# MODELING THE RELIABILITY OF TRANSPORT UNDER EXTREME CONDITIONS OF OPERATION AS A QUEUING SYSTEM WITH PRIORITIES 

M.D. Katsman<br>The Joint-Stock Company of Railway Transport of Ukraine "Ukrzaliznytsia", Kyiv, Ukraine mdkatsman@gmail.com<br>V.I. Matsiuk<br>-<br>National University of Life and Environmental Sciences of Ukraine, Kyiv, Ukraine vimatsiuk@gmail.com<br>V.K. Myronenko<br>-<br>State University of infrastructure and technologies, Kyiv, Ukraine pupil7591@gmail.com


#### Abstract

The article presents a simulation model of a queuing system (QS) with a queue and relative priority, which can be used to manage the reliability of transport systems under resource constraints. The developed simulation model combines agent and discrete-event simulation principles and allows studying queuing systems in terms of establishing regularities: probabilities (service, failure, pushout), time delays (waiting in a queue, under service), queue sizes, order of queue formation upon arrival of consumers of different priority. As a result of the research, dependencies were obtained for the probability of servicing higher priority consumers depending on the intensities of their arrival and service; probabilities of servicing lower priority consumers depending on the intensity of service and servicing of higher priority consumers; the probability of "pushing out" lower priority consumers from the QS by higher priority consumers depending on service intensities and the arrival of high priority consumers.


Keywords: transport system, agent modeling, simulation modeling, queuing systems, absolute priority, arrival and service intensity, service and push-out probabilities, consumer service probabilities

## 1. Introduction

In mathematics and applied research, the queuing theory is widely used in modeling the functioning of real systems, information [1], computing, energy, medical [2], biological [3], transport [4][5] and others.

In the field of transport, the application of the queuing theory methods is quite traditional.

Recently, the events of the aggressive war of Russia, which it is waging against Ukraine during 202223 , force us to formulate new practical problems that are solved with the use of these methods, in particular, in queuing system (QS) with priorities.

Military operations in Ukraine have repeatedly aimed at destroying the railway infrastructure, especially on electrified lines. For historical and technological reasons, almost 4 times more electric locomotives than diesel locomotives were used for transportation on Ukrainian railways. With such a disproportion, diesel locomotives cannot replace the entire fleet of electric locomotives, to serve all of transportation traffic on electrified lines, in the event that the power supply is stopped on these lines. Likewise, it is not possible to transfer all trains from destroyed/damaged electrified lines to non-electrified lines, due to the defined geography of transportation between consignors and consignees.

Then there were "problems with priorities" - for example, to which alternative route the train of a certain type should be directed, if the capacity of the route and the number of locomotives (diesel or electric) are limited. At the same time, the priority of dispatching the trains can be determined according to different criteria, separately for passenger and cargo service.

The priorities of passenger transportation can be assigned, for example, as a special military personnel transportation, the first term evacuation, and for the rest of passenger trains, it is possible, ranking them by the cost of all tickets sold up to an hour divided by the time remaining to the planned train arrival to final destination according to the timetable. In the event of cargo transportation, the priorities may be assigned to cargoes and trains especially important for the state and defense purposes, while the rest of cargoes/trains can be prioritized according to the value of cargoes and the time available for their delivery to the destination station. After ranking the priorities, from the highest to the lowest, the decision should be made on the appropriate use of locomotive fleet and railway line capacity on alternative routes. The priming and flexibility of these decisions are only growing, if the mathematical apparatus for optimizing the QS with priorities is applied.

In such QSs, if they have consumers (customers, trains, cargoes, shippers etc.) with different priorities, then the consumers with higher priority are served earlier. Priorities can be absolute. It QSs with absolute priorities, servicing the consumers of the lower priority is interrupted, when higher priority consumers arrive. While the lower priority consumer, for which the servicing which was interrupted, turns back to the queue. It is only serviced again when there are no consumers of higher priority in the queue. In QSs with relative priorities, the servicing of a consumer is always finished once it has started, even if a higher priority consumer arrives at that time.

Special attention should be paid to QSs with low rates of both arrival and servicing of higher priority customers. That is, the time intervals between high priority consumer arrivals and the time periods for servicing such consumers are long.

For example, with all the current non-predictability of warfare, it is known that the time intervals between infrastructure-destroying strikes and the duration of its recovery are relatively long compared to the intervals between trains and the duration of transportation in normal, peaceful conditions. Then the priority is given to transportation related to infrastructure restoration, and it is to ensure these transportations that locomotives and other resources are directed.

Each priority class of consumers can have a separate list of ranked consumers and its own queue. Consumers from the lower priority list are served only after the last consumer from the higher priority list has been served. For all arrivals, it is assumed that the processes of receipt of requirements are independent, Poisson and do not depend on service durations.

QSs with priorities have been sufficiently studied for a long time. With the help of Laplace transformations, dependences on the determination of the characteristics of such QSs for various service disciplines were obtained [6]. During the study of QS with different priorities - priorities without interruption of service and with interruption of service and additional after-service dependences on finding the mean waiting time in the queue, the average time spent in the system
for k-priority consumers were obtained [7].
In the monograph [8], a partially modified QS was studied using the MathLab Simulinc simulation model development environment (with the SimEvent and StateFlaw libraries). It is shown that with the help of dynamic priorities, the priority of a non-priority customer is increased once, while the probability of serving these customers increases. However, such a characteristic of QS as the probability of serving the flow of arriving consumers decreases, since non-priority consumers are served longer than priority consumers.

With the help of simulation model development environment (AnyLogic, with Java SE libraries) and tools of queuing theory, agent and discrete-event principles, technological risks and failures in transport and logistics systems were studied [9] [10]. As a result, the regularity of the influence of the number of service channels on the mean service rate with an unchanged total arrival and service rates in the system was established. However, the priority of applications was not considered in these studies.

In [11], methods of queuing theory of Markov and non-Markov types are applied to simulate the resistance of security personnel to a malicious group with a random number of criminals in the group and various ways of organizing the actions of such personnel. And in research [12], mathematical models of the queuing system are considered, simulating the processes of abnormal situations during railway transportation of dangerous goods, as well as the processes of elimination of ecologically dangerous consequences of such events.

In this work, a QS with a queue and relative priority is investigated, with customers of higher and lower priorities arriving at rates $\lambda_{\mathrm{H}}, \lambda_{\mathrm{L}}$, the customer servicing times of are exponential with rates $\mu_{\mathrm{H}}, \mu_{\mathrm{L}}$. A higher-priority customer that arrives to the QS "pushes " a lower-priority customer out after its service has finished and takes a place in front of it if it is in the queue for servicing. A lower priority customer that has been pushed out leaves the QS unserved if there are no places in the queue and is queued if a place exists.

The peculiarity of the proposed experiment, conducted for the purpose of studying the functioning of the model, is that its input receives a stream of events that have negative consequences that significantly affect the functioning of this model (for example, reduce the intensity of service in the system). The flow of these "negative" events (the higher priority flow) has low intensity compared to the lower priority flow and takes a significant amount of time to service the requirements of that flow.

The complexity of the implementation of the queuing theory analytical methods is determined by the voluminous forms of mathematical description for such a kind of QSs. Therefore, it is advisable to use computer simulation methods when solving the scientific and applied problems.

## 2. Methods

### 2.1 Development of simulation model for a QS with priority customers

The model is developed on the basis of discrete-event and agent principles. When developing the model, standard blocks of the Process Modeling Library were used.


Figure 1: The main window of the simulation model in the AnyLogic environment

For the possibility of prioritizing customers based on the Entity base class of agents, a MyAgent population of agents (customers) was created with the priority parameter added. During modeling, each new requirement will be assigned a priority level:
priority $=2.0$ - higher priority customers with the value of the parameter;
priority $=1.0$ - lower priority customers with the parameter value.
The simulation of demand arrival and service starts with source_1 and source_2 blocks, which form the lower and higher priority demands, respectively.

When the generated agents (customers) exit from the source blocks, the priority parameter is assigned the value of the priority level of the customers: The simulation of customers arrival and service starts with source_1 and source_2 blocks, which form the lower and higher priority customers, respectively. When the generated agents (customers) exit from the source blocks, the priority parameter is assigned the value of the priority level of the requirements:
for lower priority agents (customers):

```
"agent.priority = 1;";
```

for higher priority agents (customers):
"agent.priority = 2;".

Next, the agents (customers) enter the general queue q_Big. This element is used to form a general queue of customers that will be accepted by the QS and processed in it. Queue q_Big is not an element of the QS under study and is outside the QS.

The queue in the q_Big element is formed according to the general principle of FIFO ("first in, first out"), taking into account the priority of customers and the model time of customers generation. That is, the FIFO principle is used within the customers of higher and lower priorities. To verify the process of queue formation, all information about the presence of customers is displayed in a text field: the higher the entry in the visual representation of the formation of the queue (Fig. 2), the closer the customer is to the exit from the $q_{-}$Big block. Information about all customers is presented in the form:

[^0]forming the higher priority | customers queue |
| ---: |\(\quad\left\{\begin{array}{l}59|2.0| 46.25755548555329 <br>

62|2.0| 48.1663569634813 <br>
63|2.0| 48.453920530276484 <br>
65|2.0| 49.55600857118395 <br>
66|2.0| 50.80820062272476 <br>
67|2.0| 51.056665195592 <br>
69|2.0| 52.57789533847726 <br>
23|1.0| 18.74180565204156 <br>
24|1.0| 19.891553009613112 <br>
27|1.0| 22.500494831050226 <br>
30|1.0| 22.946753825482073 <br>
31|1.0| 23.632296483761976 <br>
32|1.0| 25.13147214439839 <br>
35|1.0| 27.67875914479366\end{array}\right.\)

Figure 2: Visual display of queue formation in the q_Big block
Since the number of places in the queue is limited in the QS itself, the mechanism of separation of requirements by priority and formation of separate queues from requirements of higher and lower priorities, respectively (blocks $q_{2} 2$ and $q_{-} 1$ ) is used. Separation of requirements is carried out by the sO_Big element. Passage of requirements from the general $q_{-}$Big queue to the QS queue is carried out using a block of the hold type (hold_big, Fig. 1). The algorithm is implemented through Java code using the built-in function fun_hold_big():

```
"if ((q_2.size() + q_1.size()) < places){
hold_big.unblock();
}>.
```

The function fun_hold_big() is implemented every time any customer is received for the $q_{-}$Big queue or the delay block. The fun_hold_big() function algorithm checks the total number of requests in the QS queue, and if the number of customers is less than the set number of places in the queue (places), the hold_big block is unblocked (hold_big.unblock() procedure).

The model also implements the algorithm of passing agents (requirements) into the middle of the system "by priority" and pushing agents (requirements) of lower priority out of the system by agents (requirements) of higher priority. The specified algorithm is implemented using blocks of the hold type (hold_1) and the additional function fun_Logic_HIghPr() with Java code:

```
<if ((q_2.size() + q_1.size()) == places && q_1.size() > 0){
if (q_Big.size() > 0 && q_Big.get(0).priority == 2){
    q_1.remove(q_1.get(0));
    source_out.inject(1);
    hold_big.unblock();
}
}>.
```

According to the algorithm, if an agent (customer) of higher priority arrives at the QS and there are no free places in the QS queue, the agent (customer) of lower priority is "pushed out" of the QS queue (block q_1) and is replaced by a corresponding agent (customer) of a higher priority from the general queue (block $q_{-} B i g$ ).

To simulate the processing of agents (customers), a Delay block with one location (one server device) is used. Agents (customers) of higher priority are served first and proceed immediately from the $q_{-} 2$ block to the Delay block without delay.

The arrival of lower priority agents (customers) is regulated functionally using the hold_1 block and the fun_Logic_LowPr() function with Java code:

$$
\begin{aligned}
& \text { «if }\left(\mathrm{q} \_2 . \operatorname{size}()==0 \text { \&\& q_1.size }()>0\right)\{ \\
& \text { hold_1.unblock( }) \text {; } \\
& \} » .
\end{aligned}
$$

When the agent (customer) passes the hold_1 block using Java code, the hold_1 block is blocked: «hold_1.block();»

During simulation, data is collected on the probability of servicing or pushing out the customers according to the general principle:

$$
\xi=\frac{\sum N_{i, \text { success }}}{\sum N_{i}}
$$

where $\sum N_{i, s u c c e s s}$ is the number of i - type events that were processed in the system or pushed out of the system;
$\sum N_{i}$ is the total number of i - type events generated during the simulation.
In addition, statistical information is generated regarding the average size of queues and the time that requests are in service or waiting in the QS.

### 2.2 Results of experiments and discussion of the results

To study the regularities of queue formation and service probabilities in the system, a series of experiments on the sensitivity of the model were conducted, where the variable parameter is the arrival flow of higher priority customers $\left(\lambda_{H} \in[0,001 ; 0,5]\right)$ at different values of the service rate $\left(\mu_{\mathrm{H}}\right.$ $\in[0,001 ; 0,5])$. Other raw data of the experiments are presented in Table 1.

Table 1: Input parameters of the experiments

| Modeling parameter | Symbol | Range of <br> values | Distribution <br> density |
| :--- | :---: | :---: | :---: |
| Higher priority customers arrival rate (High) | $\lambda_{\mathrm{H}}$ | $0,001-0,5 ;$ <br> step 0,001 | exponential |
| Lower priority customers arrival rate (Low) | $\lambda_{\mathrm{L}}$ | $0,5-$ const | exponential |
| Higher priority customers service rate (High) | $\mu_{\mathrm{H}}$ | $0,001-0,5 ;$ <br> крок 0,1 | exponential |
| Lower priority customers service rate (Low) | $\mu_{\mathrm{L}}$ | $0,5-$ const | exponential |
| Number of servers | $\quad 4$ |  |  |
| Number of places in the QS queue | 4 |  |  |
| Number of priorities |  | 2 |  |
| Model time unit | hour |  |  |
| Model time duration | 6 months (4392 |  |  |
|  |  | hrs.) |  |

The main simulation results are shown in Fig. 3-8.


Figure 3: Probability of servicing higher priority consumers depending on arrival and service rates

The data of fig. 3 indicate that the probability of servicing higher priority customers decreases as the intensity of servicing higher priority customers approaches 0.5 value. Moreover, the "breaking point" at the beginning of the decrease of the probability from 1.0 begins at the moment of equilibrium of the intensities of arrival and service of higher priority customers, that is, at this point:

$$
\lambda_{H}=\mu_{H} .
$$

The descending part of the dependence of the probability of servicing higher priority customers (Fig. 4) is closely described by a power function of the form:

$$
y=k x^{-c},
$$

where $k$ are $c$ coefficients;

$$
\begin{array}{ll}
\text { for } \mu_{\mathrm{H}}=0,1: & y=0.0994 x^{-0.998}, \\
\text { for } \mu_{\mathrm{H}}=0,2: & y=0.2032 x^{-0.981}, \\
\text { for } \mu_{\mathrm{H}}=0,3: & y=0.3067 x^{-0.97} .
\end{array}
$$

The result of approximating the periods of descending probability of service of higher priority customers depending on the intensities of service and arrival is presented in Fig. 4.


Figure 4: Approximation by power-law dependences of periods of descending probability of servicing higher priority customers depending on the rates of arrival and service

From those obtained in fig. 4 data can be recorded:

$$
\left\{\begin{array}{c}
P_{\mu_{\mathrm{H}}}=\mu_{\mathrm{H}} \lambda_{\mathrm{H}}^{-1}, \lambda_{\mathrm{H}}>\mu_{\mathrm{H}}, \\
P_{\mu_{\mathrm{H}}}=1, \lambda_{\mathrm{H}} \leq \mu_{\mathrm{H}} .
\end{array}\right.
$$

If we round the value of k to tenths, we get:

$$
P_{\mu_{\mathrm{H}}}=\mu_{\mathrm{H}} \lambda_{\mathrm{H}}^{-C}, \quad \lambda_{\mathrm{H}} \geq \mu_{\mathrm{H}},
$$

then finally:

$$
\begin{aligned}
& P_{0,1}=0,1 \lambda_{\mathrm{H}}^{-1}, \lambda_{\mathrm{H}} \in[0,1, \ldots, 0,5], \\
& P_{0,2}=0,2 \lambda_{\mathrm{H}}^{-1}, \lambda_{\mathrm{H}} \in[0,2, \ldots, 0,5], \\
& P_{0,3}=0,3 \lambda_{\mathrm{H}}^{-1}, \lambda_{\mathrm{H}} \in[0,3, \ldots, 0,5] .
\end{aligned}
$$

The dependence of higher priority customers servicing rate $\mu_{\mathrm{H}}$ on the probability $\mathrm{P}_{\mathrm{H}}$ and higher priority customers arrival rate $\lambda_{\mathrm{H}}$ is presented in the table. 2.

Table 2: Dependence of $\mu_{H}$ value on $P_{H}$ and $\lambda_{H}$ values

| $P_{\mathrm{H}}$ | $\lambda_{\boldsymbol{H}}$ | 0,1 | 0,2 | 0,3 | 0,4 | 0,5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.00 | $\mu_{\mathrm{H} 1,0}$ | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 |
| 0.90 | $\mu_{\mathrm{H} 0,9}$ | 0.09 | 0.18 | 0.27 | 0.36 | 0.45 |
| 0.80 | $\mu_{\mathrm{H} 0,8}$ | 0.08 | 0.16 | 0.24 | 0.32 | 0.40 |
| 0.70 | $\mu_{\mathrm{H} 0,7}$ | 0.07 | 0.14 | 0.21 | 0.28 | 0.35 |
| 0.60 | $\mu_{\mathrm{H} 0,6}$ | 0.06 | 0.12 | 0.18 | 0.24 | 0.30 |
| 0.50 | $\mu_{\mathrm{H} 0,5}$ | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 |
| 0.40 | $\mu_{\mathrm{H} 0,4}$ | 0.04 | 0.08 | 0.12 | 0.16 | 0.20 |
| 0.30 | $\mu_{\mathrm{H} 0,3}$ | 0.03 | 0.06 | 0.09 | 0.12 | 0.15 |
| 0.20 | $\mu_{\mathrm{H} 0,2}$ | 0.02 | 0.04 | 0.06 | 0.08 | 0.10 |
| 0.10 | $\mu_{\mathrm{H} 0,1}$ | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |

Graphs of dependence of $\mu_{\mathrm{H}}$ values on $P_{\mathrm{H}}$ and $\lambda_{\mathrm{H}}$ rate for $P_{\mathrm{H}}=1.0 ; P_{\mathrm{H}}=0.7 ; P_{\mathrm{H}}=0.5$ and $P_{\mathrm{H}}=0.3$ are presented in Fig. 5.


Figure 5: Graph of the dependence of $\mu H$ values on $\lambda H$ values at $P H=1.0 ; P H=0.7$;

$$
P H=0.5 \text { and } P H=0.3
$$

Table 2 and fig. 5 show that to increase the value of $P_{\mathrm{H}}\left(\lambda_{\mathrm{H}}, \mu_{\mathrm{H}}\right)$ it is necessary to increase the values of $\mu_{\mathrm{H}}$. Thus, to increase $P_{\mathrm{H}}$ by 0.1 , it is necessary to increase $\mu_{\mathrm{H}}$ by $1 \%$ at $\lambda_{\mathrm{H}}=0.1$, at $\lambda_{\mathrm{H}}=0.2-$ by $2 \%$, etc., at $\lambda_{\mathrm{H}}=0.5$ the increase is $5 \%$ from the previous values of $\mu_{\mathrm{H}}$.

The graph of $P_{L}$ probabilities of servicing lower-priority customers depending on service $\mu_{\mathrm{H}}$ and arrival $\lambda_{\mathrm{H}}$ intensities of higher-priority customers is presented in Fig. 6.


Figure 6: Probability of servicing $P_{L}$ requirements of lower priority depending on higher priority customers arrival and servicing rate

From Fig. 6, it can be seen that the probability of servicing lower-priority customers has a decreasing nature, while the "sharpness" of the fall is inversely proportional to the intensity of servicing higher-priority customers.

Consider the function $P_{L}(t)$ depending on the intensity $\mu_{\mathrm{H}}$ and the time $t_{\mathrm{H}}=1 / \lambda_{\mathrm{H}}$ of the interval between the customers of the highest priority of the incoming flow. The function $P_{L}(t)$ is a function of the probability of failure-free service of lower priority consumers, which is linear in nature. The mean time of failure-free service of these consumers is equal to the area bounded by the line $P_{L}(t)$ and the coordinate axes $P_{L}(t)$ and $t_{\mathrm{H}}$. The dependence of the probability value $P_{L}(t)$ of servicing lower priority consumers on the intensity $\mu_{\mathrm{H}}$ of servicing higher priority consumers and the duration of the time interval $t_{H}$ between these consumers in the arrival flow is shown in Fig. 7.

Indeed, with an increase in the value of $\mu \mathrm{H}$, the average service duration decreases, at the same time, with an increase in the value of $\lambda_{\mathrm{H}}$, the interval between consumers in the arrival flow of higher priority consumers decreases, which leads to an increase in the average number of such consumers, and therefore to a higher load on the service channel, which in turn, prevents consumers of lower priority from entering the service channel and reduces the service probability $P_{L}$.


Figure 7: Graphs of the dependence of the probability of $P_{L}\left(t_{H}, \mu_{H}\right)$ on the values of $\mu_{H}$ and $t_{H}$

Fig. 7 shows that when the value of $t_{\mathrm{H}}$ decreases at $P_{L}\left(t_{\mathrm{H}}, \mu_{\mathrm{H}}\right)=$ const, or the value of $P_{L}\left(t_{\mathrm{H}}\right.$, $\mu_{\mathrm{H})}$ at $t_{\mathrm{H}}=$ const, the value $\overline{t_{L}}=0.5 P_{L}\left(t_{H}, \mu_{H}\right) t_{H}$ decreases. To increase the value of $P_{L}\left(t_{\mathrm{H}}, \mu_{\mathrm{H}}\right)$ at a certain value of $t_{\mathrm{H}}$, it is necessary to increase the value of $\mu_{\mathrm{H}}$, that is, to provide more intensive service of higher priority customers.

Fig. 8 shows graphs of the dependence of the probability of pushing out $P_{L P}$ consumers of lower priority.


Figure 8: Probability $P_{L P}$ of "pushing out" from the QS lower priority consumers by high priority consumers depending on high priority consumers arrival and service rates

The processes that are associated with the increase of $P_{L P}$ are described in detail above, the decrease of $P_{L P}$ after reaching its maximum is associated with a decrease in the number of consumers of lower priority and the load of the service channel. The value of $\lambda_{H, \mathrm{extr}}$ corresponds to the maximum value of $P_{L P}$. To determine the value of $\lambda_{H, \text { extr }}$, dependences of the probability of $P_{L P}$ were obtained, approximated by a polynomial of the sixth degree, of the form:

$$
y=a_{6} x^{6}+\ldots+a_{1} x+C
$$

The coefficients of the polynomials approximating the probabilities of $P_{L P}$ of the value $\lambda_{H, \text { extr }}$ are presented in the table. 3.

Table 3: Coefficients of polynomials approximating the probabilities of "pushing out" the QS of low-priority consumers

| Coefficient of the <br> polynomial | $\mu_{\mathrm{H}}=0.1$ | $\mu_{\mathrm{H}}=0.2$ | $\mu_{\mathrm{H}}=0.3$ | $\mu_{\mathrm{H}}=0.4$ | $\mu_{\mathrm{H}}=0.5$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{a}_{6}$ | -1767609 | 97550.53 | 4593.061 | 1980.917 | 983.323 |
| $\mathrm{a}_{5}$ | 431830.5 | -54236 | -3453.6 | -2276.31 | -1392 |
| $\mathrm{a}_{4}$ | -42096 | 10862.98 | 857.173 | 966.0881 | 721.876 |
| $\mathrm{a}_{3}$ | 1791.001 | -1046.51 | -114.584 | -212.099 | -184.777 |
| $\mathrm{a}_{2}$ | -34.143 | 48.5078 | 8.249455 | 23.6418 | 23.30407 |
| $\mathrm{a}_{1}$ | 2.292621 | 1.042772 | 1.748402 | 0.770417 | 0.615751 |
| $\mathrm{a}_{0}$ | -0.00172 | 0.004007 | -0.0002 | 0.017116 | 0.019575 |
| Extremum, $\boldsymbol{\lambda}_{\boldsymbol{H}, \mathrm{extr}}$ | 0.071 | 0.135 | 0.198 | 0.2628 | 0.328 |

Knowing the extrema of the probabilities of pushing out lower priority consumers and the corresponding rates of arrival flows and servicing higher priority consumers is of practical importance, for example, for making decisions about the distribution of trains of different priorities (repair, passenger, freight, other types) on different nodes and routes of the transport network.

## Conclusions

1. Taking into account the practical experience of functioning and ensuring the reliability of the railway transport system in extreme conditions of military operations, destruction and restoration of infrastructure, shortage of time and fleet of vehicles, it is proposed to formalize the functioning of this system as a queuing system (QS) with priorities, with variable parameters of arrival and servicing the consumers of higher and lower priorities, which allows making more informed decisions for the management of such systems.
2. The necessity of using, along with analytical methods, simulation modeling tools has been proven, in particular, a simulation model has been developed and implemented, which combines agent and discrete-event simulation principles and allows studying QSs when consumers of different priority are arriving, namely in the part of establishing regularities: probabilities (service, refusal, push-out), time delays (waiting in a queue, under service), queue sizes, order of queue formation when customers of different priority arrive.
3. The resulting dependencies:

- the probability of servicing higher priority consumers depending on arrival and servicing rates;
- probabilities of servicing lower priority consumers depending on arrival and servicing rates of higher priority consumers;
- probability of "pushing" lower priority consumers out the QS by higher priority consumers, depending on arrival and service intensities of higher priority consumers;
- values of the intensity of service of higher priority consumers $\mu_{\mathrm{H}}$ on the probability $P_{\mathrm{H}}$ and the intensity of their arrival $\lambda_{\mathrm{H}}$;
- values of the probability of $P_{\mathrm{L}}$ on the values of the intensity of service $\mu_{\mathrm{H}}$ of the consumers of the highest priority and the duration of the interval $t_{H}$ between these consumers in the arrival flow.

4. Experimental data of modeling the probability of servicing higher priority customers depending on their arrival and service rates, as well as the approximation of these data by empirical dependencies allow us to propose ratios that simplify the calculations of QSs with priorities, namely:

$$
\left\{\begin{array}{c}
P_{\mu_{H}}=\mu_{H} \lambda_{H}^{-1}, \quad \lambda_{H}>\mu_{H} \\
P_{\mu_{H}}=1, \quad \lambda_{H} \leq \mu_{H}
\end{array}\right.
$$

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[^0]:    "requirement generation serial number $\mid$ priority level of the requirement $\mid$ model time of requirement generation".

