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A NEW ZERO-TRUNCATED DISTRIBUTION AND ITS APPLICATIONS TO COUNT DATA

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Abstract

Numerous disciplines, including engineering, public health, sociology, psychology, and epidemiology, are particularly interested in the analysis and modelling of zero truncated count data. As a result, we suggest a novel and straightforward structural model in this study called zero truncated new discrete distribution. We examine its statistical properties including probability mass function, cumulative function, and moments. The parametric estimation of the zero-truncated new discrete distribution is explained by Maximum Likelihood Estimation method and, to investigate its performance, a simulation study is proposed. The importance of the distribution is evaluated using two real-world data sets as well as one simulated data set and the model comparison is made on the basis of AIC and BIC criterions.

Keywords: truncation, zero-truncated distribution, simulation maximum likelihood estimation, goodness-of-fit

1. Introduction

The truncation of probability distributions is a significant statistical phenomenon that is employed in numerous domains, including medicine, reliability theory, industry, queueing systems, and many others. When a range of probability values for the variables is either ignored or unobservable, probability models are truncated. For instance, if we want to see how many journal articles across different disciplines have been published, how many tickets were given out to teenagers based on their academic performance, how many children have ever been born to a sample of mothers who are over 40, etc. Typically, the Zero Truncated Poisson(ZTP), Zero Truncated Negative Binomial(ZTNB), and Zero Truncated Poisson Lindley(ZTPL) distributions are used to model the aforementioned scenarios. For the examination of gall-cell counts and amount of eggs in flower heads, Finney and Varley [7] used the ZTP distribution. Brass [2] used the ZTNB distribution to simulate the number of children ever born to a sample of mothers above the age of 40. The ZTNB distribution was used in a regression model by Lee *et al.* [15] to examine the over-dispersed data of ischemic stroke hospitalizations. ZTNB distribution was also used by Phange and Loh [18] to analyze the prevalence of rare species and hospital stay. In 1990, Creel and Loomis [5] used the ZTP distribution and applied it to deer hunting data set which was collected in California. Also, Lindsay [14] analyzed the postal survey data using ZTP distribution. Further, ZTPL distribution was introduced by Ghitany *et al.* [9] in 2008 and a real count data set was analyzed.

The zero truncated models outlined above are seen to have neither a high peak nor a heavy tail. These models do not perform well when over and under-dispersion problems are present in the data. Some other researchers who have worked in this field are Coleman and James [4], Mathews and Appleton [17], they have discussed various applications of zero-truncated distributions particularly ZTP distribution. Further, a detailed discussion was given by Best *et al.* [3] in 2007 on the applications including goodness-of-fit of ZTP distribution. Hassan *et al.*[10] in 2008 obtained the Bayes estimator and reliability function of the ZTP and also derived its recurrence relations. In 2008, Ghitany *et al.* [9] introduced the ZTPL distribution and also derived the method of moments (MoM) and maximum likelihood estimators (MLE) of the parameter including their large sample properties and simulation procedure. In addition to this, the effectiveness of the MoM and ML estimators have been compared, it has been found that the ML estimators are more effective than the MoM estimators. In 2017, Shanker & Shukla [21] introduced a Zero Truncated Two Parameter Poisson Lindley(ZTTPPL) distribution using compounding technique by mixing Size Biased Poisson(SBPD) with a continuous distribution. They have showed that ZTTPPL model gives better fitting than ZTPD in case of biological science data. Simon and Shanker [24] in 2018, obtained a Zero Truncated Discrete Lindley(ZTDL) distribution and analyzed that both the methods i.e., MoM and MLE gave the same estimates of the distributions parameter. In 2020, Kiani [13] proposed a new model, named as Zero Truncated Two Parameter Discrete Lindley(ZTTPDL) distribution. The distribution has some embedded models and represents a two-component mixture of a Zero Truncated Geometric(ZTG) distribution and a ZTNBD with certain parameters. Another distribution named as Poisson Ishita distribution(PID) was obtained by Shukla & Shanker [25] which was further studied by Kamlesh *et al.* [27] in 2020. They obtained the zero truncation of the PID model. Similarly, Shanker [22] in 2017 obtained a zero truncation for Poisson-Akash model of Shanker [23]. Sium in 2020 [26] obtained a zero-truncated model of discrete Akash distribution and obtained its different structural properties.
Now some of the aforementioned shortcomings served as the motivation for the proposal of a Zero Truncated New Discrete (ZTND) distribution with a basic structure and only one parameter. The proposed model is equi-dispersed, over-dispersed as well as under-dispersed. Additionally, the model's adaptability is examined by looking at how well it agrees with some real data sets as shown by checking its p-value, and some criterions like AIC and BIC. For this purpose, three data sets including one simulated data are examined , and it has been found that the proposed model, which displays minimum values for certain statistics, is the most appropriate distribution among the others.

2. Zero Truncated New Discrete Distribution

Sanjay *et al.*[12] in 2021, obtained a new discrete version of the failure model which was given by Siddiqui in 2016 [20] by using the cdf and reliability function of the Continuous Distribution (CD).

Let y be a random variable following new discrete distribution [12] then pmf is given as:

$$P_0(y:\theta) = \frac{\theta-1}{\theta^{(y+1)}} \qquad ; y = 0, 1, 2, 3...; \qquad ; \theta > 1 \tag{1}$$

Now, the formula given below can be used to determine the pmf of ZTND distribution.

$$P(y;\theta) = \frac{P_0(y;\theta)}{1 - P_0(0;\theta)} \tag{2}$$

Using equation (1) and (2) we get the pmf of ZTNDD as:

$$P(y;\theta) = \frac{\theta-1}{\theta^y} \quad ; y = 1, 2, 3, ... \quad \theta > 1 \tag{3}$$

The cdf of ZTNDD is provided as:

$$F(y;\theta) = \frac{\theta^y - 1}{\theta^y}$$

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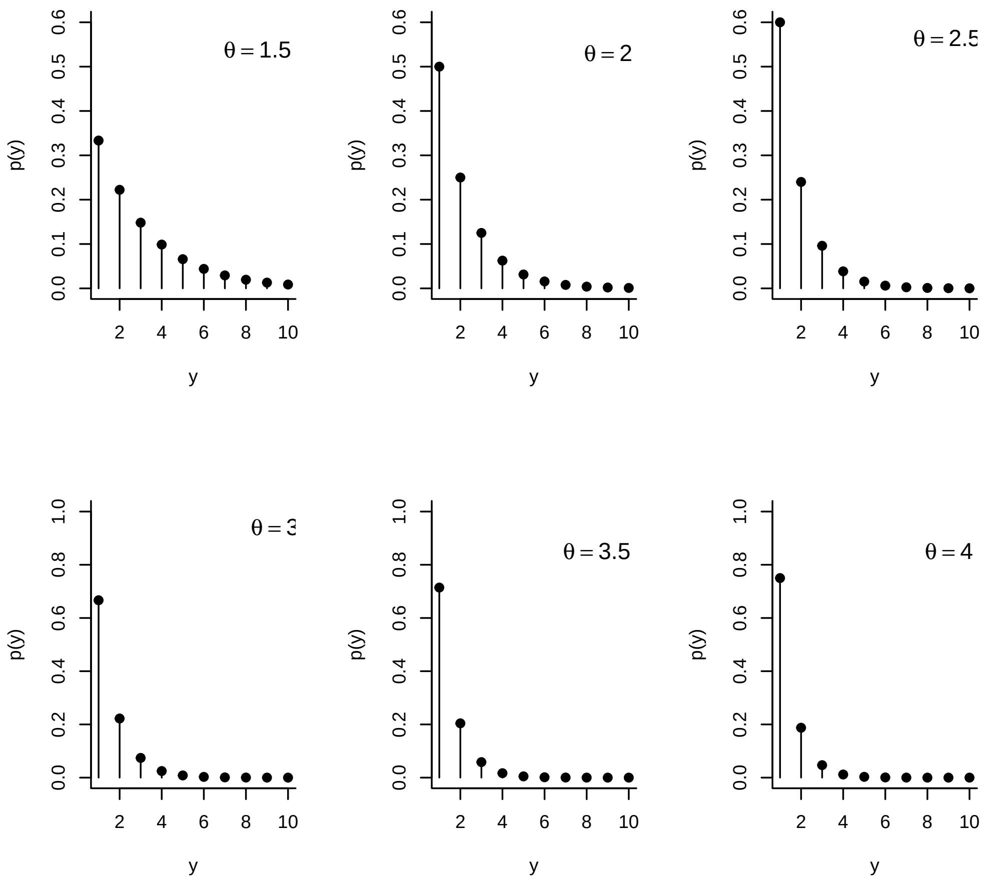

Figure 1: *PMF plots of ZTNDD*

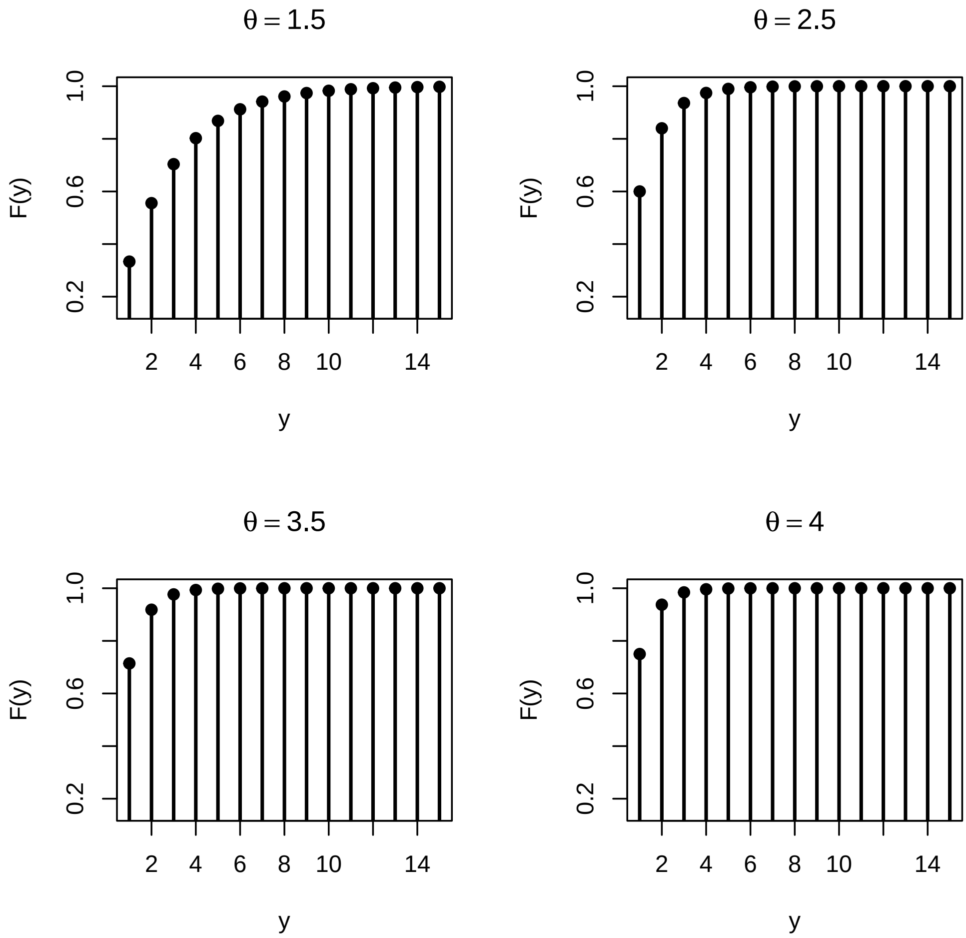

Figure 2: *CDF plots of ZTNDD*

3. Structural Properties and Generating Functions

In this part, we acquired various structural properties including factorial moments and raw moments. Also, MGF is obtained and is given in next subsection.

3.1. Raw Moments/Moments about Origin

The first four raw moments of the proposed model are:

$$m_1' = \frac{\theta}{\theta - 1}$$

$$m_2' = \frac{\theta^2 + \theta}{(\theta - 1)^2}$$

$$m_3' = \frac{\theta(\theta^2 + 4\theta + 1)}{(\theta - 1)^3}$$

$$m_4' = \frac{\theta(\theta^3 + 11\theta^2 + 11\theta + 1)}{(\theta - 1)^4}$$

3.2. Factorial Moments

The four factorial moments are:

$$m^{(1)} = \frac{\theta}{\theta - 1}$$

$$m^{(2)} = \frac{2\theta}{(\theta - 1)^2}$$

$$m^{(3)} = \frac{6\theta}{(\theta - 1)^3}$$

$$m^{(4)} = \frac{24\theta}{(\theta - 1)^4}$$

3.3. Moment Generating Function(M.G.F)

From the formula given below, we can get the M.G.F of proposed model as:

$$M_y(t) = E(e^{ty}) = \sum_{y=1}^{\infty} e^{ty} P(Y = y)$$

$$\Rightarrow M_y(t) = \frac{(\theta - 1)}{\theta} \sum_{y=1}^{\infty} \frac{e^{ty}}{\theta^y}$$

$$\Rightarrow M_y(t) = \frac{(\theta - 1)e^t}{(\theta - e^t)}$$

From the above calculations, the mean and variance of the proposed model are:

$$E(y) = m_1' = \frac{\theta}{\theta - 1} \tag{4}$$

$$V(y) = \sigma^2 = \frac{\theta}{\theta^2 + 1 - 2\theta}$$

Index of Dispersion(IoD) is given as

$$IoD = \frac{1}{\theta - 1}$$

Skewness and Kurtosis are given as:

$$\sqrt{\beta}_1 = \frac{\mu_3}{\sigma^3} = \frac{(1 + \theta)^2}{\theta}$$

$$\beta_2 = \frac{\mu_4}{\sigma^4} = \frac{(\theta^2 + 7\theta + 1)}{\theta}$$

Coefficient of Variation for the proposed model is:

$$C.V = \frac{1}{\sqrt{\theta}}$$

Figure 3: *Index of Dispersion plot for ZTNDD*

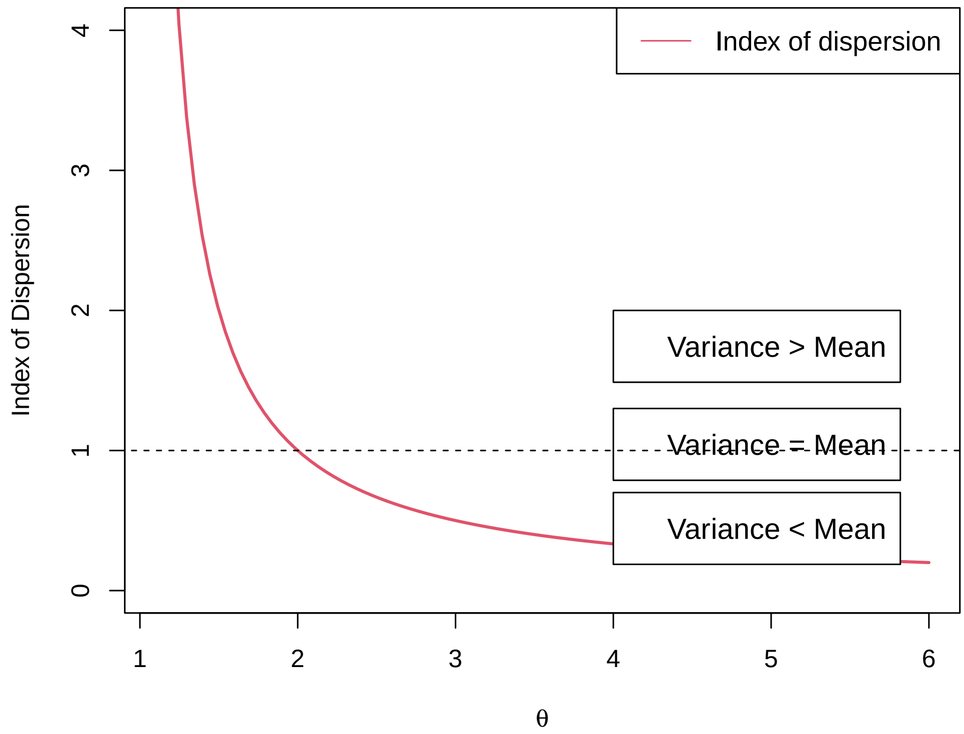

Table 1: *Behaviour of the model's descriptive statistics for various parameter values.*

θ	Mean	Variance	C.V	Skewness	Kurtosis	IoD
1.5	3.0000	6.0000	0.8165	4.1667	9.1667	2.0000
2.0	2.0000	2.0000	0.7071	4.5000	9.5000	1.0000
2.5	1.6667	1.1111	0.6324	4.9000	9.9000	0.6667
3.0	1.5000	0.7500	0.5774	5.3333	10.3333	0.5000
3.5	1.4000	0.5600	0.5345	5.7857	10.7857	0.4000
4.0	1.3333	0.4444	0.5000	6.2500	11.2500	0.3333
4.5	1.2857	0.3673	0.4714	6.7222	11.7222	0.2857
5.0	1.2500	0.3125	0.4472	7.2000	12.2000	0.2500

From the table, it is clear that our model is positively skewed and leptokurtic. Also, the model is equi-dispersed for (θ=2), underdispersed for (θ>2) and over-dispersed for (1<θ<2).

4. Parametric Estimation

One of the key problems in mathematical statistics is the parameter estimation. In this segment, we will look at the parametric estimation of the ZTNDD using the maximum likelihood estimation method as well as the method of moments.

4.1. Maximum Likelihood Method of Estimation

Let's take a sample y_1, y_2, y_3,...,y_n from ZTNDD of size n with parameter θ. The likelihood function is given by

$$L = \frac{(\theta-1)^n}{\theta^{\sum y_i}}$$

$$\Rightarrow \log L = n\log(\theta-1) - \log\theta \sum_{i=1}^{\infty} y_i$$

Differentiating above equation w.r.t. θ and equating to zero we get

$$\frac{\partial \log L}{\partial \theta} = \frac{n}{\theta-1} - \frac{\sum_{i=1}^{\infty} y_i}{\theta} = 0$$

$$\Rightarrow \hat{\theta} = \frac{\bar{y}}{\bar{y}-1}$$

4.2. Method of Moments

We must compare the first sample moment with the corresponding population moment in order to estimate the unknown parameter of the ZTND model using the technique/method of moments. Now, replacing μ by $\bar{y}$ in equation (4) we get the moment estimate of the parameter θ as:

$$\hat{\theta} = \frac{\bar{y}}{\bar{y}-1}$$

5. Reliability Analysis

This section deals with Reliability and hazard rate function of the proposed model.

5.1. Reliability Function

The likelihood that a system will continue to work after a certain amount of time is the reliability function. For the proposed model, it is given as:

$$R(y;\theta) = P(Y > y) = \frac{1}{\theta^y} \quad ; y = 1, 2, 3...$$

5.2. Hazard Rate

The hazard rate for ZTNDD is given by:

$$h(y;\theta) = \frac{P(y;\theta)}{R(y;\theta)}$$

$$\Rightarrow h(y;\theta) = \frac{(\theta-1)}{\theta^y}\theta^y = (\theta-1)$$

Table 2: *Values of Reliability function R(y;θ) for the proposed model*

y	$\theta = 1.5$	$\theta = 2.0$	$\theta = 2.5$	$\theta = 3.0$	$\theta = 3.5$	$\theta = 4.0$
1	0.667	0.500	0.400	0.333	0.286	0.250
2	0.444	0.250	0.160	0.111	0.082	0.0625
3	0.296	0.125	0.064	0.037	0.0233	0.016
4	0.198	0.063	0.026	0.012	007	004
5	0.132	0.031	0.010	0.004	0.002	–
6	0.088	0.016	0.004	0.001	–	–
7	0.059	0.008	0.002	–	–	–
8	0.039	0.004	–	–	–	–
9	0.026	0.002	–	–	–	–
10	0.017	–	–	–	–	–

5.3. Simulation Study

This part is based on extensive simulation studies to compare the effectiveness of the created estimator. Six parameter settings (θ=1.5,2.0,2.5,2.7,3.0,3.5) are taken into account from ZTNDD with sizes n=10, 25, 60, 350, and 500. The simulation process, which is described below, is based on 1000 iterations of the suggested model.

Table 3: *Simulation study of MLEs for proposed model*

Sample Size(n)	$\theta = 1.5$				$\theta = 2.0$			
	Bias	Variance	MSE	Coverage Probability (95%)	Bias	Variance	MSE	Coverage Probability (95%)
10	0.11539	0.07045	0.08376	1.00	0.57527	2.55556	2.88649	0.94
25	0.02430	0.01274	0.01332	0.98	0.10359	0.06458	0.07531	1.00
60	0.01663	0.00710	0.00738	0.96	0.00713	0.02679	0.02684	0.96
350	-0.00388	0.00099	0.00101	0.96	0.01830	0.00680	0.00713	0.96
500	0.00652	0.00083	0.00087	0.96	0.00118	0.00473	0.00473	0.90
Sample Size(n)	$\theta = 2.5$				$\theta = 2.7$			
	Bias	Variance	MSE	Coverage Probability (95%)	Bias	Variance	MSE	Coverage Probability (95%)
10	0.45452	0.96431	1.17090	0.94	0.74182	4.06066	4.61096	0.94
25	0.13630	0.12885	0.14743	0.96	0.05464	0.21413	0.21712	0.98
60	0.03798	0.04603	0.04748	0.98	0.02543	0.07962	0.08027	0.92
350	0.03642	0.01411	0.01543	0.92	-0.02979	0.02648	0.02737	0.90
500	0.02586	0.00647	0.00714	0.88	-0.01670	0.01099	0.01127	0.94
Sample Size(n)	$\theta = 3.0$				$\theta = 3.5$			
	Bias	Variance	MSE	Coverage Probability (95%)	Bias	Variance	MSE	Coverage Probability (95%)
10	0.68743	2.96483	3.4374	0.98	0.54364	3.79498	4.09053	0.96
25	0.25330	0.51149	0.57565	0.98	0.32301	0.65368	0.75802	0.98
60	0.16092	0.17636	0.20225	0.96	0.11374	0.24541	0.25835	0.92
350	0.03626	0.03187	0.03319	0.94	0.06039	0.04573	0.04938	0.96
500	-0.00804	0.00909	0.00916	0.96	-0.01819	0.02315	0.02349	0.96

It is clear from the simulation table (3) that the Bias and MSE decreases significantly as sample size increases. Additionally, when sample size grows, coverage probability gets closer to 0.95. We can therefore claim that MLE exhibits the consistency property. Therefore, we draw the conclusion that the MLE does a good job of forecasting the distribution's parameter.

6. Applications

In this part, we show that, in comparison to other competing models, our suggested model gives a better fitting when analysing two real-world datasets and one simulated data set. The first two data sets reflect European red mites, are taken from Garman [8] and Mcguire [16] respectively. These data sets are shown in Tables (4) and (7) respectively. The parameters of each of these distributions are determined using the maximum likelihood approach. The expected frequencies for fitting ZTNDD, ZTPD, ZTNBD, SBPD, SBNBD(Size Biased Negative Binomial), SBPLD(Size Biased Poisson Lindley Distribution)[1], and SBDLD(Size Biased Discrete Lindley)[24] are obtained using R studio, and the goodness of fit of the model is evaluated using Pearson's chi-square test. For each fitted model, the expected counts, chi square, and p-value are shown in Table (5 and 8). Additionally, we take into account the AIC (Akaike information criterion) and BIC criteria in order to evaluate our proposed distribution to the other competing models. The fact is lesser the AIC and BIC values, better is the distribution.

$$AIC = 2k - 2\log L \quad and \quad BIC = k\log n - 2\log L$$

where n is the sample size, k is the number of parameters in the model, and $logL$ is the log-likelihood function's maximum value for the model under consideration. Here Table (6 and 12) show that, in comparison to other competing models, the ZTND distribution has lower AIC and BIC values.

6.1. Data Set I

This data set is related to European red mites taken by Garman [8]. This data set was recently used by Rama & Simon in 2018 [13] in modelling zero truncaed discrete distribution.

Table 4: *European Red Mites[8]*

Count	1	2	3	4	5	6	7
Frequency	38	17	10	9	3	2	1

Table 5: *Expected frequencies and p-values of ZTND model and other competing models for data set I*

Count	Frequency	ZTNDD	ZTPD	ZTNBD	SBPD	SBNBD	SBPLD	SBDLD
1	38.00	37.00	29.00	36.00	25.00	37.00	32.00	30.00
2	17.00	20.00	26.00	21.00	29.00	20.00	24.00	25.00
3	10.00	11.00	15.00	11.00	17.00	11.00	13.00	14.00
4	9.00	6.00	7.00	6.00	6.00	6.00	6.00	6.00
5	3.00	3.00	2.00	3.00	2.00	3.00	3.00	3.00
6	2.00	2.00	1.00	2.00	0.00	2.00	1.00	1.00
7	1.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00
Degrees of Freedom		3.00	2.00	2.00	2.00	2.00	2.00	2.00
χ^2		2.07	10.08	2.46	20.73	2.07	6.36	8.34
p-value		0.56	0.01	0.29	<0.01	0.36	0.04	0.02

Table 6: *Different criterions for ZTND model and other competing models for data set I*

Criteria	ZTNDD	ZTPD	ZTNBD	SBPD	SBNBD	SBPLD	SBDLD
-l	118.80	122.79	119.73	127.89	119.78	120.07	121.08
AIC	239.60	247.59	241.46	257.78	241.56	242.13	244.17
BIC	241.99	249.97	246.22	260.16	246.33	244.51	246.55

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6.2. Data Set II

This data set is also related to European red mites taken from Mcguire [16]. This data set was recently used by Hassan & Mir[11] in analysing the Bayesian estimation of size biased generalized geometric series distribution.

Table 7: *European red mites [11]*

Count	1	2	3	4	5
Frequency	128	37	18	3	1

Table 8: *Expected frequencies and p-values of ZTND model and other competing models for Data set II*

Count	Frequency	ZTNDD	ZTPD	ZTNBD	SBPD	SBNBD	SBPLD	SBDLD
1	128.00	128.00	121.00	126.00	118.00	127.00	123.00	122.00
2	37.00	40.00	49.00	43.00	54.00	42.00	46.00	49.00
3	18.00	13.00	13.00	13.00	12.00	13.00	13.00	13.00
4	3.00	4.00	3.00	4.00	2.00	4.00	3.00	3.00
5	1.00	1.00	0.00	1.00	0.00	1.00	1.00	1.00
Degrees of Freedom		2.00	1.00	1.00	1.00	1.00	1.00	1.00
χ^2		2.35	5.59	2.99	10.77	2.73	3.43	4.70
p-value		0.31	0.02	0.08	0.00	0.10	0.06	0.03

Table 9: *Different criterions for ZTND model and other competing models for data set II*

Criteria	ZTNDD	ZTPD	ZTNBD	SBPD	SBNBD	SBPLD	SBDLD
-l	170.09	171.16	171.96	173.83	170.92	170.48	171.04
AIC	342.19	344.32	343.92	349.66	344.05	342.95	344.09
BIC	345.42	347.55	350.38	352.89	350.51	346.19	347.32

6.3. Simulated Data Set

For further testing our claim that our model is suitable for fitting purposes, we ran a simulation experiment. The simulated data is present in Table(10).

Table 10: *Simulated Data Set*

Count	1	2	3	4	5	6	7	8	9	10
Frequency	523	250	104	61	31	15	8	6	1	1

Table 11: *Expected frequencies and p-values of ZTND model and other competing models for Simulated data*

Count	Frequency	ZTNDD	ZTPD	ZTNBD	SBPD	SBNBD	SBPLD	SBDLD
1	523.00	514.00	425.00	510.00	388.00	514.00	457.00	439.00
2	250.00	250.00	323.00	253.00	367.00	250.00	300.00	316.00
3	104.00	122.00	164.00	123.00	174.00	122.00	145.00	152.00
4	61.00	59.00	62.00	60.00	55.00	59.00	61.00	61.00
5	31.00	29.00	19.00	29.00	13.00	29.00	24.00	22.00
6	15.00	14.00	5.00	14.00	2.00	14.00	9.00	7.00
7	8.00	7.00	1.00	7.00	0.00	7.00	3.00	2.00
8	6.00	3.00	0.00	3.00	0.00	3.00	1.00	1.00
9	1.00	2.00	0.00	1.00	0.00	2.00	0.00	0.00
10	1.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00
Degrees of Freedom		6	4	4	3	5	4	4
χ^2		3.90	172.81	4.86	260.35	3.90	56.42	92.80
$p-value$		0.69	<0.01	0.30	<0.01	0.56	<0.01	<0.01

Table 12: *Loglikelihood, AIC and BIC values of proposed model and other competing models for Simulated*

Criteria	ZTNDD	ZTPD	ZTNBD	SBPD	SBNBD	SBPLD	SBDLD
$-l$	1348.836	1418.286	1349.320	1477.478	1348.845	1373.345	1387.746
AIC	2699.672	2838.572	2702.641	2956.956	2701.690	2748.690	2777.492
BIC	2704.580	2843.480	2712.456	2961.864	2711.506	2753.598	2782.399

7. Discussion and Conclusion

From Table(1), it is evident that our model is positively skewed and leptokurtic in nature. Also, from the index of dispersion plot(3), it is obvious that our model can accommodate equi-dispersed, under-dispersed as well as over-dispersed data. From Table(3), it can be observed that the Bias and MSE decreases with increase in sample size and also as sample size increases the coverage probability approaches to 0.95.
From table(5,8, and 11), it can be observed that our model has highest p-value among all the competing models in all the three data sets. Also, the information criterion is used to validate our model, and it can be seen from table(6,9, and 12) that our model has lowest values of AIC, BIC criterions which agrees with the fact that lesser the values of AIC and BIC, better is the model.
In this paper we have introduced a zero truncated model namely zero truncated new discrete distribution(ZTNDD). We have derived the structural properties and generating functions of the truncated model including Moments about origin, Factorial moments, Moment generating function. Also, MLE is calculated, and for checking the behaviour of MLE, we have carried out a simulation study. We have fitted two real life data sets and one simulated data set, we compared our model with several other discrete models and it has been found that our model performs better than all models.

Author contributions All the authors have contributed equally.

Data Availability The data are given in the manuscript.

Declarations

Ethical statements Three datasets are used in the application section and two of them are taken from the literature.

Conflict of interest The authors have no conflict of interest.

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