A NEW CONTROL CHART FOR PROCESS DISPERSION BASED ON RANKED SET SAMPLING

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Abstract

In this paper, we propose a new control chart based on Downton's estimator (D) for monitoring the process dispersion using ranked set sampling design and runs rules. The performance of the proposed control chart is compared with the originally proposed D chart based on simple random sampling method when underlying process distribution is normal and non-normal. The average run length is used to evaluate the performance of the proposed control chart. It is observed that the proposed control chart is efficient in detecting shifts in process dispersion as compared with the chart using simple random sampling method. The performance of the proposed chart is further investigated using runs rules. The efficiency of the designed runs rules RSS-D chart is compared with its existing counterparts and is found to be superior.

Keywords: Control chart, Average run length, Process dispersion, Ranked set sampling, Runs rules, Downton estimator.

1. Introduction

In statistical process control (SPC), control charts are the most popular tools used to monitor the quality characteristics in an industrial process. Shewhart R chart and S chart are the commonly used control charts for monitoring the process dispersion. The R chart is based on the sample range (R) whereas S chart is based on sample standard deviation (S). These charts are based on subgroups of sample size n using simple random sampling (SRS) method. Both R and S charts are easy to implement and are effective in the detection of large shifts, but are less sensitive in the detection of small and moderate shifts in the process dispersion (Montgomery[1]).Several alternatives have been proposed in the literature to improve the performance of Shewhart type control charts. One recent approach is the construction of control charts based on ranked set sampling (RSS) scheme. The RSS scheme was first suggested by McIntyre [2]. This scheme ensures increased precision through ranking of observations. He pointed out that this sampling scheme works superior to standard simple random sampling (SRS) for estimation of the population mean. Takahasi and Wakimoto [3] laid the necessary mathematical formulation for this sampling scheme. Several researchers have studied applications of ranked set sampling in different streams. Kvam and Samaniego [4] showed that ranked set sampling may occur naturally in survival analysis. Kvam and Samaniego [5] studied the applications of this sampling scheme in reliability. Recently, RSS scheme has got considerable attention in the construction of control charts. Salazar and Sinha [6] first suggested control charts for monitoring process mean using RSS scheme. It was shown that the RSS based control charts perform better than the charts based on SRS. Muttlak and Al-Sabah [7] developed control charts for process mean based on different RSS schemes and showed that the proposed RSS based control charts perform better than the classical SRS charts. Al-Naseer and Al-Rawwash [8] developed the mean control chart using robust RSS method. Al-Omari and Al-Naseer [9] suggested a new quality control chart for mean using robust extreme ranked set sampling (RERSS) method. It was shown that RERSS chart performs better than all other charts based on SRS and RSS methods. Al-Omari and Haq [10] developed Shewhart-type control charts to improve the monitoring the process mean by using the double quartile-ranked set sampling, quartile double ranked set sampling and double quartile extreme ranked set sampling methods. Al-Rawwash et al. [11] developed control charts of the sample mean based on several sampling techniques including RSS as well as some of its recent modifications assuming that the underlying distribution is normal with mean μ and variance σ^2. Mehmood et al. [12] discussed the details of different ranked set strategies and their applications in an industrial process using control charts. Abbasi [13] investigated the performance of the mean chart based on various sampling schemes under normal and non-normal process distributions. Tahir et al. [14] designed and investigated dispersion control charts based on different estimates of process dispersion under different RSS strategies for normal and non-normal processes.

Most of the papers published so far on the RSS based control charts have concentrated on monitoring the process location and few control charts has been developed for monitoring the process dispersion. Motivated by the attractive features of RSS scheme, in this paper, we propose a D chart based on RSS scheme (denoted as RSS-D chart) for monitoring process dispersion. It is shown that the proposed RSS-D chart performs uniformly better than the D chart without RSS scheme for detecting increasing shifts in the process dispersion. Although the RSS scheme improves the performance of the D chart in detecting shifts in process dispersion, further improvement can be achieved if runs rules scheme is implemented to the RSS-D chart.In case of SRS, a number of authors have defined runs rules schemes for different control charts to improve the detecting ability for small and moderate shifts in process parameters. Klein [15] proposed two control charts based on runs rule as efficient alternatives to Shewhart control chart. These control charts were named as 2-out-of-2 runs rule control chart and 2-out-of-3 runs rule control chart. In 2-out-of-2 runs rule control chart, if two successive points fall beyond a special control limit, then process is declared to be out of control. In 2-out-of-3 runs rule control chart, if two points among successive three points fall beyond a special control limit, then process is declared to be out of control. This special limit is determined in such a way that the in-control ARL equals with specified value ARL_0. This chart has been proven to perform better than Shewhart chart.Antzoulakos and Rakitzis [16] have studied the control charts with supplementary runs rules for monitoring the process standard deviation. In general, the use of runs rules increases a chart's sensitivity but also produces more false alarms. To keep the same rate of false alarm, control limits have to be adjusted (Cheng and Chen [17]). Although runs rules have been widely used in practice to increase the sensitivity of control charts, to our best knowledge, little research has been done on the RSS based control charts.

The purpose of this paper is to improve the performance of the D chart proposed by Abbasi and Miller [18] to detect shifts in process dispersion by using RSS scheme and further to implement runs rule scheme to proposed RSS-D chart. The rest of the paper is organized as follows: In section2, we outline the process of constructing the D control chart based on SRS (denoted as SRS-D chart) for monitoring the process dispersion. The description of ranked set sampling scheme used in this study is given in section 3. The charting structure of the proposed RSS-D chart is presented in section 4. RSS based D-chart with runs rules is discussed in section 5.The performance evaluation of proposed RSS-D chart and RSS-D chart with runs rules and its comparative study with SRS-D control chart is given in section 6. In section 7, an illustrative example based on simulated data sets is provided for a better understanding about implementation of the proposed control chart. A real life application of the proposed control chart is illustrated in section 8. Conclusions are given in the final section 9.

2. D Chart for Process Dispersion Based on SRS

Let$X_1, X_2, \ldots, X_n$ be a simple random sample of size n from a normal process whose mean is μ and standard deviation is σ. Further, let$X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the order statistics corresponding to this sample. Downton [19] proposed following statistic as an estimator for process standard deviation σ .

$$D = \frac{2\sqrt{\pi}}{n(n-1)} \sum_{i=1}^{n} \left[i - \frac{1}{2}(n+1) \right] X_{(i)} \quad (1)$$

which is an unbiased estimator of σ for normally distributed process data. Abbasi and Miller [18] proposed the control chart based on D to monitor the process variability. It was shown that for normally distributed process, the D chart is equally efficient to the Shewhart S chart for detecting shifts in the process standard deviation. Since the distribution of D is not symmetric at least for small to moderate values of n, they have used probability limits instead of three-sigma control limits in the construction of D chart. The probability limits of D chart are computed by using quantile points of the distribution of$= \frac{D}{\sigma}$. The upper control limit (UCL) of the D chart is given as

$$UCL = Z_{1-\alpha}\overline{D} \text{ with } P(Z \geq Z_{1-\alpha}) = 1 - \alpha \quad (2)$$

where α is the specified probability of making Type-I error, Z_α is α^{th}quantile point of the distribution of Z. The process dispersion is then monitored by plotting values of the D statistic on the chart with UCL given by Eq. (2). If $D < UCL$, the process is regarded as in-control, otherwise there is an increase in process standard deviation and process is deemed out-of-control.

To design the D chart based on simple random sample (SRS), let $X_1, X_2, \ldots, X_n$be independent and identically distributed$N(\mu_0, \sigma_0^2)$ random sample of size$n$, where $\mu_0$and σ_0^2 are the in-control mean and variance, respectively. As the concern of this paper is monitoring process dispersion, let us assume that process standard deviation has a shift of sizeδ. Hence, the new standard deviation for the process becomes $\sigma_1 = \delta\sigma_0$, where $\sigma_1$is out-of-control value of process standard deviation. For positive shift ($\delta > 1$), that is, for increase in σ, $k^+\sigma_0$is the upper control limit and process is declared to be out-of-control if $D > k^+\sigma_0$. For negative shift ($\delta < 1$), that is for decrease inσ, $k^-\sigma_0$ is the lower control limit and process is declared to be out-of-control if $D < k^-\sigma_0$. The values of k^+ and k^- are found for fixed in-control ARL value based on simulations.

A commonly used performance measure for a control chart is its ARL. For a given shift in process parameter, the ARL is the average number of points plotted on a control chart until an out-of-control signal is obtained. In case of Shewhart control chart, which signals as soon as a point falls beyond the control limits, $ARL = {}^{1}/_{p}$, where$p$ is the probability that a point exceeds the control limits.

For positive shift δ,

$$p = Pr(D > k^+\sigma_0 / \sigma = \delta\sigma_0)$$

$$= Pr\left(\frac{D}{\delta\sigma_0} > \frac{k^+\sigma_0}{\delta\sigma_0} / \sigma = \delta\sigma_0\right)$$
$$= Pr\left(Z > \frac{k^+}{\delta}\right)$$
$$= 1 - F\left(\frac{k^+}{\delta}\right)$$

In-control ARL is the reciprocal of the detecting power p.

$$ARL(\delta) = \frac{1}{p} = \frac{1}{1 - F\left(\frac{k^+}{\delta}\right)} \quad (3)$$

For negative shift δ,

$$p = Pr(D < k^-\sigma_0 / \sigma = \delta\sigma_0)$$

$$= Pr\left(\frac{D}{\delta\sigma_0} < \frac{k^-\sigma_0}{\delta\sigma_0} / \sigma = \delta\sigma_0\right)$$
$$= Pr\left(Z < \frac{k^-}{\delta}\right)$$
$$= F\left(\frac{k^-}{\delta}\right)$$

where $F(.)$ is distribution function of Z.

In-control ARL is the reciprocal of the detecting power p.

$$ARL(\delta) = \frac{1}{p} = \frac{1}{F\left(\frac{k^-}{\delta}\right)} \quad (4)$$

3. Ranked Set Sampling Scheme

The mechanism of ranked set sampling (RSS) is as follows. Let X be the characteristic under study. In order to get a RSS sample of size$n$, total of n^2 units are drawn from the population. These n^2 units are randomly arranged in n sets each of size$n$. Let$\{X_{i,k}, i = 1,2, \ldots, n\}$be the k^{th} set, where= 1,2, …, n . The observations in the k^{th} set $\{X_{i,k}, i = 1,2, \ldots, n\}$are ranked either by visual magnitude of characteristic X or by using some other variable, which is significantly correlated with$X$. If the ranking of $X$is exact, then the sampling scheme is called RSS with perfect ranking, and if ranking is not exact, then the sampling scheme is called RSS with imperfect sampling. In the present study, we have assumed perfect RSS scheme. Let k^{th}ordered set be $\{X_{[i,k]},\ i = 1,2, \ldots, n\}$, where $X_{[i,k]},\ i = 1,2, \ldots, n;\ k = 1,2, \ldots, n$ is the i^{th} order statistic in k^{th}set. These sets are called the ranked sets. Then $i^{th}, i = 1,2, \ldots, n$ order statistic is picked from $i^{th}, i = 1,2, \ldots, n$ ranked set. That is, first order statistics from first ranked set, second order statistics from second ranked set and so on. Thus, a final selected sample of size n is $\{X_{[1,1]}, X_{[2,2]}, \ldots, X_{[n,n]}\}$, which is termed as a ranked set sample (RSS).

4. RSS Based D Chart for Process Dispersion

In this section, a working mechanism of RSS-D chart is described. A subgroup sample from the underlying process is obtained using RSS scheme of sampling. It is assumed that ranking is perfect. Using this scheme, we generate subgroups of size n and repeat them for n cycles from the distribution that is under study. Downton statistic based on RSS sample$\{X_{[1,1]}, X_{[2,2]}, \ldots, X_{[n,n]}\}$ of size n is given as

$$D = \frac{2\sqrt{\pi}}{n(n-1)} \sum_{i=1}^{n} \left[i - \frac{1}{2}(n+1)\right] X_{[i,i]} \quad (5)$$

Since the exact distribution of D statistic based on ranked set sampling scheme is not known, simulation approach is used to compute ARL values of the proposed RSS-D chart. The control limits for RSS-D chart are found outusing extensive simulation. 50000 ranked set samples of size n were simulated and quantile points of D statistic based on RSS scheme were realized. Using these quantile points, the values of $k^+$and $k^-$are evaluated using equations (3) and (4) respectively for positive and negative shifts in process standard deviation σ. The estimated ARL values of the proposed RSS-D chart along with standard error of the chart are found with 50000 simulations for each shift of magnitude δin process standard deviationσ. The ARL performance of the RSS-D chart under both normal and non-normal process distributions is presented in Section 6.

5. RSS-D Chart with Runs Rule

In this section, we study the performance of the proposed RSS-D chart using the following runs rules. Note that we would usually use only one of these rules at a time. Each rule would have different control limits.

The process is declared to be out of control when

(i) Two consecutive points plot outside the control limits (2-of-2-rule).

(ii)Any two of three successive points plot outside the control limit (2-of-3-rule)

The ARL performance of the RSS-D chart using these runs rules has been evaluated under both normal and non-normal process distributions using a simulation study in Section 6.

6. Performance Evaluation and Comparison

6.1.Performance evaluation under normality

This section evaluates and compares the performance of RSS-D chart and RSS-D chart using runs rules and D chart with simple random sampling. To compare the performance of these charts under normality, in-control observations are generated from $N(\mu_0, \sigma_0^2)$ distribution. Without loss of generality, we take $\mu_0 = 0$ and $\sigma_0^2 = 1$. For out-of-control process, observations are generated from $N(\mu_0, \sigma_1^2)$distribution. Without RSS scheme, the values of the control chart statistic D for the D chart are computed using Eq. (1) for subgroups of size n. With the RSS scheme, the values of the control chart statistic D for the RSS- D chart are computed using Eq. (5) based on RSS sample of size n. For increase in standard deviation (that is, $\sigma_1 > \sigma_0$), where $\sigma_1 = \delta\sigma_0$, the magnitude of shift in process standard deviation considered are $\delta = 1.0, 1.1, 1.2, 1.3, 1.4, 1.5$ and 2.0, while for decrease in process standard deviation (that is, $\sigma_1 < \sigma_0$),the magnitude of shift in process standard deviation considered are $\delta = 1.0, 0.9, 0.8, 0.7, 0.6, 0.5$ and 0.1. All charts have calibrated in-control ARL of 200 and subgroup size of $n = 5, 8$ and 10 are considered. The values of k^+ and k^- are obtained by designing the chart to have an in-control ARL of 200. The out-of-control ARL values of the charts are found with 50000 simulations for each of shift of magnitude δ in the process standard deviation σ.

ARL values for positive shifts and negative shifts in process standard deviation with different sample sizes as$n = 5, 8, 10$ are given in Table 1 and Table 2 respectively.

Table 1: *ARL comparison for process dispersion with positive shift under normal distribution.*

Sample n	Shift δ	SRS-D	RSS-D	RSS-D (2-out-of-2)	RSS-D (2-out-of-3)
5	1.0	200	200.27(1.816)	200.39 (1.795)	200.25 (1.817)
	1.1	66.65	62.43 (0.561)	60.35 (0.540)	55.36 (0.491)
	1.2	29.67	25.90 (0.230)	25.76 (0.223)	23.22 (0.197)
	1.3	15.56	13.48 (0.117)	14.52 (0.119)	12.78 (0.102)
	1.4	9.59	8.35 (0.071)	9.50 (0.074)	8.26 (0.061)
	1.5	6.49	5.66 (0.048)	6.87 (0.050)	6.11 (0.042)
	2.0	2.31	2.05 (0.014)	3.22 (0.016)	2.98 (0.013)
	k^+	2.068	1.9985	1.5377	1.60286
8	1.0	200	199.65(1.822)	200.03 (1.779)	200.85(1.823)
	1.1	54.00	45.14 (0.407)	40.46 (0.358)	37.27 (0.326)
	1.2	20.62	15.75 (0.138)	14.63 (0.122)	13.14 (0.105)
	1.3	10.40	7.47 (0.064)	7.61 (0.057)	6.80 (0.048)
	1.4	6.05	4.33 (0.035)	4.91 (0.033)	4.44 (0.027)
	1.5	4.13	2.92 (0.022)	3.69 (0.021)	3.46 (0.018)
	2.0	1.55	1.27 (0.005)	2.24 (0.006)	2.19 (0.004)
	k^+	1.78	1.6296	1.33955	1.43986
10	1.0	200	199.55(1.852)	200.03 (1.809)	199.49 (1.792)
	1.1	48.33	36.44 (0.333)	32.07 (0.282)	29.49 (0.252)
	1.2	17.45	11.56 (0.101)	10.67 (0.085)	9.58 (0.074)
	1.3	8.27	5.20 (0.042)	5.39 (0.037)	5.02 (0.032)
	1.4	4.86	3.02 (0.023)	3.66 (0.021)	3.39 (0.017)
	1.5	3.28	2.08 (0.014)	2.88 (0.013)	2.74 (0.010)
	2.0	1.34	1.10 (0.003)	2.06 (0.003)	2.05 (0.002)
	k^+	1.676	1.508285	1.27474	1.307509

Table 2: *ARL comparison for process dispersion with negative shift under normal distribution.*

Sample n	Shift δ	SRS-D	RSS-D	RSS-D (2-out-of-2)	RSS-D (2-out-of-3)
5	1.0	201	200.64 (1.851)	200.77 (1.816)	200.98 (1.852)
	0.9	135.62	176.49 (1.592)	118.89 (1.055)	116.73 (1.067)
	0.8	86.12	152.17 (1.386)	62.21 (0.548)	62.80 (0.564)
	0.7	51.93	128.76(1.173)	30.37 (0.265)	30.30 (0.265)
	0.6	29.34	99.68 (0.917)	14.24 (0.118)	14.27 (0.117)
	0.5	15.42	72.75 (0.667)	6.58 (0.047)	6.53 (0.048)
	0.1	1.00	1.42 (0.007)	2.00(0.00)	2.00(0.00)
	k^-	0.240	0.1184	0.483308	0.483291
8	1.0	200	200.47(1.824)	199.59(1.786)	200.79(1.832)
	0.9	102.77	101.45(0.911)	61.40(0.548)	62.08(0.562)
	0.8	51.32	47.47(0.425)	20.01(0.169)	19.89(0.167)
	0.7	23.99	20.34(0.182)	7.24(0.0543)	7.22(0.054)
	0.6	10.72	8.29(0.071)	3.37(0.018)	3.40(0.019)
	0.5	4.63	3.20(0.024)	2.22(0.006)	2.23(0.006)
	0.1	1.00	1.00(0.00)	2.00(0.00)	2.00(0.00)
	k^-	0.388	0.4425	0.674418	0.67443
10	1.0	201.00	200.82(1.818)	201.01(1.859)	201.59 (1.823)
	0.9	91.63	72.29(0.658)	43.57(0.390)	43.54(0.379)
	0.8	39.72	24.76(0.220)	11.52(0.093)	11.81(0.095)
	0.7	16.37	8.50(0.072)	4.19(0.026)	4.23(0.026)
	0.6	6.77	3.06(0.023)	2.39(0.008)	2.40(0.008)
	0.5	2.81	1.43(0.007)	2.02(0.001)	2.02(0.002)
	0.1	1.00	1.00(0.00)	2.00(0.00)	2.00(0.00)
	k^-	0.449	0.549	0.7365543	0.736105

Following are the findings from Table 1 and Table 2.

- It is observed that with an increase in sample size n, the out-of-control ARL values of all the charts under investigation decreases. This shows that large sample sizes help control chart statistic to quickly detect the shifts in process dispersion of the production process.
- For a fixed value of sample size n, with an increase or decrease in the shift of process dispersion, the out-of-control ARL values of all the charts under investigation decreases. The same trend is observed in standard errors of the estimated ARL values.
- For increase in process dispersion, the proposed RSS-D chart consistently produces smaller out-of-control ARL values than SRS-D chart for the entire range of shifts in the process dispersion.
- The RSS-D chart with runs rule shows a better performance than SRS-D chart for smaller shifts ($\delta \leq 1.2$) and smaller sample size ($n = 5$), while for larger sample size ($n = 8$ or 10), chart shows better performance for small to moderate shifts.
- The RSS-D chart with runs rule shows better performance than RSS-D chart only for small shifts in process dispersion.
- For decrease in process dispersion, for smaller sample size ($n = 5$), the out-of-control ARL values of the proposed RSS-D chart are greater than that of SRS-D chart. As sample size increases, the RSS-D chart becomes more efficient than the SRS-D chart.
- RSS-D chart with runs rule shows better performance than RSS-D chart and SRS-D chart for decreasing shifts in process dispersion.
- The performance of RSS-D chart with 2-out-of-2 and 2-out-of-3 runs rule is similar.

6.2. Performance evaluation under non-normality

In this section, we study the performance of the RSS-D and RSS-D chart by implementing runs rules schemes under non-normal distributions and it is compared with the D chart based on SRS scheme. In order to study the performance of these charts under non-normality, we considered heavy-tailed symmetric and skewed distributions. Specifically, we simulated observations in the heavy-tailed symmetric case from double exponential distribution and in the skewed case from gamma distribution. The probability density functions of these non-normal distributions are given as

Double exponential distribution: The double exponential distribution, denoted by Double Exponential(μ, λ) has the probability density function,

$$f(x;\mu,\lambda) = \frac{1}{2\lambda} e^{-\frac{1}{\lambda}|x-\mu|}, -\infty < x < \infty; \lambda > 0, -\infty < \mu < \infty \quad (6)$$

where μ and λ are respectively the location and scale parameters. The mean and variance of a double exponential distribution are μ and $2\lambda^2$ respectively.

Gamma distribution: The gamma distribution, denoted by $Gamma(\alpha, \beta)$ has the probability density function,

$$f(x;\alpha,\beta) = \frac{\beta^\alpha}{\Gamma\alpha} x^\alpha e^{-\beta x}, x > 0;\ \alpha, \beta > 0 \quad (7)$$

where α and β are respectively the shape and scale parameters. The mean and variance of a gamma distribution are α/β and α/β^2respectively.

To study the performance of the RSS-Dcontrol chart under non-normality, we have considered the process data from double exponential distribution with location parameter $\mu = 0$ and scale parameter $\lambda = 1$ and gamma distribution with shape parameter $\alpha = 2$ and scale parameter $\beta = 1$. Simulation approach is used to compute ARL values of these charts. The ARL values of the charts are found with 50000 simulations for each shift of magnitude δ in the process standard deviation. The standard errors of the estimated ARL values of these charts are given (in parentheses) along with ARL values. The value ofk^+isdetermined based on 100 iterations of 50,000 simulations from the underlying process with no shift, such that in-control ARL is 200. In this study we consider the case of increase in standard deviation only.

Tables 3 and 4 present the ARL values of the charts under investigation to detect increase in process standard deviation for different magnitudes for in-control ARL of 200 and sample sizes $n = 5,8$ and 10, when underlying process distributions are gamma and double exponential.

Table 3: *ARL comparison for process dispersion with positive shift under gamma distribution.*

Sample n	Shift δ	SRS-D	RSS-D	RSS-D (2-out-of-2)	RSS-D (2-out-of-3)
5	1.0	200.23 (0.897)	200.61 (0.904)	199.50 (1.808)	200.26 (1.807)
	1.1	93.56 (0.415)	92.95 (0.41)	77.59 (0.703)	74.34 (0.668)
	1.2	50.63 (0.226)	48.38 (0.215)	37.54 (0.327)	34.74 (0.300)
	1.3	30.75 (0.135)	28.33 (0.126)	20.92 (0.178)	19.58 (0.163)
	1.4	20.19 (0.089)	17.99 (0.078)	13.36 (0.11)	12.38 (0.098)
	1.5	14.04 (0.060)	12.17 (0.052)	9.32 (0.073)	8.65 (0.063)
	2.0	4.46 (0.018)	3.533 (0.013)	3.59 (0.02)	3.41 (0.017)
	k^+	2.452	2.4051	1.635	1.73
8	1.0	200.32 (0.897)	201.30 (0.897)	201.5 (1.814)	200.46 (2.786)
	1.1	80.14 (0.357)	74.60 (0.330)	58.25 (0.517)	55.35 (0.788)
	1.2	38.73 (0.171)	22.76 (0.099)	23.21 (0.198)	21.53 (0.280)
	1.3	21.50 (0.094)	13.52 (0.058)	11.69 (0.095)	10.85 (0.084)
	1.4	13.31 (0.057)	8.81 (0.037)	7.16 (0.054)	6.76 (0.047)
	1.5	9.00 (0.038)	6.27 (0.026)	4.977 (0.033)	4.71 (0.029)
	2.0	2.75 (0.010)	1.82 (0.005)	2.364 (0.008)	2.34 (0.006)
	k^+	2.0315	1.8919	1.411	1.468

10	1.0	201.12 (0.892)	199.84 (0.887)	200.5 (1.814)	201.10 (1.809)
	1.1	74.03 (0.329)	65.79 (0.293)	47.66 (0.421)	46.55 (0.405)
	1.2	33.71 (0.149)	26.84 (0.119)	17.43 (0.148)	16.57 (0.134)
	1.3	17.92 (0.078)	12.85 (0.055)	8.418 (0.064)	7.98 (0.058)
	1.4	10.88 (0.046)	7.18 (0.029)	5.129 (0.035)	4.97 (0.031)
	1.5	7.11 (0.030)	4.49 (0.018)	3.708 (0.021)	3.65 (0.019)
	2.0	2.21 (0.007)	1.39 (0.003)	2.137 (0.005)	2.12 (0.003)
	k^+	1.885	1.717	1.33	1.378

Table 4: *ARL comparison for process dispersion with positive shift under double exponential distribution.*

Sample n	Shift δ	D	RSS-D	RSS-D (2-out-of-2)	RSS-D (2-out-of-3)
5	1.0	200.61 (0.889)	199.86 (0.891)	200.00 (1.825)	200.00 (1.801)
	1.1	94.25 (0.422)	91.15 (0.407)	83.25 (0.745)	79.02 (0.709)
	1.2	51.29 (0.228)	48.36 (0.214)	41.50 (0.370)	38.04 (0.333)
	1.3	31.22 (0.137)	28.46 (0.124)	24.08 (0.206)	21.82 (0.182)
	1.4	20.65 (0.089)	18.37 (0.079)	15.44 (0.129)	14.10 (0.115)
	1.5	14.54 (0.063)	12.64 (0.054)	11.20 (0.090)	9.88 (0.076)
	2.0	4.75 (0.019)	3.85 (0.014)	4.25 (0.026)	3.91 (0.021)
	k^+	2.521	2.5075	1.745	1.798
8	1.0	200.95 (0.893)	200.36 (0.901)	200.31 (1.778)	200.45 (1.817)
	1.1	83.00 (0.369)	75.99 (0.336)	62.75 (0.557)	60.27 (0.525)
	1.2	40.38 (0.178)	34.99 (0.154)	26.41 (0.227)	24.44 (0.208)
	1.3	22.65 (0.099)	18.41 (0.080)	13.94 (0.115)	12.95 (0.102)
	1.4	14.22 (0.061)	10.96 (0.046)	8.69 (0.068)	7.99 (0.058)
	1.5	9.65 (0.04)	7.20 (0.030)	6.088 (0.043)	5.67 (0.038)
	2.0	2.95 (0.01)	2.11 (0.007)	2.67 (0.011)	2.59 (0.009)
	k^+	2.095	1.975	1.457	1.523
10	1.0	199.84 (0.895)	199.48 (0.894)	200.40 (1.808)	200.33 (1.813)
	1.1	75.13 (0.332)	67.76 (0.301)	53.10 (0.476)	50.23 (0.439)
	1.2	34.70 (0.153)	28.44 (0.130)	20.59 (0.179)	19.13 (0.161)
	1.3	18.79 (0.082)	14.20 (0.060)	10.51 (0.085)	9.65 (0.073)
	1.4	11.49 (0.049)	8.19 (0.034)	6.33 (0.046)	6.00 (0.040)
	1.5	7.66 (0.032)	5.26 (0.021)	4.50 (0.029)	4.25 (0.025)
	2.0	2.38 (0.008)	1.61 (0.004)	2.28 (0.007)	2.26 (0.005)
	k^+	1.938	1.792	1.371	1.424

Following are the important findings based on Table 3 and Table 4.

- When underlying process distribution is gamma, the ARL values of RSS-D chart and RSS-D charts with runs rules are smaller than that of the SRS-Dchart for all sample sizes n and entire range of shifts considered. The RSS-D charts with runs rules show a better performance than RSS-D and SRS-D charts. The RSS-D chart with 2-out-of-3 runs rule is very effective for detecting shifts than 2-out-of-2 chart.
- When underlying process distribution is double exponential, we see that the ARL values of RSS-D chart and RSS-D charts with runs rules are smaller than that of the SRS-D chart for all sample sizes n and entire range of shifts considered. The RSS-D charts with runs rules show a better performance than RSS-D and SRS-D charts. The RSS-D chart with 2-out-of-3 runs rule is very effective for detecting shifts than 2-out-of-2 chart.
- The ARL performance of RSS-D chart and SRS-D chart under non-normal process distributions is higher than that for the normal process distribution with sample sizes$n =$ 5,8 and 10

- In general, under skewed and heavy tailed process distributions, the RSS-D chart is more efficient than the SRS-D chart for monitoring shifts in process dispersion.

7. Illustrative Example

In this section, we provide an example to illustrate the proposed chart. For this purpose, two datasets are generated which will be later referred as dataset 1 and dataset 2.

Dataset 1 contains 40 simple random samples each of size $n = 5$, out of which first 10 samples are generated from $N(0,1)$referring to in-control situation and remaining 30 samples are generated from $N(0,1.2)$ referring to a shift in a process standard deviation. The D-statistics of these 40 samples are computed and are presented in Table 5. The graphical display of the control chart applied to data set 1 with UCL=1.676 is given in Figure 1.

Table 5: *D-values for simulated data with SRS.*

In-control Process		Out of control process with Shift (δ) =1.2					
No.	D-value	No.	D-value	No.	D-value	No.	D-value
1	1.04679	11	1.285956	21	0.884477	31	1.100131
2	1.165552	12	1.513122	22	1.113954	32	1.534435
3	1.468238	13	0.722581	23	0.887508	33	1.196817
4	0.4986	14	1.658094	24	1.431795	34	1.590498
5	0.574144	15	1.40786	25	1.294181	35	1.173697
6	1.231162	16	1.263608	26	1.52249	36	0.677852
7	0.84646	17	1.518884	27	1.379621	37	0.991026
8	0.728805	18	0.896195	28	1.349834	38	1.412045
9	0.767625	19	1.431129	29	1.029293	39	1.136405
10	0.920877	20	1.032453	30	1.981427	40	1.138973

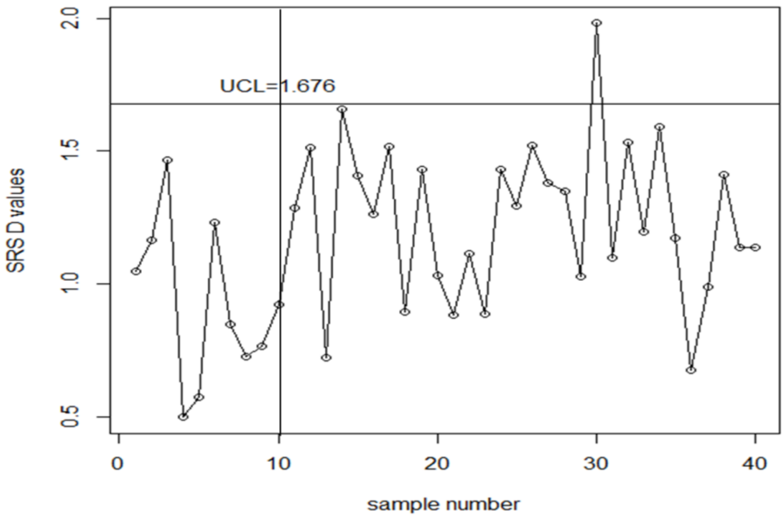

Figure 1: *SRS-D control chart.*

From Figure 1, we can see that D control chart gives first out of control signal at point no. 30, which is the only out of control signal.

Dataset 2 consists of 40 ranked set samples each of size$n = 10$ of which first 10 samples are generated from $N(0,1)$ referring to in-control situation and remaining 30 RSS samples are generated from $N(0,1.2)$ referring to a shift in a process standard deviation. The D-statistics based on these 40 ranked samples are computed and are presented in Table 6. The graphical display of the control chart applied to data set 2 with UCL=1.5082 is given in Figure 2.

Table 6: *D-values for simulated data with RSS*

In-control Process		Out of control process with Shift (ẟ) =1.2					
No.	D-value	No.	D-value	No.	D-value	No.	D-value
1	0.76489	11	0.955223	21	1.658042	31	1.41809
2	1.11167	12	1.340428	22	1.200549	32	1.12013
3	0.928814	13	1.440937	23	1.057834	33	1.22928
4	1.029337	14	0.856949	24	1.827535	34	1.1807
5	1.046913	15	1.241251	25	1.452098	35	1.476113
6	1.197435	16	1.410242	26	0.963128	36	1.039491
7	1.258457	17	0.76041	27	1.096964	37	0.93336
8	1.291416	18	1.293803	28	0.947557	38	1.461749
9	1.262475	19	1.189676	29	1.917295	39	1.562749
10	0.871265	20	1.502025	30	1.601436	40	0.895917

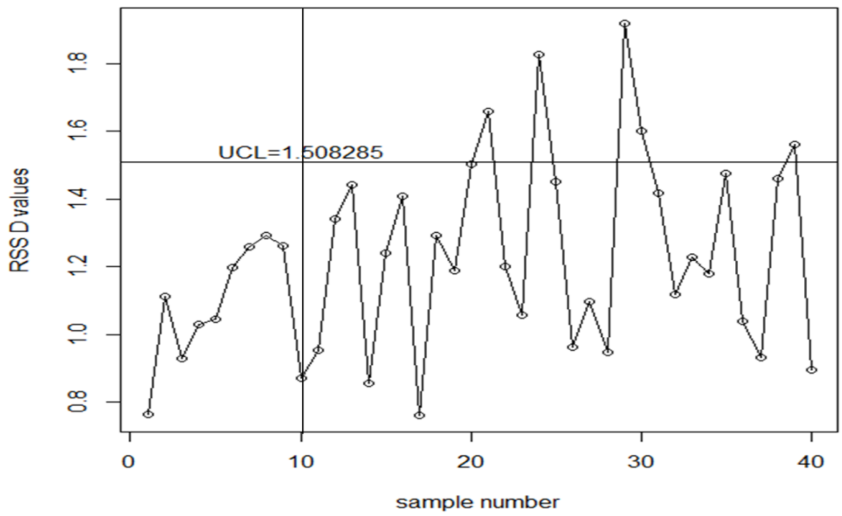

Figure 2: *RSS-D control chart*

From Figure 2, we can see that RSS D chart gives first out of control signal at point no.21 and total out of control signals given are05. These signals are at point numbers 21, 24, 29, 30, 39.

From the above two figures, it is clear that RSS D chart signals earlier than D chart, also giving more number of signals than D chart. The above example clearly indicates that the proposed RSS-D chart performs better than D chart without RSS.

8. A Real Life Application

The use of RSS and other sampling schemes based on ranking of observations in SPC has increased in the recent years. As pointed out by Nawaz and Han [20], many a times, the actual measurement process has the constraints on the time and cost. Therefore, it is advisable to adopt such sampling designs which provide greater efficiency with relatively small sample sizes. Instead of taking physical measurements, sometimes it is easier to rank the sampling units on the basis of size, weight and volume by visual inspection or judgment. For example, in an automatic bottle filling plant, the actual quantification of filled volume of liquid in bottles, is costly and time consuming. Instead, it is easier and inexpensive torank the bottles on the basis of the amount of filled volume of liquid in bottles by visual inspection before actual measurement.

Abujiya and Muttlak [21] constructed control charts for mean using double ranked set sampling. To demonstrate the performance of proposed chart, they used a real dataset, which was originally taken by Muttlak and Al-Sabah [22]. The dataset consisted of 54 subgroups, each of size three. The subgroup data were collected using the SRS and RSS schemes. The data were taken from a Pepsi Cola production company in Khobar, Saudi Arabia. The quality characteristic understudy was considered to be the unfilled part of the bottle, which was measured with the help of an instrument. By visual inspection of levels of the soft drinks, the bottles in each set were ranked. Then the samples were realized with newly proposed sampling techniques. Haq and Munir [23] also used the same dataset to demonstrate the performance of improved CUSUM charts for monitoring process mean based on RSS.The detection abilities of the CUSUMcharts with RSS and ordered RSS schemes were compared.

Nawaz and Han [20] considered a real dataset taken from Tüfekci [24] for monitoring process location by using new RSS-based memory control charts. Since the original data was not collected from thequality control prospective, therefore, the available data was considered as a population and RSS samples were drawn from it to demonstrate the process monitoring.

In this section, we consider dataset on inside diameter measurements (mm) for automobile engine piston rings, given in Montgomery [1]. The dataset consists of total 40 subgroups each of size 5 collected using SRS method. Based on this dataset, we estimated the population mean (μ) and population standard deviation (σ) as$\hat{\mu} = 74.00361$ and $\hat{\sigma} = 0.010144$. To compare the performance of proposed RSS-D control chart with SRS-D chart, 8 RSS subgroups are formed using given 40 subgroups assuming that the ranking is perfect. To have fair comparison, out of available 40 SRS samples, 8 equidistant subgroups are selected. Using the estimated values of parameters, 22 SRS and 22 RSS samples are generated with a shift of size $\delta = 1.3$ in process dispersion. These values are tabulated in Table VII. The graphical display of the control charts applied to these data sets with UCL=2.068 for SRS and UCL= 1.9985 for RSS is given in Figure 3.

Table 7: *D-values for SRS and RSS samples of Inside Diameter Measurements (mm) for Automobile Engine Piston Rings.*

D values for SRS Samples							
In-control Process		Out of control process with Shift (ᵟ) =1.3					
No.	D-value	No.	D-value	No.	D-value	No.	D-value
1	1.624979	9	1.373168	17	0.613449	25	2.546284
2	0.90859	10	1.278336	18	1.419153	26	2.014164
3	0.297039	11	0.993755	19	1.312875	27	1.862361
4	0.803753	12	0.417964	20	0.786504	28	1.006743
5	1.310467	13	0.778037	21	2.358553	29	0.645239
6	1.799708	14	1.593989	22	1.268609	30	1.791942
7	1.118265	15	1.286196	23	1.984099		
8	1.45025	16	0.627962	24	0.736714		

D values for RSS Samples							
In-control Process		Out of control process with Shift (ᵟ) =1.3					
No.	D-value	No.	D-value	No.	D-value	No.	D-value
1	1.048373	9	0.749542	17	0.50747	25	2.456585
2	1.048373	10	1.469715	18	2.005007	26	1.333404
3	0.873645	11	1.827723	19	1.612167	27	1.005756
4	1.467723	12	1.485399	20	0.765135	28	1.214323
5	1.415304	13	1.57744	21	1.68864	29	0.801106
6	0.995955	14	1.691586	22	1.294557	30	1.63209
7	1.57256	15	2.131071	23	1.493243		
8	1.555087	16	1.51146	24	1.607816		

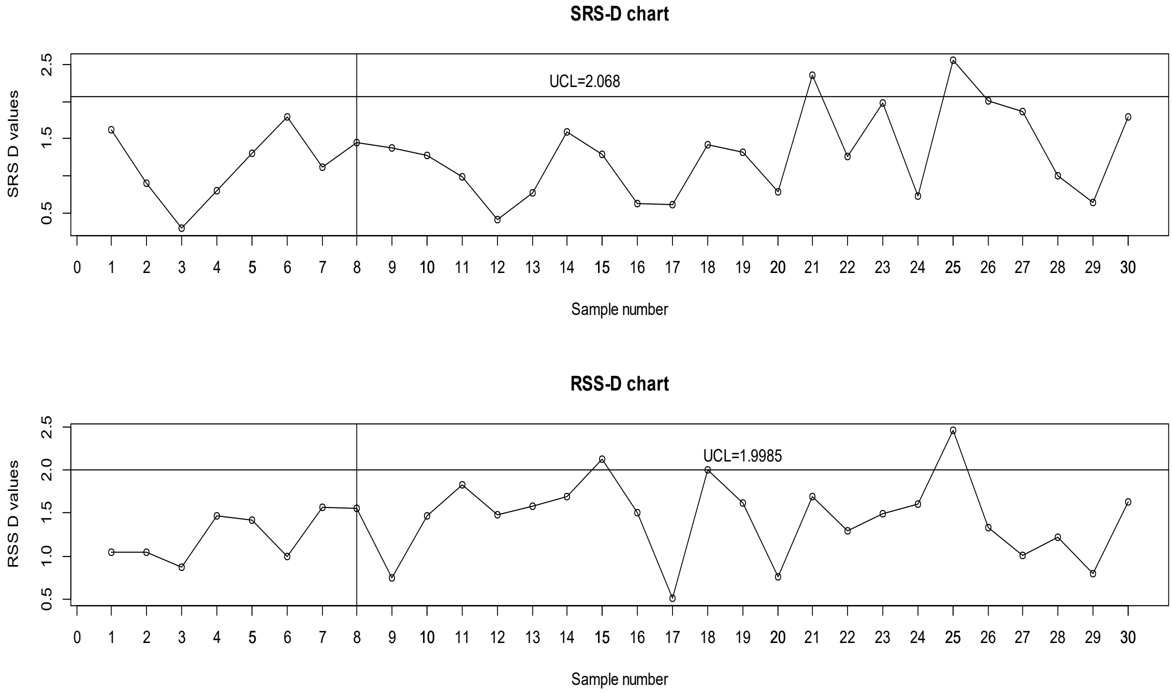

Figure 3: *SRS-D and RSS-D control chart for a real dataset.*

From Figure 3, we can see that the SRS D chart gives first out of control signal at point no 21 and total out of control signals given are02, which are at point numbers 21,25. Whereas, RSS D chart gives first out of control signal at point no. 15 and total out of control signals given are 03. These signals are at point numbers 15, 18, 25.

9. Conclusions

In this paper, a control chart based on Downton's estimator D is developed using ranked set sampling scheme for monitoring the process dispersion of normally and non-normally distributed processes. The performance of the proposed chart is investigated in terms of ARL and is compared with originally proposed D chart without RSS for monitoring the process dispersion. The proposed chart is found to be performing well for entire range of shifts in process dispersion under both normal and non-normal process distributions. The efficiency of proposed chart is further enhanced using two runs rules schemes. The chart with runs rules is found to be performing well for small and moderate shifts under normal process and performing uniformly better for entire range of shifts in dispersion under heavy tailed and skewed process distributions.

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