PROFITABILITY ANALYSIS OF A FOOD INDUSTRIAL SYSTEM HAVING MAKE-AND-PACK PRODUCTION STRATEGY WITH PRIORITY BASIS REPAIR

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Abstract

The present investigation is concerned with the profitability analysis of a food industrial system where production is based on the make-and-pack strategy. The system is assumed to have two subsystems: first subsystem is for making while the second is for packing the product so formed. As per the gathered information about the production procedure in the food industrial plant, the priority of repair is given to the making subsystem over the packing subsystem. Here, failure of either subsystem leads to a complete breakdown of the system. Also, two types of failures are considered in the packing subsystem i.e. minor failures and major failures. Two kinds of repairers (operator and fitter) are appointed to tackle the failures in the subsystems. For minor and major failures in the packing subsystem, the operator and fitter respectively are responsible for repairs. However, any failure in making subsystem is repaired by the combined efforts of the operator and fitter. Reliability characteristics such as mean time to system failure (MTSF), system availability, and expected busy period of the repair persons are studied by employing the semi-Markov process and regenerative point technique. The system profitability is graphically analyzed concerning to failure rates of both subsystems.

Keywords: make-and-pack production process, priority basis repair, regenerative point technique, semi-Markov process

1. Introduction

Globalization has opened up new opportunities and challenges to the manufacturing industries which force producers to seek out more efficient ways to manufacture their products. The producers adopt various strategies to obtain the most reliable product and as a result, the production process is becoming more multifarious. So, for the smooth handling of the production process, it is divided into some stages i.e. known as two-stage production or three-stage production, etc. From the existing literature, it is visible that Johnson [1] was the first one who used the term two-stage production in which he considered one machine in each stage. Afterward, a number of significant studies came into existence regarding two-stage production with consideration of more than one machine in the second stage as taken by Gupta and Tunc [2] and Honkomp *et al.* [3]. From the general point of view of the production, this two-stage production was specifically named the make-and-pack process. Usually, a make-and-pack production process is coupled with two stages: the first stage is responsible for the making or formation of the product while the second stage is responsible for the packaging or packing of the product so formed. This mode of production makes workability more flexible by managing the processing rates of the packaging lines with the respect that of formulation lines. It is frequently confronted in many production industries such as the paper industry, pharmaceutical industry, food industry, chemical industry, etc.

Make-and-pack production process is a well-known concept in the food processing industry. Akkerman *et al.* [4] examined the effect of some capacity and time constraints on the performance of a two-stage food production system. Sel *et al.* [5] discussed the planning and scheduling of make-and-pack production under lifetime uncertainty. Klanke *et al.* [6] analyzed the short-term scheduling of the make-and-pack process for minimizing the total production makespan of the schedule. It is apparent from the extensive literature review that most of the research on make-and-pack process is limited to the scheduling of this production process. But there is another side rather than the scheduling of this process which can be equally responsible for the financial loss in manufacturing industries and that is the failure in the systems used for production. As if the system will fail and did not come into operation timely, then the company can face a great loss in terms of money and reputation. So, the present paper is an effort to develop the reliability model for a food industrial system considering two subsystems one for making and another for the packing stage. In reliability analysis, a modeling approach is usually adopted to understand and predict the system's behavior in given situations with the help of probabilistic concepts. Also, the priority of repair and two types of repairmen are taken from the point of making the system more available so that profit can be maximized.

As far as reliability analysis is concerned, it has been a very compelling topic for a long time. It has been dealing with various types of industrial systems since the 1960s. A lot of research has been done and appreciated on the reliability analysis of various systems as seen in the literature. The various types of systems viz. single-unit systems or two or more unit industrial systems have been studied in the literature considering various factors affecting the system performance such as preventive maintenance of the units, perfect/imperfect switchover of the units, priority basis repair, two or more types of repairmen, standby units, inspections, replacement of the failed unit or its repair in the online or offline mode and plenty of factors are also there. These all factors were sized up to see the feasibility of profit maximization. Gaver [7] made derivations regarding MTSF and the availability of the type of systems that are composed of two paralleled subsystems. He considered situations for when there is waiting time to repair for another subsystem or not when both fail simultaneously. Pandey *et al.* [8] discussed the stochastic modeling of a powerloom plant consisting of two units having mechanical failures along with the concept of two additional failures due to poorly trained weavers; common cause failure and human error. Mathew *et al.* [9] studied a two-unit system of continuous casting plant with three types of failures such as repairable, replaceable, or requiring reconditioning/ reinstallation. Taj *et al.* [10] have examined a single-machine subsystem involved in a cable plant by considering its three types of maintenance strategies. Kumari *et al.* [11] investigated the profit of the butter-oil (ghee) manufacturing system through the supplementary variable technique. Singh *et al.* [12] evaluated the reliability metrics of a complex repairable system having two subsystems connected in series with imperfect switching. Saini *et al.* [13] analyzed a redundant system with non-identical units; one original and another duplicate cold standby unit where priority was given to the original unit over the repair of the duplicate unit. Bashir *et al.* [14] proposed a model considering two units along with their controlled and uncontrolled failures in terms of repairing and replacing respectively. Andalib and Sarkar [15] discussed a repairable system with two spare units along with their repair/service by two repair persons. Sharma and Drishti [16] studied the seasonal effect on the workability of an ice-cream plant. Monika and Chopra [17] have developed the reliability model for the food industrial system by considering demand-based seasons. Some other recent reliability studies on realistic systems can be explored in Rizwan *et al.* [18], Sachdeva *et al.* [19], and Yusuf and Sanusi [20].

It is apparent from the literature that several aspects have been taken regarding the reliability modeling of industrial systems. So, on studying these aspects and by the visit of the industrial system under consideration, the concept of priority of repair of the making subsystem over the packing subsystem and two types of repairmen are considered. In all, the present study helps to fill the gap between the scheduling of the make-and-pack process of the food production systems and failures that cause a delay in food production following the financial loss. We develop a model with consideration of the two subsystems (one for making and another for packing) of the food industrial system based on the production strategy. Also, it is assumed that the packing subsystem has more than one unit working in parallel and this subsystem can have failures of two types i.e. minor and major failures. Accordingly, various measures of system effectiveness have been evaluated with the help of the regenerative point technique to see the behavior of the profit function, and also profit maximizing parameters are deduced.

2. Notations

Table 1: *Notations used throughout the paper*

Notations	Description
λ_1	Constant failure rate of the making subsystem
λ_2	Constant failure rate of the packing subsystem
$g_1(t)/G_1(t)$	pdf/cdf of repair time of making subsystem
$g_{21}(t)/G_{21}(t)$	pdf/cdf of time to complete repair of minor failures of packing subsystem
$g_{22}(t)/G_{22}(t)$	pdf/cdf of time to complete repair of major failures of packing subsystem
$q_{ij}(t)/Q_{ij}(t)$	pdf/cdf of transition time from a state 'i' to a state 'j'
$\phi_i(t)$	cdf of first passage time from a regenerative state 'i' to a failed state
A_0	Steady state availability of the system
B_0^o/B_0^f	Busy period of the operator/fitter for repair
$M_i(t)$	Probability that the system is up initially in regenerative state 'i' and is up at instant t without going through any other regenerative state
$W_i(t)$	Probability that the repair person is busy in repair of the subsystem (making/packing) initially in regenerative state 'i' and is engaged at time t without visiting to any other regenerative state
m_{ij}	The unconditional mean time taken by the system to visit any regenerative state 'j' when the time is measured from the time of entrance into state 'i'
μ_i	Mean sojourn time, i.e., the expected spent time in a regenerative state 'i' before visiting any other state
*/©	Symbol for Laplace transform / Laplace convolution
**/Ⓢ	Symbol for Laplace Stieltjes transform/ Laplace Stieltjes convolution

Table 2: Notations regarding the states of the system

State	Symbol	Meaning
State 0	(O_m, O_p)	Operative state of the system where both subsystems viz. making and packing subsystems are operative
State 1	(O_m, F_{urp_1})	Operative state of the system where packing subsystem is under minor repair
State 2	(F_{urm}, F_{wrp_1})	Failed state of the system where making subsystem is under repair and the packing subsystem is waiting for minor repair
State 3	(F_{urm}, D_p)	Failed state of the system where making subsystem is under repair and the packing subsystem is in down state
State 4	(D_m, F_{urp_2})	Failed state of the system where making subsystem is in down state and the packing subsystem is under major repair

This section is devoted to notations used in the present study. All notations are specified in Table 1 and Table 2.

3. Description of the system and Assumptions

3.1. Description of the system

This paper deals with a food industrial system in which production is based on the make-and-pack strategy. This system has two subsystems; one for making and another for packing the product so formed. The making subsystem has four units working in series and the packing subsystem has two units working in series which further have five and three subunits working in parallel respectively (see Figure 1). When there are failures in the making subsystem then the system will completely fail and the packing subsystem will be kept in a down state to manage the production because the packing subsystem will not have material for packing, while the packing subsystem can behave in two manners when it has failures depending on the type of failures i.e. minor or major failures. When minor failures are there, then the system will work in the same manner (as it is assumed that the failures will be repaired within negligible time or online we can say) but in case of major failures, the system will completely fail and in that case also the making subsystem will be in the down state as if not kept in the down state then the surplus product will form which will not be worked upon further due to failure of the packing subsystem. Also, when there will be a simultaneous failure in both subsystems, then the priority of repair will be given to making subsystem. It is due to the reason that the failures in the packing subsystem, at the same time, can be handled by shifting the material to other subunits working in parallel; as per the information from the officials of the food processing plant. Five states are there depicting the possibilities taken with the system (see Figure 2) which are described in Table 2.

3.2. Assumptions

The following assumptions have been taken throughout the paper discussion:

- Initially, both the making and packing subsystems are in operative condition.
- The system will be in the state of complete failure only in two cases; either there is a failure in the making subsystem or there is a major failure in the packing subsystem.

- The operator and fitter both work together for repair only when there is a failure in making subsystem as it has priority of repair.
- The failure times and repair times follow exponential and general distributions respectively.
- The repaired system works as a new one.

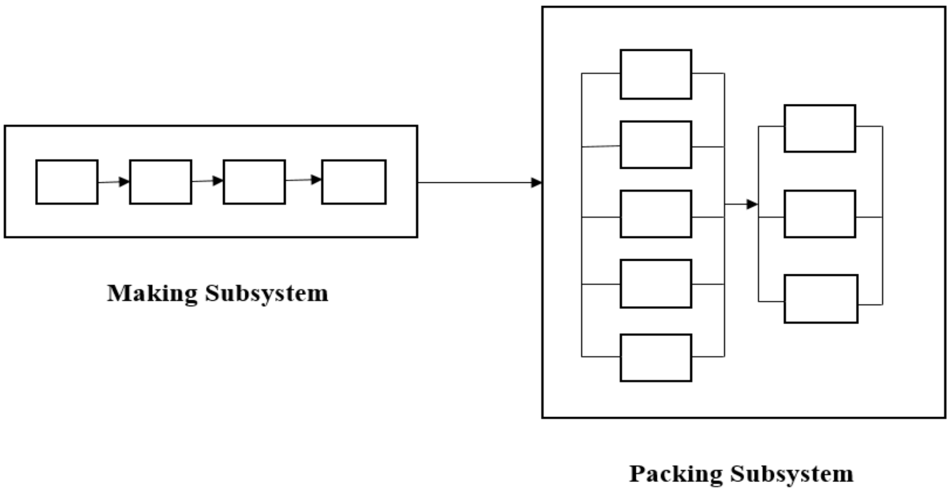

Figure 1: *System Block Diagram*

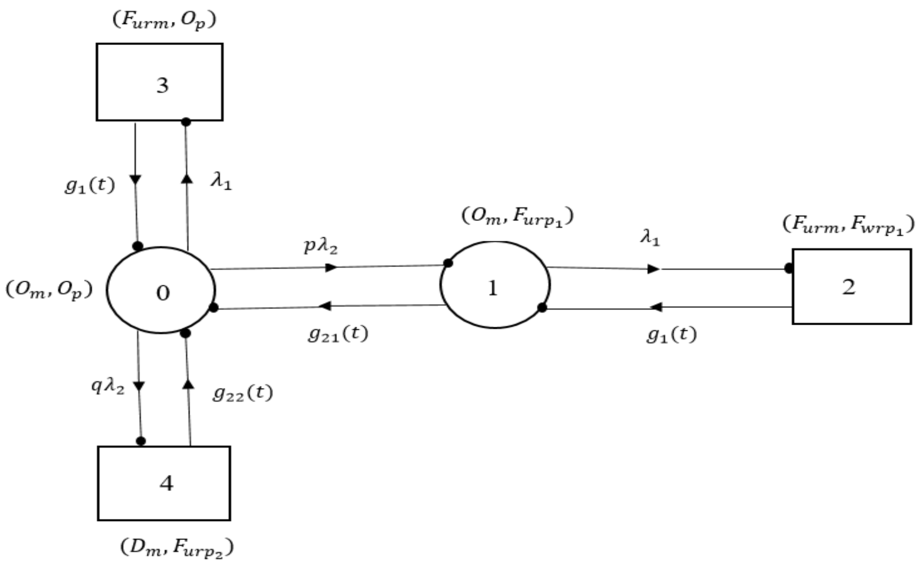

Figure 2: *State Transition Diagram*

4. Solution of Mathematical Model

The time points of entry into 0 to 4 states are the regeneration points and consequently, these states are termed as regenerative states. The states 0 and 1 are up states of the system whereas the states 2, 3, and 4 are failed states of the system. The transition probabilities for the states are mentioned as under:

$$q_{01}(t) = p\lambda_2 . e^{-(\lambda_1+\lambda_2)t}, \quad q_{03}(t) = \lambda_1 . e^{-(\lambda_1+\lambda_2)t}, \quad q_{04}(t) = q\lambda_2 . e^{-(\lambda_1+\lambda_2)t}, \quad q_{10}(t) = g_{21}(t). e^{-\lambda_1 t},$$

$$q_{12}(t) = \lambda_1 e^{-\lambda_1 t} . \overline{G_{21}(t)}, \quad q_{21}(t) = g_1(t), \quad q_{30}(t) = g_1(t), \quad q_{40}(t) = g_{22}(t).$$

Further, the non-zero elements p_{ij}, where $p_{ij} = \lim_{s \to 0} q_{ij}{}^*(s)$, are

$$p_{01} = \frac{p\lambda_2}{\lambda_1+\lambda_2}, \quad p_{03} = \frac{\lambda_1}{\lambda_1+\lambda_2}, \quad p_{04} = \frac{q\lambda_2}{\lambda_1+\lambda_2}, \quad p_{10} = g_{21}^*(\lambda_1), \quad p_{12} = 1 - g_{21}^*(\lambda_1), \quad p_{21} = p_{30} = p_{40} = 1.$$

From these, we can conclude that

$$p_{01} + p_{03} + p_{04} = 1, \quad p_{10} + p_{12} = 1.$$

Corresponding to the states 0 to 4, the mean sojourn times, μ_i are evaluated as:

$$\mu_0 = \frac{1}{\lambda_1+\lambda_2}, \quad \mu_1 = \frac{1-g_{21}^*(\lambda_1)}{\lambda_1}, \quad \mu_2 = -g_1^{*\prime}(0), \quad \mu_3 = -g_1^{*\prime}(0), \quad \mu_4 = -g_{22}^{*\prime}(0).$$

Further, the unconditional mean times, m_{ij} used in the solution of the model are as given below:

$$m_{01} = \frac{p\lambda_2}{(\lambda_1+\lambda_2)^2}, \quad m_{03} = \frac{\lambda_1}{(\lambda_1+\lambda_2)^2}, \quad m_{04} = \frac{q\lambda_2}{(\lambda_1+\lambda_2)^2}, \quad m_{10} = -g_{21}^{*\prime}(\lambda_1), \quad m_{12} = g_{21}^{*\prime}(\lambda_1) + \frac{1-g_{21}^*(\lambda_1)}{\lambda_1},$$

$$m_{21} = -g_1^{*\prime}(0), \quad m_{30} = -g_1^{*\prime}(0), \quad m_{40} = -g_{22}^{*\prime}(0).$$

Furthermore, we get

$$m_{01} + m_{03} + m_{04} = \mu_0, \quad m_{10} + m_{12} = \mu_1, \quad m_{21} = \mu_2, \quad m_{30} = \mu_3, \quad m_{40} = \mu_4.$$

The expressions for reliability measures and profit have been appraised in the upcoming subsections.

4.1. MTSF

The MTSF is the metric related to the failure of the system. It provides the expected time to which the system is operational before the complete failure. For determining the MTSF of the system, the failed states have been considered as the absorbing states. The resulting iterative relations for $\phi_i(t)$ are given below

$$\phi_0(t) = Q_{01}(t) Ⓢ \phi_1(t) + Q_{03}(t) + Q_{04}(t) \quad (1)$$

$$\phi_1(t) = Q_{10}(t) Ⓢ \phi_0(t) + Q_{12}(t) \quad (2)$$

Taking Laplace Stieltjes transform of equations (1-2) and solving them, we have

$$\phi_0^{**}(s) = \frac{N_1(s)}{D_1(s)}$$

where,

$$N_1(s) = \begin{vmatrix} q_{03}^*(s) + q_{04}^*(s) & -q_{01}^*(s) \\ q_{12}^*(s) & 1 \end{vmatrix} \text{ and } D_1(s) = \begin{vmatrix} 1 & -q_{01}^*(s) \\ -q_{10}^*(s) & 1 \end{vmatrix}$$

Assuming that the considered system initiates its journey from the state '0', the MTSF is calculated as:

$$\text{MTSF} = \lim_{s \to 0} \frac{(1-\phi_0^{**}(s))}{s} = \frac{N_1}{D_1}$$

where,

$N_1 = p_{01}.\mu_1 + \mu_0$ and $D_1 = 1 - p_{01}.p_{10}$.

4.2. Availability

Availability is a key performance indicator of a production system. This metric is used in the production process as it keeps an eye on the continuous operations of the system which results in terms of maximum profit for the manufacturers.

To determine the system availability, let us define $A_i(t)$ as the probability that starting from the regenerative state 'i', the system is in the up state at the instant 't'. Thus, the iterative relations will be:

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{03}(t)©A_3(t) + q_{04}(t)©A_4(t) \tag{3}$$

$$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{12}(t)©A_2(t) \tag{4}$$

$$A_2(t) = q_{21}(t)©A_1(t) \tag{5}$$

$$A_3(t) = q_{30}(t)©A_0(t) \tag{6}$$

$$A_4(t) = q_{40}(t)©A_0(t) \tag{7}$$

where,

$M_0(t) = e^{-(\lambda_1+\lambda_2)t}$ and $M_1(t) = e^{-\lambda_1 t}.\overline{G_{21}(t)}$.

Taking Laplace transform of equations (3-7) and then solving them further, the steady-state availability is evaluated as

$$A_0 = \lim_{s \to 0} \left[s.\frac{N_2(s)}{D_2(s)} \right] = \frac{N_2}{D_2}$$

where,

$$N_2(s) = \begin{vmatrix} M_0^*(s) & -q_{01}^*(s) & 0 & -q_{03}^*(s) & -q_{04}^*(s) \\ M_1^*(s) & 1 & -q_{12}^*(s) & 0 & 0 \\ 0 & -q_{21}^*(s) & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} \text{ and}$$

$$D_2(s) = \begin{vmatrix} 1 & -q_{01}^*(s) & 0 & -q_{03}^*(s) & -q_{04}^*(s) \\ -q_{10}^*(s) & 1 & -q_{12}^*(s) & 0 & 0 \\ 0 & -q_{21}^*(s) & 1 & 0 & 0 \\ -q_{30}^*(s) & 0 & 0 & 1 & 0 \\ -q_{40}^*(s) & 0 & 0 & 0 & 1 \end{vmatrix}$$

which gives

$N_2 = \mu_0 . p_{10} + \mu_1 . p_{01}$ and $D_2 = p_{10} . \mu_0 + p_{01} . \mu_1 + (p_{12} . p_{01} + p_{03} . p_{10}) . \mu_2 + p_{10} . p_{04} . \mu_4$.

4.3. Busy period

In our study, we have two repairers i.e., operator and fitter. They have been assigned separate repair duties based on the failures in the subsystems. Their busy periods' expressions are evaluated in the next two subsections.

4.3.1. Operator's busy period for repair

To calculate the operator's busy period with the system, we have the following iterative relations for $B_i^o(t)$, where $B_i^o(t)$ is defined as the probability when the operator is fully engaged in repair at an instant '*t*', provided that the system moved in regenerative state '*i*' at *t* = 0.

$$B_0^o(t) = q_{01}(t)©B_1^o(t) + q_{03}(t)©B_3^o(t) + q_{04}(t)©B_4^o(t) \quad (8)$$

$$B_1^o(t) = W_1(t) + q_{10}(t)©B_0^o(t) + q_{12}(t)©B_2^o(t) \quad (9)$$

$$B_2^o(t) = W_2(t) + q_{21}(t)©B_1^o(t) \quad (10)$$

$$B_3^o(t) = q_{30}(t)©B_0^o(t) \quad (11)$$

$$B_4^o(t) = q_{40}(t)©B_0^o(t) \quad (12)$$

where,

$W_1(t) = e^{-\lambda_1 t} . \overline{G_{21}(t)}$ and $W_2(t) = \overline{G_1(t)}$.

Further, we have taken the Laplace transform of the aforementioned equations (8-12). The obtained busy period of the operator in the steady state is

$$B_0^o = \lim_{s \to 0} \left[s . \frac{N_{31}(s)}{D_{31}(s)} \right] = \frac{N_{31}}{D_{31}}$$

where,

$$N_{31}(s) = \begin{vmatrix} 0 & -q_{01}^*(s) & 0 & -q_{03}^*(s) & -q_{04}^*(s) \\ W_1^*(s) & 1 & -q_{12}^*(s) & 0 & 0 \\ W_2^*(s) & -q_{21}^*(s) & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} \text{ and}$$

$$D_{31}(s) = \begin{vmatrix} 1 & -q_{01}^*(s) & 0 & -q_{03}^*(s) & -q_{04}^*(s) \\ -q_{10}^*(s) & 1 & -q_{12}^*(s) & 0 & 0 \\ 0 & -q_{21}^*(s) & 1 & 0 & 0 \\ -q_{30}^*(s) & 0 & 0 & 1 & 0 \\ -q_{40}^*(s) & 0 & 0 & 0 & 1 \end{vmatrix}$$

which gives

$N_{31} = p_{01}.(\mu_1 + \mu_2.p_{12})$ and $D_{31} = p_{10}.\mu_0 + p_{01}.\mu_1 + (p_{12}.p_{01} + p_{03}.p_{10}).\mu_2 + p_{10}.p_{04}.\mu_4$.

4.3.2. Fitter's busy period for repair

For the fitter's busy period with the system, the following recursive relations can be obtained where $B_i^f(t)$ is defined as the probability when the fitter is fully engaged at an instant 't', provided that the system moved in regenerative state 'i' at $t = 0$.

$$B_0^f(t) = q_{01}(t) © B_1^f(t) + q_{03}(t) © B_3^f(t) + q_{04}(t) © B_4^f(t) \tag{13}$$

$$B_1^f(t) = q_{10}(t) © B_0^f(t) + q_{12}(t) © B_2^f(t) \tag{14}$$

$$B_2^f(t) = W_2(t) + q_{21}(t) © B_1^f(t) \tag{15}$$

$$B_3^f(t) = W_3(t) + q_{30}(t) © B_0^f(t) \tag{16}$$

$$B_4^f(t) = W_4(t) + q_{40}(t) © B_0^f(t) \tag{17}$$

where,

$W_2(t) = \overline{G_1(t)}$, $W_3(t) = \overline{G_1(t)}$ and $W_4(t) = \overline{G_{22}(t)}$.

The Laplace transform of the equations (13-17) is considered, and we obtain the busy period of the fitter as:

$$B_0^f = \lim_{s \to 0}\left[s.\frac{N_{32}(s)}{D_{32}(s)}\right] = \frac{N_{32}}{D_{32}}$$

where,

$$N_{32}(s) = \begin{vmatrix} 0 & -q_{01}^*(s) & 0 & -q_{03}^*(s) & -q_{04}^*(s) \\ 0 & 1 & -q_{12}^*(s) & 0 & 0 \\ W_2^*(s) & -q_{21}^*(s) & 1 & 0 & 0 \\ W_3^*(s) & 0 & 0 & 1 & 0 \\ W_4^*(s) & 0 & 0 & 0 & 1 \end{vmatrix} \text{ and}$$

$$D_{32}(s) = \begin{vmatrix} 1 & -q_{01}^*(s) & 0 & -q_{03}^*(s) & -q_{04}^*(s) \\ -q_{10}^*(s) & 1 & -q_{12}^*(s) & 0 & 0 \\ 0 & -q_{21}^*(s) & 1 & 0 & 0 \\ -q_{30}^*(s) & 0 & 0 & 1 & 0 \\ -q_{40}^*(s) & 0 & 0 & 0 & 1 \end{vmatrix}$$

which gives

$N_{32} = p_{10}.p_{04}.\mu_4 + p_{03}.p_{10}.\mu_2 + p_{12}.p_{01}.\mu_2$ and

$D_{32} = p_{10}.\mu_0 + p_{01}.\mu_1 + (p_{12}.p_{01} + p_{03}.p_{10}).\mu_2 + p_{10}.p_{04}.\mu_4.$

4.4. Profit

The main purpose of every organization is to gain profit to survive in this competitive era. Reliability analysis helps us to provide profitable strategies with the aid of measures of system effectiveness. The profit earned by the system can be evaluated by the following expression

Profit = $C_0.A_0 - \left(C_{ro}.B_0^o + C_{rf}.B_0^f\right)$

where, C_0 is revenue per unit up time of the system and C_{ro}, C_{rf} refer to the respective service costs per unit time for which the operator and fitter are busy.

5. Numerical Calculation

Based on gathered data from the food industrial plant, we have estimated the rates and probabilities involved in the model. The estimated rates and probabilities are given in Table 3.

Table 3: *Rates and probabilities*

Failure rate of making subsystem	0.08
Failure rate of packing subsystem	0.04
Repair rate of making subsystem	1.64
Repair rate of packing subsystem due to minor failures	5.37
Repair rate of packing subsystem due to major failures	1.76
Probability that there are minor failures in packing subsystem	0.22
Probability that there are major failures in packing subsystem	0.78

The considered system has been assessed by assuming the repair time for making subsystem as well as packing subsystem for both minor and major failures as exponentially distributed with parameters β_1, β_{21}, and β_{22} respectively, so that $g_1(t) = \beta_1 e^{-\beta_1 t}$, $g_{21}(t) = \beta_{21} e^{-\beta_{21} t}$, $g_{22}(t) = \beta_{22} e^{-\beta_{22} t}$.

Correspondingly, we have $p_{10} = \frac{\beta_{21}}{\lambda_1+\beta_{21}}$, $p_{12} = \frac{\lambda_1}{\lambda_1+\beta_{21}}$, $\mu_1 = \frac{1}{\lambda_1+\beta_{21}}$, $\mu_2 = \frac{1}{\beta_1}$ and $\mu_4 = \frac{1}{\beta_{22}}$. The costs involved are as; C_0 = 100000, C_{ro} = 1600, and C_{rf} = 3000. Using the rates mentioned in Table 3, we have obtained measures of system effectiveness as MTSF = 8.996875124, A_0 = 0.937665183, B_0^o = 0.001552568, and B_0^f = 0.062334817.

6. Result and Discussion

We have noticed that although the system performance is governed by the various parameters involved, yet the aspect of failure rates of both subsystems is the base of our study. The impact of both subsystems' failure rates on the MTSF, system availability, and overall profit is examined graphically. From Figure 3, it is observed that MTSF decreases with the increase in the failure rate of the making as well as the packing subsystem. The availability of the system is effected by the failure rates of both subsystems which is evident from Figure 4. Here, from the trend of the availability of the system with the failure rates of the subsystems, it is clear that availability declines with the rise in failure rates of the making subsystem and the packing subsystem.

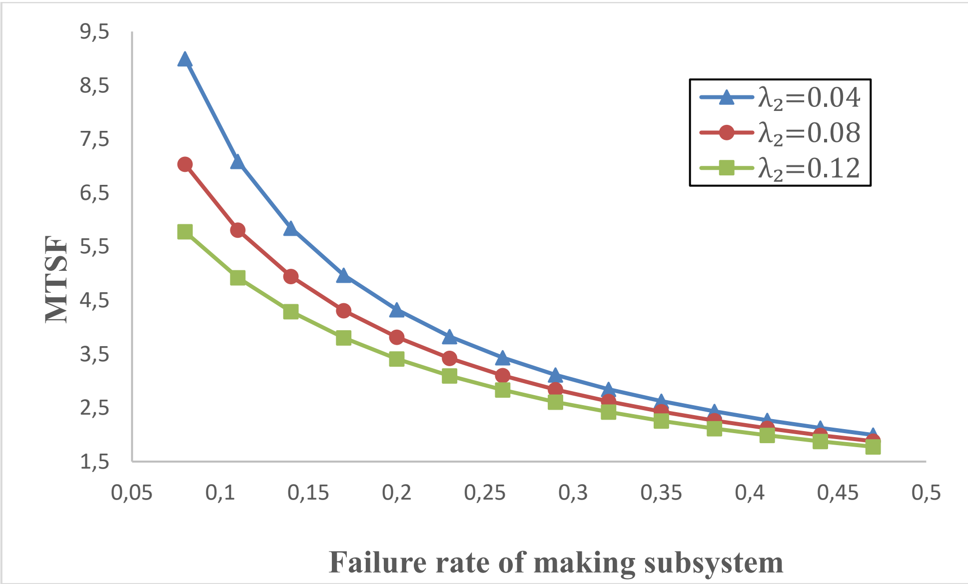

Figure 3: *MTSF v/s Failure rate of making subsystem for various values of* λ_2

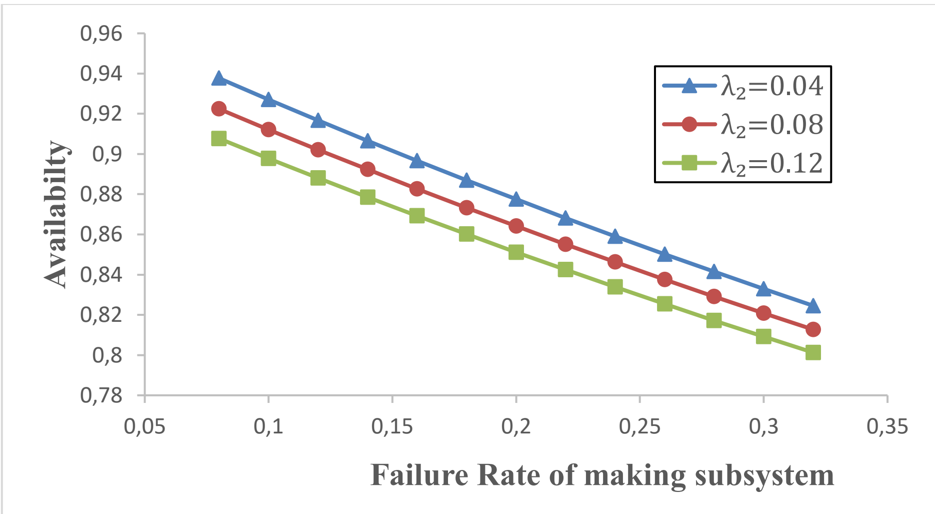

Figure 4: *Availability v/s Failure rate of making subsystem with various values of* λ_2

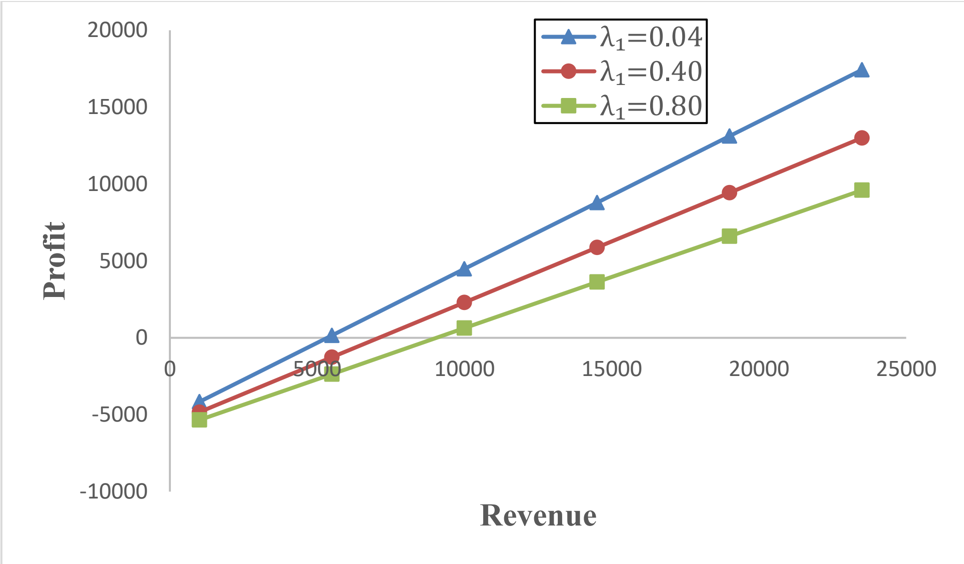

Figure 5: *Profit v/s system revenue for various values of λ_1*

For studying the effect of failure rates on system profit, we have fixed some parameters such as β_1 = 1.64, β_{21} = 5.37, β_{22} = 1.76, p = 0.22, q = 0.78, C_{ro} = 1600 and C_{rf} = 3000. To examine the effect of λ_1 on system profit, we have varied the parameter λ_1 as 0.04, 0.40, and 0.80. Figure 5 indicates that for the varying λ_1 as 0.04, 0.40, and 0.80, the profit is positive or zero or negative when the system revenue i.e., C_0 is greater or equal or lesser than 5339.423, 7096.091, and 9047.954 respectively. Further, we have explored the effect of λ_2 on profit, by varying the parameter λ_2 as 0.04, 0.40, and 0.80. It can be seen in Figure 6 that for λ_2 as 0.04, 0.40, and 0.80, the profit is positive or zero or negative when the system revenue i.e., C_0 is greater or equal or lesser than 5534.609, 7096.091, and 9772.342 respectively.

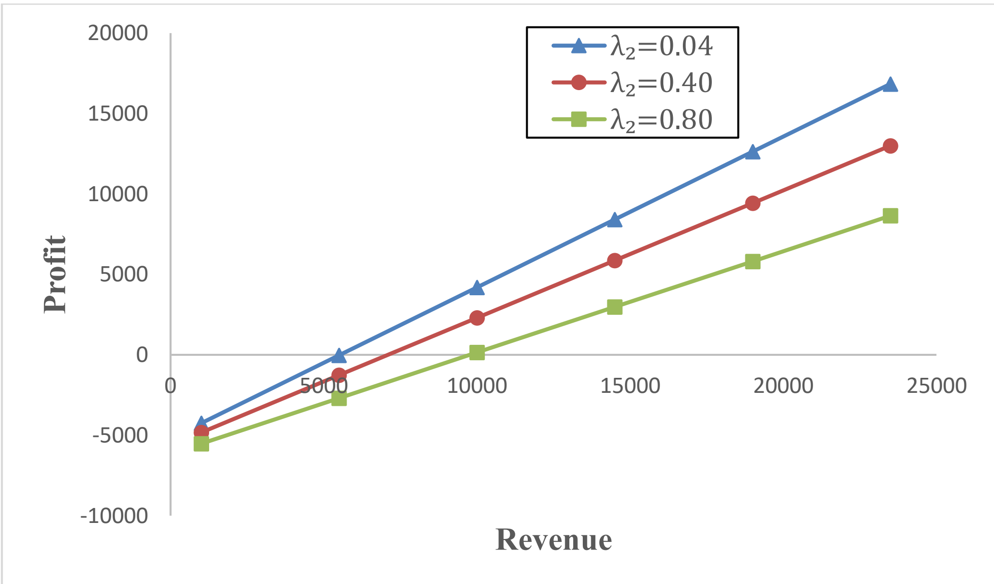

Figure 6: *Profit v/s system revenue for various values of λ_2*

7. Conclusion

In the present paper, we have figured out the MTSF, availability, and profitability of the food industrial system that is based on the make-and-pack production process in the aspect of the failure rates of the subsystems. The graphical interpretation confirms that both reliability metrics, MTSF, and system availability reduce with the surge in time and failure rates of both making and packing subsystems. It is also found true that the profit of the studied system increases with the increase in the system revenue and decreases when the failure rates of the subsystems increase. So, it is concluded that for lower values of failure rates, the expected profit is high as compared to higher values of failure rates. As far as the subsystems are concerned, it is graphically observed that for the same value of failure rate i.e., λ_1, λ_2 = 0.04, more revenue is needed for the packing subsystem as compared to the making subsystem for profitability. In other words, we can say that for the same number of failures in making as well as packing subsystems, the packing subsystem for some values of revenue results in loss to the system and at the same time making subsystem provides profit with those values. Therefore, priority control of the failures of packing subsystem is more advisable for the system's profitability as when failures will be controlled then revenue will be more. Such valuations can help to assess the failure times of the subsystems that can be afforded by the manufacturers. The developed model can be compared to the case when there is no priority basis repair to see the difference in earned profits.

Conflict of Interest Declaration: The authors have no conflicts of interest to declare.

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