THE APPLICATION OF QUEUEING THEORY FOR MAXIMIZING SYSTEM SIZE USING ENCOURAGED ARRIVAL

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Abstract

Customers are frequently drawn in by lucrative deals and discounts offered by businesses. These interested customers are referred to as encouraged arrivals. The major goal of this study is to evaluate the performance of the automobile assembly line in order to decrease waiting time by coordinating the activities at each workstation using stopwatch time study approach. The novelty of this research is to convert poisson arrival to encouraged arrival with some discounts (like.,10%, 20%). The queuing problem is represented by the notation M/M/1: FCFS/∞/∞ in Kendall's notation. It is a single channel, multi-server service with infinite system capacity and an infinite number of calling population. Data concerning the system's encouraged arrival and service distribution were established. These data were used to calculate the system performance parameter. The finding of the study was used to predict the system's performance and effectiveness and to make logical recommendations for possible future improvements. According to the results, it is possible to conclude that increasing the level of automation reduces part waiting time, decreasing the cost of waiting. When compared to the poisson arrival system, the size of the Markovian encouraged arrival queuing system is increased as shown in the table. Little's law is verified that system size and queue size is same as in length. Little's law is used to predicts lead time based on production rate and work-in-process. Here it is verified as shown in table.

Keywords: Encouraged arrival, Assembly Line, Queuing Analysis, Stopwatch time study, Steady state Solution.

1. Introduction

In public areas like post offices, banks and gas stations, waiting is a common occurrence. Not just people, but also machinery and moving vehicles at traffic lights experience the phenomena of waiting. Bottlenecks arise and assume the shape of lines when resources are limited and unable to fulfil demand. In an assembly line, to make a final product as soon as possible, pieces are systematically attached to a product utilizing the well-planned logistics. The sequential organization of employees, equipment or parts is the main goal of assembly lines. In flow-oriented production system, the productive units performing the operations are repeatedly linked to a service.

The work components are often conveyed along the line via, a transportation system such as a conveyor belt where they are delivered to stations one after another. The different types of assembly lines include single-model, batch-model and mixed-model lines. A few product models are manufactured in batches, one at a time on the same line with time allocated for a changeover so that the line is set up for the production of a new system. The procedure for choosing the configuration of the goods to be produced on the line must minimize the number of workstations, balance the delay and fulfil other placement requirements such as production rate, variety, minimal distance moved, division of labor and quality.

Introduction to congestion theory in telephone systems was discussed in [1] effective implementation of cycle time reduction strategies. For semiconductor back-end manufacturing considered in [2]. The goal of traditional assembly line balancing procedures is to get us to the point of subdividing work so that the amount of time that stations are out of balance is kept to a minimum is studied in [3]. Queuing analysis to analyses patient load in outpatient and inpatient services to facilitate more realistic resource planning is found in [4]. The Application of queuing theory in multi-stage production line is studied in [5]. Discussion of operational transport analysis methods and the practical application of queuing theory to stationary traffic considered in [6]. Modelling and analysis of manufacturing systems considered in [7]. Queuing theory in solving automobile assembly line problems in [8]. Improving effectiveness and efficiency of assembly line with a stopwatch time study and balancing activity elements was discussed in [9]. Queuing theory and manufacturing systems modelling and analysis are done and they are developed. Few among them were found in [10 and 11]. Parallel tasks and stations are considered by Bard (1989), as is dead time, which is the time required for transporting workpieces from one station to the next while no tasks can be executed.

Maximization of system size in solving automobile assembly line problem using encouraged arrival proposed in this work. An introduction is described in Section 1. The Markovian queue with encouraged arrival for mathematical model formulation is described in Section 2. Numerical illustrations are provided in Section 3. Results and discussion are given in Section 4. Section 5 contains the Conclusion.

2. Mathematical Model

The mathematical model predicates to satisfy the following conditions:

i) Customers arrive one by one to an encouraged arrival discipline process with rate $\lambda(1+\chi)$, where χ represents the customer's previous or observed data. If a previous firm gave discounts and percentages, the number of consumers observed values ranging from χ = 0.1 and χ = 0.2 respectively.
ii) Service time is symmetrically and exponentially distributed.
iii) Customers adhere to the first in, first out principle.

2.1 Steady State Solution:

We obtain the following system of differential difference equations.

$$\frac{d}{dt}P_0(t) = -\lambda(1+\chi)P_0(t) + \mu P_1(t) \tag{1}$$

$$\frac{d}{dt}P_n(t) = \lambda(1+\chi)P_{n-1}(t) - \{\lambda(1+\chi) + \mu\}P_n(t) + \mu P_{n+1}(t) \quad n \geq 1 \tag{2}$$

In the steady state, as $t \to \infty$, $P_n(t) = P_n$ and therefore $P_n^1(t) = 0$ as $t \to \infty$ then, the equations are,

$$0 = -\lambda(1+\chi)P_0 + \mu P_1 \tag{3}$$

$$0 = \lambda(1+\chi)P_{n-1} - \{\lambda(1+\chi) + \mu\}P_n + \mu P_{n+1} \tag{4}$$

Now the value of P_n is obtained as, $P_n = \rho^n P_0$

$$P_n = \left(\frac{\lambda(1+\chi)}{\mu}\right)^n P_0 \tag{5}$$

The value P_0 can be computed by using the obvious requirement, that the sum of all probabilities must be equal to 1.

$$\sum_{n=0}^{\infty} P_n = P_0 \sum_{n=0}^{\infty} \rho^n$$

$$= P_0 \frac{1}{1-\rho}$$

$$\sum_{n=0}^{\infty} P_n = 1 \tag{6}$$

Where, $P_0 = 1 - \rho$

It is clear that the traffic rate ρ must be less than 1, otherwise the sum of probabilities would not be 1 (not even limited). From (6) to (5) gives the general formula for P_n:

$$P_n = \rho^n (1-\rho)$$

$$= \left(\frac{\lambda(1+\chi)}{\mu}\right)^n$$

$$P_n = \left(1 - \frac{\lambda(1+\chi)}{\mu}\right) \tag{7}$$

The equation (7) represent a very important result used to obtain all the characteristics of the M/M/1 system.

3. Numerical Illustration

The performance of the M/M/1 queueing system is analysed numerically concerning the parameters Values $\lambda(1+\chi)$, μ, χ represent discounts values 10% and 20% of the table and figure. The following table 1 displays the parameters for the number of customers in the queueing system for various values of $\lambda(1+\chi)$, μ.

Labor level

An important component to take into account when conducting an assessment study of an automobile assembly plant is the number of personnel engaged in productive operations on the assembly line. Depending on how much automation is used throughout the line, different people are required. The encouraged arrival method is more efficient than the Poisson process.

Output of the Problem Evaluation Analysis

The Estimation Analysis was done by calculating the system performance parameters such as idle system, length in system, length in queue, waiting time in system, waiting time in queue and system utilization.

Table 1: *Encouraged arrival 10% and utilization factor for Markovian model*

Work Station	χ	Encouraged Arrival rate $\lambda(1+\chi)$/Min	Mean Service Time (μ)/min	Utilization factor $\frac{\lambda(1+\chi)}{\mu}$
1	0.1	12.1	16.9	0.72
2	0.1	14.3	17.8	0.80
3	0.1	13.2	17.8	0.74
4	0.1	11.0	16.4	0.67
5	0.1	11.0	18.1	0.61
6	0.1	14.3	19.2	0.74
7	0.1	12.1	17.2	0.70
8	0.1	11.0	19.8	0.56
9	0.1	13.2	17.6	0.75
10	0.1	11.0	16.9	0.65
11	0.1	14.3	18.1	0.79
12	0.1	11.0	17.8	0.62
13	0.1	12.1	17.1	0.71
14	0.1	14.3	18.9	0.76
15	0.1	13.2	18.6	0.71
16	0.1	15.4	20.0	0.77

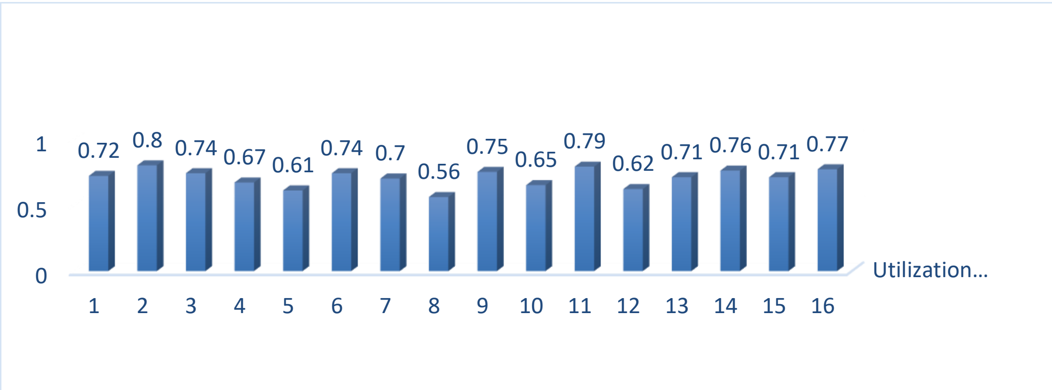

Figure 1: *Usage of workstation graph*

The utilization factor for each workstation is displayed in **Figure 1.** It is clear from the graph that certain workstations are operating below capacity. This might be as a result of the low amount of automation or labor at such a station.

3.1 Encouraged Arrival 10% of the Queuing System:

In station 1:

$\lambda(1+\chi) = 11(1+0.1) = 12.1$ parts/min, $\mu = 16.9$ parts/min

(i) Traffic intensity $\rho = \frac{(\lambda(1+\chi))}{\mu} = \frac{12.1}{16.9} = 0.72$

(ii) Idle system = 1 - ρ = 1 – 0.72 = 0.28

(iii) Length in system $L_s = \frac{(\lambda(1+\chi))}{\mu-\lambda(1+\chi)} = \frac{12.1}{16.9-12.1} = \frac{12.1}{4.8} = 2.52$

(iv) Length in queue $L_q = \frac{(\lambda(1+\chi))2}{\mu(\mu-\lambda(1+\chi))} = \frac{(12.1\,)2}{16.9(4.8)} = \frac{146.41}{81.12} = 1.81$

(v) Waiting time in system $w_s = \frac{1}{\mu-\lambda(1+\chi)} = \frac{1}{16.9-12.1} = \frac{1}{4.8} = 0.21 \times 60 = 12.6$ sec.

(vi) Waiting time in queue $w_q = \frac{(\lambda(1+\chi))}{\mu(\mu-\lambda(1+\chi))} = \frac{12.1}{16.9(4.8)} = \frac{12.1}{81.12} = 0.14 \times 60 = 8.4$ sec

Table 2: *Encouraged arrival for 10% discounts and utilization factor for a Markovian queuing.*

Stations	χ	$\lambda(1+\chi)$	μ	ρ	1-ρ	L_s	L_q	W_S	W_q
1	0.1	12.1	16.9	0.72	0.28	2.52	1.81	12.6	8.4
2	0.1	14.3	17.8	0.80	0.20	4.09	3.28	17.4	13.8
3	0.1	13.2	17.8	0.74	0.26	2.87	2.13	13.2	9.6
4	0.1	11.0	16.4	0.67	0.33	2.04	1.37	11.4	7.2
5	0.1	11.0	18.1	0.61	0.39	1.55	0.94	8.4	5.4
6	0.1	14.3	19.2	0.74	0.26	2.92	2.17	14.4	9.0
7	0.1	12.1	17.2	0.70	0.30	2.37	1.67	11.4	8.4
8	0.1	11.0	19.8	0.56	0.44	1.25	0.69	6.6	3.6
9	0.1	13.2	17.6	0.75	0.25	3.0	2.25	13.8	10.2
10	0.1	11.0	16.9	0.65	0.35	1.86	1.21	10.20	6.6
11	0.1	14.3	18.1	0.79	0.21	3.76	2.97	15.6	12.6
12	0.1	11.0	17.8	0.62	0.38	1.62	0.99	9.0	5.4
13	0.1	12.1	17.1	0.71	0.29	2.42	1.71	12.0	8.4
14	0.1	14.3	18.9	0.76	0.24	3.11	2.35	13.2	9.6
15	0.1	13.2	18.6	0.71	0.29	2.44	1.74	11.4	7.80
16	0.1	15.4	20.0	0.77	0.23	3.35	2.58	13.2	10.2

In M/M/1 automobile assembly line problem with 10% encouraged arrival for length in system and queue as well as waiting time in system and queue.

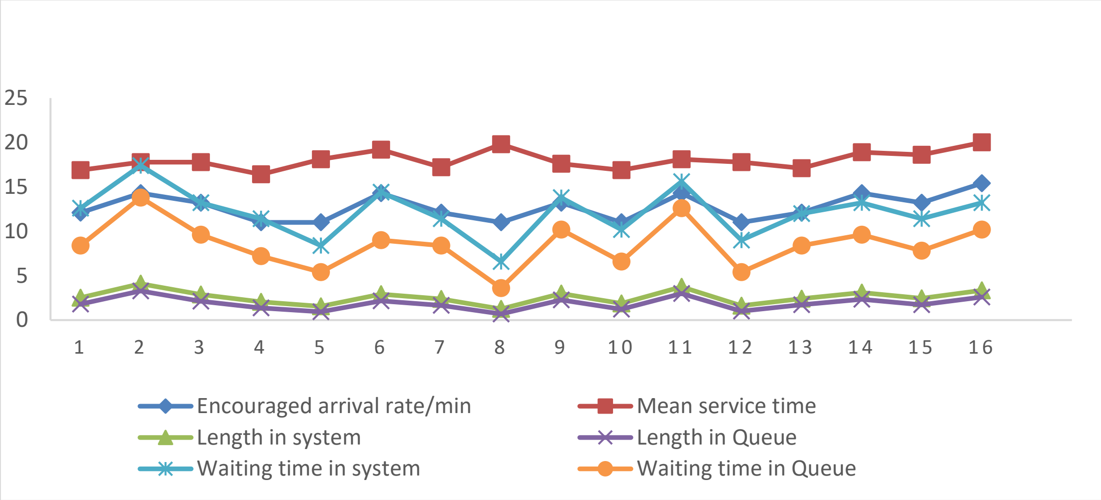

Figure 2: *Encouraged arrival of 10% discounts system and queue Size.*

Table 3: *Encouraged Arrival 10% discounts comparing with the Poisson Arrival (PA) model.*

Stations	(λ)	$\lambda(1+\chi)$	PA (L_s)	EA (L_s)	PA (L_q)	EA (L_q)	PA (W_s)	EA (W_s)	PA (W_q)	EA (W_q)
1	11	12.1	1.86	2.52	1.21	1.81	10.2	12.6	6.62	8.4
2	13	14.3	2.71	4.09	1.98	3.28	12.5	17.4	9.13	13.8
3	12	13.2	2.07	2.87	1.39	2.13	10.3	13.2	6.97	9.6
4	10	11.0	1.56	2.04	0.95	1.37	9.4	11.4	5.72	7.2
5	10	11.0	1.23	1.55	0.68	0.94	7.4	8.4	4.09	5.4
6	13	14.3	2.10	2.92	1.42	2.17	9.7	14.4	6.55	9.0
7	11	12.1	1.77	2.37	1.12	1.67	9.7	11.4	6.19	8.4
8	10	11.0	1.02	1.25	0.52	0.69	6.1	6.6	3.09	3.6
9	12	13.2	2.14	3.0	1.46	2.25	10.7	13.8	7.31	10.2
10	10	11.0	1.45	1.86	0.86	1.21	8.7	10.20	5.15	6.6
11	13	14.3	2.55	3.76	1,83	2.97	11.8	15.6	8.45	12.6
12	10	11.0	1.28	1.62	0.72	0.99	7.7	9.0	4.32	5.4
13	11	12.1	1.80	2.42	1.16	1.71	9.8	12.0	6.33	8.4
14	13	14.3	2.20	3.11	1.51	2.35	10.2	13.2	6.99	9.6
15	12	13.2	1.82	2.44	1.17	1.74	9.1	11.4	5.87	7.80
16	14	15.4	2.33	3.35	1.63	2.58	10.0	13.2	7.00	10.2

Table 3 demonstrate that measuring each station has a substantial influence on the efficacy and efficiency of production operations in encouraged arrival by increasing system size and waiting time when compared to the poisson arrival for 10% discount by utilizing stopwatch time study methods.

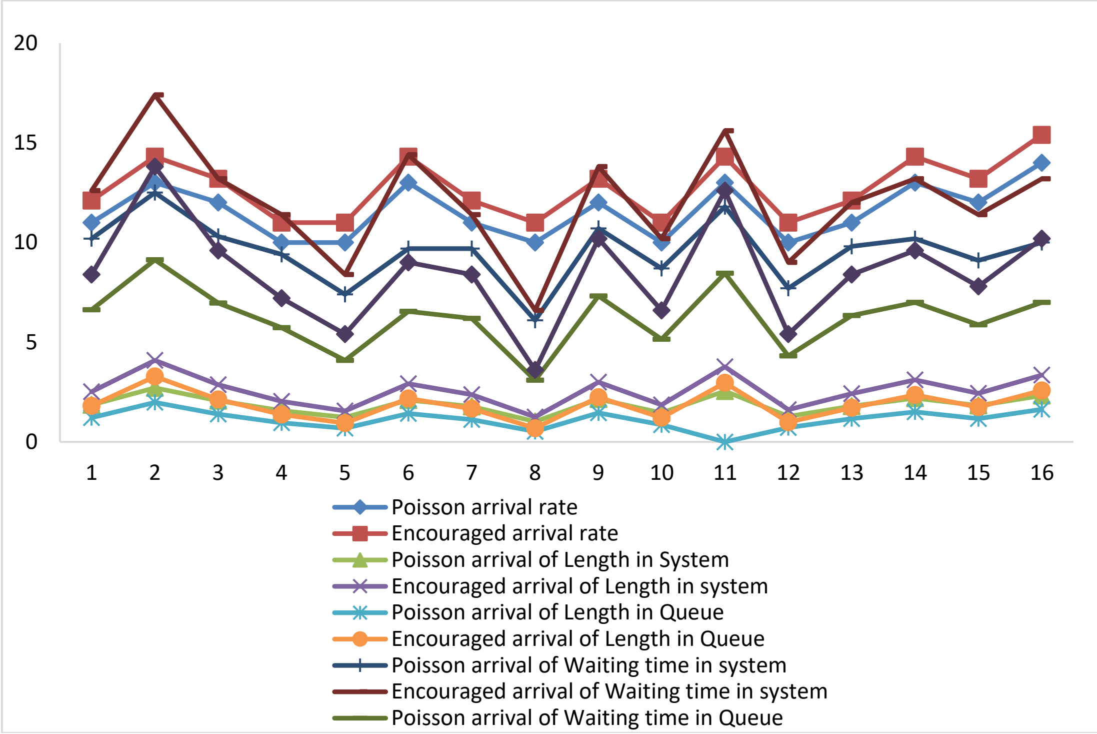

Figure 3: *Comparison of poisson arrival to encouraged arrival in systems and queues for 10% discounts*.

Table 4: *Verification of Little's law*

Stations	$\lambda(1+\chi)$	L_q	L_s	W_q	W_s	$L_q = \lambda(1+\chi)W_q$	$L_s = \lambda(1+\chi)\, W_s$
1	12.1	1.81	2.52	0.14	0.21	1.7	2.54
2	14.3	3.28	4.09	0.23	0.29	3.28	4.14
3	13.2	2.13	2.87	0.16	0.22	2.11	2.90
4	11.0	1.37	2.04	0.12	0.19	1.32	2.09
5	11.0	0.94	1.55	0.09	0.14	0.99	1.54
6	14.3	2.17	2.92	0.15	0.204	2.15	2.92
7	12.1	1.67	2.37	0.14	0.19	1.69	2.29
8	11.0	0.69	1.25	0.06	0.11	0.66	1.21
9	13.2	2.25	3.0	0.17	0.23	2.24	3.04
10	11.0	1.21	1.86	0.11	0.17	1.21	1.87
11	14.3	2.97	3.76	0.21	0.26	3.0	3.72
12	11.0	0.99	1.62	0.09	0.15	0.99	1.65
13	12.1	1.71	2.42	0.14	0.20	1.7	2.42
14	14.3	2.35	3.11	0.16	0.22	2.28	3.15
15	13.2	1.74	2.44	0.13	0.19	1.72	2.51
16	15.4	2.58	3.35	0.17	0.22	2.62	3.38

3.2 Encouraged arrival 20% of the queuing system:

Table 5: *Encouraged arrival for 20% discounts and utilization factor for a Markovian model.*

Stations	χ	$\lambda(1+\chi)$	(μ)	(ρ)	(1-ρ)	(L_s)	(L_q)	(W_S)	(W_q)
1	0.2	13.2	16.9	0.78	0.22	3.57	2.79	16.2	12.6
2	0.2	15.6	17.8	0.87	0.13	7.09	0.98	27.0	23.4
3	0.2	14.4	17.8	0.80	0.20	4.24	3.43	17.4	14.4
4	0.2	12.0	16.4	0.73	0.27	2.73	1.99	13.8	10.20
5	0.2	12.0	18.1	0.66	0.34	1.97	1.30	9.6	6.6
6	0.2	15.6	19.2	0.81	0.19	4.33	3.52	16.8	15.6
7	0.2	13.2	17.2	0.77	0.23	3.3	2.53	15.0	11.4
8	0.2	12.0	19.8	0.61	0.39	1.54	0.93	7.8	4.8
9	0.2	14.4	17.6	0.82	0.18	4.5	3.68	13.2	15.6
10	0.2	12.0	16.9	0.71	0.29	2.45	1.74	12.0	8.4
11	0.2	15.6	18.1	0.86	0.14	6.24	5.38	24.0	20.4
12	0.2	12.0	17.8	0.67	0.33	2.07	1.39	10.20	7.2
13	0.2	13.2	17.1	0.77	0.23	3.38	2.61	15.6	11.4
14	0.2	15.6	18.9	0.83	0.17	4.73	3.90	18.0	15.0
15	0.2	14.4	18.6	0.77	0.23	3.43	2.65	14.4	10.8
16	0.2	16.8	20.0	0.84	0.16	5.25	4.41	18.6	15.6

In M/M/1 automobile assembly line problem using the stopwatch time study approach method with 20% encouraged arrival for length in system and queue, as well as waiting time in system and queue.

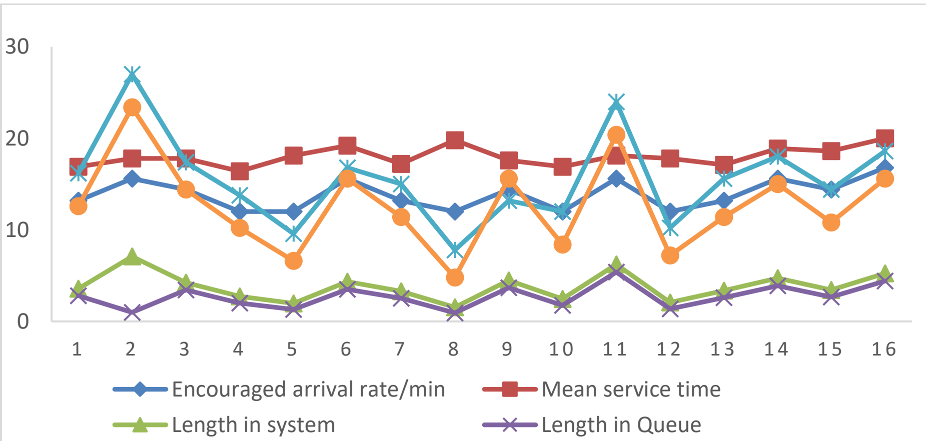

Figure 4: *Shows that the encouraged arrival of 20% discounts systems and queues size.*

The (table 6) demonstrate that measuring each station has a substantial influence on the efficacy and efficiency of production operations in encouraged arrival by increasing system size and waiting time when compared to the poisson arrival for 20% discount by using stopwatch time study approach.

Table 6: *Encouraged arrival 20% discounts model comparing with the Poisson arrival (PA) model.*

Stations	(λ)	$\lambda(1+\lambda)$	PA (L_s)	EA (L_s)	PA (L_q)	EA (L_q)	PA (W_s)	EA (W_s)	PA (W_q)	EA (W_q)
1	11	13.2	1.86	3.57	1.21	2.79	10.2	16.2	6.62	12.6
2	13	15.6	2.71	7.09	1.98	0.98	12.5	27.0	9.13	23.4
3	12	14.4	2.07	4.24	1.39	3.43	10.3	17.4	6.97	14.4
4	10	12.0	1.56	2.73	0.95	1.99	9.4	13.8	5.72	10.20
5	10	12.0	1.23	1.97	0.68	1.30	7.4	9.6	4.09	6.6
6	13	15.6	2.10	4.33	1.42	3.52	9.7	16.8	6.55	15.6
7	11	13.2	1.77	3.3	1.12	2.53	9.7	15.0	6.19	11.4
8	10	12.0	1.02	1.54	0.52	0.93	6.1	7.8	3.09	4.8
9	12	14.4	2.14	4.5	1.46	3.68	10.7	13.2	7.31	15.6
10	10	12.0	1.45	2.45	0.86	1.74	8.7	12.0	5.15	8.4
11	13	15.6	2.55	6.24	1,83	5.38	11.8	24.0	8.45	20.4
12	10	12.0	1.28	2.07	0.72	1.39	7.7	10.20	4.32	7.2
13	11	13.2	1.80	3.38	1.16	2.61	9.8	15.6	6.33	11.4
14	13	15.6	2.20	4.73	1.51	3.90	10.2	18.0	6.99	15.0
15	12	14.4	1.82	3.43	1.17	2.65	9.1	14.4	5.87	10.8
16	14	16.8	2.33	5.25	1.63	4.41	10.0	18.6	7.00	15.6

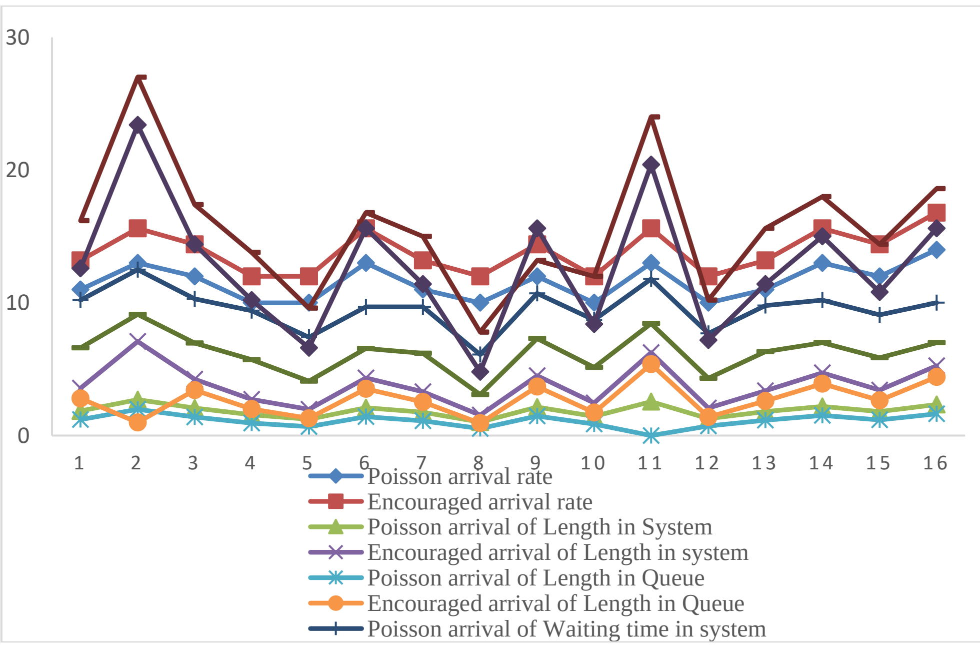

Figure 5: *Comparison of poisson arrival to encouraged arrival in systems and queues for 20% discounts.*

Table 7: *Verification of Little's law:*

Station	$\lambda(1+\chi)$	L_q	L_s	W_q	W_s	$L_q = \lambda(1+\chi)W_q$	$L_s = \lambda(1+\chi)\ W_s$
1	13.2	2.79	3.57	0.21	0.27	2.77	3.56
2	15.6	0.98	7.09	0.39	0.45	7.02	7.02
3	14.4	3.43	4.24	0.24	0.29	3.46	4.18
4	12.0	1.99	2.73	0.17	0.23	2.04	2.76
5	12.0	1.30	1.97	0.11	0.16	1.92	1.92
6	15.6	3.52	4.33	0.26	0.28	4.06	4.37
7	13.2	2.53	3.3	0.19	0.25	2.51	3.3
8	12.0	0.93	1.54	0.08	0.13	0.96	1.56
9	14.4	3.68	4.5	0.26	0.31	3.74	4.5
10	12.0	1.74	2.45	0.14	0.20	1.7	2.4
11	15.6	5.38	6.24	0.34	0.4	5.30	6.24
12	12.0	1.39	2.07	0.12	0.17	1.44	2.04
13	13.2	2.61	3.38	0.19	0.26	2.51	3.43
14	15.6	3.90	4.73	0.25	0.30	3.9	4.7
15	14.4	2.65	3.43	0.18	0.24	2.6	3.46
16	16.8	4.41	5.25	0.26	0.31	4.37	5.21

4. Results and Discussions

This research provides recommendation to improve the standard of service to be more effective and efficient. The stopwatch time study approach in [9] was used to determine the average service time and distribution of encouraged arrival rates for each workstation. The degree of variation in the materials or components used to make the cars, as well as the manufacturing processes, affect service times. The timelines for the vehicle under consideration's arrival and assembly are summarized in the table. In comparison to the Poisson arrival system, the size of the Markovian encouraged arrival queuing system increased.

5. Conclusion

Based on the findings, the queuing problem in an automobile assembly line to improve the standard of service to be more effective and efficient. The encouraged arrival and service distribution data for the system were determined. These data were used to calculate the system performance parameter. The result shows that increasing automation will lead to quicker processing times for parts and lower lead costs. The company will benefit greatly from this study because it will make it easier for management to plan future production by providing them with all the data pertaining to the performance of the company's assembly line.

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