BAYESIAN INTERVAL ESTIMATION FOR THE PARAMETERS OF POISSON TYPE LENGTH BIASED EXPONENTIAL CLASS MODEL

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Abstract

In this research paper, two sided Bayesian interval is proposed for Poisson type length biased exponential class software reliability growth model. The failure intensity function, mean time to failure function and likelihood function are derived. Bayesian interval estimation has been done for the parameters using non informative priors. The performance of proposed Bayesian interval is obtained by using Monte Carlo simulation technique. Average length and coverage probability of Bayesian interval for the parameters are calculated. From the obtained intervals it is concluded that Bayesian interval of parametersperform better for appropriate choice of execution time and certain values of parameters.

Keywords: Length biased exponential distribution, Non informative priors, Software reliability growth model (SRGM), Bayesian interval, average length, coverage probability.

1. Introduction

This paper considers Poisson type length biased exponential class model according to classification scheme of Musa and Okumoto [9]. Poisson execution time models are based on the premise that execution time is the best time domain for expressing reliability. Execution is the most practical measure of the failure inducing stress being placed on a program. Musa et al [8] have suggested that it is convenient to divide the program into number of runs. The run depends on the function executed by program. The time required for run is depends upon size of run. As the size of run varies the number of failure observed in single run may vary. Fisher [2] defined length biased and further formulated by Rao [12] Gupta and Keating [6]developed relationship between survival function, the failure rate and mean residual life function using length biased distribution. Patil and Rao [10] have given a table for some distribution and their size biased forms. Rao and Cunha [13] estimated credible interval and confidence interval through MLE for lognormal distribution and also compared average length and coverage probability of the calculated interval. Tamak [16] estimated reliability of web application using Goel-Okumoto Software Reliability Growth models (SRGM). Shreshtha and Kumar [14] computed MLE and Bayesian estimate for Rayleigh distribution using gamma prior. Singh et al [15] introduced length biased distribution as Software Reliability Growth models (SRGM). Fitrilia et al [3] estimate the failure rate by E-Bayesian estimation method. E-Bayesian estimation is an expectation of Bayes estimation, in order to obtain Bayes estimation expectations is by calculating the mean of Bayes estimator. Rabie and Li [11] studied the Bayesian and E-Bayesian approaches under squared error and LINEX loss functions. Also construct Confidence intervals for maximum likelihood estimates, as well as credible intervals for the E-Bayesian and Bayesian estimates. Andure and Ade [1] proposed length biased quasi lindley distribution and discussed different properties of proposed distribution. Gupta et al [5] obtained Bayesian and non-Bayesian estimators under symmetric (squared error) and asymmetric (linex and precautionary) loss functions using a non-informative prior. And compared risk efficiencies of Bayes estimators with maximum likelihood estimators.

In this paper, it is considered that the time to failure of an individual fault following length biased exponential distribution and the failure experienced by time t is distributed as Poisson type. In this model it is assumed that the software failures are independent of each other but depend on length of the time interval which contains the software failures.

The structure of the paper is such that section 2 presents derivation of failure intensity and expected number of failures using length biased exponential distribution, derivation of likelihood function, selection of priors, and derives joint and marginal posterior distribution of model. Section 3 presents derivation of two sided Bayesian interval for the parameters θ_0 and θ_1. Results and discussion is given in the section 4 while concluding remarks are provided in section 5.

2. Model Formulation

Consider that software is tested for its performance and observed the time of failure occurs during software system performance. Let the number of failures present in software be θ_0,and t_e be the execution time i.e. time during which CPU is busy and m_e be the number of failures observed up to execution time t_e. Consider time between the failures t_i (i=1,2,………m_e) follows the exponential distribution with parameter θ_1. The length biased exponential distribution is given as

$$f^*(t) = \begin{cases} t\theta_1^2 e^{-\theta_1 t} & ,t > 0, \theta_1 > 0, E[t] \neq 0 \\ 0 & otherwise \end{cases} \tag{1}$$

Where $f^*(t)$ denotes the length biased exponential distribution.

The failure intensity function is obtained by using equation (1)

$$\lambda(t) = \theta_0 t\theta_1^2 e^{-\theta_1 t} \qquad ,t > 0, \theta_0 > 0 \tag{2}$$

Where θ_0 express the number of failures and θ_1 express the for failure rate.

The mean failure function i.e. expected number of failures at time t_e can be obtained by using equations (2)

$$\mu(t_e) = \theta_0\theta_1^2 I_1 \tag{3}$$

Where, $I_1 = \int_0^{t_e} t_i e^{-\theta_1 t_i}\, dt$ and by solving (see Gradshteyn and Ryzhik [4] p. 357) we get,

$$\mu(t_e) = \theta_0 \left[1 - (1 + \theta_1 t_e)e^{-\theta_1 t_e}\right], t > 0, \theta_0 > 0, \theta_1 > 0 \tag{4}$$

Behavior of failure intensity and expected number of failure of length biased exponential class model has been studied by Singh et al [15]. They have compared the maximum likelihood estimates i.e. MLE's and Bayesian estimators on the basis of risk efficiencies.

Now for a system, considering that m_e software failures are observed at times t_i , i = 1,2,……,m_e up to execution time is t_e ($\geq t_{m_e}$) and the likelihood function of parameters θ_0 and θ_1 with the help of failure intensity and mean failure function can be obtained as (cf. Singh et al [15])

$$L(\theta_0, \theta_1) = \theta_0^{m_e} \theta_1^{2m_e} \left[\prod_{i=1}^{m_e} t_i\right] e^{-T\theta_1} e^{-\theta_0 \left[1-(1+\theta_1 t_e) e^{-\theta_1 t_e}\right]} \tag{5}$$

3. Bayesian interval Estimation of parameters θ_0 and θ_1

Bayesian estimation is done by combining prior information with information obtained from sample data. While testing the software, the experimenter have very little knowledge relative to the total number of failures present in the software.. Here insufficient prior information is not available about parameters θ_0 and θ_1, hence non- informative priors are considered. The following non informative prior distributions g (θ_0) and g (θ_1) are considered for parameters θ_0 and θ_1 as follows:

$$g(\theta_0) \propto \begin{cases} {\theta_0}^{-1} & , \theta_0 \epsilon\, [0, \infty) \\ 0 & , otherwise \end{cases} \tag{6}$$

and

$$g(\theta_1) \propto \begin{cases} {\theta_1}^{-1} & , \theta_1 \epsilon\, [0, \infty) \\ 0 & , otherwise \end{cases} \tag{7}$$

The joint posterior of θ_0 and θ_1 given $\underline{t}$ (=t_i, i = 1, 2,………, me) is obtained by using equations (5),(6) and (7) is as follows:

$$\pi(\theta_0, \theta_1 | \underline{t}) = D^{-1} \theta_0^{m_e-1} \theta_1^{2m_e-1} e^{-T\theta_1} e^{-\theta_0}\, e^{[\theta_0 (1+\theta_1 t_e) e^{-\theta_1 t_e}]} \qquad m_e < \theta_0 < \infty , 0 < \theta_1 < \infty \quad (8$$

Where, D is normalizing constant.

$$D = \sum_{k=0}^{\infty} \sum_{r=0}^{k} \binom{k}{r} \left[\frac{\Gamma(2m_e + r)\Gamma(m_e + k, m_e)\; t_e^r (T^*)^{-2m_e - r}}{k!}\right]$$

Where, $T^* = T + kt_e$, $\quad T = \sum_{i=1}^{m_e} t_i$

The marginal posterior distribution of θ_0 given $\underline{t}$ is,

$$\pi(\theta_0 | \underline{t}) = D^{-1} \sum_{k=0}^{\infty} \sum_{r=0}^{k} \binom{k}{r} \left[\frac{t_e^r\, \Gamma(2m_e+r)(T^*)^{-2m_e-r}}{k!}\right] \left[\theta_0^{m_e+k-1} e^{-\theta_0}\right], \theta_0 > m_e \tag{9}$$

The marginal posterior of θ_1, say $\pi(\theta_1|\underline{t})$ can be obtained as

$$\pi(\theta_1 | \underline{t}) = D^{-1} \sum_{k=0}^{\infty} \left[\frac{\Gamma(m_e+k, m_e)}{k!}\right] \left[\theta_1^{2m_e-1} (1+\theta_1 t_e)^k e^{-\theta_1 T^*}\right], \theta_1 > 0 \tag{10}$$

A symmetric 100(1- α) % two sided Bayes probability interval (θ_L, θ_U) is given as

$$\int_{-\infty}^{\theta_L} \pi(\theta | \underline{t})\, dt = \alpha/2$$

$$\int_{\theta_U}^{\infty} \pi(\theta | \underline{t})\, dt = \alpha/2$$

Where $\pi\,(\theta|\underline{t})$ is the marginal posterior distribution of θ for details see Martz and Waller [7]

Now, to obtain two sided Bayes interval for the parameter θ_0 and θ_1 by integrating equation (9) and (10) w.r.t. θ_0 and θ_1 respectively can be given as:

$$\tilde{\theta}_{0L} = D^{-1} \sum_{k=0}^{\infty} \sum_{r=0}^{k} \binom{k}{r} \left[\frac{t_e^r\, \Gamma(2m_e+r)(T^*)^{-2m_e-r}}{k!}\right] \Gamma(m_e + k, \theta_{0*})$$

$$\tilde{\theta}_{0U} = D^{-1} \sum_{k=0}^{\infty} \sum_{r=0}^{k} \binom{k}{r} \left[\frac{t_e^r\, \Gamma(2m_e+r)(T^*)^{-2m_e-r}}{k!}\right] \Gamma(m_e + k, \theta_0^*)$$

$$\tilde{\theta}_{1L} = D^{-1} \sum_{k=0}^{\infty} \sum_{r=0}^{k} \binom{k}{r} \left[\frac{\Gamma(m_e+k, m_e)}{k!}\right] t_e^r (T^*)^{-2m_e-r}\, \gamma(2m_e + r, T^* \theta_{1*})$$

$$\tilde{\theta}_{1U} = D^{-1} \sum_{k=0}^{\infty} \sum_{r=0}^{k} \binom{k}{r} \left[\frac{\Gamma(m_e+k, m_e)}{k!}\right] t_e^r (T^*)^{-2m_e-r}\, \Gamma\,(2m_e + r, T^* \theta_1^*)$$

Where, $\Gamma(m_e + k, \theta_0^*)$, $\Gamma\,(2m_e + r, T^* \theta_1^*)$ are incomplete gamma functions.

$$\int_u^\infty x^{v-1}\, e^{-\mu x} dx = \mu^{-v}\Gamma(v,\mu u) \qquad [Re\, u > 0, Re\, \mu > 0, n > 0]$$

is known as incomplete gamma function. The details about the incomplete gamma function can be seen from Gradshteyn and Ryzhik [7]. Therefore, $(\tilde{\theta}_{0L}, \tilde{\theta}_{0U})$ and $(\tilde{\theta}_{1L}, \tilde{\theta}_{1U})$ are the Bayesian interval estimates for parameter θ_0 and θ_1 respectively.

4. Results and Discussion

Here, two sided Bayesian interval are obtained for the parameter total number of failures i.e. θ_0 and the parameter θ_1. The interval estimate is the posterior probability distribution about the parameter. To study the performance of two sided Bayesian interval, sample of different sizes (say m_e) was generated from the length biased exponential distribution and it is repeated 1000 times. Monte Carlo simulation technique is used for simulation and the result is presented in the tables given below. The credible interval depends upon the values of execution time i.e. t_e and number of failures experienced at times t_i, i = 1, 2…, m_e. Average length and coverage probability of the proposed Bayesian interval have been calculated for different values of parameters θ_0 and θ_1 and for certain execution time. Here, it is assumes that two sided Bayesian interval maintains the credible level if the estimated coverage probability is found in between the range of 0.940 to 0.960 i.e. $(1-\alpha)\pm 0.01$ where, $\alpha = 0.05$.

Here tables (1) to (4) give average length and coverage probability for Bayesian interval for parameter θ_0 i.e. total number of failures. Average length have been obtained by assuming parameter θ_0 (=1(1)5), $\theta_1(= 0.6(0.1)1)$. From tables, we can see that Average length is computed for Bayesian interval for parameter θ_0 is shorter. Average length increases as the values of θ_0 increases for fixed execution time i.e. t_e. It is also observes that as the values of θ_1 increases average length decreases. From table, we can see that coverage probability increases as the θ_0 increases and coverage probability decreases as the θ_1 increases. From the table it also observes that as execution time increases average length decreases.

Table (5) to (8) gives average length and coverage probability for Bayesian interval for parameter the θ_1. The Average length have been calculated by assuming values θ_0 (=1(1)5) and $\theta_1(= 0.6(0.1)1)$. Average length increases as the values of θ_0 increases for fixed execution time i.e. t_e and it is decreases as θ_1 increases. From table, it can be seen that coverage probability increases as the θ_0 increases and coverage probability decreases as parameter θ_1 increases. From the table it also observes that average length decreases as execution time increases.

Table 1: *Average length and coverage probability of Bayesian interval* $(\tilde{\theta}_{0L}, \tilde{\theta}_{0U})$ of θ_0 *calculated for different values of parameters* θ_0 *and* θ_1 *when execution time* $t_e = 5$

θ_1 \ θ_0	1	2	3	4	5
0.6	2.12412 (0.994)	2.95254 (0.994)	5.07429 (0.994)	5.39784 (0.995)	6.45182 (0.995)
0.7	2.06795 (0.993)	2.18189 (0.994)	4.17136 (0.994)	5.07228 (0.995)	6.69461 (0.995)
0.8	1.90843 (0.993)	2.98570 (0.993)	3.93268 (0.994)	4.72848 (0.994)	6.42171 (0.995)
0.9	1.42513 (0.993)	2.26186 (0.993)	3.56985 (0.994)	4.40744 (0.994)	5.06234 (0.995)
1	1.40031 (0.993)	2.04029 (0.993)	2.28113 (0.993)	3.85033 (0.994)	4.29281 (0.994)

The values in the parenthesis are coverage probability of true value of parameter.

Table 2: *Average length and coverage probability of Bayesian interval* $(\tilde{\theta}_{0L}, \tilde{\theta}_{0U})$ of θ_0 *calculated for different values of parameters* θ_0 *and* θ_1 *when execution time* t_e *= 7*

θ_1 \ θ_0	1	2	3	4	5
0.6	0.723187 (0.994)	1.544598 (0.994)	1.58816 (0.994)	3.681955 (0.995)	4.526929 (0.995)
0.7	0.476033 (0.993)	1.222565 (0.994)	1.236041 (0.994)	2.442999 (0.994)	3.921342 (0.995)
0.8	0.470561 (0.993)	0.847506 (0.993)	1.191867 (0.994)	1.889722 (0.994)	2.520656 (0.995)
0.9	0.463198 (0.993)	0.71856 (0.993)	1.092366 (0.994)	1.828802 (0.994)	2.504829 (0.994)
1	0.375691 (0.993)	0.570903 (0.993)	0.795695 (0.993)	1.412542 (0.994)	2.446384 (0.994)

***The values in the parenthesis are coverage probability of true value of parameter.**

Table 3: *Average length and coverage probability of Bayesian interval* $(\tilde{\theta}_{0L}, \tilde{\theta}_{0U})$ of θ_0 *calculated for different values of parameters* θ_0 *and* θ_1 *when execution time* t_e *= 10*

θ_1 \ θ_0	1	2	3	4	5
0.6	0.267986 (0.993)	0.324152 (0.993)	0.465934 (0.993)	0.57678 (0.994)	1.019986 (0.994)
0.7	0.260624 (0.993)	0.322248 (0.993)	0.353603 (0.993)	0.431857 (0.993)	0.906414 (0.994)
0.8	0.260624 (0.993)	0.282712 (0.993)	0.34624 (0.993)	0.421758 (0.993)	0.868347 (0.994)
0.9	0.260624 (0.993)	0.275349 (0.993)	0.338877 (0.993)	0.412504 (0.993)	0.801439 (0.994)
1	0.257986 (0.993)	0.267986 (0.993)	0.316789 (0.993)	0.402406 (0.993)	0.718558 (0.993)

***The values in the parenthesis are coverage probability of true value of parameter.**

Table 4: *Average length and coverage probability of Bayesian interval* $(\tilde{\theta}_{0L}, \tilde{\theta}_{0U})$ of θ_0 *calculated for different values of parameters* θ_0 *and* θ_1 *when execution time* t_e *= 12*

θ_1 \ θ_0	1	2	3	4	5
0.6	0.260624 (0.993)	0.260624 (0.993)	0.309427 (0.993)	0.368328 (0.993)	0.667052 (0.993)
0.7	0.258620 (0.993)	0.275349 (0.993)	0.280105 (0.993)	0.34624 (0.993)	0.591506 (0.993)
0.8	0.256715 (0.993)	0.267986 (0.993)	0.275349 (0.993)	0.331515 (0.993)	0.402406 (0.993)
0.9	0.253609 (0.993)	0.264624 (0.993)	0.267986 (0.993)	0.319525 (0.993)	0.380318 (0.993)
1	0.252286 (0.993)	0.260624 (0.993)	0.260624 (0.993)	0.309427 (0.993)	0.365592 (0.993)

***The values in the parenthesis are coverage probability of true value of parameter.**

Table 5: *Average length and coverage probability of Bayesian interval* $(\tilde{\theta}_{1L}, \tilde{\theta}_{1U})$ of θ_1 *calculated for different values of parameters* θ_0 *and* θ_1 *when execution time* $t_e = 5$

θ_1 \ θ_0	1	2	3	4	5
0.6	0.00605 (0.994)	0.01269 (0.994)	0.02099 (0.994)	0.02412 (0.995)	0.02947 (0.995)
0.7	0.00543 (0.993)	0.01146 (0.994)	0.01713 (0.994)	0.02134 (0.994)	0.02700 (0.995)
0.8	0.00474 (0.993)	0.00859 (0.993)	0.01511 (0.994)	0.02037 (0.994)	0.02694 (0.995)
0.9	0.00387 (0.993)	0.00710 (0.993)	0.01386 (0.994)	0.01912 (0.994)	0.02613 (0.994)
1	0.00348 (0.993)	0.00616 (0.993)	0.01001 (0.993)	0.01895 (0.993)	0.02599 (0.993)

The values in the parenthesis are coverage probability of true value of parameter

.**Table 6:** *Average length and coverage probability of Bayesian interval* $(\tilde{\theta}_{1L}, \tilde{\theta}_{1U})$ of θ_1 *calculated for different values of parameters* θ_0 *and* θ_1 *when execution time* $t_e = 7$

θ_1 \ θ_0	1	2	3	4	5
0.6	0.00049 (0.994)	0.00158 (0.994)	0.00252 (0.994)	0.00572 (0.994)	0.00908 (0.994)
0.7	0.00046 (0.993)	0.00106 (0.994)	0.00229 (0.994)	0.00498 (0.994)	0.00890 (0.994)
0.8	0.00041 (0.993)	0.00093 (0.993)	0.00149 (0.993)	0.00401 (0.994)	0.00771 (0.994)
0.9	0.0007 (0.993)	0.00082 (0.993)	0.00139 (0.993)	0.00248 (0.994)	0.00358 (0.994)
1	0.0002 (0.992)	0.00067 (0.993)	0.00109 (0.993)	0.00143 (0.993)	0.00245 (0.994)

The values in the parenthesis are coverage probability of true value of parameter.

Table 7: *Average length and coverage probability of Bayesian interval* $(\tilde{\theta}_{1L}, \tilde{\theta}_{1U})$ of θ_1 *calculated for different values of parameters* θ_0 *and* θ_1 *when execution time* $t_e = 10$

θ_1 \ θ_0	1	2	3	4	5
0.6	0.000066 (0.992)	0.000102 (0.992)	0.001163 (0.993)	0.00227 (0.992)	0.00265 (0.993)
0.7	0.000066 (0.992)	0.000071 (0.992)	0.00025 (0.992)	0.000219 (0.992)	0.00184 (0.993)
0.8	0.000065 (0.992)	0.000067 (0.992)	0.00022 (0.992)	0.000207 (0.992)	0.00159 (0.993)
0.9	0.000063 (0.992)	0.000060 (0.992)	0.00021 (0.992)	0.000130 (0.992)	0.000249 (0.993)
1	0.000062 (0.992)	0.000052 (0.992)	0.000041 (0.992)	0.000062 (0.992)	0.000109 (0.993)

The values in the parenthesis are coverage probability of true value of parameter.

Table 8: *Average length and coverage probability of Bayesian interval* $(\tilde{\theta}_{1L}, \tilde{\theta}_{1U})$ of θ_1 *calculated for different values of parameters* θ_0 *and* θ_1 *when execution time* $t_e = 12$

θ_1 \ θ_0	**1**	**2**	**3**	**4**	**5**
0.6	0.000063 (0.992)	0.000594 (0.992)	0.000603 (0.992)	0.00064 (0.992)	0.00151 (0.993)
0.7	0.0000532 (0.992)	0.0000599 (0.992)	0.0000608 (0.992)	0.000541 (0.992)	0.000179 (0.992)
0.8	0.0000330 (0.992)	0.0000466 (0.992)	0.0000607 (0.992)	0.000116 (0.992)	0.000462 (0.992)
0.9	0.0000204 (0.992)	0.0000229 (0.992)	0.0000385 (0.992)	0.0000602 (0.992)	0.000220 (0.992)
1	0.0000128 (0.991)	0.0000193 (0.992)	0.0000359 (0.992)	0.0000596 (0.992)	0.000124 (0.992)

The values in the parenthesis are coverage probability of true value of parameter.

5. Conclusion

In this paper, two-sided Bayesian interval has been obtained for length biased exponential class model with parameters i.e. total number of failures θ_0 and scale parameter θ_1. Bayesian interval is obtained and studied on the basis of average length and coverage probability. It is found that Bayesian interval has shorter average length and high coverage probability. As execution time increases average length decreases and coverage probability increases. From results it is concluded that the proposed Bayesian interval preferred for parameter total number of failures i.e. θ_0 and θ_1.

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