SINGLE SAMPLING PLANS FOR VARIABLE INSPECTION BASED ON BIRNBAUM – SAUNDERS DISTRIBUTION

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Abstract

Sampling inspection by the method of variables is a well-known category of product control in which the discretion of acceptance or rejection is made hinge on some specific rule, which is framed according to the measurement of a quality characteristic under study. In this scenario, the quality feature under study is considered a continuous random variable that can be demonstrated using any continuous type statistical distribution. Birnbaum – Saunders distribution is a continuous probability distribution, which is having numerous applications in various fields. Application of Birnbaum – Saunders distribution is considered in this paper for establishing acceptance sampling inspection plans for variables based on the examination of units from a single sample. Numerical illustrations are given to demonstrate the application of proposed sampling plan. In addition, the results of numerical illustrations are explained with the help of simulated data.

Keywords: Sampling inspection plans, variable sampling, Birnbaum – Saunders distribution, non-normality, skewness.

I. Introduction

Sampling inspection plans or acceptance sampling plans play a cardinal role in determining the quality of products by reviewing the sample of units taken from the lot of products. More often, the product control methodologies are grouped into two categories, namely, sampling inspection by the method of attributes and sampling inspection by the method of variables. In sampling inspection by the method of attributes, selection of one or more samples is made from the manufactured lot of items and the sampled units are classified into defective (non-conforming) or non-defective (conforming) according to some prescribed decision criteria. The choice of accepting or rejecting the lot is taken based on some explicit criteria.

In sampling inspection by the method of variables, a sample of manufactured commodities is picked from the lot, and then the quality feature of interest is measured and recorded. The decision about the lot acceptance or lot disposition is interpreted according to some specific criteria, which are framed based on the measurement of the quality variable under study. This method is applicable to the cases where the quality feature under study is a continuous random variable, which is quantifiable on a continuous scale.

A peculiar aspect of variable sampling is that the quality feature under study is a continuous random variable which can be demonstrated using any continuous type probability distribution from the literature of statistical theory. A noteworthy advantage of variable sampling is that the decision is made about the lot quality in accordance with the exact measurements of the quality variable. It provides more accurate information about the product quality than compared to the sampling inspection by attributes. Variable sampling plans can be used with same level of protection as attribute sampling plans, but with lesser number of sampling units, showing its efficiency with respect to the sample size.

Under normality assumption of the quality feature, many researchers have devised the variable sampling plans. When the normality assumption is sheered or when the quality variable shows some kind of skewness, the standard variable sampling plans based on normal distribution are no longer applicable. In such situations, the need for the application of non-normal distributions or skewed distributions arises. Many researchers have initiated the works on variable sampling plans for non-normal populations. Zimmer and Burr [1] considered Burr distribution for constructing variable sampling plans based on the measures of skewness and kurtosis. Takagi [2] designed sampling procedures for variable inspection based on non-normal distributions when the population variance is unknown. Geetha and Vijayaraghavan [3] considered Pareto distribution for the construction of variable sampling plans. Seifi and Nezhad [4] constructed variable sampling plan for resubmitted lots under Bayesian approach. Other major developments in variables sampling plans include the works of Srivastava [5], Owen [6], Duncan [7], Guenther [8, 9], Yeh [10, 11], Yeh *et.al.* [12], Aslam *et.al.* [13], Balamurali and Usha [14], Yen and Cheng [15], Wu *et.al.* [16-18], Geetha and Pavithra [19], and Rao *et.al.* [20].

The aspects of construction and assessment of variable sampling plans are quite easy when the quality feature under study stick to follow normality assumption. In industrial applications, the normality assumption of the quality variable may be sheered or the quality feature may exhibit non-normal patterns. In such situations, designing of sampling inspection plans by the method of variables becomes unwieldy.

In this paper, the designing procedure of sampling inspection plans by the method of variables based on a single sample is examined when the distribution of the quality feature of the manufactured commodity under study shows a similar pattern of Birnbaum – Saunders distribution.

II. Birnbaum – Saunders Distribution

One among the important probability distributions that have potential applications in life testing and reliability is the Birnbaum – Saunders distribution. It is a two - parameter continuous, unimodal and positively skewed probability distribution that is specifically used in the studies relating to modeling of fatigue life of metals, which are subject to periodic stress. [See, Balakrishnan and Kundu [21]]. It is also known as the fatigue-life distribution and has been widely applied to fatigue and reliability studies. Birnbaum and Saunders [22] derived an ingenious probability model to describe lifetimes associated with materials exposed to fatigue and tension. Birnbaum and Saunders [23, 24] formalized the fatigue-life distribution which is named after them. Since then, extensive work was carried out by many researchers on this distribution to provide generalizations, estimation and inferential procedures.

According to Leiva [25], its field of application has been extended beyond the context of fatigue and reliability analysis and is a model in situations where the accumulation of a certain factor forces a quantifiable characteristic to exceed a critical threshold. A detailed account of description, analysis and applications of the Birnbaum and Saunders distribution has been given in the phenomenal paper by Balakrishnan and Kundu [21]. Another unparalled contribution on this distribution has been made by Leiva [25] in his book which provides an overview of the distribution, its probabilistic and statistical features, and its extension to regression analysis, diagnostics, etc.

The Birnbaum – Saunders distribution has a failure rate function which can take the three forms of failure rates, viz., increasing and decreasing failure rate, and unimodal or inverse bathtub. [See, Johnson, Kotz and Balakrishnan [26] and Nelson [27]]. When a probability distribution for life-time variable has a failure rate function that takes various shapes, it is the natural choice to adopt the distribution in practice. Considering the importance of Birnbaum and Saunders distribution, as pointed out by many research in the literature, its application in sampling inspection by variables is now considered.

Let X be a random variable representing the quality feature of a material and follows Birnbaum – Saunders distribution. The probability density function and cumulative distribution function of X are, respectively, given by

$$f(x;\theta,\delta)=\frac{\sqrt{x/\theta}+\sqrt{\theta/x}}{2\delta x}\varphi\left(\frac{\sqrt{x/\theta}-\sqrt{\theta/x}}{\delta}\right); x>0,\theta>0,\delta>0 \tag{1}$$

and

$$F(x;\theta,\delta)=\Phi\left(\frac{\sqrt{x/\theta}-\sqrt{\theta/x}}{\delta}\right); x>0,\theta>0,\delta>0 \tag{2}$$

where δ and θ are the shape and scale parameters, respectively, and φ and Φ are the density and distribution functions of standard normal distribution, respectively.
The mean and variance of BSD are given by,

$$E(x)=\theta\left(1+\frac{\delta^2}{2}\right), \tag{3}$$

and

$$V(x)=\theta^2\delta^2\left(1+\frac{5}{4}\delta^2\right), \tag{4}$$

The reliability function and hazard function for specified time x under Birnbaum – Saunders distribution are, respectively, given by

$$R(x;\theta,\delta)=\Phi\left(\frac{\sqrt{\theta/x}-\sqrt{x/\theta}}{\delta}\right) \tag{5}$$

and

$$Z(x;\theta,\delta)=\frac{\dfrac{\sqrt{x/\theta}+\sqrt{\theta/x}}{2\delta x}\varphi\left(\dfrac{\sqrt{x/\theta}-\sqrt{\theta/x}}{\delta}\right)}{\Phi\left(\dfrac{\sqrt{\theta/x}-\sqrt{x/\theta}}{\delta}\right)}. \tag{6}$$

The coefficient of variation of BSD is,

$$\gamma(x) = \delta\left(\frac{\sqrt{5\delta^2 + 4}}{\delta^2 + 2}\right), \tag{7}$$

The coefficients of skewness and kurtosis of a random variable X which follows BSD are given by,

$$\alpha_3 = \frac{16\delta^2\left(11\delta^2 + 6\right)^2}{\left(5\delta^2 + 4\right)^3} \tag{8}$$

$$\alpha_4 = 3 + \frac{6\delta^2\left(93\delta^2 + 40\right)}{\left(5\delta^2 + 4\right)^2} \tag{9}$$

It can be identified that the coefficients of variation, skewness and kurtosis only depends on the shape parameter of BSD. It is also possible to note that, as the shape parameter, δ move towards zero, the BSD exhibit a symmetrical pattern around scale parameter, θ (the median of the BSD) and the variability decreases. Furthermore, notice that, as the shape parameter increases, the BSD has heavier tails. This means that δ modifies the skewness and kurtosis of the distribution.

III. Sampling Inspection by Variables

Acceptance sampling plans for variables based on a single sample can be stated based on the conditions mentioned hereafter.

i. The random variable depicting the quality feature is quantifiable on a continuous scale that is having a recognized model of statistical distribution.
ii. Each single item subjected to quality monitoring has provided with a single specification limit say, lower specification limit (LSL), L, or upper specification limit (USL), U. If a measurement on a particular item goes beyond the specification, the item is considered as an unsatisfactory component.

The working process of an acceptance sampling plan based on variable inspection for a single sample is described in the following manner:

Step 1: Randomly select a small group of elements of size n from the submitted population of manufactured items. Measure the quality variable of interest for each unit in the selected group and record the measurements.

Step 2: Approve the submitted population of manufactured commodities as accepted, if $\bar{x} + k\sigma \le U$ or $\bar{x} - k\sigma \ge L$; and reject, otherwise. The conditions are chosen according to the given specification. If σ is unknown, s, an unbiased estimate of population standard deviation, is used in the place of σ.

The sample size n and acceptance constant k constitute the parameters of the acceptance sampling plans for variables based on a single sample.

IV. Operating Characteristic Function

The effectiveness of any acceptance sampling plan can be evaluated using an important measure, called operating characteristic (OC) function. It gives the probability of acceptance of a lot with a specified proportion of faulty items. It is denoted by $P_a(p)$, where p is the fraction of defective or nonconforming items in the bunch of commodities. When USL is provided, the proportion of nonconforming items and probability of accepting the lot are, respectively, given by

$$p = P(X > U \mid \mu) \tag{10}$$

and

$$P_a(p) = P(\bar{x} + k\sigma \leq U \mid \mu) \tag{11}$$

When σ is unknown, the sample standard deviation, *s*, which is an unbiased estimate of σ, is used.

A plot of probability of acceptance against the proportion of defective or nonconforming items is known as the operating characteristic (OC) curve, which is commonly used for comparing the efficiencies of sampling plans. A common procedure for designing a sampling plan is described by specifying two points on the OC curve, *viz.*, $(p_0, 1-\alpha)$ and (p_1, β), p_0 and p_1 denote the acceptable quality level (*AQL*) and limiting quality level (*LQL*), respectively; α and β are producer's risk and consumer's risk, respectively. The *AQL* and *LQL* are, respectively, defined by

$$AQL = p_0 = P(X > U \mid \mu_0) \tag{12}$$

and

$$LQL = p_1 = P(X > U \mid \mu_1), \tag{13}$$

where μ_0 and μ_1 are the means of probability distribution of the quality variable under study which results in *AQL* and *LQL*. The producer's risk and consumer's risk can be obtained using the given requirements of *AQL* and *LQL* as

$$\alpha = P(\bar{x} + k\sigma > U \mid \mu_0) \tag{14}$$

and

$$1 - \beta = P(\bar{x} + k\sigma > U \mid \mu_1) \tag{15}$$

When σ is unknown, the estimate s is used for the evaluation of α and β.

The probability of acceptance and the proportion of nonconforming items are defined based on the underlying distribution of the quality feature under study. They are obtained using the cumulative probability distribution function of BSD.

V. Designing Procedure of Variable Single Sampling Plan

In most of the practical scenarios, the quality feature of the product under study will show some kind of deviations from the normality assumption. In such situations, the standard variable sampling plans cannot be utilized. It is, also, the fact that population parameters of the statistical distribution, which is used to model the quality variable, are unidentifiable in most of the cases and hence, they are estimated using the sample statistics. If the distribution of quality variable under consideration is deviating from normality, the development phase of unknown σ sampling plans becomes more cumbersome.

Let $F(x;\theta,\delta)$ be the cumulative distribution function of BSD, which is also considered as the distribution function of the quality feature. From equation (12), it is easily observed that the acceptable quality level p_0 can be found using the cumulative distribution function of quality parameter, which can be expressed as

$$AQL = p_0 = 1 - F_x(x;\mu_0), \tag{16}$$

where μ_0 is the mean of BSD, which results in an acceptable quality level.

Similarly, from equation (13) the limiting quality level p_1 can be expressed as

$$LQL = p_1 = 1 - F_x(x;\mu_1), \tag{17}$$

where μ_1 is the mean of BSD, which results in limiting quality level.

Also, from equations (14) and (15), the producer's risk and consumer's risk can be obtained using the cumulative distribution function $F(x;\theta,\delta)$ as

$$\alpha = 1 - F_x(\bar{x};\mu_0) \tag{18}$$

and

$$1-\beta = 1 - F_x(\bar{x};\mu_1) \tag{19}$$

When the underlying distribution of quality parameter is normal, the designing of variable sampling plan includes the process of determining the standard normal deviate value K_p^*. For designing a variable sampling plan for non-normal distributions, the value of K_p^* corresponds to the deviate value of underlying distribution. Hence, for obtaining the parameters of a variable sampling plan under BSD, the deviate values are obtained as follows.

$$K_{p_0}^* = \frac{x_{p_0} - M}{S}, \tag{20}$$

where x_{p_0} is the value of x for which the upper tail area of BSD is p_0 and $K_{p_0}^*$ is the standardized value of x_{p_0}. Here, *M* and *S* denotes, respectively, the mean and standard deviation of the BSD.

Similarly, the deviate value corresponding to the limiting quality level is obtained as

$$K_{p_1}^* = \frac{x_{p_1} - M}{S}, \tag{21}$$

where x_{p_1} is the value of x for which the upper tail area of BSD is p_1 and $K_{p_1}^*$ is the standardized value of x_{p_1}.

The optimum values of the parameters of a variable sampling plan for non-normal populations is given by Zimmer and Burr (1963) as

$$n_U = \left[\frac{K_\alpha + K_\beta}{K_{p_0}^* - K_{p_1}^*}\right]^2 \tag{22}$$

and

$$k_U = \frac{K_\alpha K_{p_1}^* + K_\beta K_{p_0}^*}{K_\alpha + K_\beta}. \tag{23}$$

Similarly, the constants of a variable sampling plan when the lower specification limit is specified, can be determined as

$$n_L = \left[\frac{K_\alpha + K_\beta}{K_{1-p_0}^* - K_{1-p_1}^*}\right]^2 \tag{24}$$

$$k_L = \frac{K_\alpha K^*_{1-p_1} + K_\beta K^*_{1-p_0}}{K_\alpha + K_\beta} \tag{25}$$

In 1972, Takagi introduced an approach for obtaining the sample size and the acceptance constant of single sampling inspection plans by variables based on a wide range of non-normal distributions and proposed an expansion component in the context of the measures of skewness and kurtosis. According to Takagi, when σ is unknown, $\bar{x} \pm ks$ will follow normal distribution asymptotically with parameters, $\mu_y = \mu \pm k\sigma$ and $\sigma_y^2 = \frac{\sigma^2}{n}\left[1 + \frac{k^2}{4}(\alpha_4 - 1) \pm k\alpha_3\right]$, where μ, σ, α_3 and α_4 represent the mean, standard deviation, skewness and kurtosis of the underlying probability distribution, respectively. The operating characteristic function given in (11) can be rewritten, approximately, as

$$P_a(p) = P\left(Z \le \frac{U - \mu_y}{\sigma_y} \mid \mu\right) \tag{26}$$

where μ_y and σ_y denote the mean and standard deviation of $\bar{x} \pm ks$ and Z is the standardized variable.

Let us denote $K_p^* = (U - \mu)/\sigma$. Then, for the designated points of acceptable and limiting quality levels, one may have

$$\mu_0 = U - K^*_{p_0}\sigma \tag{27}$$

and

$$\mu_1 = U - K^*_{p_1}\sigma. \qquad 28)$$

Let α and β denote the producer's risk and consumer's risk, respectively. Then, the normal deviates corresponding to the specified risks are given by

$$K_\alpha = \frac{U - \mu_{x0}}{\sigma_x} == \frac{U - (\mu_0 + k\sigma)}{\sqrt{(\sigma^2/n)\left[1 + (k^2/4)(\alpha_4 - 1) + k\alpha_3\right]}} = \frac{U - (\mu_0 + k_U\sigma)}{\sigma\sqrt{e_U/n_U}} \tag{29}$$

and

$$-K_\beta = \frac{U - \mu_{x1}}{\sigma_x} = \frac{U - (\mu_1 + k\sigma)}{\sqrt{(\sigma^2/n)\left[1 + (k^2/4)(\alpha_4 - 1) + k\alpha_3\right]}} = \frac{U - (\mu_1 + k_U\sigma)}{\sigma\sqrt{e_U/n_U}}. \tag{30}$$

Using the expressions of μ_0, μ_1, K_α and K_β one can obtain the following:

$$K^*_{p_0} = k_U + K_\alpha\sqrt{\frac{e_U}{n_U}} \tag{31}$$

$$K^*_{p_1} = k_U - K_\beta\sqrt{\frac{e_U}{n_U}} \tag{32}$$

Solving the equations (31) and (32), the explicit expressions for the parameters of sampling inspection plan can be obtained as

$$n_U = e_U \left[\frac{K_\alpha + K_\beta}{K^*_{p_0} - K^*_{p_1}} \right]^2 \tag{33}$$

and

$$k_U = \frac{K_\alpha K^*_{p_1} + K_\beta K^*_{p_0}}{K_\alpha + K_\beta}, \tag{34}$$

where K_α and K_β denote the well-known standard normal deviates exceeding the probabilities α and β respectively, and $e_U = 1 + (k_U^2/4)(\alpha_4 - 1) + k_U \alpha_3$ is known as the expansion factor. The expansion factor is utilized in order to gain information about the parameters of known σ plans with the relations $n'_U = n_U / e_U$ and $k'_U = k_U$. In a similar manner, the constants of variable sampling plan, when the lower specification limit is specified, can be determined as

$$n_L = e_L \left[\frac{K_\alpha + K_\beta}{K^*_{1-p_0} - K^*_{1-p_1}} \right]^2 \tag{35}$$

and

$$k_L = \frac{K_\alpha K^*_{1-p_1} + K_\beta K^*_{1-p_0}}{K_\alpha + K_\beta}, \tag{36}$$

where $e_L = 1 + (k_L^2/4)(\alpha_4 - 1) + k_L \alpha_3$, making use of which the parameters of a known σ plan can be obtained as $n'_L = n_L / e_L$ and $k'_L = k_L$.

VI. Numerical Illustrations

I. Numerical Illustration 1

The thickness of silicon wafers is a vital quality feature in microelectronic circuits. The manufacturing firm specified the upper specification limit for the thickness as 0.02 mm. From the history, it was ascertained that the thickness of silicon wafer follows BSD having parameters $\delta = 0.25$ and $\theta = 0.0125$.

The mean and standard deviation were determined as $\mu = 0.01289$ and $\sigma = 0.0032$. Under the given conditions, for the specified values of $p_0 = 0.01, p_1 = 0.05, \alpha = 0.05$ and $\beta = 0.10$, it is required to obtain a single sampling plan by variables. Corresponding to the specified quality levels and the risks, one obtains the following: $K^*_{p_0} = 2.864411, K^*_{p_1} = 1.8225, K_\alpha = 1.644854$ and $K_\beta = 1.281552$. Thus, the optimum values of the parameters n and k of the sampling plan are determined from (21) and (22) as $\mathrm{n} = 7.89$ or 8 and $\mathrm{k} = 2.2788$.

In order to make the comparison of the results obtained in the illustration, data have been simulated based on 5000 runs using R programming. The simulated data of 8 observations from BSD having the specified parameters provide the sample average was found to be $\bar{x} = 0.01118$.

It can be observed that $\bar{x} + k\sigma = 0.0185$ which falls below the upper specification limit, $\mathrm{U} = 0.02$. Hence, the lot would be accepted.

II. Numerical Illustration 2

A manufacturing company produces bottles for various purposes. For a particular make of bottle, a lower specification on the bursting strength of bottle is at 200 psi. The lot is accepted, if 1% or less of the bottles burst within this limit, with probability 0.95 (i.e., $p_0 = 0.01$, $\alpha = 0.05$) whereas if 6% or more of the bottles have bursting strength below this limit, the lot is rejected with probability 0.90 (i.e., $p_1 = 0.06$, $\beta = 0.10$).

Let X be the quality variable which follows BSD having the parameters, $\delta = 0.1$ and $\theta = 275$. The mean, variance, skewness and kurtosis are obtained as $\mu = 276.375, \sigma^2 = 765.7031, \alpha_3 = 01.1994$ and $\alpha_4 = 3.1497$, respectively. For the specified quality levels, say $p_0 = 0.01$ and $p_1 = 0.06$, and the associated producer's and consumer's risks, $\alpha = 0.05$ and $\beta = 0.10$, one can obtain the parameters of the sampling plan by variables using the expressions (21) and (22). Corresponding to $p_0 = 0.01$ and $p_1 = 0.06$, the deviate values $K^*_{p_0}$ and $K^*_{p_1}$ are obtained, from (19) and (20), as 2.1083 and 1.4793, respectively. Corresponding to the specified $\alpha = 0.05$ and $\beta = 0.10$, the values of K_α and K_β are obtained as 1.644854 and 1.281552. Given the lower specification limit and other requirements, the parameters of single sampling plan by variables are obtained from equations (21) and (22) as $n = 21.65$, which when rounded becomes 22 and $k = 1.7548$.

A simulation study is carried out for comparing the results arrived in the above illustration. Assume that the standard deviation is known. The simulated results are based on 5000 runs using R programming. By simulation based on 5000 runs, the randomly generated sample of 22 observations from BSD having the specified parameters $\delta = 0.1$ and $\theta = 275$ yields the sample mean, *viz*., $\bar{x} = 276.3843$.

It is known that the acceptance criterion under a variable sampling plan with specified lower specification limit is given as $\bar{x} - k\sigma \geq L$. It can be seen that $\bar{x} - k\sigma = 227.8266$, which is greater than the lower specification limit, i.e., U = 200. Hence, the lot would be accepted.

VII. Conclusion

Normal distribution and its wide range of properties play a prominent role in the theory and applications of statistics. Acceptance sampling plans for variable inspection consider normal distribution as an important model for the quality variable. Even though applications of normal distribution are a plenty in various domains, there are situations, where non-normality arises in the real-life data. In this paper, a Birnbaum – Saunders distribution (BSD) is considered for designing an acceptance sampling plan by variables based on the information acquired from a single sample. The strategy for the choice of variable single sampling inspection plans when the quality feature shows the behavior of BSD is discussed, when the producer's and consumer's requirements are specified. The numerical illustrations are given to demonstrate how the proposed plan could be executed in practical application. The simulation based on 5000 runs for generating samples from BSD has been done utilizing R programming and the simulated results are compared with the results arrived in the illustrations.

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