown of flow rate q^* from 0.4 to 1.4 are fully coincide in practice. For Francis hydraulic turbines in the range of water discharge from 0.4 to 0.7 the power value p^* as a function of flow rate q^* is a little bit less than for above turbines, which indicates that the efficiency of this turbine for small flow rates is lower than the one of Pelton's and axial flow turbines (the less flow rate the less output power value). The curves of $p^*=f(q^*)$ change for above hydraulic turbines of small HPSs are shown in Fig. 1 (further "*" indexes are removed).

And also it needs to point to the circumstance, that all above mentioned correlations were obtained for constant (rated) rotational frequency of hydraulic turbines. Thus if the hydraulic turbines were jointed with uncontrolled electric generators – synchronous or asynchronous, whose rotational frequency is constant in steady-state mode, then (8) equation becomes automatically the one of driving torque, developed by hydraulic turbine as:

$$m_{ht} = \frac{p^*}{n^*} = p_{ht} \tag{9}$$

where n^*=1.

3. Results and discussion

The issues of operating modes study of hydraulic units with adjustable rotational frequency are presented in [3, 4].

Let's consider the equations of state of hydraulic unit, composed of Francis turbine and classical synchronous generator with electromagnetic excitation, operating to electric network.

The equations of synchronous generator with electromagnetic excitation are presented in [5]. Thus with consideration for the equations of hydraulic turbine the general equations will be presented in the form of:

$$\left.\begin{aligned}
&p\Psi_{ds} = U_{ds} - \omega_r \cdot \psi_{qs} - r_s \cdot i_{ds} \\
&p\psi_{qs} = U_{qs} + \omega_r \cdot \psi_{ds} - r_s \cdot i_{qs} \\
&p\psi_{dr} = -\frac{r_{dr}}{x_{dr}} \cdot \psi_{dr} + \frac{r_{dr} \cdot x_{ad}}{x_{dr}} \cdot i_{ds} + \frac{r_{dr} \cdot x_{ad}}{x_{dr}} \cdot i_{df} \\
&p\psi_{qr} = \frac{r_{qr}}{x_{qr}} \cdot \psi_{qr} + \frac{r_{qr} \cdot x_{aq}}{x_{qr}} \cdot i_{qs} \\
&p\psi_{df} = \frac{r_{df}}{x_{ad}} \cdot U^{*}_{df} - r_{df} \cdot i_{df} \\
&p\omega_r = \frac{1}{T_j} \cdot m_{ht} - \frac{1}{T_j} \cdot m_{em} \\
&i_{ds} = \frac{x_{dr}}{\Delta d} \cdot \psi_{ds} - \frac{\Delta d_1}{\Delta d} \cdot i_{df} - \frac{x_{ad}}{\Delta d} \cdot \psi_{dr} \\
&i_{qs} = \frac{x_{qr}}{\Delta q} \cdot \psi_{qs} - \frac{x_{aq}}{\Delta q} \cdot \psi_{qr} \\
&i_{df} = \frac{x_{dr}}{\Delta d_2} \cdot \psi_{df} - \frac{\Delta d_1}{\Delta d_2} \cdot i_{ds} - \frac{x_{ad}}{\Delta d_2} \cdot \psi_{dr} \\
&m_{em} = \psi_{ds} \cdot i_{qs} - \psi_{qs} \cdot i_{ds} \\
&\omega_r = p\theta - 1 \\
&m_{ht} = k_{tran} \cdot \left(- A \cdot q^5 - B \cdot q^4 + C \cdot q^3 + D \cdot q^2 + E \cdot q\right)
\end{aligned}\right\} \qquad (10)$$

where $U_{ds} = -U_s \cdot \sin\theta,\ U_{qs} = U_{qs} \cdot \cos\theta$

$\Delta d = x_{ds} \cdot x_{dr} - x_{ad}^2; \Delta q = x_{qs} \cdot x_{qr} - x_{aq}^2$

$\Delta d_1 = x_{dr} \cdot x_{ad} - x_{ad}^2; \Delta d_2 = x_{dr} \cdot x_{df} - x_{ad}^2$

U^{*}_{df} – excitation voltage as a fraction of excitation voltage of no-load operation [6]; m_{ht} – driving torque of hydraulic turbine; k_{tran} – index of transfer of hydraulic turbine's basic units to the basic units of synchronous generator; q – water discharge in relative units. .

If to automatize the system, then it becomes necessary to link the water discharge q, which determines identically a value of turbine's driving torque m_{ht}, with the output power of generator p_{get}, i.e. besides the generator's equations it needs to take into account the equation of controller. For Francis hydraulic turbines and propeller (axial flow) ones a water discharge is controlled with the help of opening of wicket gates, which in one's turn are controlled by servomotors. A time constant of control tract is rather significant and reaches 1–2 seconds.

If to take for simplicity of analysis the inertia governor as a controller (link of first order), then its equation will be in the form of:

$$U_{out} = \frac{K}{T_p + 1} U_{in} \qquad (11)$$

As it was noted, as the output power of controller U_{out} the angle of opening of wicket gates is used, which determine a water discharge q flowing through turbine, and the input power is the active power at the output of generator, which is set either by dispatcher or by a value of load diagram p_{gset}.

Thus the equations of controller and expressions for current value of active power on the terminals of generator will add to the equations of generator and hydraulic turbine (10):

$$\left.\begin{aligned} pq &= \frac{k_{12}}{T_p}(p_{g_{set}}) - \frac{q}{T_p} \\ p_{gen} &= U_{ds}\cdot i_{ds} + U_{qs}\cdot i_{qs} \end{aligned}\right\} \tag{12}$$

where p – symbol of differentiation with respect to synchronous time $\tau=314\cdot t$, T_p – time constant of controller in [rad], k_{12} – amplification factor (of transfer) of controller, p_{gset} – set value of power at generator's output.

In accordance with Fig. 1 k_{12} can be determined by approximation of curve $q=f(p)$, in this process at least two approximations are needed: up to $q\leq 1$ $k_{12}=k_1$ (for example for axial flow turbines $k_1\approx 1$), and for $q\geq 1$ $k_{12}=k_2$, (rarely carried out mode).

$$k_{12} = \begin{vmatrix} k_1 & npu & q\leq 1 \\ k_2 & npu & q\geq 1 \end{vmatrix} \tag{13}$$

Let's perform the approximate calculation of small hydraulic unit with Francis turbine, which parameters are equal to: rated head H_{rat} =50 *m*, rated water discharge for diameter of hydraulic turbine D=2.45 *m* is equal to Q =56 m^3/s, then the power of small HPS in nominal conditions will be equal to:

$$P_{rat.ht} = 9.81\cdot 0.9\cdot 56\cdot 50 = 24.72\,\text{MW}$$

Rotational frequency of hydraulic turbine with reduced rotational frequency equal to $n^1_{rat} = 130$ *rpm* [2] is determined by the formula [3].

$$n_{ht} = \frac{n^1_{rat}\cdot\sqrt{H}}{D} = \frac{130\sqrt{50}}{2.45} \approx 375\,\text{rpm}$$

The generator is chosen with the power of P_{rat}=25 MW, total power of generator *S*, which is taken as a basis one, is equal to S_{rat}=31.25 *MW* with rated revolutions equal to n_{rat}=375 *rpm*. In this case an index of transfer is k_{trat}=0.8.

The parameters of steady-state mode of the system are given in Table 4.

Table 4. Parameters of steady-state mode of the system

q	rel.unit	0.4	0.6	0.8	1
p	rel.unit	0.33	0.59	0.816	1
m_{ht}	$0.8\cdot m_{ht}$	0.264	0.47	0.65	0.8
$p_{g_{set}}$	rel.unit	0.258	0.46	0.642	0.787
$q_{st\text{-}state}$	rel.unit	0.328	0.585	0.816	1

Two first rows of the table were determined in accordance with Fig. 1 for Francis hydraulic turbine (i.e. data of Table 1). Third row displays a value of driving torque of hydraulic turbine m_{ht} with taking into account the index of transfer k_{trat}=0.8 according to (9) expression. The data of set active power at the output of generator p_{gset} (in relative units) were placed in the fourth row. And finally in the fifth row the data of controller output were placed, i.e. the values $q_{st\text{-}state}$ in steady-state mode (equation (12))

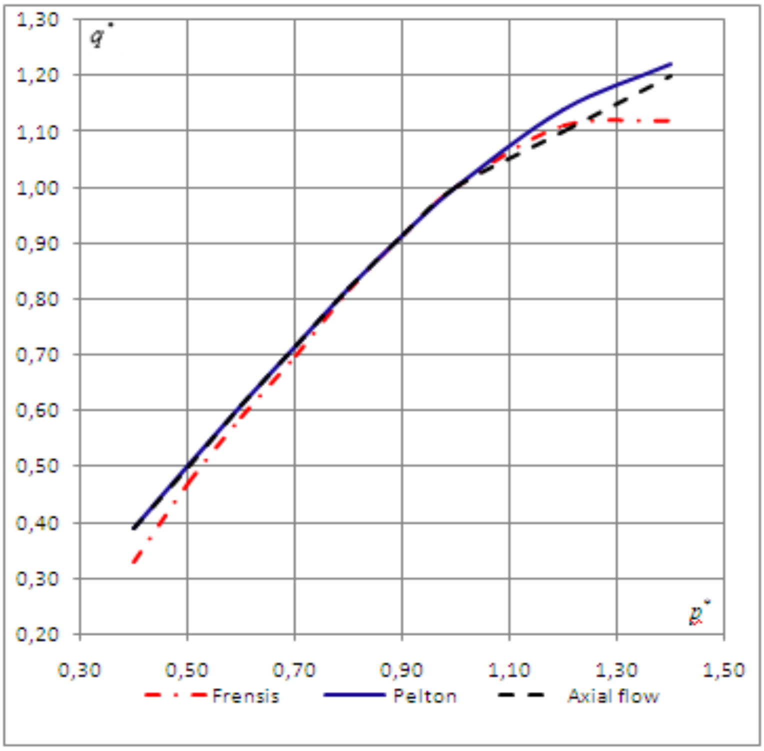

Figure 1. The relationship between turbine output power and water flow rate

In this section, we design a tracking controller for a The chosen parameters of controller are the following: time constant of controller T_{gen}=1.5 (471.22 rad.), amplification factor (of transfer) $k_{12}=k_1$=1.27.

The fluktogrammas of mode parameters' change of small HPS's generator with automatic control system of water discharge (and therefore of the hydraulic turbine's power) are presented in Fig. 2. The data of generator and controller and algorithm of modeling are given in Appendix. The fluktogramma of change of generator's electromagnetic torque $m_{em}=f(\tau)$ is given in Fig. 2(*a*). According to the modeling algorithm at the first stage the asynchronous start is carried out without load and with short-circuited excitation winding within the period up to τ=5000 radian. A slip *s* changes in accordance with Fig 2(*b*). At 5000 radian the open-circuit voltage $U^*_f=1$ is supplied to the excitation winding, and the machine locks in synchronism, the rotational frequency of generator sets at the mark $\omega=1$ (Fig.2, *c*).

At 15000 radian the set value of active power p_{set1}=0.787 inputs to the equation of controller, a torque of hydraulic turbine in this process is formed according to expression $m_{ht}=k_{tran}\cdot q_1=-0.8\cdot q_1$, here with a value of water discharge *q* sets at a level of $q_1=1$ (Fig.2, *d*) and the relevant to it value of driving torque of turbine is equal to $m_{ht}=-0.8$ (minus sign indicates the generator mode) (Fig.2, *a*). Further in accordance with the set values of output power of generator p_{set2}=0.642, p_{set3}=0.46, p_{set4}=0.258 the *q* values, accordingly equal to q_2=0.815, q_3=0.584 and q_4=0.327 (Fig.2, *d*), are set at the output of controller. In accordance with these *q* values the torque values m_{ht} and m_{em} automatically form, the values of last one are accordingly equal to $m_{em1}=-0.799$, $m_{em2}=-0.652$, $m_{em3}=-0.467$ and $m_{em4}=-0.262$. These values of electromagnetic torque are corresponded to the values of active powers at the generator's output $p_{gen1}=-0.784$, $p_{gen2}=-0.64$, $p_{gen3}=-0.45$ and $p_{gen4}=-0.254$ (Fig. 2, *e*). Comparing the values of current powers of generator $p_{gen1}\div p_{gen4}$ with the set values it can be stated, that the error amount because of approximation and offset of chosen type of controller doesn't exceed 3%.

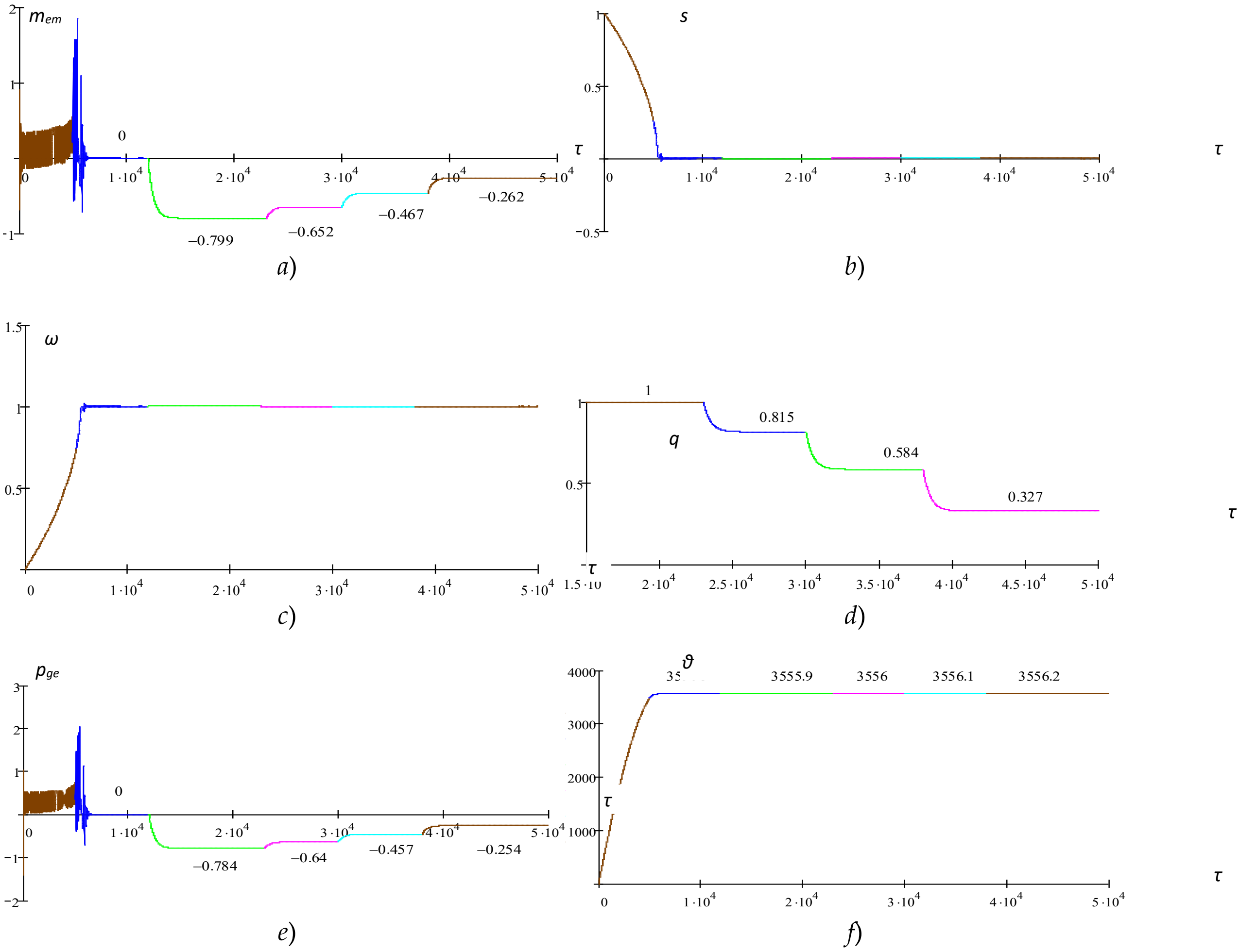

Figure 2. Fluctograms of operating parameters variations of small hydroelectric power generator

The curve of change of synchronous machine's interior angle θ is given in Fig.(2, *f*); for almost rated active load (p_{gen1}=–0.784) this angle is equal to θ_{rat}=0.4 radian (~23 deg.), and for minimum active load (p_{gen4}=–0.254) is equal to θ_4=0.1 radian (~5.7 deg.).

In conclusion it needs to note again, that the controller in these researches is chosen as the relaxation circuit of first order, which is a simplification, needs for more visual demonstration of system's principle of operation. The choice of more complicated controller will not influence on the idea and algorithm of construction of general modeling system.

Conclusions

1. The mathematical model is developed for study of operating modes of small HPSs' hydroelectric units, equipped with Frensis, Pelton and axial flow turbines (Kaplan turbines with antideflection mounting of blades). For three above mentioned types of turbines the analytic expression was obtained in a form of multinomial, which connects the power values of turbines, expressed in relative units, with a value of flow rate of energy carrier – water; the structure of multinomial is invariable and only the values of its factors change according to turbines' types.
2. Collaborate research of hydraulic turbines with synchronous generators on full equations and also of the controller has demonstrated the working capacity, accuracy and effectiveness of developed mathematical model, which allows its using for oriented calculations of operating modes of small HPSs both at the designing stage and in operating conditions.

Appendices

Appendix 1. Parameters of Synchronous generator

x_{ds}=0.986
x_{qs}=0.63
x_{aq}=0.435
x_{qr}=0.7
x_{df}=1.1
T_j=1000 rad. (~1.5 *s*.)

x_{ad}=0.787
x_{dr}=1
r_s=0.02
r_{qr}=0.019
r_{dr}=0.028
U_s=1

Calculated value Δd=0.367; Δd_1=0.167; Δd_2=0.48; Δq=0.255.

Controller parameters: gain ratio k_{12}=k_1=1.27; time constant T_{gen}=471.22 rad.

Appendix 2. Algorithm of the mathematical model

$$D(t,Y)=\begin{bmatrix} -\sin(Y_6)+Y_1-Y_1\cdot Y_5-0.02\cdot i_{sd} \\ \cos(Y_6)-Y_0+Y_0\cdot Y_5-0.02\cdot i_{sq} \\ -0.028\cdot Y_2-0.022(3.23\cdot Y_0-1.12\cdot Y_4-1.65\cdot Y_2)+0.002\cdot i_{df} \\ -0.027\cdot Y_3-0.0117\cdot i_{sq} \\ 0.057\cdot U^*_{df}-0.045\cdot i_{df} \\ 0.001\cdot(-k_{tran}\cdot Y_7)-0.001[Y_0\cdot(i_{sq})-Y_1\cdot(\cdot i_{sd})] \\ Y_5 \\ 0.0027\cdot(p_{set})-0.002126\cdot Y_7 \end{bmatrix} \quad D(t,Y)=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$Y_0=\Psi_{ds}$; $Y_1=\Psi_{qs}$; $Y_2=\Psi_{dr}$; $Y_3=\Psi_{qr}$; $Y_4=\Psi_{df}$; $Y_5=s$; $Y_6=\theta$; $Y_7=q$.

$$\omega = p\theta - 1 = s - 1$$
$$i_{ds} = 3.23\cdot Y_0 - 1.12\cdot Y_4 - 1.65\cdot Y_2$$
$$i_{qs} = 2.745\cdot Y_1 - 1.7\cdot Y_3$$
$$i_{df} = 2.47\cdot Y_4 - 1.12\cdot Y_0 - 1.07\cdot Y_2$$

p_{set} – active power preset value in generator output

References

[1]N.G.Voros, C.T.Kiranoudis, Z.B.Maroulis. Short-cut design of small hydroelectric plans.http: //bsesrv 214. bse.vt.edu /Grisso/ Ethiopia /Books Resources/ Hydro-electric.

[2] G.I.Krivchenko, Hydraulic machines (turbines, pumps). Moscow, "Energy" journal pg. 236. 1978.

[3] R.I.Mustafayev, L.G.Hasanov, M.M.Musayev, E.M.Mamedov, Kh.İ.Nabiyev. "Modeling and analysis of operating regimes of hydraulic units of small hydropower plant with double fed machine", Electromechanics, No. 6, pp. 79-86, Moscow, Russian, 2015.

[4] R.I.Mustafayev, L.G.Hasanov, M.M.Musayev. "Modeling and study of hydropower units of small hydro power plants with frequency-controlled permanent magnet synchronous generators", HEI of CISEnergetics,No. 2, pp. 102-117, Minsk, Belarus, 2016.

[5] R.I.Mustafayev, L.H. Hasanova. "Modeling and study state of working of wind-electric installations synchronous generator under frequency steering", Electricity, No. 7, pp. 34-40, Moscow, Russian, 2010.

[6] N.I.Sokolov. Application of analog computers in power systems. Under editorship of. Prof. Publishing house "Energy" Moscow, Russia, p. 408, 1964.

[7] V.L.Anchimyuk. Automatic control theory. Publishing "Graduate school", p.350. Minsk, 1979.