PROJECT CHARACTERISTICS WITH TRIANGULAR FUZZY NUMBER

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Abstract

The Critical Path Method (CPM) is required to plan, organize, and arrange for major project networks. A clear estimate of the time duration will help in the successful execution of the CPM. However, the time duration cannot be precisely specified in real life. As a result, there is always uncertainty about the duration of activities, leading to the invention of the fuzzy critical path method. This study proposes a simple method for critical path analysis and project characteristics in a project network with a triangular fuzzy number. Furthermore, this study defines the most critical path and the relative path degree of criticality, both theoretically valid and practical. The suggested method can determine the critical path, project characteristics and critical degree of an activity as shown by an example discussed in some earlier studies. The proposed approach is very simple to apply and does not require knowing the explicit form of the membership functions of the fuzzy activity times.

Keywords: Activity duration, critical degree, critical path method, project characteristics, triangular fuzzy number,

1. Introduction

The project manager needs to complete the required work in time, but recognition of project obstruction plays the main task. Projects may be complicated to handle in various scenarios. To meet this challenge, the CPM has been proven to be a practical implementation in efficiently controlling projects when the activity durations are predictable and known. CPM's objective is to recognize essential activities in the project if resources can be focused on to shorten project duration.

The effective outcome of CPM needs the provision of clearly defined time frames for an individual task. Anyhow, this condition is sometimes difficult to meet in practice since many tasks will be performed initially. To meet such realistic conditions, Zadeh, 1965 [21] familiarized the theory of a fuzzy set. Because of ambiguity regarding the activities period in a network design, the Fuzzy Critical Path Method (FCPM) has been presented in the 1970s. Several techniques have been developed in past years for determining the fuzzy critical path.

Dubois, et al., 1978 [6] calculated the earliest, latest and slack times and criticality levels of activities using fuzzy numbers to demonstrate the network topology and series-parallel graphs. Gazdik [7] initially determined the activity durations and critical paths using fuzzy arithmetic operations. Atanassov et al. 1989 [3] defined membership value, non-membership value, and another element called hesitation; it has been used in various fields. Rommelfanger et al., 1994[16] calculated the activity's earliest, latest and slack times in a project network using fuzzy intervals and expanded all concepts of classical network procedure using the suggested methodology. In order to allocate renewable resources, the FPS (Fuzzy Project Scheduling) method was used. Using time parameters, understand durations, beginning timings, and due dates of certain activities. L-R fuzzy numbers are utilized to model the uncertainty of these parameters. Constructing optimistic and pessimistic schedules based on \propto -cut levels that have been specified are possible. A fuzzy project was a planning decision support system that applies L-R fuzzy numbers to evaluate activity times. Hapke et al., 1994 [8] proposed fuzzy project scheduling technique is employed in a software project scheduling context to allocate resources across related tasks. Scheduling is made more accurate by using a novel resource allocation approach that accounts for and includes uncertainty in the scheduling process. Loterapong et al., 1994 [12] proposed the fuzzy set theory approach to distribute resources to minimize unexpected project delays, use available resources, and minimize resource shortages. Three criteria are used to measure performance compared to the Minimum Late Finish (MINLFT) and the Minimum Slack (MINSLK) rules project duration, resource utilization, and resource interruption. Chang et al., 1995 [5] introduced the fuzzy Delphi method effectively determines the project length and the critical degree for each direction in a project. Attempted to substitute probabilistic or deterministic assumptions in the project network review with probabilistic ones and decrease the complexity caused by inexact and incomplete knowledge about activity times by representing activity times as fuzzy numbers. Chanas et al., 2002 [4] introduced the idea of criticality in the network having fuzzy activity times. It computed the path degree of criticality using two alternative approaches. Stefan et al., 2002 [20] evaluated the criticality of a network in terms of path and activity duration periods, where intervals or fuzzy intervals describe activity duration times. Sireesha et al., 2010 [19] adapted fuzzy triangular numbers to solve project scheduling problems. This technique computes every activity's total float and the network's critical path beyond performing any forward or backward computations. Oladeinde et al. 2013 [14] utilized an altered fuzzy backward pass technique with a recursive relation to obtaining the most recent start and finish times, which enabled them to overcome negative fuzzy numbers. Khalaf 2013 [11] suggested a new mechanism for dealing with fuzzy project scheduling problems by applying a ranking function. They also suggested a novel technique for estimating every activity's fuzzy cumulative, free, and independent slack. Jaya Gowri et al. [9] proposed an algorithm to tackle the problem in an intuitionistic fuzzy environment. The Triangular intuitionistic fuzzy number is defuzzified using graded mean integration representation. Now the intuitionistic fuzzy number is converted to a crisp number. Then applied, the proposed algorithm to find the critical path. Adilakshmi et al. 2021 [17] find the critical path and project duration with the fuzzy hexagonal number. In their paper, the activity times are represented as a hexagonal fuzzy number and obtained a new ranking function in hexagonal fuzzy number by the centroid of centroid method. By using the ranking function, hexagonal fuzzy numbers are transformed into crisp values. Priyadharshini et al. 2022 [15] developed a different algorithm, namely the maximum edge distance method, to find the optimal path in an intuitionistic fuzzy weighted directed graph with its edge weights as an intuitionistic triangular fuzzy number. The method is an alternative way to identify the critical path in the fuzzy environment. Adilakshmi et al. 2022 [1] find the critical path and project duration with the pentagonal fuzzy number. In their paper, the activity times are represented as a pentagonal fuzzy

number and obtained a new ranking function in pentagonal fuzzy number by the centroid of centroid method. Then applying a ranking function to PFNs is transformed into crisp values. Mahesh et al. [13] developed a new Fuzzy linear programming-based method using a single MF, called modified logistics MF. The modified MF logistics and its modifications taking into account the characteristics of the parameter are from the analysis process. This MF was tested for useful performance by modeling using FLP. Adilakshmi et al. 2022 [2] improved Dijkstra's algorithm for finding the critical path and project duration with the triangular fuzzy number. Their paper represents the activity times as a triangular fuzzy number. Then applied an algorithm to find the critical path.

This paper suggested measuring a fuzzy network's Critical Degree and project Characteristics utilizing tabular representation with Triangular fuzzy activity times and a new subtraction arithmetic operation.

2. Preliminaries

In this section, we will look at a few key definitions.

2.1. Fuzzy Set [16]

As stated in Zadeh's paper, the formalization of a fuzzy set is:

Let *X* be a space of points (objects), with a generic element of *X* denoted by *x*. Thus, $X = \{x\}$. A fuzzy set (class) *A* in *X* is characterized by a membership (characteristic function) function $\mu_A(x)$, which associates with each point in *X* a real number in the interval [0,1], with the value of $\mu_A(x)$ at *x* representing the "grade of membership" of *x* in *A*. When *A* set in the ordinary sense of the term, its membership function can take on only two values, 0 and 1, $\mu_A(x) = 1$ or 0 according to *x* does or does not belong to *A*.

2.2. Fuzzy Number [8]

It is a Fuzzy set of the following conditions:

- Convex fuzzy set
- Normalized fuzzy set.
- Its membership function is piece-wise continuous.
- It is defined in the real number.

Fuzzy numbers should be normalized and convex. Here the condition of normalization implies that the maximum membership value is 1.

2.3. Triangular Fuzzy Number (TFN) [8]

The TFN indicated by $\tilde{A} = (a, b, c)$ and its membership function is given by;

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & -\infty < x \le a \\ \frac{x-a}{b-a}, & a \le x < b \\ \frac{c-x}{c-b}, & b \le x < c \\ 0, & c \le x < \infty \end{cases}$$

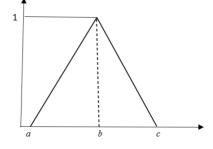


Figure 1: Graphical representation of Triangular fuzzy number

2.4. Arithmetic Operations of Triangular fuzzy number [4]

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two Triangular fuzzy numbers then; $k\tilde{A} = k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3)$ $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ $\tilde{A} \bigcirc \tilde{B} = (a_1 + a_1, a_2 + a_2, a_3 + a_3)$ $\tilde{A} \bigcirc \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$ $\tilde{A} \oslash \tilde{B} = (a_1 * b_1, a_2 * b_2, a_3 * b_3)$ $\tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{a_2}, \frac{a_2}{b_2}, \frac{a_3}{b_3}\right)$ Example: Let $\tilde{A} = (3,6,9)$ and $\tilde{B} = (2,4,6)$ then $\tilde{A} \oplus \tilde{B} = (5, 10, 15)$ $\tilde{A} \ominus \tilde{B} = (1,2,3)$ $\tilde{A} \otimes \tilde{B} = (6, 24, 54)$ $\tilde{A} \oslash \tilde{B} = (1.5, 1.5, 1.5)$ Remark: Some authors defined $\tilde{A} \ominus \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$. How is it possible? Here I consider one example. Let $\tilde{A} = (2,4,6) \& \tilde{B} = (2,4,6)$, Here both \tilde{A} and \tilde{B} are same TFNs. Now $\tilde{A} \ominus \tilde{B} = (2 - 6, 4 - 4, 6 - 2)$ = (-4,0,4).It is a completely wrong output since both \tilde{A} and \tilde{B} are the same. According to my definition;

$$\tilde{A} \ominus \tilde{B} = (2 - 2, 4 - 4, 6 - 6) = (0,0,0)$$

3. Proposal Method

Fuzzy numbers are acceptable representations in project networks since activity periods are frequently challenging to predict or define exactly. In this chapter, the length of every activity is expressed as a positive TFN.

The below steps to follow to find project critical path and characteristics.

Step1: Construct a network diagram G(V, E, T) with the set of given vertices and edges, where $V = \{v_1, v_2, ..., v_n\}$ be set of *n* vertices, *E* be set of edges which represent activities, and *T* be set of fuzzy triangular numbers as activity times.

Step2: Construct a square table with $(n - 1) \times (n - 1)$ order."i" indicate the number of rows, and the column number is indicated by *j*, where i = 1,2,3...,n - 1; j = 2,3,...,n.Two numbers (i, j) describes each entry in the table, where indicates the row number and *j* indicates the column number.

Step3: Set the entries \tilde{T}_{ij} , the highest fuzzy time needed from source node to tail node $i \rightarrow j$. Insert the times \tilde{t}_{ij} in the table's first row. Then add the preceding time of the second node and t_{2j} to complete the elements in the second row i = 2. Then proceed by adding the preceding time of the third node and t_{3j} to the elements of the third row i = 3, and so on. If we get more than one path, then we take the maximum value. Continuing this procedure on down the rows up to the ending row is attained.

Step4: To determine the critical path, go backward from the known endpoint. Begin by picking the element from the end column j = n with the highest fuzzy value. This value represents the project completion time. The row number corresponding to the highest value in the end column provides the preceding activity's end node index; hence, the next activity head node on the critical path is noted. If column-n has the greatest number in row k, column k is checked, and the greatest number in that column is needed. This step is continued until the initial node is reached when a list of all actions along the fuzzy critical path is obtained.

Step5: Create a new $(n - 1) \times (n - 1)$ order table to compute free slack times. Choose the entry with the largest value $\tilde{T}_{end} = \tilde{T}_{kj}$ where k = 2, 3, ..., n - 1; j = n.

The free slack of activity (i, j) is calculated using $F\tilde{S}_{ij} = max{\tilde{T}_{kj}} - \tilde{T}_{ij}$.

Where $\tilde{T}_{ij} = t_{ij} + the maximum fuzzy completion time in column j.$

Step6: Form another $(n-1) \times (n-1)$ order table for computing total slack time using $T\tilde{S}_{ij} = F\tilde{S}_{ij} + min\{T\tilde{S}_{jk}\}$ for all k > j.

In the last row $T\tilde{S}_{ij} = F\tilde{S}_{ij}$ is taken to begin the computations.

Step7: Compute the earliest and latest activity periods by using;

Earliest start time $(E\tilde{S}_{ij}) = \tilde{E}_i$

Earliest finish time
$$(E\tilde{F}_{ii}) = \tilde{E}_i + \tilde{t}_{ii}$$

Latest finish time $(L\tilde{F}_{ii}) = \tilde{L}_i$

Latest start time $(L\tilde{S}_{ij}) = \tilde{L}_j - \tilde{t}_{ij}$ Total Float $(T\tilde{F}_{ij}) = L\tilde{S}_{ij} - E\tilde{S}_{ij}$ or $L\tilde{F}_{ij} - E\tilde{F}_{ij}$

4. Application

Consider a site preparation and concrete slab foundation that includes nine different activities. The network table is shown in Table 1, and the network diagram is presented in Figure 1, where every activity duration is presented as a TFN.

Activity	Activity description	TFN
1→2	Removal tress	(6,12,18)
1→3	Site clearing	(13,25,33)
2→3	General excavation	(15,20,29)
2→4	Placing Formwork and reinforcement for concrete	(2,11,20)
2→5	Excavation for utility frame work	(7,16,25)
3→5	Grading general area	(17,25,33)
4→5	Installing sewer line	(8,9,10)
4→6	Pouring concrete	(15,21,31)
5→6	Installing other utilities	(4,11,14)

Table1: Project Activity duration with TFN

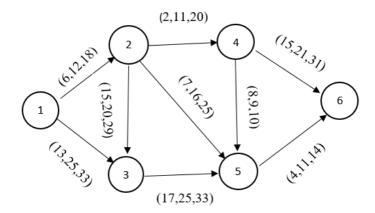


Figure2: *Fuzzy project network with activity time as TFN*

The tabular representation of Fuzzy activity times is presented in Table 2.

(i,j)	2	3	4	5	6
1	(6,12,18)	(13,25,33)			
2		(15,20,29)	(2,11,20)	(7,16,25)	
3				(17,25,33)	
4				(8,9,10)	(15,21,31)
5					(4,11,14)

Table 2: Tabular description of Fuzzy activity times with TFN

5. Results

5.1. Project critical path

In section 3 explained the procedure. From this procedure, calculated fuzzy critical path. Table3 represents maximum fuzzy time durations.

_	Table 3: Tubular representation of waximum Fuzzy activity duration (T_{ij})					
	(i,j)	2	3	4	5	6
	1	(6,12,18)	(13,25,33)			
	2		(21,32,47)	(8,23,38)	(13,28,43)	
	3				(38,57,80)	
	4				(16,32,48)	(23,44,69)
	5					(42,68,94)

Table 3: Tabular representation of Maximum Fuzzy activity duration (\tilde{T}_{ij})

Using the following ranking formula, calculated rank of maximum fuzzy time duration (\tilde{T}_{ij}) and represented in Table 4.

Let (a, b, c) be a TFN, then $\mathcal{R}(a, b, c) = \frac{a+2b+c}{4}$.

(i,j)	2	3	4	5	6
1	12	24			
2		33	23	28	
3				58	
4				32	45
5					68

Table 4: *Tabular representation of Rank of* \tilde{T}_{ij}

From Table 4,

In the 6th column; $\mathcal{R}{\tilde{T}_{56}} > R{\tilde{T}_{46}} \Longrightarrow \tilde{T}_{56} > \tilde{T}_{46}$

As a result, the sixth row has the greatest fuzzy value in the last column.

Therefore, the last activity's head node index is 5, and the last activity's tail node index is 6.

In the 5th column; $\mathcal{R}{\tilde{T}_{35}} > R{\tilde{T}_{45}} > R{\tilde{T}_{25}} \Longrightarrow \tilde{T}_{35} > \tilde{T}_{45} > \tilde{T}_{25}$

Therefore, the 3rdrow is the highest fuzzy value in the 5th column.

Thus, the last activity's head node index is 3, and the last activity's tail node index is 5.

In the 4th column;

 \tilde{T}_{24} is the only fuzzy number in the column.

Therefore, the 2nd row is the highest fuzzy value in the 4th column.

Thus, the last activity's head node index is 2, and the last activity's tail node index is 4. In the 3rd column;

In the 3rd column, $\mathcal{R}{\tilde{T}_{23}} > R{\tilde{T}_{13}} \Longrightarrow \tilde{T}_{23} > \tilde{T}_{13}$

Therefore, the 3rd row is the highest fuzzy value in the 3rd column.

Thus, the last activity's head node index is 2, and the last activity's tail node index is 3.

In the 2nd column, \tilde{T}_{12} is the only fuzzy number.

Therefore, 1st row is the highest fuzzy value in 2nd column.

Thus, the last activity's head node index is 1, and the last activity's tail node index is 2.

The head node on the fuzzy critical path of the next to last activity is identified if the column's highest fuzzy number is in row k, then column k is checked, and the highest fuzzy number in that column is needed. Then, the last activity's head node index is 1, and the last activity's tail node index is 2.

The tabular representation of identified critical path is presented in Table 5.

(<i>i</i> , <i>j</i>)	2	3	4	5	6
1	(6,12,18)				
2		(21,32,47)			
3				(38,57,80)	
4					
5					(42,68,94)

 Table 5: Maximum Fuzzy duration of Critical path

Therefore, the project critical path is $1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 6$ and the project end period is 68 days.

5.2. Project Characteristics

In section 3, explained earliest, latest and total floats formulas. Adapting those formulas, computed earliest, latest and total float of every activity.

5.2.1. Fuzzy Earliest and Latest Times of events

Table 6 shows the fuzzy events earliest and latest times.

Event (i)	\tilde{E}_i	\tilde{L}_i
1	(0,0,0)	(0,0,0)
2	(6,12,18)	(6,12,18)
3	(21,32,47)	(21,32,47)
4	(8,23,38)	(27,47,63)
5	(38,57,80)	(38,57,80)
6	(42,68,94)	(42,68,94)

Table 6: Earliest and Latest times of Fuzzy Network events with TFN

Table 7 represents activities earliest; latest and total float time of every activity.

$i \rightarrow j$	E Ŝ _{ij}	$E\tilde{F}_{ij}$	LŜ _{ij}	L <i>Ĩ</i> ij	Total float($T\tilde{F}_{ij}$)
1→2	(0,0,0)	(6,12,18)	(0,0,0)	(6,12,18)	(0,0,0) *
1→3	(0,0,0)	(13,25,33)	(8,7,14)	(21,32,47)	(8,7,14)
2→3	(6,12,18)	(21,32,47)	(6,12,18)	(21,32,47)	(0,0,0) *
2→4	(6,12,18)	(8,23,38)	(25,36,43)	(27,47,63)	(19,24,25)
2→5	(6,12,18)	(13,28,43)	(31,41,55)	(38,57,80)	(25,29,37)
3→5	(21,32,47)	(38,57,80)	(21,32,47)	(38,57,80)	(0,0,0) *
4→5	(8,23,38)	(16,32,48)	(30,48,70)	(38,57,80)	(22,25,32)
4→6	(8,23,38)	(23,44,69)	(27,47,63)	(42,68,94)	(19,24,25)
5→6	(38,57,80)	(42,68,94)	(38,57,80)	(42,68,94)	(0,0,0) *

 Table 7: Earliest, latest and total float times of every activity with TFN

5.3. Fuzzy slack times

Slack time is an important factor in project management and planning. There are two kinds of slacks for each activity, i.e., fuzzy free slack $(F\tilde{S}_{ij})$ and fuzzy total slack $(T\tilde{S}_{ij})$.

5.3.1. Fuzzy free slack time

An activity's fuzzy free slack time $(F\tilde{S}_{ij})$ is the duration of activity that could be prolonged, or its beginning can be delayed without impacting the beginning period of the immediately following activity. By default, every value in the \tilde{T}_{ij} table reflects the maximal duration needed to achieve and complete the task(i, j); thus, the gap within the maximum column and particular element in that column indicates how much free slack time is feasible for that particular activity.

In section 3, the method to find Fuzzy free slack times using a tabular representation is explained. Fuzzy free slack time calculations

 $F\tilde{S}_{12} = (6,12,18) - (6,12,18) = (0,0,0)$ $F\tilde{S}_{13} = (21,32,47) - (13,25,33) = (8,7,14)$ $F\tilde{S}_{23} = (21,32,47) - (21,32,47) = (0,0,0)$ $F\tilde{S}_{24} = (8,3,28) - (8,23,38) = (0,0,0)$ $F\tilde{S}_{25} = (38,57,80) - (13,28,43) = (25,29,37)$
$$\begin{split} & F\tilde{S}_{35} = (38,57,80) - (38,57,80) = (0,0,0) \\ & F\tilde{S}_{45} = (38,57,80) - (16,32,48) = (22,25,32) \\ & F\tilde{S}_{46} = (42,68,94) - (23,44,69) = (19,24,25) \\ & F\tilde{S}_{56} = (42,68,94) - (42,68,94) = (0,0,0) \end{split}$$

The calculated free slack times of every activity are represented in Table 8.

$i \rightarrow j$	2	3	4	5	6
1	(0,0,0)	(8,7,14)			
2		(0,0,0)	(0,0,0)	(25,29,37)	
3				(0,0,0)	
4				(22,25,32)	(19,24,25)
5					(0,0,0)

Table 8: Tabular representation of free slack times

5.3.2. Total Slack Fuzzy Times

The total slack time is the delay or an activity extension(i, j)period that can occur without affecting the project's completion.

In section 3, explained how to calculate total slack time for each activity.

Total slack time calculations for fuzzy project network

$$T\tilde{S}_{56} = (0,0,0) + (0,0,0) = (0,0,0)$$

 $T\tilde{S}_{46} = (19,24,25) + (0,0,0) = (0,0,0)$

 $T\tilde{S}_{45} = (22,25,32) + (0,0,0) = (22,25,32)$

 $T\tilde{S}_{35} = (0,0,0) + (0,0,0) = (0,0,0)$

 $T\tilde{S}_{25} = (25,29,37) + (0,0,0) = (25,29,37)$

 $T\tilde{S}_{24} = (0,0,0) + (22,25,32) = (22,25,32)$ $T\tilde{S}_{22} = (0,0,0) + (0,0,0) = (0,0,0)$

$$TS_{23} = (0,0,0) + (0,0,0) = (0,0,0)$$

 $T\tilde{S} = (0,7,14) + (0,0,0) = (0,7,14)$

 $TS_{13} = (8,7,14) + (0,0,0) = (8,7,14)$ $T\tilde{S}_{12} = (0,0,0) + (0,0,0) = (0,0,0)$

Table 9 displays the total fuzzy slack periods of every activity in the fuzzy project network.

$i \longrightarrow j$	2	3	4	5	6
1	(0,0,0)	(8,7,14)			
2		(0,0,0)	(19,24,25)	(25,29,37)	
3				(0,0,0)	
4				(22,25,32)	
5					(0,0,0)

Table 9: Tabular representation of total float of each activity	
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The above table shows that activities $1 \rightarrow 2$, $2 \rightarrow 3$, $3 \rightarrow 5$, $5 \rightarrow 6$ have zero slack time. Therefore, the critical path is $1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 6$.

5.4. Critical degree

In the PERT, an activity considers a critical activity; its total float is zero. That is, the total float reduces, and then criticality increases. In the Fuzzy methodology, if $C\tilde{D}_{ij} = 1$, the activity (i, j) is considered as a critical activity.

Let $T\tilde{F}_{ij} = (a_i, b_i, c_i)$ be the total float of fuzzy activity(*i*, *j*), then the Critical Degree of activity is described as follows:

$$C\widetilde{D}_{ij} = \begin{cases} 1, & b_i \leq 0\\ -a_i \\ \overline{b_i - a_i}, & a_i < 0 < b_i \\ 0, a_i \geq 0 \end{cases}$$

Table 10 provides the critical degree calculations for each activity.

Table 10: Critical Degree of Each Activity							
Activity	Total float	Critical degree					
1→2	(0,0,0)	1*					
1→3	(8,7,14)	0					
2→3	(0,0,0)	1*					
2→4	(19,24,25)	0					
2→5	(25,29,37)	0					
3→5	(0,0,0)	1*					
4→5	(22,25,32)	0					
4→6	(19,24,25)	0					
5→6	(0,0,0)	1*					

We noticed from the above table that activities $1\rightarrow 2$, $2\rightarrow 3$, $3\rightarrow 5$, $5\rightarrow 6$ have critical degree 1. According to the critical degree definition, activities $1\rightarrow 2$, $2\rightarrow 3$, $3\rightarrow 5$, and $5\rightarrow 6$ is considered as

Therefore, the critical path is $1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 6$.

6. Conclusion

This paper presented various methods to determine the critical path with a triangular fuzzy number. Earlier work on network planning employing fuzzy sets theory has developed approaches for project scheduling. However, these techniques do not directly enable backward-pass computations in the same way they do forward-pass calculations. Project parameters like earliest, latest times, total float, slack times and critical degree are computed with TFNs. One significant set of this technique uses primary arithmetic fuzzy operations to get relevant determinable outputs. We presented a novel strategy in this paper: the Fuzzy tabular representation.

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critical activities.

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