TRADE CREDIT FINANCING SCHEME ON RETAILER'S ORDERING QUANTITY FOR IMPERFECT QUALITY ITEM WITH LEARNING EFFECTS AND STOCKING STRATEGIES

A. R. Nigwal[1], U. K. Khedlekar[2], N. Gupta[3] and L. Sharma[4]

•

[1,4]Department of Mathematics, Ujjain Engineering College, Ujjain, 456010, Madhya Pradesh India,
[2,3]Department of Mathematics and Statistics, Dr. Harisingh Gour Vishwavidyalaya,
Sagar, 470003, Madhya Pradesh, India, (A Central University)
[1]arnw@rediffmail.com

Abstract

Today's life is a age of modern life, and in the modern life for any kind of business setup, customer service, pricing, stocking strategies and trade credit financing schemes are effective, essential and survival parameters to grow the business. In this paper, we have developed, an economical order quantity model for imperfect quality product by considering retailer's stock sensitive demand of product under trade credit financing policy. Further in this paper we have studied the Learning effect on screening process on every batch of imperfect quality product. Under the trade credit financing scheme, we have considered that, the supplier proposes to the retailer, a fixed credit time period for payment and retailer also offers to his customers to a fixed credit time period of payment.

Finally an appropriate total profit function per unit time has been derived under the various trade credit financing periods of payment including various expenditure and other related parameters. A sensitivity analysis has been done to verify the optimum results and also a numerical example has been given to verify the model's outputs.

Keywords: Learning Effect, Stocking, Imperfect quality items, Trade credit policy. Screening process

1. Introduction

Generally, it has been analyzed, that a large number of consumer goods displayed in a shelf at shopping center or supermarket are connected with on sale items to induce more sales and profits. For any kind of an item, increment of shelf space induces more consumers to buy it. This possible because of its visibility, prominence or variety. Conversely, low stocks of certain paved goods might raise the feeling that they are not good and fresh. Therefore, demand is often based on inventory-level. In the last some decade, a considerable literature has been written in the operational research area on how inventory-level-dependent demand affects inventory control policies. Wu *et al.* [1] developed a inventory model for determining the optimal ordering quantity for non-instantaneous deteriorating products considering with stock-level dependent demand. They suggested that this may more beneficial for those situations in which backlogging parameter increases and decreases the order quantity. In (2005), Teng and Chang [2] Giri, and Bardhan, [3] formulated two layer supply chain coordination policy for deteriorating items with price stock level depended market demand of single product under revenue sharing contract. They concluded that the centralized system is the best strategy instead of decentralized system. developed EPQ models for deteriorating items considering with selling price and stock level dependent demand. They proposed appropriate decision for managerial activities. Ray and Chaudhuri,

[4], developed an EOQ model assuming a deterministic stock-dependent demand, incorporating with shortage, inflation and time discounting. They show through the numerical example that inventory backlogging is beneficial from both (retailer and supplier) organizational as well as economic viewpoints; Giri, and Chaudhuri, [5] designed a model considering with deterministic stock-dependent demand rate of perishable products incorporating time dependent nonlinear holding cost of the products. Parlar and Wang, [6] designed a quantity discounting decisions model for supplier and buyer relationship in which they start with Stackelberg equilibrium of the problem. They concluded quantity discount policy can be very effective in obtaining the maximum profit increase that the supplier and the retailer can possibly obtain together in certain cases. A deterministic inventory model is developed by Pal *et al.*, [7] assuming that the demand rate is stock-dependent and the items deterioration rate is constant. They highlighted the various problems related stocking of goods and further they optimized the profit function with respect to decision variable time and economical order quantity. Muth, and Spremann [8] provided a classical square root formula on the class of economical lot sizing problem considering with learning effects on the production process. Salameh *et al.* [9] developed a economical production inventory (EMQ) model which was formulated under the learning curve effect on the finite production rate. Cheng [10] Formulated an economical manufacturing quantity (*EOM*) under the influence of learning process. The order size is considered as to be large enough to allow the manufacturing learning phenomenon to manifest itself. The set-up cost is also assumed to reduce as a result of learning over the life of the product. Salameh and Jaber [11] developed a traditional (*EOQ/EPQ*) model for imperfect quality items using the (*EOQ/EPQ*) formulas. They also assumed that at the end of 100 percent screening work the poor-quality items are sold as a single batch with lower price.

Jaber and Guiffrida [12] Provided an (*EPQ*) model with rework for imperfect quality items using (*WLC*) wright learning curve. For this they proposed two different cases, first one is learning process adopted in production, no learning process in reworks and second one is learning process adopted in production and rework both. Eroglu and Ozdemir [13] developed an economical order quantity (*EOQ*) model in which they considered that each ordered lot contains some defective items incorporating with shortages at retailers end. They analyzed, how to affects optimal solution by increasing rate of percentage of defective items. They also assumed that, after 100 percent screening of each lot, the good and defective items are separated into two collection of imperfect quality and scrap items.

Jaber *et al.* [14] extended the work of Salameh and Jaber [11] by introducing the assumption that the percentage defective items per lot decreases under the learning curve (*LC*) effects, which was experimentally certified and validated by actual data of automotive industry. Jaber *et al.* [15] investigated the quality learning curve (*QLC*) for the assumption that the manufacturing process is interrupted due to maintain the quality to bring the process in control again. In this article they developed two various cases, first one is learning process is adopted in production, no learning process is adopted in reworks and second one is learning process is adopted in production and reworks both. Pan [16] analyzed the effect of learning curve on setup cost for their (*CRI*) model. They also assumed that the controllable lead time with the mixture of backorder and partial lost sales. Lin [17] investigated the market survey and manufacturing problem for a monopolist firm for quality and cumulative sales dependent demand. They also assumed that per unit production cost reduces with the cumulative manufacturing and learning effect.

Yoo *et al.* [18] focused on the problem that not only imperfect production process is possible but also inspection processes are always not perfect, due to generating defects and inspection errors. For this they developed a profit-maximizing economical manufacturing (*EMQ*) model by incorporating imperfect quality production and two-way imperfect inspection both. Sui *et al.* [19] provided a model for Vendor-Managed Inventory (*VMI*) system in place of traditional retailer-managed inventory applying with learning curve approach in which the supplier makes decisions of inventory management for the retailer.

Khan *et al.* [20] extended the paper of Salameh and Jaber's [11] model by introducing a new

case where there is learning in inspection. The model is more realistic than Salameh and Jaber's [11] model in the that they considered situations of lost sales and back-orders.
Wahab and. Jaber [21] developed a model for the optimal lot sizes of an item with imperfect quality which is extension of Salameh and Jaber [11] by incorporating different holding cost for good and defective items. Jaber and Khan [22] presented a model to develop a combination of performance of average processing time and process yield with respect to the number of equal batches. For the development of model, they changed the learning curve parameters in production system and rework both.

Das *et al.* [23] introduced a production-inventory model (EPQ) for deteriorating items in an indefinite conditions characterized by inflation and timed value of money by considering with static demand. They also considered that the planning interval of the business activity time is random in nature and follows exponential distribution function with a known mean. Khan *et al.* [24] extended the model of Salameh and Jaber [11] by introducing the inspection error in the time of the screening process and the probability of inspection errors is assumed to be known.
Konstantaras *et al.* [25] developed an economical order quantity (*EOQ*) model for imperfect quality items considering with shortages. They also assumed that the fraction of perfect quality in each shipment increases with respect to learning effect. A new and more advance inventory model for imperfect quality items has been developed by Jaggi *et al.* [26] under the situations of permissible delay in payments. Shortages are also allowed and fully backlogged, which are fulfilled during screening process. In this model It has been assumed that screening rate is enough greater than the demand rate.

Teng *et al.* [27] also proposed an economical order quantity (*EPQ*) model from the retailer's' point of view to determine his/her optimal production lot size (EPQ) and trade credit financing period simultaneously. For this they assumed that (i) trade credit financing scheme encourage not only sales but also opportunity cost and default risk, and (ii) production cost reduces with respect to learning curve effect. Kumar *et al.* [28] proposed the effect of learning on the economical ordering policy (EPQ) for deteriorating items incorporating shortages and partially backlogging. They also assumed that due to impact learning process the ordering cost is partly constant and partly decreasing in each cycle. Further they also considered the two-level storage cost for replenishment inventor.

Givi *et al.* [29] introduced a Human Reliability Analysis (*HRA*) model that estimates the human error rate while performing a collectively job under the domination of learning-forgetting and fatigue-recovery. This model is enable to quantify the human error rate dynamically with time. Agi and Soni [30] proposed a deterministic demand inventory model for jointly pricing and inventory control of a perishable product considering both physical deterioration and freshness condition degradation. They considered the market demand of product as price and stock sensitive. They suggested that when the primary demand is high, then the retailer would be interested in greedy the benefit of this higher primary demand by increasing the retailing price and accelerating the inventory turnover by reducing a cycle time in place of pricing strategies. Sarkar, and Sumon [31] extended an inventory model for deteriorating items with stock-level dependent market demand. This model has been studied in that situations in which backlogging rate and deterioration rate are time varying with respect to time. Further a sensitivity analysis is analyzed of the model's outputs with respect to key parameters. Jayaswal *et al.* [32] introduced trade credit financing inventory model for imperfect quality items under the effects of learning on ordering policy. They derived average profit function per cycle time by incorporating various expenditure costs and related parameters for the retailers and the optimization process is also shown by a numerical example. Yadav *et al.* [33], developed two layer supply chain model to study the effect of imperfect quality items under the asymmetric information with market expenditure sensitive demand.
Soni, and Shah [34] developed an economical production quantity (EPQ) model for retailer's by considering partially constant and partially stock stock sensitive demand. Further they also consider a new progressive credit period. They concluded which credit period is more beneficial for business activity. Benyong and Feng [35] developed a two layer supply chain inventory model

with revenue sharing contract and service requirement under the unpredictability of supply and demand. They formulated the buyer's and supplier's optimal coordination and service requirement situations. They demonstrated how the service requirement's impacts the buyer's and supplier's decisions. Nigwal *et al.*[36] designed an EPQ model on retailer's order quantity using learning effect on screening process under trade credit financing scheme. Nigwal *et al.*[37] developed three stage price dependent trade credit policy for supplier, manufacturer and retailer.

Generally, in the traditional economical order quantity (*EOQ*) models, it is assumed that the retailer pay to supplier as soon as the product is received. But in the practice, supplier to stimulate sales of his products,he offers to the retailer a certain permissible delay period of payment and after end of this delayed period he charges the interest. In this chapter we have considered a two stage trade credit financing periods, in which firstly supplier offers to the retailer a permissible delay period of payment and the retailer also offers to his customers a permissible delay period of payment without interest. Further, we have also assumed that every manufacturing system may manufacture some defective and good items both. The defective items may be detected by the screening process after delivery of imperfect quality items' batches.To separate the good and defective items we apply screening process on each batches of imperfect quality items on retailer's end. Furthermore we have applied learning effects on screening process.

Learning curve (*LC*) or Experience curve (*EC*) was derived first by Wright [38] in 1936. It is a mathematical tool which relates the learning variables and cumulative quantity of units. In this chapter we study the impact of learning on screening process on imperfect quality items. Sigmoid function is the ideal shape of all other learning curves and in this paper we use Sigmoid function which is formulated as $\alpha(n) = \left(\frac{\alpha}{g+e^{\beta n}}\right)$, where $\alpha(n)$ represents defective percentage rate of item in the single batch and n represents number of batches. β, $g > 0$ and $a > 0$ are the learning curve parameters.

Table 1: Comparative table for contribution of different authors:

Authors	Learning Effects	Screening	Trade Credit Financing	Pricing	Stock level Strategies
Wright (1936)	✓	×	×	×	×
Muth, and Spremann (1983)	✓	×	×	×	×
Salameh *et al.*(1993)	✓	×	×	×	×
Pal *et al.*(1993)	×	×	×	✓	×
Parlar and Wang (1994)	×	×	×	✓	×
Cheng (1994)	✓	×	×	×	×
Ray, and Chaudhuri(1997)	×	×	×	✓	✓
Giri, and Chaudhuri. (1998)	×	×	×	✓	×
Salameh and Jaber (2000)	✓	✓	×	×	×
Jaber *et al.*(2004)	✓	✓	×	×	×
Teng and Chang (2005)	×	×	×	✓	✓
Wu *et al.*(2006)	×	×	×	✓	✓
Eroglu and Ozdemir(2007)	✓	✓	×	×	×
Jaber and Guiffrida (2008)	✓	✓	×	×	×
Pan (2008)	✓	×	×	×	×
Lin (2008)	✓	×	×	✓	×
Soni, and Shah (2008)	×	×	✓	✓	✓
Jaber *et al.* (2008)	✓	✓	×	×	×
Yoo *et al.*	✓	✓	×	×	×
Sui, *et al.* (2010)	✓	✓	×	×	×
Khan *et al.* (2010)	✓	✓	×	×	×
Wahab and. Jaber (2010)	✓	✓	×	×	×
Jaber and Khan (2010)	✓	×	×	×	×
Das *et al.* (2010)	✓	×	×	×	×
Khan *et al.* (2011)	×	✓	×	×	×
Giri, and Bardhan (2012)	×	×	×	✓	✓
Sarkar, and Sumon (2013)	×	×	×	✓	✓
Konstantaras *et al.* (2012)	✓	✓	×	×	×
Jaggi *et al.* (2013)	×	✓	✓	×	×

Authors	Learning Effects	Screening	Trade Credit Financing	Pricing	Stock level Strategies
Teng *et al.* (2013)	✓	×	×	×	×
Kumar *et al.* (2013)	✓	×	×	×	×
Givi *et al.* (2015)	✓	×	×	×	×
Benyong and Feng (2017)	×	×	×	✓	✓
Jayaswal *et al.* (2019)	✓	✓	✓	×	×
Maher and Soni (2020)	×	×	×	✓	✓
Nigwal *et al.* (2022)	×	×	✓	✓	✓
Nigwal *et al.* (2022)	✓	✓	✓	✓	×
This paper	✓	✓	✓	×	✓

2. The Mathematical Model

I. Notations and Assumptions:

ϕ_n : Lot size of the n^{th} batch,

D : Demand rate of items in units per unit of time for perfect quality items, Where, $D = a + bI(t)$,

S_c : Setup cost per order,

ϕ_n : Initial stock of inventory,

C_p : Purchasing cost per unit of an item,

h : Holding cost of items per unit time,

p : Retailing price per unit of perfect quality items,

v : Retailing price (On discounted rate) per unit of defective items ($p > v$),

$\alpha(n)$: Percentage rate of defective items per batch,

T_n : Length of cycle for shipment per order,

χ : Screening rate of items per unit time ($D < \chi$),

C_s : Screening cost per unit items,

τ_n : Screening time of ϕ_n batch in planing time T_n, where, $\tau_n = \frac{\phi_n}{\chi} < T_n$,

I_e : Interest rate per unit \$ earned by retailer,

I_p : Interest rate per unit \$ paid by retailer,

TSR : Sells revenue,

TE : Total cost,

$\Pi(\phi_n)$: Retailer's total profit per unit time,

L : Length delay period of payment offered per cycle time by supplier to the retailer,

M : Length delay period of payment offered per cycle time by retailer to customers,

The following assumptions are assumed during the development of model:

- The supplier provides a fixed and predetermined credit period to settle the accounts to the supplier,
- For infinite supply rate, selling price p and optimal lot size ϕ_n, are decision variable,

- No scrape item will be obtain during the screening process,
- Screening procedure and demand of items occurs simultaneously ($D < \chi$).,
- It has been assumed that each lot size contains perfect and imperfect items both,
- It has been assumed that the price of the perfect quality items is greater than the imperfect quality items,
- It has been assumed that the earned interest rate is less than the payable interest rate,
- It has been that the retailer offers a permissible delay period of payment to his customers without interest to stimulate the sales,
- We has been assumed that a limited but maximum amount of stock displayed in a supermarket without leaving a negative impact on customers,
- It has been assumed that L, $M \in [0, T_n]$, only.
- During the formulation of profit, T_n is approximated by second term, because $b < 1$ and a is very large, therefore $\frac{b^2}{a^2} \approx 0$.

II. The Mathematical formulation of model

The inventory level of perfect quality items at any time t, is governed by the following differential equation:

$$\frac{dI(t)}{dt} = -(a + bI(t)),\ 0 \leq t \leq T_n, \tag{1}$$

with the boundary conditions: $I(0) = (1 - \alpha(n))\phi_n$, and $I(T_n) = 0$
solution of this equation : Where A is arbitrary constant, to remove the constant using the conditions$I(0) = (1 - \alpha(n))\phi_n$, then solutions becomes

$$I(t) = \frac{a}{b}(e^{-bt} - 1) + (1 - \alpha(n))\phi_n e^{-bt} \tag{2}$$

at time $t = T_n$, the T_n can be determined by the following formula

$$T_n = \frac{log(1 - \alpha(n))\phi_n}{a} \tag{3}$$

and according to the assumptions the screening time τ_n is given by the following formula

$$\tau_n = \frac{\phi_n}{\chi} \tag{4}$$

The Sales revenue $SR = p(1 - \alpha(n))\phi_n + v\alpha(n)\phi_n$,Ordering Cost $= O_c$,Purchasing Cost $= P_c\phi_n$, Screening Cost $= S_c\phi_n$, Inventory Holding Costs $= h\left[\frac{1}{a}(1 - (e^{bT_n}T_n b)) + \frac{(1-\alpha-(n))\phi_n}{b}(1 - e^{-bT_n})\right]$ and $h\left[\frac{\phi_n^2\alpha(n)}{\chi}\right]$ Now the total expenditure per cycle is given by:

$$TE = S_c + p_c\phi_n + s_c\phi_n + = h\left[\frac{a}{b}\left(-\frac{e^{-bT_n}}{b} - T_n\right) + \frac{(1 - \alpha(n))\phi_n e^{-bT_n}}{b} + \frac{a}{b^2} + \frac{(1 - \alpha(n))\phi_n}{b}\right] \tag{5}$$

At a time of each replenishment a fixed and certain credit period of payment L is provided by supplier to the retailer and similarly a fixed and certain period of payment M is also provided by retailer to their customers. Where M, L$\in (0, T_n)$ and $\tau_n \neq T_n$ There are four different cases available for retailer and their customers.

(1) $\tau_n \geq L \geq M$ (2) $\tau_n \geq M \geq L$ (3) $L \geq M \geq \tau_n$ (4) $M \geq L \geq \tau_n$

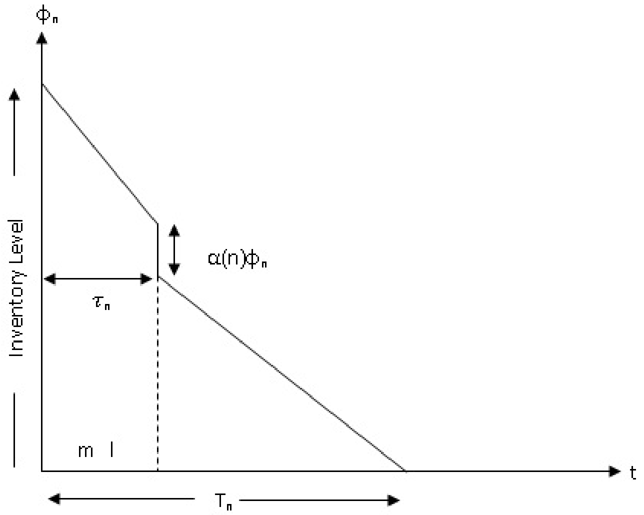

Figure 1: *Inventory Level Chart*

the retailer's whole profit $\Pi_i(\phi_n)$, i=1,2,3, 4 per unit of time can be defined as:

$$\Pi_i(\phi_n) = TSR_i\text{-}TE_i\text{+(Earned Interest)-(Paid Interest)}, \quad \text{where } i\text{=1, 2, 3, 4} \tag{6}$$

Case 1: $\tau_n \geq L \geq M$

As assumed credit periods, firstly we consider L is greater than M as per depicted in Figure 1, the earned interest and paid interest for this case is estimated as follows:

Earned Interest by retailer:

$$EI_r = I_e p[a + b(1-\alpha(n))\phi(L-M)] \tag{7}$$

Paid Interest by retailer:

$$PI_r = I_p C_p[a + b(1-\alpha(n))\phi_n b(L - \frac{(1-\alpha(n))}{a})\phi_n] + C_p I_p \alpha(n)\phi_n(\frac{\phi_n}{\chi} - L) \tag{8}$$

and total profit function per unit time may be defined as follow:

$$\Pi_1(\phi_n) = \frac{TSR_1\text{-}TE_1\text{+(Earned Interest)-(Paid Interest)}}{T_n} \tag{9}$$

Hence the total profit function per unit time is:

$$\begin{aligned}\Pi_1(\phi_n) &= pa + \frac{v\alpha(n)a}{(1-\alpha(n))} + \frac{I_e p a^2}{(1-\alpha(n))\phi_n} + ba(L-M)I_e p \\ &- \frac{(C_s + C_p\phi_n + S_c\phi_n)a}{(1-\alpha(n))\phi_n} - \frac{2hb}{a} - h(1-\alpha(n))\phi_n - \frac{h\phi_n\alpha(n)}{(1-\alpha(n))\chi} \\ &- I_p C_p\left[\frac{a}{(1-\alpha(n))\phi_n} + ab\left(L - \frac{(1-\alpha(n))}{a})\phi_n\right)\right] + \frac{C_p I_p \alpha(n) a}{(1-\alpha(n))}\left(\frac{\phi_n}{\chi} - L\right). \end{aligned} \tag{10}$$

Theorem 2.1. *Retailer's profit function is an optimum at retailer's ordering quantity ϕ_n^*, where ϕ_{n1}^* is given by the following equation:*

$$\phi_{n1}^* = \sqrt{\frac{C_s a - I_e p a^2 + C_p I_p a^2}{h(1-\alpha(n))^2 + \frac{h\alpha(n)}{\chi} - C_p I_p b(1-\alpha(n))^2 + \frac{C_p I_p \alpha(n) a}{\chi}}}. \tag{11}$$

Proof. On differentiating the equations (10) with respect to ϕ_n, we get

$$\begin{aligned}\frac{d\Pi_1(\phi_n)}{d\phi_n} &= -\frac{I_e pa^2}{(1-\alpha(n))\phi_n^2} + \frac{C_s a}{(1-\alpha(n))\phi_n^2} - h(1-\alpha(n)) - \frac{h\alpha(n)}{\chi(1-\alpha(n))} \\ &+ \frac{C_p I_p a^2}{(1-\alpha(n))\phi_n^2} + C_p I_p b(1-\alpha(n)) - \frac{C_p I_p \alpha(n) a}{(1-\alpha(n))\chi} \end{aligned} \tag{12}$$

As per optimality condition, on equating to zero the above equation (12), yields

$$-I_e pa^2 + C_s a - h(1-\alpha(n))^2\phi^2 - \frac{h\alpha(n)\phi^2}{\chi} + C_p I_p\left(a^2 + b(1-\alpha(n))^2\phi^2 - \frac{\alpha(n)a\phi^2}{\chi}\right) = 0$$

On solving the above equation we obtain the equation (11) □

Theorem 2.2. *As per optimality condition, at the point of optimality, the second derivative $\frac{d^2\Pi_1}{d\phi_n^2}$ is always negative if*

$$\frac{2I_e pa^2}{(1-\alpha(n))\phi_n^3} - \frac{2C_s a}{(1-\alpha(n))\phi_n^3} - \frac{C_p I_p \alpha(n) a^2}{(1-\alpha(n))^3} < 0 \tag{13}$$

Proof. On differentiating again the equation (12) with respect to ϕ_n, we get the second order derivative is

$$\frac{d^2\Pi_1}{d\phi_n^2} = \frac{2I_e pa^2}{(1-\alpha(n))\phi_n^3} - \frac{2C_s a}{(1-\alpha(n))\phi_n^3} - \frac{C_p I_p \alpha(n) a^2}{(1-\alpha(n))^3} \tag{14}$$

As per assumptions all the terms $\frac{2I_e pa^2}{(1-\alpha(n))\phi_n^3}$, $\frac{2C_s a}{(1-\alpha(n))\phi_n^3}$ and $\frac{C_p I_p \alpha(n) a^2}{(1-\alpha(n))^3}$ are always positive, and therefore by numerical analysis

$$\frac{2I_e pa^2}{(1-\alpha(n))\phi_n^3} - \frac{2C_s a}{(1-\alpha(n))\phi_n^3} - \frac{C_p I_p \alpha(n) a^2}{(1-\alpha(n))^3} < 0.$$

□

Case 2: $\tau_n \geq M \geq L$

As assumed credit periods, we consider M is greater than L as per depicted in the Figure 2, the earned interest and paid interest for this case is estimated as follows:

Earned Interest by retailer:

$$EI_r = 0 \tag{15}$$

Paid Interest by retailer:

$$PI_r = I_p C_p[a + b(1-\alpha(n))\phi_n b(L - \frac{(1-\alpha(n))}{a})\phi_n] + C_p I_p \alpha(n)\phi_n(\frac{\phi_n}{\chi} - L) \tag{16}$$

and total profit function per unit time may be defined as follow:

$$\Pi_2(\phi_n) = \frac{TSR_2\text{-}TE_2\text{+(Earned Interest)-(Paid Interest)}}{T_n} \tag{17}$$

Hence the total profit function per unit time is:

$$\begin{aligned}\Pi_2(\phi_n) &= pa + \frac{\nu\alpha(n)a}{(1-\alpha(n))} - \frac{(C_s + C_p\phi_n + S_c\phi_n)a}{(1-\alpha(n))\phi_n} - \frac{2hb}{a} - h(1-\alpha(n))\phi_n \\ &- \frac{h\phi_n\alpha(n)}{(1-\alpha(n))\chi} - I_p C_p\left[\frac{a^2}{(1-\alpha(n))\phi_n} + ab\left(\frac{(1-\alpha(n))\phi_n}{a} - M\right)\right] \\ &- \frac{C_p I_p \alpha(n) a}{(1-\alpha(n))}\left(\frac{\phi_n}{\chi} - L\right). \end{aligned} \tag{18}$$

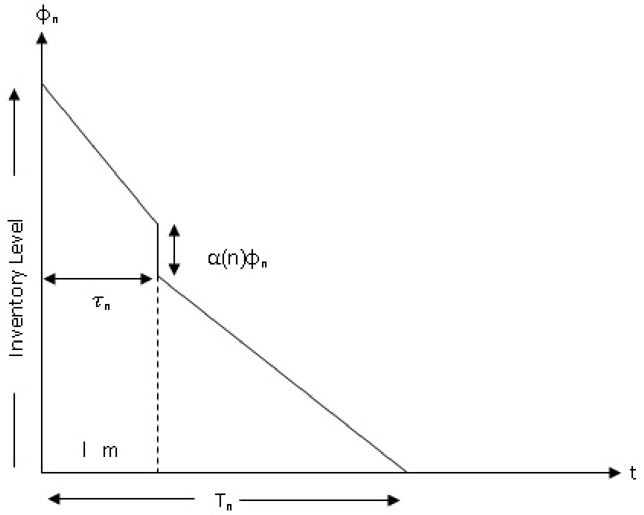

Figure 2: *Inventory Level Chart*

Theorem 2.3. *Retailer's profit function is an optimum at retailer's ordering quantity ϕ^*_{n2}, where ϕ^*_n is given by the following equation:*

$$\phi^*_{n2} = \sqrt{\frac{C_s a + C_p I_p a^2}{h(1-\alpha(n))^2 + \frac{h\alpha(n)}{\chi} + C_p I_p b(1-\alpha(n))^2 + \frac{C_p I_p \alpha(n) a}{\chi}}}. \tag{19}$$

Proof. On differentiating the equations (18) with respect to ϕ_n, we get

$$\begin{aligned} \frac{d\Pi_2(\phi_n)}{d\phi_n} &= \frac{C_s a}{(1-\alpha(n))\phi_n^2} - h(1-\alpha(n)) - \frac{h\alpha(n)}{\chi(1-\alpha(n))} \\ &+ \frac{C_p I_p a^2}{(1-\alpha(n))\phi_n^2} - C_p I_p b(1-\alpha(n)) - \frac{C_p I_p \alpha(n) a}{(1-\alpha(n))\chi} \end{aligned} \tag{20}$$

As per optimality condition, on equating to zero the above equation (20), yields

$$C_s a - h(1-\alpha(n))^2\phi^2 - \frac{h\alpha(n)\phi^2}{\chi} + C_p I_p \left(a^2 - b(1-\alpha(n))^2\phi^2 - \frac{\alpha(n) a \phi^2}{\chi}\right) = 0$$

On solving the above equation we obtain the value of ϕ^*_{n2} given in the equation (19) □

Theorem 2.4. *As per optimality condition, at the point of optimality, the second derivative $\frac{d^2\Pi_2}{d\phi_n^2}$ is always negative if*

$$\frac{2C_s a}{(1-\alpha(n))\phi_n^3} + \frac{C_p I_p a^2}{(1-\alpha(n))^3} > 0 \tag{21}$$

Proof. On differentiating again the equations (20) with respect to ϕ_n, we get the second order derivative is

$$\frac{d^2\Pi_2}{d\phi_n^2} = -\frac{2C_s a}{(1-\alpha(n))\phi_n^3} - \frac{C_p I_p a^2}{(1-\alpha(n))^3} \tag{22}$$

As per article's assumptions all the terms $\frac{2C_s a}{(1-\alpha(n))\phi_n^3}$ and $\frac{C_p I_p a^2}{(1-\alpha(n))^3}$ are always positive, and therefore

$$-\frac{2C_s a}{(1-\alpha(n))\phi_n^3} - \frac{C_p I_p a^2}{(1-\alpha(n))^3} < 0.$$

□

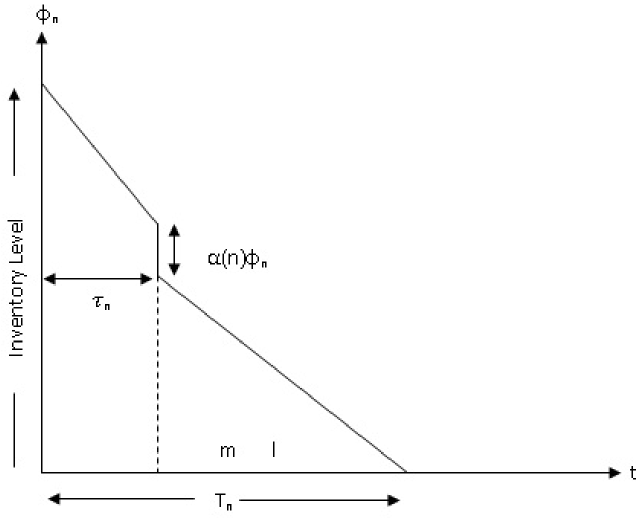

Figure 3: *Inventory Level Chart*

***Case* 3:** $L \geq M \geq \tau_n$

As assumed credit periods, we consider L is greater than τ and M as pr depicted in the Figure 3, the earned interest and paid interest for this case is estimated as follows:

Earned Interest by retailer:

$$EI_r = I_e p[a + b(1-\alpha(n))\phi(M-L)] + \nu I_e \alpha(n)\phi(L-\tau) \tag{23}$$

Paid Interest by retailer:

$$PI_r = I_p C_p \left[a + b(1-\alpha(n))\phi_n \left(\frac{(1-\alpha(n))\phi_n}{a} - L\right)\right] \tag{24}$$

and total profit function per unit time may be defined as follow:

$$\Pi_3(\phi_n) = \frac{TSR_3\text{-}TE_3\text{+(Earned Interest)-(Paid Interest)}}{T_n} \tag{25}$$

Hence the total profit function per unit time is:

$$\begin{aligned}\Pi_3(\phi_n) &= pa + \frac{\nu\alpha(n)a}{(1-\alpha(n))} + \frac{\nu I_e \alpha(n)a}{(1-\alpha(n))}\left(L - \frac{\phi_n}{\chi}\right) - \frac{(C_s + C_p\phi_n + S_c\phi_n)a}{(1-\alpha(n))\phi_n} - \frac{2hb}{a} \\ &- I_p C_p \left[a - ab\left(L + \frac{a^2 L}{1-\alpha(n)\phi_n}\right) + b((1-\alpha(n))\phi_n\right] - h(1-\alpha(n))\phi_n \\ &- \frac{h\phi_n\alpha(n)a}{(1-\alpha(n))\chi} + \frac{C_p I_p \alpha(n)a}{(1-\alpha(n))}\left(\frac{\phi_n}{\chi} - L\right).\end{aligned} \tag{26}$$

Theorem 2.5. *Retailer's profit function is an optimum at retailer's ordering quantity ϕ^*_{n3}, where ϕ^*_{n3} is given by the following equation:*

$$\phi^*_{n3} = \sqrt{\frac{C_s a + C_p I_p L a^2}{h(1-\alpha(n))^2 + \frac{h\alpha(n)a}{\chi} + C_p I_p b(1-\alpha(n))^2 + \frac{\nu\alpha(n)I_e a}{\chi}}}. \tag{27}$$

Proof. On differentiating the equations (26) with respect to ϕ_n, we get

$$\begin{aligned}\frac{d\Pi_3(\phi_n)}{d\phi_n} &= -\frac{I_e p a^2}{(1-\alpha(n))\phi_n^2} + \frac{C_s a}{(1-\alpha(n))\phi_n^2} - h(1-\alpha(n)) - \frac{h\alpha(n)}{\chi(1-\alpha(n))} \\ &+ \frac{C_p I_p a^2}{(1-\alpha(n))\phi_n^2} + C_p I_p b(1-\alpha(n)) - \frac{C_p I_p \alpha(n)a}{(1-\alpha(n))\chi}\end{aligned} \tag{28}$$

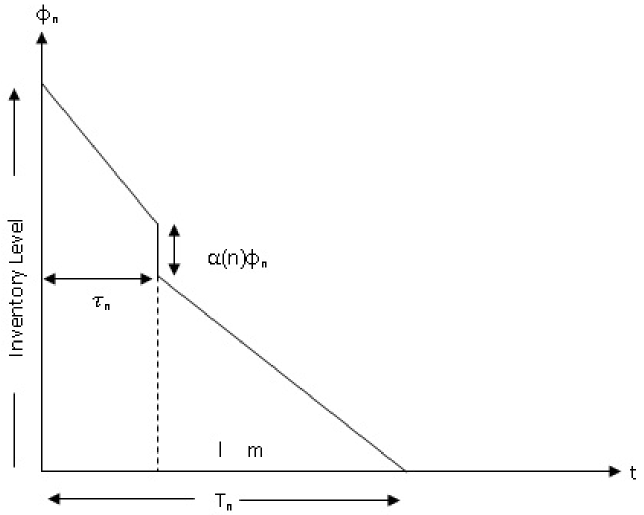

Figure 4: *Inventory Level Chart*

As per optimality condition, on equating to zero the above equation (28), yields

$$C_s a + a^2 C_p I_p L - h(1-\alpha(n))^2\phi_n^2 - \frac{h\alpha(n)\phi_n^2 a}{\chi} - C_p I_p b(1-\alpha(n))^2\phi_n^2 - \frac{\nu I_e \alpha a \phi^2}{\chi} = 0 \quad (29)$$

On solving the above equation we obtain the equation (27) □

Theorem 2.6. *As per optimality condition, at the point of optimality, the second derivative* $\frac{d^2\Pi_3}{d\phi_n^2}$ *is always positive if*

$$\frac{2C_s a}{(1-\alpha(n))\phi_n^3} + \frac{2C_p I_p a^2 L}{(1-\alpha(n))\phi_n^3} > 0 \quad (30)$$

Proof. On differentiating again the equations (28) with respect to ϕ_n, we get the second order derivative is

$$\frac{d^2\Pi_1}{d\phi_n^2} = -\frac{2C_s a}{(1-\alpha(n))\phi_n^3} - \frac{2C_p I_p a^2 L}{(1-\alpha(n))\phi_n^3} \quad (31)$$

As per assumptions all the terms $\frac{2C_s a}{(1-\alpha(n))\phi_n^3}$ and $\frac{2C_p I_p a^2 L}{(1-\alpha(n))\phi_n^3}$ are always positive, and therefore,

$$-\frac{2C_s a}{(1-\alpha(n))\phi_n^3} - \frac{2C_p I_p a^2 L}{(1-\alpha(n))\phi_n^3} < 0.$$

□

Case 4: $M \geq L \geq \tau_n$

As assumed credit periods, we consider M is greater than L and τ as per depicted in the Figure 4, the earned interest and paid interest for this case is estimated as follows:

Earned Interest by retailer:

$$EI_r = I_e p\left[a + b(1-\alpha(n))\phi_n\right] L + \nu I_e \alpha(n)\phi_n (L - \tau_n) \quad (32)$$

Paid Interest by retailer:

$$PI_r = I_p C_p \left[a + b(1-\alpha(n))\phi_n \left(\frac{(1-\alpha(n))\phi_n}{a} - L\right)\right] \quad (33)$$

and total profit function per unit time may be defined as follow:

$$\Pi_4(\phi_n) = \frac{TSR_4\text{-}TE_4\text{+(Earned Interest)-(Paid Interest)}}{T_n} \quad (34)$$

Hence the total profit function per unit time is:

$$\begin{aligned}\Pi_4(\phi_n) &= pa + \frac{\nu\alpha(n)a}{(1-\alpha(n))} - C_pI_p\left(b(1-\alpha(n))\phi_n - aL\right) - \frac{(C_s + C_p\phi_n + S_c\phi_n)a}{(1-\alpha(n))\phi_n} \\ &- \frac{2hb}{a} - h(1-\alpha(n))\phi_n - \frac{h\phi_n\alpha(n)a}{(1-\alpha(n))\chi} - I_epLa\left[\frac{a}{(1-\alpha(n))\phi_n} + b\right] \\ &+ \frac{\nu I_e\alpha(n)a}{(1-\alpha(n))}\left(L - \frac{\phi_n}{\chi}\right) - C_pI_Pa\left(1 + \frac{aL}{(1-\alpha(n))\phi_n}\right). \end{aligned} \quad (35)$$

Theorem 2.7. *Retailer's profit function is an optimum at retailer's ordering quantity ϕ^*_{n4}, where ϕ^*_{n4} is given by the following equation:*

$$\phi^*_{n4} = \sqrt{\frac{C_sa - I_epa^2L - C_pI_pa^2L}{h(1-\alpha(n))^2 + \frac{h\alpha(n)a}{\chi} + C_pI_pb(1-\alpha(n))^2 + \frac{\nu\alpha(n)}{\chi}}}. \quad (36)$$

Proof. On differentiating the equations (35) with respect to ϕ_n, we get

$$\begin{aligned}\frac{d\Pi_4(\phi_n)}{d\phi_n} &= \frac{C_sa}{(1-\alpha(n))\phi_n^2} - h(1-\alpha(n)) - \frac{h\alpha(n)a}{\chi(1-\alpha(n))} \\ &- \frac{I_epa^2L}{(1-\alpha(n))\phi_n^2} - C_pI_pb(1-\alpha(n)) - \frac{\nu\alpha(n)a}{(1-\alpha(n))\chi} - \frac{I_pC_pL}{(1-\alpha(n))\phi_n^2}. \end{aligned} \quad (37)$$

As per optimality condition, on equating to zero the above equation (37), yields

$$C_sa - h(1-\alpha(n))^2\phi^2 - I_epa^2L - \frac{\alpha(n)\phi^2}{\chi}(ah + I_e\nu) - C_pI_p\left(a^2 + b(1-\alpha(n))^2\phi^2\right) = 0 \quad (38)$$

On solving the above equation we obtain the equation (36) □

Theorem 2.8. *As per optimality condition, at the point of optimality, the second derivative $\frac{d^2\Pi_4}{d\phi_n^2}$ is always negative if*

$$\frac{2I_epa^2L}{(1-\alpha(n))\phi_n^3} - \frac{2C_sa}{(1-\alpha(n))\phi_n^3} - \frac{2C_pI_pa^2L}{(1-\alpha(n))\phi_n^3} < 0 \quad (39)$$

Proof. On differentiating again the equation (37) with respect to ϕ_n, we get the second order derivative is

$$\frac{d^2\Pi_1}{d\phi_n^2} = \frac{2I_epa^2L}{(1-\alpha(n))\phi_n^3} - \frac{2C_sa}{(1-\alpha(n))\phi_n^3} - \frac{2C_pI_pa^2L}{(1-\alpha(n))\phi_n^3} \quad (40)$$

As per assumptions all the terms $\frac{2I_epa^2L}{(1-\alpha(n))\phi_n^3}$, $\frac{2C_sa}{(1-\alpha(n))\phi_n^3}$ and $\frac{2C_pI_pa^2L}{(1-\alpha(n))\phi_n^3}$ are always positive, and therefore by numerical analysis

$$\frac{2I_epa^2L}{(1-\alpha(n))\phi_n^3} - \frac{2C_sa}{(1-\alpha(n))\phi_n^3} - \frac{2C_pI_pa^2L}{(1-\alpha(n))\phi_n^3} < 0.$$

□

3. Numerical Examples

Case-1:

We have considered the following data set of input parameters is given as: $\alpha = 160$ units/unit time, $\beta = 1$, $S_c = \$0.5$, $h = \$0.8$ unit/ unit time, $C_p = \$150$ /unit, $v = \$45$ per/unit, $\chi = 5000$ units, $C_s = \$100$/unit, $I_e = 0.003$/unit time, $I_p = \$0.004$/unit time, $\alpha(n) = 0.1599$, $n = 1$, $a = 455$, $b = 0.25$, $g = 999$ $L = 0.06$/unit time, $M = 0.05$/unit time.

Following the proposed restrictions for this case we may get the optimal ordering quantity (OOQ) $\phi_n = 339$ units per unit time, $p = 190$ and after substituting these optimum values ϕ_n, and p into the equation (10) we get the retailer's profit $\Pi(\phi_n) = 18266$, screening time $\tau_n = 0.06$ per unit time, and time interval is $T_n = 0.250$ in year.

Case-2:

We have considered the following data set of input parameters is given as: $\alpha = 160$ units/unit time, $\beta = 1$, $S_c = \$0.5$, $h = \$0.8$ unit/ unit time, $C_p = \$150$ /unit, $v = \$45$ per/unit, $\chi = 5000$ units, $C_s = \$100$/unit, $I_e = 0.003$/unit time, $I_p = \$0.004$/unit time, $\alpha(n) = 0.1599$, $n = 1$, $a = 455$, $b = 0.25$, $g = 999$ $L = 0.06$/unit time, $M = 0.07$/unit time.

Following the proposed restrictions for this case we may get the optimal ordering quantity (O OQ) $\phi_n = 423$ units per unit time, $p = 190$ and after substituting these optimum values ϕ_n, and p into the equation (18) we get the retailer's profit $\Pi(\phi_n) = 17174$, screening time $\tau_n = 0.08$ per unit time, and time interval $T_n = 0.363$ in year.

Case-3:

We have considered the following data set of input parameters is given as: $\alpha = 160$ units/unit time, $\beta = 1$, $S_c = \$0.5$, $h = \$0.8$ unit/ unit time, $C_p = \$190$ /unit, $\nu = \$45$ per/unit, $\chi = 5000$ units, $C_s = \$100$/unit, $I_e = 0.003$/unit time, $I_p = \$0.004$/unit time, $\alpha(n) = 0.1599$, $\eta = 1$, $a = 455$, $b = 0.25$, $g = 999$ $L = 0.09$/unit time, $M = 0.07$/unit time.
Following the proposed restrictions for this case we may get the optimal ordering quantity (O OQ) $\phi_n = 244$ units per unit time, $p = 190$ and after substituting these optimum values ϕ_n, and p into the equation (26) we get the retailer's profit $\Pi(\phi_n) = 39979$, screening time $\tau_n = 0.049$ per unit time, and time interval $T_n = 0.219$ in year.

Case-4:

We have considered the following data set of input parameters is given as: $\alpha = 160$ units/unit time, $\beta = 1$, $S_c = \$0.5$, $h = \$0.8$ unit/ unit time, $C_p = \$150$ /unit, $v = \$45$ per/unit, $\chi = 5000$ units, $C_s = \$100$/unit, $I_e = 0.003$/unit time, $I_p = \$0.004$/unit time, $\alpha(n) = 0.1599$, $n = 1$, $a = 455$, $b = 0.25$, $g = 999$ $L = 0.06$/unit time, $M = 0.09$/unit time.
Following the proposed restrictions for this case we may get the optimal ordering quantity (O OQ) $\phi_n = 181$ units per unit time, $p = 190$ and after substituting these optimum values ϕ_n, and p into the equation (35) we get the retailer's profit $\Pi(\phi_n) = 17364$, screening time $\tau_n = 0.036$ per unit time, and time interval $T_n = 0.164$ in year.

4. Sensitivity Analysis

A perusal of Table 2, shows that if the learning ability of the workers is 1 then 16 shipments will be required for the workers to acquire the proficiency. And, if the learning ability of the workers is 1.2 then 16 shipments will be required for the workers to acquire proficiency of screening process. And again, if the learning ability of the workers is 1.4 then 13 shipments will be required for the workers to acquire proficiency of screening process. In addition, as we increase the learning ability of the workers, the lot size, profit function, screening time and planning time increase, while the number of defective items and number of shipments decrease.The same situations are applying for table numbers 3, 4 and 5 as well. A learning efficiency of worker's not only reduces the number of shipments but also increases the profit per unit time. Table 6 shows the comparative study of various cases. Observation of tables 6, 7, 8 and 9 reveals that Case 3. gives better results per unit time and there is no considerable effect of Beta on outputs of various cases.

Table 2: Impact of Learning Rate and No. of Shipments on Outputs (Case 1)

No. of Shipment (n)	ϕ_n	% of good item	τ_n	T_n	Profit	No of Shipment Required for Perfection
Learning Rate $\beta = 1$						
1	339	84.03%	0.068	0.253	9140	
4	336	84.81%	0.067	0.252	9670	
7	307	92.37%	0.061	0.251	14298	
10	284	99.31%	0.057	0.250	17930	
13	282	99.96%	0.056	0.250	18249	n=16
16	282	100.00%	0.056	0.250	18265	
19	282	100.00%	0.056	0.250	18266	
22	282	100.00%	0.056	0.250	18266	
25	282	100.00%	0.056	0.250	18266	
Learning Rate $\beta = 1.2$						
1	339	84.04%	0.068	0.253	9147	
4	332	85.72%	0.066	0.252	10269	
7	291	97.06%	0.058	0.251	16813	
10	282	99.90%	0.056	0.250	18219	
13	282	100.00%	0.056	0.250	18265	n=13
16	282	100.00%	0.056	0.250	18266	
19	282	100.00%	0.056	0.250	18266	
22	282	100.00%	0.056	0.250	18266	
25	282	100.00%	0.056	0.250	18266	
Learning Rate $\beta = 1.4$						
1	282	99.99%	0.056	0.250	18260	
4	282	100.00%	0.056	0.250	18266	
7	282	100.00%	0.056	0.250	18266	
10	282	100.00%	0.056	0.250	18266	
13	282	100.00%	0.056	0.250	18266	n=4
16	282	100.00%	0.056	0.250	18266	
19	282	100.00%	0.056	0.250	18266	
22	282	100.00%	0.056	0.250	18266	
25	282	100.00%	0.056	0.250	18266	

Table 3: Impact of Learning Rate and No. of Shipments on Outputs (Case 2)

No. of Shipment (n)	ϕ_n	% of good item	τ_n	T_n	Profit	No of Shipment Required for Perfection
Learning Rate $\beta = 1$						
1	500	84.03%	0.100	0.361	8047	
4	495	84.81%	0.099	0.361	8577	
7	456	92.37%	0.091	0.362	13206	
10	426	99.31%	0.085	0.363	16838	
13	423	99.96%	0.085	0.363	17157	n=16
16	423	100.00%	0.085	0.363	17173	
19	423	100.00%	0.085	0.363	17174	

No. of Shipment (n)	ϕ_n	% of good item	τ_n	T_n	Profit	No of Shipment Required for Perfection
22	423	100.00%	0.085	0.363	17174	
25	423	100.00%	0.085	0.363	17174	
Learning Rate $\beta = 1.2$						
1	500	84.04%	0.100	0.361	8054	
4	490	85.72%	0.098	0.361	9176	
7	435	97.06%	0.087	0.362	15721	
10	423	99.90%	0.085	0.363	17127	
13	423	100.00%	0.085	0.363	17173	n=13
16	423	100.00%	0.085	0.363	17174	
19	423	100.00%	0.085	0.363	17174	
22	423	100.00%	0.085	0.363	17174	
25	423	100.00%	0.085	0.363	17174	
Learning Rate $\beta = 1.4$						
1	500	84.05%	0.100	0.361	8062	
4	481	87.40%	0.096	0.361	10250	
7	426	99.16%	0.085	0.363	16767	
10	423	99.99%	0.085	0.363	17168	
13	423	100.00%	0.085	0.363	17174	n=13
16	423	100.00%	0.085	0.363	17174	
19	423	100.00%	0.085	0.363	17174	
22	423	100.00%	0.085	0.363	17174	
25	423	100.00%	0.085	0.363	17174	

Table 4: Impact of Learning Rate and No. of Shipments on Outputs (Case 3)

Learning Rate $\beta = 1$						
No. of Shipment (n)	ϕ_n	% of good item	τ_n	T_n	Profit	No of Shipment Required for Perfection
1	288	84.03%	0.058	0.217	35175	
4	285	84.81%	0.057	0.217	35454	
7	263	92.37%	0.053	0.218	37890	
10	246	99.31%	0.049	0.219	39802	
13	244	99.96%	0.049	0.219	39970	n=16
16	244	100.00%	0.049	0.219	39979	
19	244	100.00%	0.049	0.219	39979	
22	244	100.00%	0.049	0.219	39979	
25	244	100.00%	0.049	0.219	39979	
Learning Rate $\beta = 1.2$						
1	288	84.04%	0.058	0.217	35179	
4	282	85.72%	0.056	0.217	35770	
7	251	97.06%	0.050	0.218	39214	
10	244	99.90%	0.049	0.219	39955	
13	244	100.00%	0.049	0.219	39979	n=13
16	244	100.00%	0.049	0.219	39979	
19	244	100.00%	0.049	0.219	39979	
22	244	100.00%	0.049	0.219	39979	
25	244	100.00%	0.049	0.219	39979	
Learning Rate $\beta = 1.4$						
1	288	84.05%	0.058	0.217	35183	
4	277	87.40%	0.055	0.217	36335	
7	246	99.16%	0.049	0.219	39765	
10	244	99.99%	0.049	0.219	39976	
13	244	100.00%	0.049	0.219	39979	n=13

No. of Shipment (n)	ϕ_n	% of good item	τ_n	T_n	Profit	No of Shipment Required for Perfection
16	244	100.00%	0.049	0.219	39979	
19	244	100.00%	0.049	0.219	39979	
22	244	100.00%	0.049	0.219	39979	
25	244	100.00%	0.049	0.219	39979	

Table 5: Impact of Learning Rate and No. of Shipments on Outputs (Case 4)

No. of Shipment (n)	ϕ_n	% of good item	τ_n	T_n	Profit	No of Shipment Required for Perfection
Learning Rate $\beta = 1$						
1	213	84.03%	0.043	0.163	8238	
4	211	84.81%	0.042	0.163	8767	
7	195	92.37%	0.039	0.164	13396	
10	182	99.31%	0.036	0.164	17028	
13	181	99.96%	0.036	0.164	17347	$n = 16$
16	181	100.00%	0.036	0.164	17364	
19	181	100.00%	0.036	0.164	17364	
22	181	100.00%	0.036	0.164	17364	
25	181	100.00%	0.036	0.164	17364	
Learning Rate $\beta = 1.2$						
1	213	84.04%	0.043	0.163	8244	
4	209	85.72%	0.042	0.163	9366	
7	186	97.06%	0.037	0.164	15911	
10	181	99.90%	0.036	0.164	17317	
13	181	100.00%	0.036	0.164	17363	n=13
16	181	100.00%	0.036	0.164	17364	
19	181	100.00%	0.036	0.164	17364	
22	181	100.00%	0.036	0.164	17364	
25	181	100.00%	0.036	0.164	17364	
Learning Rate $\beta = 1.4$						
1	213	84.05%	0.043	0.163	8252	
4	205	87.40%	0.041	0.163	10440	
7	182	99.16%	0.036	0.164	16957	
10	181	99.99%	0.036	0.164	17358	
13	181	100.00%	0.036	0.164	17364	n=13
16	181	100.00%	0.036	0.164	17364	
19	181	100.00%	0.036	0.164	17364	
22	181	100.00%	0.036	0.164	17364	
25	181	100.00%	0.036	0.164	17364	

Table 6: Comparative Table for Case 1

β	No. of ship. required for proficiency	τ_n	T_n	Profit
1	16	0.056	0.363	18265
1.2	16	0.056	0.363	18266
1.4	4	0.056	0.363	18266

Table 7: Comparative Table for Case 2

β	No. of ship. required for proficiency	τ_n	T_n	Profit
1	16	0.085	0.363	17173
1.2	13	0.085	0.363	17173
1.4	13	0.085	0.363	17174

Table 8: Comparative Table for Case 3

β	No. of ship. required for proficiency	τ_n	T_n	Profit
1	16	0.049	0.219	39979
1.2	13	0.049	0.219	39979
1.4	13	0.049	0.219	39979

Table 9: Comparative Table for Case 4

β	No. of ship. required for proficiency	τ_n	T_n	Profit
1	16	0.036	0.164	17364
1.2	13	0.036	0.164	17364
1.4	13	0.036	0.164	17364

5. Conclusion

In this article we have optimized the retailer's ordering quantity for imperfect quality items with learning effects on screening process under the trade credit financing scheme. The main focus of this study is that how affects the retailer's ordering quantity when stocking strategies is beneficial for market situations. The various interval of credit periods have been analyzed and verified through the different numerical examples. A comparative study has been done through the numerical examples. and we have concluded that Case 3. is more beneficial for this type of trade credit financing strategies. This article suggests that, those item which sale depends on stocking may earn more and more profit by increasing T_n, ϕ_n, and τ_n. Article also suggests that, in the financing policy keep always $\tau_n > M > L$ for better outputs. This article may be extended by incorporating the rework process on defective items. One can also extended this article by incorporating procurement cost on ordering size of items. One can also extended this article by incorporating expected quantity of defective items.

References

[1] Wu K. S., Liang Y. O., and Chih T. Y. (2006). An Optimal Replenishment Policy for Non-Instantaneous Deteriorating Items with Stock-Dependent Demand and Partial Backlogging. International Journal Production Economics, **101**(2): 69 - 84.

[2] Teng, J. T. and Chang T. C. (2005). Economic Production Quantity Models for Deteriorating Items with Price- and Stock-Dependent Demand. Computers & Operations Research, **32**(2): 297- 308.

[3] Giri, B. C., and Bardhan S. (2012). Supply Chain Coordination for a Deteriorating Item with Stock and Price-Dependent Demand under Revenue Sharing Contract. International Transactions Operational Research, **19**(5): 753 -768.

[4] Ray, J., and Chaudhuri K. S. (1997). An EOQ Model with Stock-Dependent Demand, Shortage, Inflation and Time Discounting. International Journal Production Economics, **53**(2): 71 - 80.

[5] Giri, B. C., and Chaudhuri K. S. (1998). âĂIJDeterministic Models of Perishable Inventory with Stock-Dependent Demand Rate and Nonlinear Holding Cost.âĂİ European Journal Operational Research, **105**(3): 67 - 74.

[6] Parlar, M., and Qinan W. (1994). Discounting Decisions in a Supplier-Buyer Relationship with a Linear BuyerâĂŹs Demand. IIE Transactions, (Institute of Industrial Engineers). 26(2): 34 - 41.

[7] Pal, S., Goswami A. , and Chaudhuri K. S. (1993). A Deterministic Inventory Model for Deteriorating Items with Stock-Dependent Demand Rate. International Journal Production Economics, **32**(3): 91 - 99.

[8] Muth E. J. and Spremann K. (1983). NoteâĂŤLearning Effects in Economic Lot Sizing. Management Science, **29**: 264-269.

[9] Salameh M. K. Mohamed-Asem U. A. M. and Mohamad Y. J. (1993). Mathematical modelling of the effect of human learning in the finite production inventory model. Applied Mathematical Modelling, **17**: 613-615.

[10] Cheng T. C. E.(1994). An economic manufacturing quantity model with learning effects. International Journal Production Economics, **33**: 257-264.

[11] Salameh M. K. and Jaber M. Y. (2000). Economic production quantity model for items with imperfect quality, International Journal Production Economics, **64**: 59-64.

[12] Jaber M. Y. and Guiffrid A. L. (2004). A Learning curves for processes generating defects requiring reworks, European Journal Operations Research, **159**: 663-672.

[13] Eroglua A, and Ozdemirb G. (2007). An economic order quantity model with defective items and shortages. International Journal Production Economics, **106**:, 544-549.

[14] Jaber M. Y. and Guiffrid A. L. (2008). Learning curves for imperfect production processes with reworks and process restoration interruptions. European Journal Operations Research, **189**: 93-104.

[15] Jaber M.Y., Goyal S. K. and Imran M. (2008). Economic production quantity model for items with imperfect quality subject to learning effects. International Journal Production Economics, **115**: 143-150.

[16] Pan J. C. (2008). The Learning Effect on Setup Cost Reduction for Mixture Inventory Models with variable Lead Time, Asia Pacific Journal Operations Research, **25**: 513-529.

[17] Lin P. C. (2008). Optimal pricing, production rate, and quality under learning effects. Journal of Business Research, **61**: 1152-1159.

[18] Yoo H. S., Kim D. S. and Park M. S. (2009), Economic production quantity model with imperfect-quality items, two-way imperfect inspection and sales return, International Journal Production Economics,**121**: 255-265.

[19] Sui Z., Gosavi A. and Lin L. (2010). A Reinforcement Learning Approach for Inventory Replenishment in Vendor-Managed Inventory Systems With Consignment Inventory. Engineering Management Journal, **22**: 44-53.

[20] Khan M., Jaber M. Y., and Bonney M. (2011). An economic order quantity (*EOQ*) for items with imperfect quality and inspection errors. International Journal Production Economics, **133**: 113 -118.

[21] Wahab M. I. M. and Jaber M.Y. (2010). Economic order quantity model for items with imperfect quality, different holding costs, and learning effects: A note. Computers & Industrial Engineering. **58**: 186-190.

[22] Jaber M. Y. and Khan M.(2010), Managing yield by lot splitting in a serial production line with learning, rework and scrap. International Journal Production Economics, **124**; 32-39.

[23] Das D. Roy A. and Kar S. (2010). A Production-Inventory Model for a Deteriorating Item Incorporating Learning Effect Using Genetic Algorithm. Hindawi Publishing Corporation Advances in Operations Research., Article ID 146042: 26 pages.

[24] Khan M., Jaber M.Y. and Wahab M.I.M (2010). Economic order quantity model for items with imperfect quality with learning in inspection. International Journal Production Economics, **124**: 87-96.

[25] Konstantaras I., Skouri K. and Jaber M. Y. (2012). Inventory models for imperfect quality items with shortages and learning in inspection. Applied Mathematical Modelling, **36**: 5334-5343.

[26] Jaggi C. K., Goyal S. K. and Mittal M. (2013). Credit financing in economic ordering policies for defective items with allowable shortages. Applied Mathematics and Computation, **219**: 5268-5282.

[27] Teng J. T., Lou K. R. and Wang L. (2014), Optimal trade credit and lot size policies in economic production quantity models with learning curve production costs.International Journal Production Economics, **155**, 318-323.

[28] Kumar N., Singh S.R. and Kumari R. (2013): Learning effect on an inventory model with two-level storage and partial backlogging under inflation. International Journal of Services and Operations Management, **16**: 105-122.

[29] Givi Z. S., Jaber M. Y. and Neumann W. P., (2015). Modelling worker reliability with learning and fatigue. Applied Mathematical Modelling,, **39**: 5186-5199.

[30] Agi Maher. A. N. and Soni Hardik N. (2020). Joint Pricing and Inventory Decisions for Perishable Products with Age- Stock, and Price-Dependent Demand Rate. Journal Operational Research Society., **71**(1): 85 - 99.

[31] Sarkar B. and Sumon S. (2013). An Improved Inventory Model with Partial Backlogging, Time Varying Deterioration and Stock-Dependent Demand. Economic Modelling, 30(1): 24 - 32.

[32] Jayaswal M. K. , Sangala I., Mittal M. and Malik S. (2019). Effects of learning on retailer ordering policy for imperfect quality items with trade credit financing. Uncertain Supply Chain Management, **7**: 49-62.

[33] Yadav R., Prateek S,, Mittal M. and Mehta S.(2018). Effects of Imperfect Quality Items in the Asymmetric Information Structure in Supply Chain Model. Uncertain Supply Chain Management, **6**: 287-298.

[34] Soni H. and Shah N. H. (2008). Optimal Ordering Policy for Stock-Dependent Demand under Progressive Payment Scheme. European Journal of Operational Research, 184(1): 91âĂŞ100.

[35] Benyong H. and Feng Y. (2017). Optimization and Coordination of Supply Chain with Revenue Sharing Contracts and Service Requirement under Supply and Demand Uncertainty. International Journal Production Economics, 183(October 2016): 185-193.

[36] Nigwal A. R., Khedlekar U. K. and Gupta N.,(2022), Learning Effects On Retailer Ordering Policy For Imperfect Quality Items Under Trade Credit Financing With Pricing Strategies, Jnanabha, 52(2): 6-22.

[37] Nigwal A. R., Khedlekar U. K. Sharma L. and Gupta N.(2022). Trade Credit Policies for Supplier, Manufacturer, and Retailer: An Imperfect Production-Inventory System with Rework. Journal of Mathematical & Fundamental Sciences, 54(1): 76-108.

[38] Wright T. P. (1936). Factors affecting the cost of airplanes. Journal of the Aeronautical Sciences, **3**: 122-128.