RELIABILITY ANALYSIS OF THIN -WALLED PRESSURE VESSELS USING ADVANCED FIRST ORDER SECOND MOMENT (AFOSM) METHOD

K. Sunitha, T. Sumathi Uma Maheshwari, M. Tirumala Devi
•
Department of Mathematics, Kakatiya University, Warangal, Telangana, India-506009
kadiresunitha79@gmail.com, tsumathiuma@gmail.com, oramdevi@yahoo.com

Abstract

Pressure vessels are highly used in commercial purposes and industries such as boiling, softening and hot water storing tanks. Pressure vessels are subjected to its internal and external pressure. In this paper, thin-walled pressure vessels are taken for analysis. Pressure, radius, thickness and strength of the material are considered as random variables. The random variables follow normal distribution. The reliability index of the pressure vessel made with different materials such as 6061 aluminum alloy and SA 516 70 stainless steel has been found. Reliability analysis has been done for the pressure vessel by using AFOSM with MATLAB. It is observed that strength of the materials influences more on reliability of the vessels.

Keywords: Thin-walled cylinder, reliability, stress, strength, normal distribution, pressure, thickness, inner radius of cylinder, mean, variance.

1. Introduction

Chemical Industry requires the handling and storing the large quantities of material such as liquids and gases in containers or vessels and therefore pressure vessels play an important role in the industry. Pressure vessels are leak-proof containers that store liquid or gas. Pressure vessels [1] of various sizes and shapes have been produced for different purposes, constructing a required pressure vessel is engineer's task. Engineer's aim is to design a high-level performance of a vessel. Reliability is the probability of a design, should satisfy certain needs under the prescribed environment for certain period. Reliability methods are used to get specified reliability of a structure. Finding reliability of the design is inescapable. In 1981, Cornell [2] called structural reliability is a healthy adolescent.

L. Cizelj et al. [3] applied First Order Reliability Method (FORM) and Second Order Reliability Method (SORM) in the safety assessment of steam generator tubes with through-wall axial stress corrosion cracks. Gunjan Agarwal and Baidurya Bhattacharya [4] studied partial safety factor design of rectangular partially prestressed concrete beams in ultimate flexural limit state. Antanas Kudzys and Romualdas Kliukas [5] discussed the reliability index of the design of reinforced concrete structures of annular cross sections. P. Hari Prasad et al. [6] used FORM for the reliability analysis of a section of a structural beam and reliability index was found. Zheng Yulong et al. [7] used Hasofer-Lind method for thin -walled pressure vessels, compared with Second Moment Method and found that thickness of the pressure vessel was small. Devaraju A. and pazhanivel K. [8] studied reliability of thin -walled pressure vessels with ANSYS software.

In this paper, thin-walled pressure vessel is considered for the analysis. Found reliability index of the pressure vessel which is made with different materials such as 6061 aluminum alloy and SA 516 70 stainless steel. Reliability analysis has been done for the pressure vessel by using AFOSM with MATLAB.

2. Methodology

If the system has a deterministic strength X, and a randomly developed stress Y then the reliability of the system is the probability that the resistance is greater than the stress, $P(X > Y)$. Failure probability is the probability that the stress is greater than the strength $P(Y > X)$. In the specific case of a Gaussian random variable, the stress Y can be reduced into standard normal variable y,

$$Y = \mu_Y + y\sigma_Y \Rightarrow y = \frac{Y-\mu_Y}{\sigma_Y} \tag{1}$$

where μ_Y is the mean of Y and σ_Y is the standard deviation of Y, then the reliability of the system

$$R = P(X > \mu_Y + y\sigma_Y)$$
$$R = P(\frac{(X-\mu_Y)}{\sigma_Y} > y)$$
$$R = P(\beta > y) \tag{2}$$

where β is the reliability index, in the single variable case, this inequality is a safe region. This is the set of values of y for which the structure will not fail. The probability of failure is the complement of the reliability.

$$P_f = 1 - R = 1 - P(\beta > y) \tag{3}$$

Let $f_X(x)$ and $f_Y(y)$ are the probability density functions of strength X and stress Y. Then the distribution function F is

$$F_X(y) = P(Y \leq y) = \int_{-\infty}^{y} f_X(u)du \tag{4}$$

The probability of failure becomes

$$P_f = P(Y > X) = \int_0^\infty F_X(y) f_Y(y)dy$$
$$= \int_0^\infty \int_{-\infty}^{y} f_X(u)\ f_Y(y)dudy \tag{5}$$

2.1. First Order Reliability Method

Let $(X_1, X_2, X_3 \ldots\ldots X_n)$ be the set of random variables (structural design variables). The limit state equation for the failure surface of the structure is

$$g(X_1, X_2, X_3 \ldots\ldots X_n) = 0 \tag{6}$$

Collapse of the structure or failure is defined by the failure condition as $g(X_1, X_2, X_3 \ldots\ldots X_n) < 0$. Probability of failure is $P_f = P(g(X_1, X_2, X_3 \ldots\ldots X_n) < 0)$ and reliability is $P(g(X_1, X_2, X_3 \ldots\ldots X_n) > 0)$. Methods for the determination of this probability depends on the complexity of the limit state function. The limit state function is the limit at which the performance transits from acceptable to unacceptable.

There are different types of limit states: Ultimate limit state, serviceability limit state, etc. The limit state function is

$$g = X - Y \tag{7}$$

In the above expression, X and Y are random variables. X is the strength and Y is the stress developed in the structure. If $g < 0$ it leads to breakage of the structure. i.e., failure, and if $g > 0$ then the structure is safe.
If the random variables C, S for a linear performance function described by the equation (7) are normally distributed then C, S can be reduced as

$$C' = \frac{(C-\mu_C)}{\sigma_C} \text{ and } S' = \frac{(S-\mu_S)}{\sigma_S} \tag{8}$$

$$g = C - S = (C'\sigma_C + \mu_C) - (S'\sigma_S + \mu_S) \quad = (\mu_C - \mu_S) + \quad C'\sigma_C - \ S'\sigma_S \tag{9}$$

The line in reliability analysis is the line corresponding to $g(\ C', S') = 0$ because this line separates the safe and failure region in the standardized space. From the above, the reliability index β is the shortest distance from origin of reduced variables to the line of $g(C', S'\ ) = 0$

$$\beta = \frac{(\mu_C - \mu_S)}{\sqrt{\sigma_C^2 + \sigma_S^2}} \tag{10}$$

where μ_C and μ_S are the mean values of C and S respectively; σ_C^2 and σ_S^2 are their variance values. If the random variables C and S have the log- normal distribution, then the reliability index is given by

$$\beta = \frac{(\overline{W_C} - \overline{W_S})}{\sqrt{\sigma_{W_C}^2 + \sigma_{W_C}^2}} \tag{11}$$

where $W_C = logC$, and $W_S = logS$, $\overline{W_C}$, $\overline{W_S}$ are the mean values of W_C and W_S; $\sigma_{W_C}^2$ and $\sigma_{W_S}^2$ are their variance values.

If the limit state surface is linear, then the Hasofer -Lind reliability index coincides with the reliability index computed from FOSM. However, Hasofer -Lind reliability method is used for non -linear limit state surfaces. If the limit state line is closer to the origin in the standardized coordinate system, the failure region is larger, and if it is farther away from the origin, the failure region is smaller. Thus, the position of the limit state surface relative to the origin in the standardized coordinate system is a measure of the reliability of the system. Then the probability of failure is

$$P_f = P((C - S) < 0) = P(g < 0) \tag{12}$$

$$\mathrm{P_f} = \Phi(-\beta) \Rightarrow \beta = -\Phi^{-1}(\mathrm{P_f}) \tag{13}$$

and reliability $R = 1 - P_f$, where Φ and Φ^{-1} are the cumulative distribution function and its inverse.

2.2. Advanced First Order Second Moment Method (AFOSM)

In this method, the assessment of the reliability index is based on the reduction of the limit state function to the standardized coordinate system Thus, the random variables X_i, which are normally distributed and are reduced as

$$X_i' = \frac{(X_i - \mu_{X_i})}{\sigma_{X_i}}, \quad (i = 1,2 \dots \dots n) \tag{14}$$

where X_i' is a standardized random variable with zero mean and unit standard deviation, Thus, Eq. (8) is used to transform the original limit state surface $g(X) = 0$ into a reduced limit state surface $g(X') = 0$. Where, X denotes 'original coordinate system'. In the standardized coordinate system, Hasofer- Lind reliability index β is equal to the minimum distance from the origin to the limit state surface

$$\beta = \sqrt{(x^{*\prime})^T (x^{*\prime})} \tag{15}$$

The minimum distance point on the limit state surface is called the design point. It is denoted by vector $\boldsymbol{x}^*$ in the original coordinate system and by vector $\boldsymbol{x}^{*\prime}$ in the reduced coordinate system. These vectors represent the values of the random variables. For the general case of a non-linear limit state surface, the assessment of the minimum distance can be written as an optimization problem

$$\begin{aligned} &Minimize\, D = \sqrt{x'^T x'} \\ &Subject\ to\ g(X') = 0. \end{aligned} \tag{16}$$

By using Lagrange's multipliers, the minimum distance (for n variables) could be estimated as

$$\beta = -\frac{\sum_{i=1}^{n} x_i^{*\prime} \left(\frac{\partial g}{\partial X_i{}'}\right)^*}{\sqrt{\sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i{}'}\right)^{*2}}} \tag{17}$$

where $\left(\frac{\partial g}{\partial X_i{}'}\right)^*$ is the i^{th} partial derivative evaluated at the design point ($x_i^{*\prime}, x_2^{*\prime}, x_3^{*\prime}, \dots . x_n^{*\prime}$). The design point in the reduced coordinates is

$$x_i^{*\prime} = -\alpha_i \beta \ (i = 1, 2, \dots \dots, n) \tag{18}$$

where α_i are the direction cosines along the coordinate axes $X_i{}'$. They are given by

$$\alpha_i = \frac{\left(\frac{\partial g}{\partial X_i{}'}\right)^*}{\sqrt{\sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i{}'}\right)^{*2}}} \tag{19}$$

3. Results

3.1. Computation of reliability for thin-walled cylindrical shell pressure vessel

Thickness is very important in the design for the safety of the cylinder. Thickness of the cylinders is 20th part of diameter or even less. It is assumed that thin - walled cylindrical pressure vessel with SA516-70 stainless steel, and 6061 aluminum alloy. Yield strength, radius of the cylinder, pressure, thickness of the cylinder, are taken as random variables which follow normal distribution. Let (μ_X, μ_r, μ_p, μ_t), (σ_X, σ_r, σ_p, σ_t) be the mean and standard deviation design vector of the random variables.

Table 1: Design variables and materials properties

	SA51670stainless-steel		6061aluminumalloy	
	Mean	Standard deviation	Mean	Standard deviation
Yield strength (MPa)	335	16.56	276	16.56
Radius of cylinder(mm)	2000	100	2000	100
Inside pressure(MPa)	5	0.4	5	0.4
Thickness (mm)	50	2	50	2
Joint efficiency		0.85		

Let the thin cylindrical pressure vessel limit state function is [10]

$$g = X - Y = X - \frac{p(r-(0.6)t)}{Et} \tag{20}$$

where X is the strength of the cylinder, Y is stress of the cylinder, p is pressure in the cylinder, r is radius, t is thickness of the cylinder, E is joint efficiency. Therefore, the equation of failure surface is given by

$$g = X - \frac{p(r-(0.6)t)}{Et} = 0 \tag{21}$$

using eqn. (8), the failure surface function eqn. (20) in the standardized coordinate system is given by

$$g = (X'\sigma_X + \mu_X) - \frac{(p'\sigma_p+\mu_p)(r'\sigma_r+\mu_r-(0.6)(t'\sigma_t+\mu_t))}{E(t'\sigma_1+\mu_t)} = 0 \tag{22}$$

where X', p', r', t' are reduced random variables.

Since $X' = \beta\alpha_1, p' = \beta\alpha_2, r' = \beta\alpha_3, t' = \beta\alpha_4$ then

$$(\beta\alpha_1\sigma_X + \mu_X) - \frac{(\beta\alpha_2\sigma_p + \mu_p)(\beta\alpha_3\sigma_r + \mu_r - (0.6)(\beta\alpha_4\sigma_t + \mu_t))}{E(\beta\alpha_4\sigma_1 + \mu_t)} = 0 \tag{23}$$

$$(\beta\alpha_1\sigma_X + \mu_X)E(\beta\alpha_4\sigma_1 + \mu_t) - \left(\beta\alpha_2\sigma_p + \mu_p\right)\left((\beta\alpha_3\sigma_r + \mu_r) - (0.6)(\beta\alpha_4\sigma_t + \mu_t)\right) = 0 \tag{24}$$

$$\beta^2\left(E\alpha_1\alpha_3\sigma_X\sigma_t - \alpha_2\alpha_4\sigma_p\sigma_r + (0.6)\alpha_3\alpha_4\sigma_p\sigma_t\right) - \beta\left(E\alpha_1\sigma_X\mu_t + E\alpha_3\mu_X\mu_t - \alpha_4\sigma_p\mu_r - \alpha_2\mu_p\sigma_r + (0.6)\alpha_4\sigma_p\mu_t + (0.6)\alpha_3\mu_p\sigma_t\right) + \left(E\mu_X\mu_t - \mu_p\mu_r + (0.6)\mu_p\mu_t\right) = 0 \tag{25}$$

(26)

3.1. Mean thickness vs reliability

It is observed from Table 2 that if thickness of SA 516 70 Stainless steel increases from 50 mm to 140 mm, then reliability increases from 0.7882 to 0.9998 where as in 6061 aluminum alloy, the reliability of the material increases from 0.6627 to 0.9985. The reason for this is, with the increase of thickness, the stress is lowered at a given pressure.

Table 2: Mean thickness vs reliability

	SA51670 stainless-steel			6061aluminumalloy		
μ_t	β	P_f	R_1	β	P_f	R_2
50	0.8	0.2118	0.7882	0.42	0.3372	0.6627
60	1.29	0.0985	0.9014	0.85	0.1976	0.8023
70	1.68	0.0464	0.9535	1.21	0.1131	0.8868
82	2.03	0.0211	0.9788	1.51	0.0655	0.9344
100	2.56	0.0052	0.9947	2.05	0.0201	0.9798
120	3.08	0.0019	0.9989	2.53	0.0057	0.9942
140	3.58	0.0002	0.9998	2.97	0.0014	0.9985

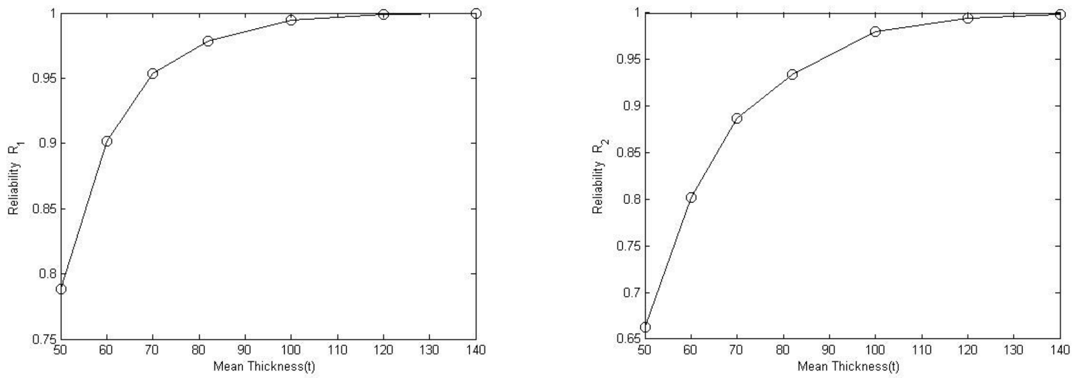

Figure 1: *Variation of reliability as a function of mean thickness*

3.2. Mean pressure vs reliability

It is observed from Table3 that if μ_p increases from 1.9 MPa to 5 MPa of SA 516 70 stainless steel, then reliability decreases from 0.9996 to 0.7882 and the reliability of 6061 aluminum alloy decreases from 0.9971 to 0.6627. By the increase of pressure, the stress will be developed in the cylinder and the increment in the stress causes low.

Table 3: Mean pressure vs reliability

	SA 51670 stainless steel			6061 aluminum alloy		
μ_p	β	P_f	R_1	β	P_f	R_2
1.9	3.35	0.0004	0.9996	2.77	0.0043	0.9971
2.5	3.19	0.0007	0.9993	2.04	0.0206	0.9793
3	2.06	0.0186	0.9804	1.59	0.0559	0.9440
4	1.38	0.0837	0.9163	0.93	0.1761	0.8238
5	0.8	0.2118	0.7882	0.42	0.3372	0.6627

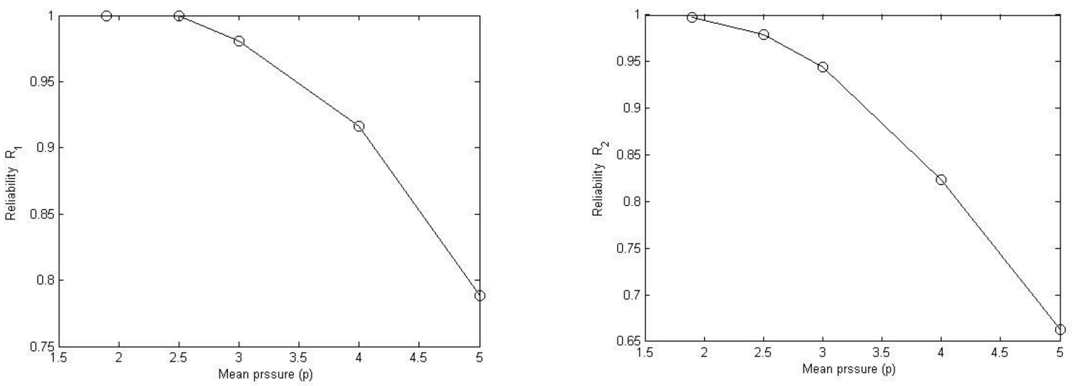

Figure 2: *Variation of reliability as a function of mean pressure*

3.3. Mean radius vs reliability

If mean radius of the design increases from 800 mm to 2000 mm, then reliability of SA 516 70 Stainless steel decreases from 0.9993 to 0.7882 and the reliability of 6061 aluminum alloy decreases from 0.9958 to 0.6627. With the increase of mean radius, the size of the cylinder increases, then the reliability decreases. The variation of reliability with mean diameter is shown below.

Table 4: Mean radius vs reliability

SA 516 70 Stainless steel				6061 aluminum alloy		
μ_r	β	P_f	R_1	β	P_f	R_2
800	3.21	0.0007	0.9993	2.64	0.0041	0.9958
1000	2.56	0.0053	0.9947	2.05	0.0201	0.9798
1200	2.07	0.0193	0.9807	1.61	0.0536	0.9463
1400	1.7	0.0446	0.9554	1.25	0.1056	0.8943
1600	1.28	0.0838	0.9162	0.94	0.1736	0.8263
1800	1.11	0.1335	0.8665	0.67	0.2514	0.7485
2000	0.8	0.2118	0.7882	0.42	0.3372	0.6627

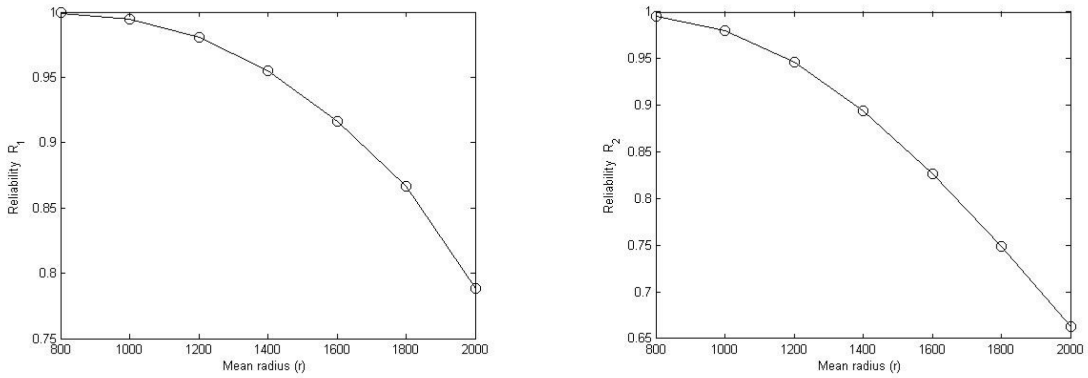

Figure 3: *Variation of reliability as a function of mean radius*

3.4. Mean strength vs Reliability

In this case, with the increment of mean strength, the deformation is lowered then the reliability of the cylinder enhances. The reliability of SA 516 70 Stainless steel increases from 0.7882 to 0.9846 with the increase of mean strength from 335 MPa to 580 MPa and in 6061 aluminum alloy, the reliability increases from 0.6628 to 0.7580 with the increase of mean strength from 276MPa to 312 MPa.

Table 5: Mean strength vs reliability

SA51670 Stainless steel				6061 aluminum alloy			
μ_c	β	P_f	R_1	μ_c	β	P_f	R_2
335	0.8	0.2118	0.7882	276	0.42	0.3372	0.6628
350	0.97	0.1669	0.8331	280	0.45	0.3263	0.6737
370	1.09	0.1378	0.8622	284	0.49	0.3120	0.6880
390	1.21	0.1131	0.8869	288	0.52	0.3015	0.6985
410	1.33	0.0917	0.9083	292	0.55	0.2911	0.7089
430	1.44	0.0749	0.9251	296	0.58	0.2809	0.7191

SA51670 Stainless steel				6061 aluminum alloy			
μ_c	β	P_f	R_1	μ_c	β	P_f	R_2
450	1.54	0.0617	0.9383	300	0.61	0.2709	0.7291
470	1.65	0.0494	0.9506	304	0.64	0.2610	0.7390
510	1.84	0.0328	0.9672	308	0.67	0.2514	0.7486
580	2.16	0.0154	0.9846	312	0.70	0.2419	0.7580

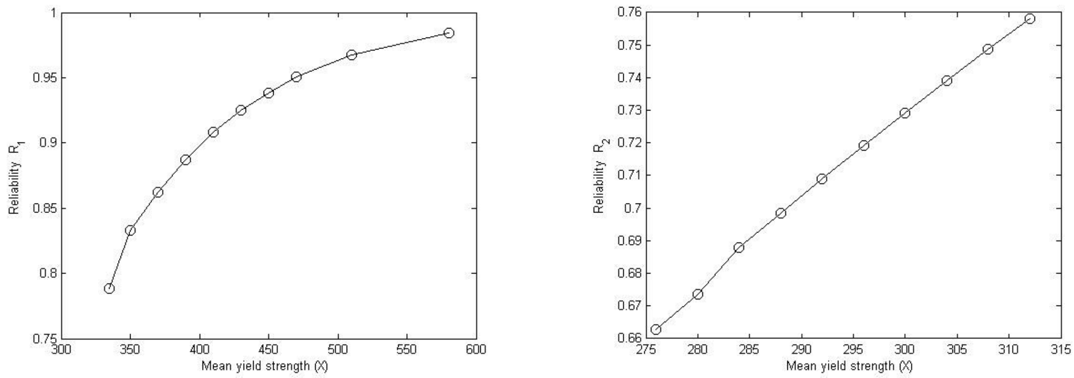

Figure 4: *Variation of reliability as a function of mean strength*

4. Conclusion

Prediction of the reliability of a pressure vessel leads to high performance of the vessel, several methods have been discussed to estimate the reliability of the vessel. In the analysis, thin-cylindrical pressure vessels with both ends closed are taken. Pressure, radius, thickness and strength of the material are considered as random variables and follow normal distribution. The reliability index of the pressure vessel made with different materials has been found. Reliability analysis has been done for the pressure vessel by using AFOSM with MATLAB.

The analysis shows that mean thickness of SA 516 70 stainless steel cylinder increases from 50 mm to 140 mm then reliability increases from 0.7882 to 0.9998 drastically and reliability of 6061 aluminum alloy cylinder increases from 0.6627 to 0.9985. If there is an increment in mean pressure of the cylinder from 1.9 MPa to 5 MPa then there is decrement in reliability from 0.9996 to 0.7882 and also a decrement in reliability of 6061 aluminum alloy cylinder from 0.6627 to 0.9971. Change in mean radius of the cylinder from 800 mm to 2000 mm causes change in reliability from 0.9996 to 0.7882. If mean strength of SA 516 70 stainless steel cylinder increases from 335 MPa to 630 MPa then reliability increases from 0.7882 to 0.9846. Thickness, diameter, pressure and strength of the cylinder influence the reliability of the pressure vessel. It is observed that strength of the materials has significant influence on reliability of the vessel.

References

[1] S. Rahman, G. Chen, R. Firmature, probabilistic analysis of off-centre cracks in cylindrical structures, *International Journal of pressure Vessels and Piping* 77(2000)3-16

[2] Cornell CA. Structural safety: some historical evidence that it is a healthy adolescent. In: *Proc. ICOSSAR' 81*, Trondheim, Norway; 1981. p. 19–31.

[3] L. Cizelj, B. Mavko, H. Riesch- Opeermann, Application of first and second order reliability methods in the safety assessment of cracked steam generator tubing, *Nuclear Engineering and Design*, 147 (1994) 359-368.

[4] Gunjan Agarwal and Baidurya Bhattacharya, Partial Safety factor design of rectangular partially pre- stressed concrete beams in ultimate flexural limit state, *Journal of structural Engineering*, Volume 37, No.4, October-November 2010 pp. 257-267.

[5] Antanas Kudzys and Romualdas Kliukas, Reliability index design in Reinforced concrete structures of annular cross sections, Modern- Building Materials and Techniques, the *10th International Conference*, 2010, Lithonia.

[6] P. Hari prasad, M. Tirumala Devi, T. Sumathi Uma Maheshwari, Reliability Analysis of Structural Beam Section using First order Reliability Method, *International Journal of Scientific Research in Science and Technology*, March-April-2018 [(4)5:01-03.

[7] Zheng Yulong, Lu Zhimin, Wang Linlin, Zhang Lin, Zhou Guangliang, The Application of Hasofer-Lind Method in Reliability Design of Thin-walled Cylindrical Shell, *Applied Mechanics and Materials*, 249-250, 303-306, 2013.

[8] Devaraju A., and pazhanivel K., A study on stress analysis for design of pressure vessel, *international journal of mechanical and production engineering*, 3(11), pp.98-101, 2015.

[9] Bhattacharya G.K. and Johnson R.A., Stress- Strength models for system reliability, *Proceedings of the symposium on reliability a fault tree analysis*, SIAM, 509-32, 1975.

[10] James R. Farr and Maan H. Jawad, GUIDEBOOK FOR THE DESIGN OF ASME SECTION VIII PRESSURE VESSELS, Fourth Edition, 2010.

[11] L.S. Srinath, Reliability Engineering, 2004.

[12] S. Ramamrutham, R. Narayan, Strength of Materials, 2018.

[13] R.S. Kurmi and N. Kurmi, Strength of Materials, S. Chand Publications, 2009.

[14] R.K. Bansal, "Strength of Materials", Laxmi Publications, 2009.

[15] K.C. Kapur and L.R. Lamberson, "Reliability in Engineering Design", John Wiley and Sons, Inc. U.K., 1997