Bayesian Analysis of ARMA and BSTS models for COVID-19 data using R and Stan

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Abstract

This study compares the performance of Bayesian ARIMA and BSTS models for COVID-19 data using Bayesian approach. Many studies in the literature have compared the BSTS model and classical ARIMA models for infectious disease modelling, and the BSTS model performs well. Apart from the literature, this study is trying to prove the Bayesian ARIMA model gives a better result than the BSTS model. This study uses a different modelling and model comparison method to compare widely used autoregressive integrated moving average (ARIMA) models with their Bayesian structural time series (BSTS) models for COVID-19 data using the Bayesian approach. It is essential to find the order of the ARIMA model before doing bayesian analysis. We find the order of the ARIMA model using measurement LOO information criteria , using the Hamiltonian Montecarlo algorithm and rstan estimate the parameters of ARIMA and BSTS models for COVID-19 data. Furthermore, compare both models using Looic and Waic values; Bayesian ARIMA models outperform in this study.

Keywords: BSTS, COVID-19, ARIMA, MCMC, LOOIC, WAIC, Stan

1. Introduction

The literature contains a variety of traditional studies that use ARIMA models. The statistical models and techniques for evaluating discrete time series are discussed in [1], along with some of the methodology's most significant applications. The class of autoregressive integrated moving average (ARIMA) models and various extensions of these models are among the models taken into consideration. [2] used the ARIMA model, a version of ARMA, and fitted the same to non-seasonal data by identifying autoregressive and moving average terms with the help of PACF and ACF. [3] describes how to find ARIMA models using the extended autocorrelation function. Over the past few decades, the Bayesian method's significance in econometrics has grown significantly. In this sense, significant references include [4], [5], [6], [7], and [8], etc. [9] clarifies the autoregressive and moving average parameters are implicitly constrained to the stationary and invertible region using a straightforward reparameterisation. [10] offers a few new transformation concepts and looks at how they fit into an effective numerical integration strategy for ARIMA models. The ARMA model can be used to model a variety of data sets due to its universal structure. Typically, the theory does not specify which model should be chosen, so it must be chosen from among a variety of competing models. The choice of an ARIMA model is vital for both statistical inference and prediction.

The bayesian structural time series (BSTS) model, which is based on [11] technique, is another model we used in this work. Create numerous layers utilising this model, such as trends, seasonality, and regression components. In addition, unlike with traditional ARIMA models, stationary time series are not required for BSTS models, since the BSTS model can deal with structural changes in time series. In contrast to classical ARIMA and machine learning models, a notable advantage of the BSTS model is its distinct interpretable structure in both observable and unobservable dynamic components.

2. Methodology

In many studies comparison between classical ARIMA and BSTS models for COVID-19 data, the BSTS model performs well in these studies [12, 13, 14]. Apart from the literature, this study is trying to prove the Bayesian ARIMA model gives a better result than the BSTS model for COVID-19 data. The study's goal is to provide a Bayesian approach to assess ARIMA models and the BSTS model, utilizing the Hamiltonian Monte Carlo (HMC) algorithm and the programming language Rstan. This is being done for the COVID-19 cumulative number of cases and the cumulative number of deaths from the date 01 March 2020, to 30 June 2021. Performing this study in two steps; first, for the analysis of Bayesian ARIMA models, it is important to find the order of the ARIMA model. We find the optimized order using the LOOIC value, and the model with the lowest LOOIC value is considered the best model [15]. In the second step, using various priors, we estimate the parameters of the BSTS model and the Bayesian ARIMA model. In the following phase, we compare the BSTS model and the Bayesian ARIMA model using various measures for the corresponding data, such as the looic and waic values [16]. The model with the lowest looic and waic values is considered the best model [17].

3. ARIMA model

ARMA model with AR coefficient ϕ_i 's and MA coefficient θ_j 's is defined as

$$y_t = \mu_0 + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t \tag{1}$$

in equation 1, p and q are order of ARMA model, y_t 's are the data points collected over time, μ_0 is the intercept, and ϵ 's are the error terms distributed according to IID normal variate with mean zero and variance σ^2. Equation 1 is denoted by ARIMA (p,q) model. Invertibility enables us to estimate the noise (ϵ_t) recursively by

$$\hat{\epsilon}_t = y_t - \mu_0 - \sum_{i=1}^{p} \phi_i y_{t-i} - \sum_{j=1}^{q} \theta_j \epsilon_{t-j}, \quad t = 1, 2, \ldots, T \tag{2}$$

As an approximation, we consider the starting values $y_t = \epsilon_t = 0$ for $t \leq 0$. Choosing $p + q \leq 2$ greatly simplifies the model, and it is frequently observed that higher order ARMA models are more difficult to justify in practise. (see, for example, [6]). A careful review of the literature also suggests that the problems of stationarity and invertibility can be easily tackled for small p and q, say $p + q \leq 2$.

Advantages of Bayesian ARIMA models:

- Flexibility: The Bayesian ARIMA model can handle a wide range of time series patterns, such as non-stationary, multi-seasonal, and multi-trend data.
- Better forecasting: The Bayesian approach to ARIMA models allows for more accurate predictions by taking into account uncertainties in the model parameters.
- Model selection: The Bayesian framework enables the use of model selection methods, such as LOOIC, which helps to determine the optimal number of AR and MA terms in the model.
- Prior information: The Bayesian ARIMA model allows for incorporating prior information or domain knowledge into the model, making it more robust and reliable.

- Model uncertainty: The Bayesian ARIMA model provides a quantification of model uncertainty, which can be useful in decision-making processes.
- Model comparison: The Bayesian ARIMA model allows for comparison with other models, and selection of the best model based on a range of criteria.

3.1. Bayesian Model Formulation of ARMA model

Let $\underline{y} : y_1, y_2, \ldots, y_T$ be the time series observations from a strictly stationary and invertible ARMA model 1. To write the approximate likelihood function for an ARMA(p, q) model with y_t's are the time series observed data is
$y_t \sim N\left(\left(\mu_0 + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + \theta_1 \epsilon_{t-1} + \ldots + \theta_q \epsilon_{t-q}\right), \sigma^2\right)$ under the assumption of $y_t = \epsilon_t = 0$ for $t \leq 0$. The conditional density of y_t given $y_{t-1}, y_{t-2}, \ldots, y_{t-p}$ can then be written as,

$$f\left(y_t \mid y_{t-1}, y_{t-2}, \ldots, y_{t-p}; \mu_0, \Phi, \Theta\right) \propto \left(\frac{1}{\sigma^2}\right) \exp\left(-\frac{1}{2\sigma^2}\left(y_t - \mu_0 - \sum_{i=1}^{p} \phi_i y_{t-i} - \sum_{j=1}^{q} \theta_j \epsilon_{t-j}\right)^2\right). \tag{3}$$

The likelihood function is defined as:

$$f\left(\underline{y} \mid \mu_0, \Phi, \Theta\right) \propto \prod_{t=p+1}^{T} f\left(y_t \mid y_{t-1}, y_{t-2}, \ldots, y_{t-p}; \mu_0, \Phi, \Theta\right) \tag{4}$$

which reduces to:

$$f\left(\underline{y} \mid \mu_0, \Phi, \Theta\right) \propto \left(\frac{1}{\sigma^2}\right)^{(T-p)/2} \exp\left(-\frac{1}{2\sigma^2} \sum_{t=p+1}^{T}\left(y_t - \mu_0 - \sum_{i=1}^{p} \phi_i y_{t-i} - \sum_{j=1}^{q} \theta_j \epsilon_{t-j}\right)^2\right) \tag{5}$$

4. BSTS model

The BSTS model is defined by a pair of equations,

$$\begin{aligned} y_t &= Z_t^{\mathrm{T}} \alpha_t + \varepsilon_t, \\ \alpha_{t+1} &= T_t \alpha_t + R_t \eta_t, \end{aligned} \tag{6}$$

where $\varepsilon_t \sim \mathcal{N}\left(0, \sigma_t^2\right)$ and $\eta_t \sim \mathcal{N}\left(0, Q_t\right)$, both error terms are independent of all other unknowns [11]. In this study, we are using a local linear trend model (a BSTS model without regression components) that assumes level and slope as random walk components.

4.1. Local linear trend model

The local linear trend model is a popular option for trend modelling because it responds rapidly to local variation, which is important when making short-term forecasts. The equation as follows:

$$y_t = \mu_t + \epsilon_t \quad \epsilon_t \sim N\left(0, \sigma_\epsilon^2\right) \tag{7}$$

the equation of the level component is:

$$\mu_{t+1} = \mu_t + \delta_t + \eta_t \quad \eta_t \sim N\left(0, \sigma_\eta^2\right) \tag{8}$$

and the equation of the slope is:

$$\delta_{t+1} = \delta_t + \zeta_t \quad \zeta_t \sim N\left(0, \sigma_\zeta^2\right) \tag{9}$$

The local linear trend is based on the supposition that both the mean and slope components follow random walks.

5. Bayesian estimation using R and Stan

5.1. Bayesian estimation of ARIMA models

In this session, use the corresponding priors to express the following stan code for the ARIMA(1,0,1) model in R:

$$\mu_0 \sim studentt(0, 1, 6)$$

$$\phi_1 \sim normal(0, 0.5)$$

$$\theta_1 \sim normal(0, 0.5)$$

$$\sigma \sim studentt(0, 1, 7)$$

For all ARIMA models, we use the same priors for μ and σ; for all AR components, we use prior as $normal(0, 0.5)$, and for all MA components, we use prior as $normal(0, 0.5)$. Stan code for the ARIMA(1,0,1) model is given below:

```
armamodel="data {
int<lower=1> N;
real y[N];
}
parameters {
real mu0;
real phi1;
real theta1;
real<lower=0> sigma;
}
model {
vector[N] nu;
vector[N] err;
psi[1] = mu0 + phi1 * mu0;
err[1] = y[1] - psi[1];
for ( n in 2:N) {
psi[n] = mu0 + phi1 * y[n-1] + theta1 * err[n-1];
err[n] = y[n] - psi[n];
}
mu0 ~ studentt(0, 1, 6);
phi1 ~ normal(0, 0.5);
theta1 ~ normal(0, 0.5);
sigma ~ studentt(0, 1, 7);
err ~ normal(0, sigma);
}
"
```

Only the ARIMA(1,0,1) model code is expressed here; other types of ARIMA models were used in this study but were not displayed owing to space limitations.

5.2. BSTS models

In this session, the Stan formulation of the Bayesian structural time series model is discussed. The appropriate priors for the parameters are discussed below:

$$u_{err} \sim N(0, 1)$$

$$v_{err} \sim N(0, 1)$$

$$\sigma_{slope} \sim N(0, 0.5)$$

$$\sigma_{level} \sim N(0, 0.5)$$

$$\sigma_{obs} \sim N(5, 10)$$

Stan code for BSTS model is:

```
\label{stan:2}
bstsmodel="
data {
int <lower=0> N;
vector[N] y;
}
parameters {
vector[N] a_err;
vector[N] b_err;
real beta;
real <lower=0> sigma_obs;
real <lower=0> sigma_slope;
real <lower=0> sigma_level;
}

transformed parameters {
vector[N] a;
vector[N] b;
a[1] = a_err[1];
b[1] = b_err[1];
for (n in 2:N) {
a[n] = a[n-1] + b[n-1] + s_level*u_err[n] ;
b[n] = b[n-1] + s_slope*v_err[n] ;
}
}

model {
a_err ~ normal(0,1);
b_err ~ normal(0,1);
sigma_slope~normal(0,0.5);
sigma_level~normal(0,0.5);
sigma_obs~normal(5,10);
for(n in 1:N){
y[n] ~ normal (a[n] ,sigma_obs);
}
}
generated quantities{
vector[N] yrepg;
real log_lik[N];
for(n in 1:N){log_lik[n]=normal_lpdf(y[n]|a[n],sigma_obs);
}
}
"
```

In this chapter for the BSTS model, we apply the same priors and stan code for the remaining data sets; when we use the same priors for all data sets, we can more precisely compare our looic and waic values.

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6. Results and discussions

During this session, we reviewed the estimated parameters obtained after executing Stan code for both Bayesian ARIMA and BSTS models.

6.1. ARIMA model Bayesian estimation

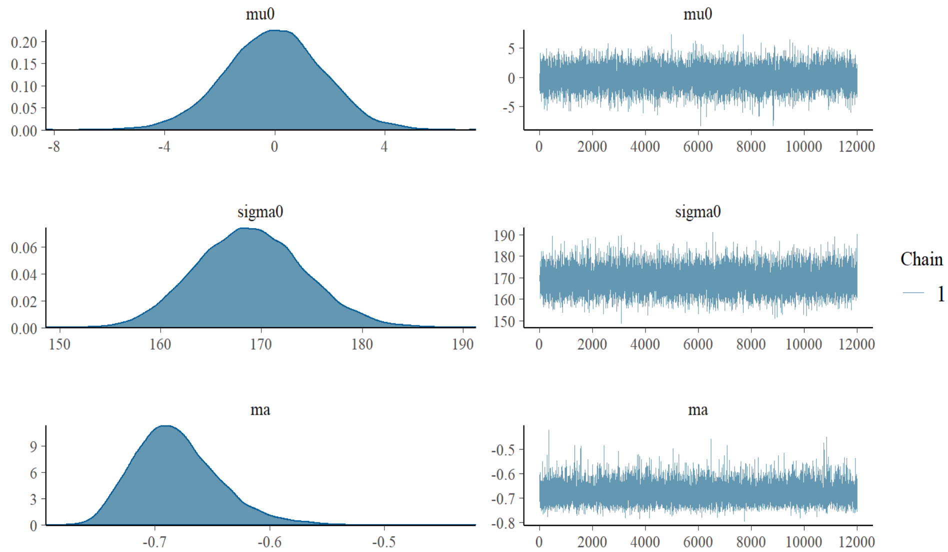
6.1.1 ARIMA model selection

Finding the AR component, MA component, and order of differencing for the model is the first step before using an ARIMA model. Table 1 lists various p and q order ARIMA models and their associated LOOIC values. We assess alternative order ARIMA models with LOOIC values for the cumulative number of cases and the cumulative number of deaths for each of the five countries. We evaluate the model with the lowest LOOIC value as the ideal model, and we employ that model in further bayesian analysis. For instance, from table 1, take the cumulative number of cases in the USA, the minimum LOOIC value is 10720.14 for the order of ARIMA (1, 2, 1), so we find our ARIMA model using the minimum LOOIC value.Similarly, for the cumulative number of deaths in the USA, the minimum LOOIC value is 7000.51 for the model ARIMA (3, 2, 2), so we use this model in further Bayesian analysis. Similarly, we ordered ARIMA for further Bayesian analysis for all other countries' cumulative cases and deaths.

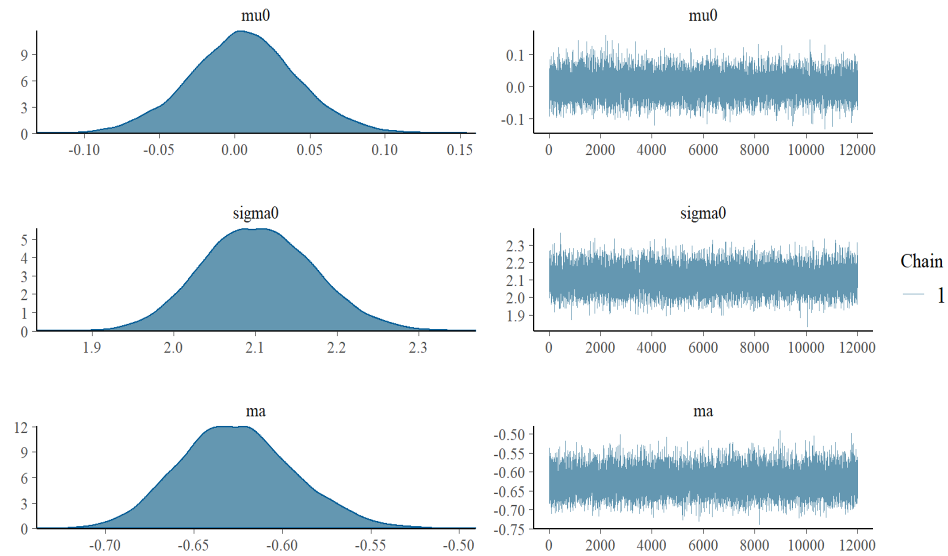
Table 1: *ARIMA model selection*

Countries	Cases model	AIC value	Deaths model	AIC value
USA	ARIMA(2,,2,2)	10721.76	ARIMA(2,2,2)	7007.206
	ARIMA(0,2,1)	10730.94	ARIMA(1,2,2)	7176.245
	ARIMA(1,2,2)	10721.83	ARIMA(2,2,1)	7131.75
	ARIMA(2,2,1)	10722.18	ARIMA(3,2,2)	7000.51
	ARIMA(1,2,1)	10720.14	ARIMA(3,2,1)	7115.77
	ARIMA(2,2,0)	10751.45	ARIMA(3,2,3)	7012.184
UK	ARIMA(2,2,2)	8921.798	ARIMA(2,2,2)	6181.108
	ARIMA(0,2,0)	8935.118	ARIMA(1,2,2)	6204.96
	ARIMA(1,2,0)	8919.275	ARIMA(2,2,1)	6237.94
	ARIMA(0,2,1)	8917.05	ARIMA(5,2,2)	5801.33
	ARIMA(1,2,1)	8918.23	ARIMA(5,2,1)	5968.22
	ARIMA(1,2,2)	8920.127	ARIMA(4,2,1)	6114.78
UAE	ARIMA(0,2,0)	6906.30	ARIMA(2,2,2)	1987.16
	ARIMA(1,2,0)	6788.24	ARIMA(0,2,1)	1984.84
	ARIMA(0,2,1)	6729.45	ARIMA(1,2,1)	1983.08
	ARIMA(1,2,1)	6732.08	ARIMA(2,2,1)	1985
	ARIMA(0,2,2)	6731.07	ARIMA(1,2,2)	1973.254
Bahrain	ARIMA(2,2,2)	5911.402	ARIMA(2,2,2)	2096.992
	ARIMA(1,2,2)	5908.39	ARIMA(0,2,1)	2096.94
	ARIMA(1,2,1)	5929.98	ARIMA(1,2,1)	2099.94
	ARIMA(2,2,1)	5931.204	ARIMA(1,2,2)	2101.53
India	ARIMA(2,2,2)	10000.67	ARIMA(2,2,2)	6895.87
	ARIMA(2,2,1)	10033.83	ARIMA(1,2,1)	6895.35
	ARIMA(3,2,2)	9941.79	ARIMA(0,2,1)	6892.32
	ARIMA(3,2,1)	9990.04	ARIMA(1,2,2)	6895.8

Using the above mentioned stan code in 5.1 and using different priors to estimate unknown parameters shown in table 2. Figure 1a and 1b show the posterior density plot and trace plot of the ARIMA model for the cumulative number of UK cases and the cumulative number of Bahrain deaths, respectively. Due to space restrctions, here is the convergence plot of two countries: the UK and Bahrain are only displayed.



(a)



(b)

Figure 1: *(a) ARIMA model for UK cases Posterior density plot, and Traceplot (b) ARIMA model for bahrain deaths Posterior density plot, and Traceplot*

Table 2 displays the anticipated outcomes for the ARIMA models. For all countries, it is simple to verify that the majority of estimated parameters across the ARIMA models are significantly different from zero. Let's use the results for deaths in the UK as an example. Except for μ_0, all other parameters are statistically significant in the ARIMA(5,2,2) model. This model's parameters have a 95% credible interval that excludes zero and demonstrates the statistical significance of the estimate. Additionally, the model's ar.1 and ma.2 components contribute favourably, but ar.2, ar.3, ar.4, and ar.5 components have statistically significant negative contributions to the ARIMA(5, 2, 2) model. For the USA's cumulative number of cases, the recommended model is ARIMA (1, 2, 2). There is one AR component, two MA components, and two other parameters, μ_0 and σ_0. All five parameters are statistically significant, which means their 95% credible interval does not contain zero, and the first MA component contributes to the model negatively. Similarly, we can interpret other countries' parameters using table 2.

Countries	Items (Models)	μ_0	σ	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	θ_1	θ_2
USA	Cases (ARIMA(1,2,1)	0.0545 (0.0286)	15711.35 (4.6476)	0.2375 (0.0006)					-0.6858 (0.0004)	
	Deaths (ARIMA(3,2,2)	0.0367 (0.0209)	360.2125 (0.1044)	0.7407 (0.0004)	-0.4641 (0.0005)	-0.2742 (0.0004)			-0.9896 (0.0001)	0.3402 (0.0004)
UK	Cases ARIMA(0,2,1)	0.0717 (0.0277)	2433.14 (0.7048)						-0.2125 (0.0004)	
	Deaths ARIMA(5,2,2)	0.0263 (0.0211)	95.9465 (0.0277)	0.1916 (0.0004)	-0.8767 (0.0003)	-0.1517 (0.0005)	-0.4477 (0.0003)	-0.5309 (0.0004)	-0.6963 (0.0004)	0.6722 (0.0003)
UAE	Cases ARIMA(0,2,1)	1.2845 (0.0371)	252.93 (0.1261)						-0.6439 (0.0005)	
	Deaths ARIMA(1,2,2)	0.0135 (0.0003)	1.8694 (0.0005)	-0.2635 (0.0020)					-0.5254 (0.0021)	-0.1067 (0.0016)
Bahrain	Cases ARIMA(1,2,2)	0.0278 (0.0279)	108.622 (0.0564)	0.4209 (0.0027)					-0.8067 (0.0026)	0.3154 (0.0010)
	Deaths ARIMA(0,2,1)	0.0054 (0.0003)	2.1048 (0.0006)						-0.6267 (0.0003)	

Table 2: *ARIMA model Bayesian estimation: estimation of the posterior means.*

6.2. BSTS model Bayesian estimation

From table 3, the majority of the posterior estimates for the BSTS model are statistically significant, which is consistent with the results from the BSTS models. For UK deaths, for instance, a posterior parameter estimate of σ_{obs} with a range of 142.04 to 155.34 implies that zero excludes from a credible interval of 95 %, indicating that the parameter is statistically significant. Similarly, for parameter σ_{slope} and σ_{level}, UK deaths have statistical significance. Similarly, for parameter σ_{slope} and σ_{level}, UK deaths have statistical significance. The parameter value of σ_{slope} is 0.02, and σ_{level} is 0.38 for the UK cumulative number of deaths. We can provide a similar justification for the parameter estimate for the other four countries. Figure 2 to 9 show the autocorrelation plot, caterpillar plot, posterior density plot, and trace plot of the BSTS model for the UK cumulative number of cases and deaths, respectively.From the trace plot, we can interpret the convergence of the Markov chain, due to space restrictions not displaying the plots of all countries.

Table 3: *BSTS model Bayesian estimation: estimation of the posterior means.*

Countries	Items	σ_{obs}	σ_{slope}	σ_{level}
USA	Cases	1948.43 (0.04)	3.25 (0.001)	0.40 (0.001)
	Deaths	284.66 (0.07)	0.03 (0.001)	0.41 (0.02)
UK	Cases	1239.84 (0.10)	11.51 (0.01)	0.39 (0.01)
	Deaths	148.45 (0.04)	0.02 (0.001)	0.38 (0.001)
UAE	Cases	218.46 (0.07)	0.03 (0.001)	0.40 (0.01)
	Deaths	2.41 (0.08)	0.02 (0.001)	0.02 (0.001)
Bahrain	Cases	107.37 (0.08)	0.03 (0.001)	0.41 (0.02)
	Deaths	2.50 (0.08)	0.01 (0.001)	0.02 (0.001)
India	Cases	1240.02 (0.10)	11.53 (0.01)	0.40 (0.01)
	Deaths	233.30 (0.09)	0.04 (0.001)	0.39 (0.001)

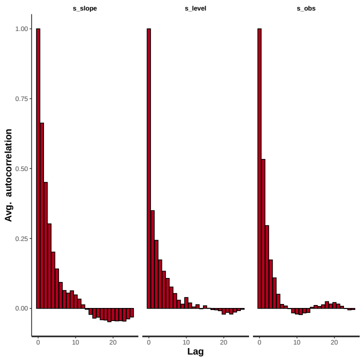


Figure 2: *BSTS model for UK cases autocorrelation plot*

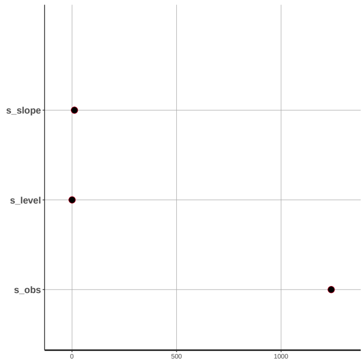


Figure 3: *BSTS model for UK cases caterpillar plot*

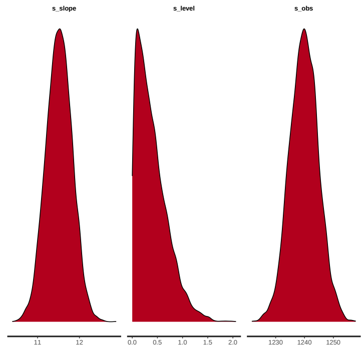


Figure 4: *BSTS model for UK cases posterior density plot*

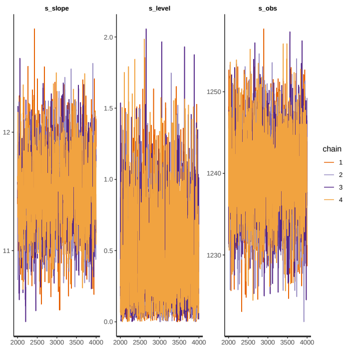


Figure 5: *BSTS model for UK cases trace plot*

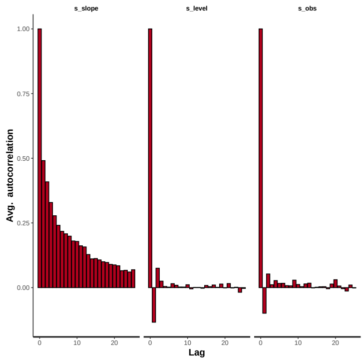


Figure 6: *BSTS model for UK deaths autocorrelation plot*

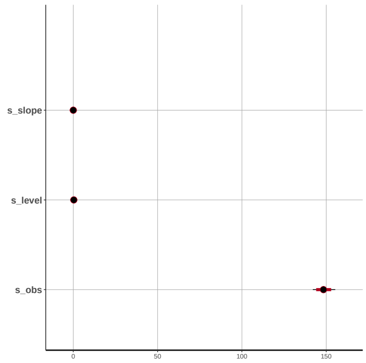


Figure 7: *BSTS model for UK deaths caterpillar plot*

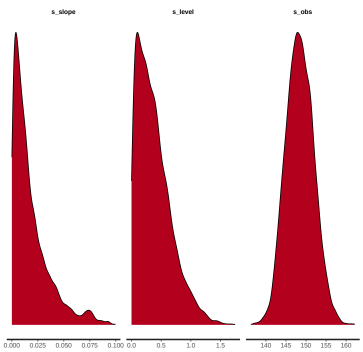


Figure 8: *BSTS model for UK deaths posterior density plot*

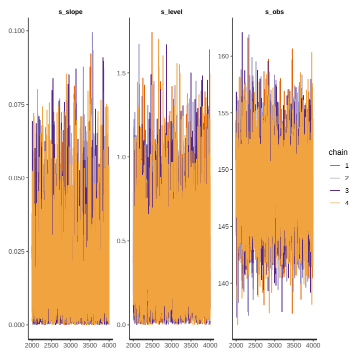


Figure 9: *BSTS model for UK deaths trace plot*

7. Conclusion

In this study, we used a Bayesian estimation to evaluate the widely used ARIMA and BSTS models while modelling COVID-19 cumulative number of cases and the cumulative number of deaths in five countries. From table 4, we discover that the ARIMA model LOOIC value is lower than that of their corresponding BSTS models. The model with the lowest LOOIC value is considered to be the best model. For example, Bahrain's cumulative number of cases LOOIC value for the Bayesian ARIMA model is 5945.3 and for the BSTS model is 6028.5. Here Looic value is the minimum for the ARIMA model and which is the best model. Similarly, for Bahrain cumulative number of deaths LOOIC value for the Bayesian ARIMA model is 2110.7, and for the BSTS model is 2273.2, therefore the minimum LOOIC value is for the ARIMA model. Therefore almost all countries' data set LOOIC value is minimum for the Bayesian ARIMA model. Hence, when compared to the corresponding BSTS model, the Bayesian ARIMA model performs the best.

Table 4: *ARIMA and BSTS model validation using LOOIC and WAIC.*

		Looic	Waic	Looic	Waic
USA	Cases	10771.5	10771.6	47343.5	47887.2
	Deaths	7221.3	7260	7665.8	7665
UK	Cases	8958.4	8958.3	24255.4	24362.8
	Deaths	5831.3	5831.2	6448.6	6448.6
UAE	Cases	6786	6800.9	7131.2	7180.2
	Deaths	1993.4	1993.5	2236	2236
Bahrain	Cases	5945.3	5945.3	6028.5	6028.5
	Deaths	2110.7	2110.9	2273.2	2273.2
India	Cases	10095.5	10096.1	24219.5	24326.3
	Deaths	6981.2	6983.5	7279.4	7361.7

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