Combined Redundancy Optimization for a System Comprising Operative, Cold Standby and Warm Standby Units

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Abstract

The company or industry can increase system reliability and provide stress-free operation by adding redundant equivalent subsystems to the active unit. The warm standby system is accessible if the operational unit malfunctions, and the cold standby system can take over. This paper aims to analyze a system comprised of one operative unit, cold standby unit, and one warm standby unit. Cold standby is activated to become warm standby when the operative unit fails, and warm standby becomes operational immediately. A minor defect causes the warm standby unit to fail, whereas a major fault causes the operational unit to fail. Such systems are used by many businesses, sectors, and facilities to prevent operational and reputational losses. Cut-off values for the failure rate, activation rate, revenue cost, and cost per repairman visit have been calculated to determine when the system is profitable. Various system performance measures have been defined by using the Markov process and regeneration point method.

Keywords: Cold Standby, Profit Analysis, Redundancy, Warm Standby

1. Introduction

The major focus of modern technology is to improve system reliability so that it can meet consumer demand. This results in the development of more expensive, sophisticated systems with significant repair costs in the event of failure. Redundancy is a desirable choice as a result. Standby redundancy is one of several strategies for implementing redundancy. It is a redundancy that has a secondary unit in case the active unit fails. Countless researchers have studied the coupling of redundant systems in the optimization of system reliability. Goel and Gupta [1] assumed three types of failures for a hot standby system. Tuteja et al.[2] investigated a redundant two-unit cold standby system with regular and expert maintenance personnel. When a system malfunctions or an ordinary repairman is unable to fix the problem, an expert repairman is contacted. Kumar et al.[3] examined a two-unit cold standby system that was repaired by an expert repairman when it failed. When the expert is working on the first unit failure and the second unit fails at the same time, the assistant repairman can fix the problem while following the expert's instructions. El-Said and EL-Sherbery [4] discussed regarding two units on cold standby with post-repair inspection. Parashar and Taneja[5] analyzed a two-unit PLC hot standby system, one as the master unit and the other as the slave unit, with minor repairs being performed by regular repairmen and major repairs being handled by specialist repairmen. Mahmoud and Moshref [6] considered standby system that experienced both hardware and human error failure. For a two unit cold standby system, Manocha and Taneja[7] modified the outcome provided by EL-Said and EL-Sherbery. By taking into account the non-regenerative state and all possible general distributions, a stochastic model has been developed by them. Malhotra and Taneja [8] identified two different cable manufacturing industry reliability models. Model 1 uses a single unit while Model 2 has a cold standby system with two units. Manocha et al. [9] created a stochastic model for a hot standby database system with two units. Database administrators handle issues with the primary or backup database systems. Adlakha et al.[10] took into consideration a two-unit cold standby communication system that was first packed and assembled as necessary. One is in use, and the other is on cold standby. Batra and Taneja[11]-[14] optimized the standby units with one or two operational units in the system. A comparative analysis is also performed to determine which of these models is preferable for system profitability. Levitin et al. [15] considered a system with the potential for shocks during data transfer and operation. Every shock shortens an element's lifespan and causes deterioration; as a result, the operating unit is immediately switched out for a cold standby unit. Malhotra et al. [16] examined the expected rest period and expected maintenance period for a two-unit cold standby system in a pharmaceutical company. Jia et al. [17] developed a stochastic model for a demand-based warm standby system with capacity storage.

It is noticeable that the combination of hot, warm, and cold standby units has not yet been investigated, despite the fact that many industrial systems have these unit backups to manage emergencies. In light of this, the present study aims to create a stochastic model with a single operational unit, warm and cold standby units. The rest of the article is organized as follows. In Section 2, system assumptions are listed. Sections 3 and 4 include notations to be applied in the study and state descriptions. Section 5 covers the transition probabilities and mean sojourn time. Sections 6, 7 and 8 describe various system performance metrics, profit functions, and their corresponding numerical assessments. In section 9, the paper is concluded with some insightful real-world applications.

2. Assumptions

Initially, we have one operating, one warm standby, and one cold standby unit. The warm standby unit becomes active immediately in the event of the operative unit malfunctioning, and the cold standby unit is activated to become the warm standby unit. Before any event is over, the activation process must be finished. The operative unit fails due to a serious problem, but warm standby fails due to a small problem. The system is still functioning if at least one unit is up and running. Each and every parameter is exponential and statistically independent.

3. Notations

The various notations for rates/probabilities are as follows:

λ/λ_1	Failure rate of operative/warm standby unit
β	Activation rate of cold standby unit
α_1/α_2	Repair rate on major/minor fault
$BJ_0(t)$	Probability that a repairman is working on a major repair to the system at time t without transitioning to another regenerative state at time t=0
$B_i(t)$	Probability that system will remain in state i while operating rather than switching to other state.
$BM_0(t)$	Probability that a repairman working on a minor repair to the system in regenerative state i at time t without switching into

another regenerative state at time t = 0

For other notations, one may refer to [11]

4. States Description

The state transition diagram (Figure 1) shows the following states at a given time point as:

State 0: (O, ws, cs); State 1: (F_{wmjr}, O, CS_a); State 2: (F_{wmnr}, O, CS_a);
State 3: (F_{mjr}, O, ws); State 4: (F_{mnr}, O, ws); State 5: (F_{mjr}, F_{wmjr}, O);
State 6: (F_{mjr}, O, F_{wmnr}); State 7: (F_{mnr}, F_{wmjr}, O); State 8: (F_{mnr}, O, F_{wmnr});
State 9: $(F_{mjr}, F_{wmjr}, F_{wmjr})$; State 10: $(F_{mjr}, F_{wmjr}, F_{wmnr})$; State 11: $(F_{mnr}, F_{wmjr}, F_{wmjr})$;
State 12: $(F_{mnr}, F_{wmjr}, F_{wmnr})$;

where,

O :	operative unit.
ws :	warm standby unit.
cs :	cold standby unit.
CS_a :	cold standby under activation.
F_{mjr} :	under major repair.
F_{mnr} :	under minor repair.
F_{wmjr} :	waiting for major repair.
F_{wmnr} :	waiting for minor repair.

5. Model Development

Figure 1 represents the transition between states of system. The operative states are {0,1,2,3,4,5,6,7,8} and failed states are {9,10,11,12}. A regenerative process is a stochastic process having time points at which the process probabilistically restarts itself, and the associated state of the system is known as the regenerative state, otherwise non-regenerative. Our analysis consistently considers the exponential distribution those results in memorylessness. All the states in the present model are, therefore, regenerative states. Hence, the defined system's possible transitions between different states and entry points into a specific state follow Markov and regenerative processes. The densities $q_{ij}(t)$, transition from state i to j is as:

$q_{01}(t) = \lambda e^{-(\lambda+\lambda_1)t}$, $q_{02}(t) = \lambda_1 e^{-(\lambda+\lambda_1)t}$, $q_{13}(t) = \beta e^{-\beta t}$
$q_{24}(t) = \beta e^{-\beta t}$, $q_{30}(t) = \alpha_1 e^{-(\lambda+\lambda_1+\alpha_1)t}$ $q_{35}(t) = \lambda e^{-(\lambda+\lambda_1+\alpha_1)t}$
$q_{36}(t) = \lambda_1 e^{-(\lambda+\lambda_1+\alpha_1)t}$, $q_{40}(t) = \alpha_2 e^{-(\lambda+\lambda_1+\alpha_2)t}$, $q_{47}(t) = \lambda e^{-(\lambda+\lambda_1+\alpha_2)t}$
$q_{48}(t) = \lambda_1 e^{-(\lambda+\lambda_1+\alpha_2)t}$ $q_{53}(t) = \alpha_1 e^{-(\lambda+\alpha_1)t}$, $q_{59}(t) = \lambda e^{-(\lambda+\alpha_1)t}$
$q_{64}(t) = \alpha_1 e^{-(\lambda+\alpha_1)t}$, $q_{6,10}(t) = \lambda e^{-(\lambda+\alpha_1)t}$ $q_{73}(t) = \alpha_2 e^{-(\lambda+\alpha_2)t}$
$q_{7,11}(t) = \lambda e^{-(\lambda+\alpha_2)t}$, $q_{84}(t) = \alpha_2 e^{-(\lambda+\alpha_2)t}$, $q_{8,12}(t) = \lambda e^{-(\lambda+\alpha_2)t}$
$q_{95}(t) = \alpha_1 e^{-\alpha_1 t}$, $q_{10,7}(t) = \alpha_1 e^{-\alpha_1 t}$, $q_{11,5}(t) = \alpha_2 e^{-\alpha_2 t}$
$q_{12,5}(t) = \alpha_2 e^{-\alpha_2 t}$

(1)

Now defining steady state probability,

$$p_{ij} = \lim_{s\to 0} q^*_{ij}(s) = \lim_{s\to 0} L[q^*_{ij}(t)] = \lim_{s\to 0} \int_0^\infty e^{-st} q_{ij}(t)dt \tag{2}$$

$p_{01} = \frac{\lambda}{\lambda+\lambda_1}$ $p_{02} = \frac{\lambda_1}{\lambda+\lambda_1}$; $p_{13} = 1$; $p_{24} = 1$
$p_{30} = \frac{\alpha_1}{\lambda+\lambda_1+\alpha_1}$ $p_{35} = \frac{\lambda}{\lambda+\lambda_1+\alpha_1}$ $p_{36} = \frac{\lambda_1}{\lambda+\lambda_1+\alpha_1}$ $p_{40} = \frac{\alpha_2}{\lambda+\lambda_1+\alpha_2}$
$p_{47} = \frac{\lambda}{\lambda+\lambda_1+\alpha_2}$ $p_{48} = \frac{\lambda_1}{\lambda+\lambda_1+\alpha_2}$ $p_{53} = \frac{\alpha_1}{\lambda+\alpha_1}$ $p_{59} = \frac{\lambda}{\lambda+\lambda_1}$
$p_{64} = \frac{\alpha_1}{\lambda+\alpha_1}$ $p_{6,10} = \frac{\lambda}{\lambda+\alpha_1}$ $p_{73} = \frac{\alpha_2}{\lambda+\alpha_2}$ $p_{7,11} = \frac{\lambda}{\lambda+\alpha_1}$

$p_{84} = \frac{\alpha_2}{\lambda+\alpha_2}$ $\quad p_{8,12} = \frac{\lambda}{\lambda+\alpha_2}$ $\quad p_{95} = 1$ $\quad p_{10,7} = 1$
$p_{11,5} = 1$ $\quad p_{12,7} = 1$

(3)

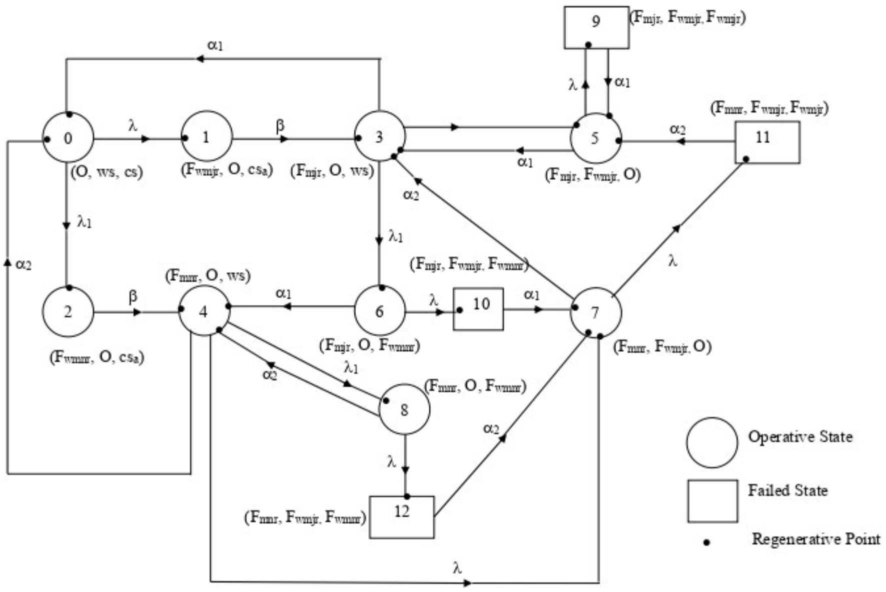

Figure 1: *State Transition Diagram*

The Mean Sojourn Time (μ_i), i.e., the stay time in particular state i,in state S_i as:

$$\mu_i = E(t) = \int_0^\infty t.(corresponding\ p.d.f.\ for\ moving\ from\ i^{th}\ state)dt. \tag{4}$$

$$\begin{aligned}
&\mu_0 = \int_0^\infty e^{-(\lambda+\lambda_1)t}dt = \frac{1}{\lambda+\lambda_1}, && \mu_1 = \int_0^\infty e^{-(\beta)t}dt = \frac{1}{\beta}\\
&\mu_2 = \int_0^\infty e^{-(\beta)t}dt = \frac{1}{\beta}, && \mu_3 = \int_0^\infty e^{-(\lambda+\lambda_1+\alpha_1)t}dt = \frac{1}{\lambda+\lambda_1+\alpha_1}\\
&\mu_4 = \int_0^\infty e^{-(\lambda+\lambda_1+\alpha_2)t}dt = \frac{1}{\lambda+\lambda_1+\alpha_2}, && \mu_5 = \int_0^\infty e^{-(\lambda+\alpha_1)t}dt = \frac{1}{\lambda+\alpha_1}\\
&\mu_6 = \int_0^\infty e^{-(\lambda+\alpha_1)t}dt = \frac{1}{\lambda+\alpha_1}, && \mu_7 = \int_0^\infty e^{-(\lambda+\alpha_2)t}dt = \frac{1}{\lambda+\alpha_2}\\
&\mu_8 = \int_0^\infty e^{-(\lambda+\alpha_2)t}dt = \frac{1}{\lambda+\alpha_2}, && \mu_9 = \int_0^\infty e^{-(\alpha_1)t}dt = \frac{1}{\alpha_1}\\
&\mu_{10} = \int_0^\infty e^{-(\alpha_1)t}dt = \frac{1}{\alpha_1}, && \mu_{11} = \int_0^\infty e^{-(\alpha_2)t}dt = \frac{1}{\alpha_2}\\
&\mu_{12} = \int_0^\infty e^{-(\alpha_2)t}dt = \frac{1}{\alpha_2},
\end{aligned} \tag{5}$$

When counting from the epoch of entry into state i, the unconditional mean time that the system needs to transit any regenerative state j is calculated mathematically as

$$m_{ij} = \int_0^\infty tq_{ij}(t)dt = q_{ij}^{'*}(0) \tag{6}$$

$m_{01} + m_{02} = \mu_0$ $\quad m_{13} = \mu_1$ $\quad m_{24} = \mu_2$ $\quad m_{30} + m_{35} + m_{36} = \mu_4$
$m_{40} + m_{47} + m_{48} = \mu_4$ $\quad m_{53} + m_{59} = \mu_5$ $\quad m_{64} + m_{6,10} = \mu_6$ $\quad m_{73} + m_{7,11} = \mu_7$

$m_{84} + m_{8,12} = \mu_8$ $\quad m_{95} = \mu_9$ $\quad m_{10,7} = \mu_{10}$ $\quad m_{11,5} = \mu_{11}$
$m_{12,7} = \mu_{12}$

(7)

6. System Performance Measures

6.1. Mean Time to System Failure (MTSF):

By using the definition of $\phi_i(t)$ and $Q_{ij}(t)$ define in section 3 and regarding the failed state as absorbing state , we have the recurrence relation for $\phi_i(t)$ from Figure 1 as
$\phi_0(t) = Q_{01}(t) \circledS \phi_1(t) + Q_{02}(t) \circledS \phi_2(t)$
$\phi_1(t) = Q_{13}(t) \circledS \phi_3(t)$
$\phi_2(t) = Q_{24}(t) \circledS \phi_4(t)$
$\phi_3(t) = Q_{30}(t) \circledS \phi_0(t) + Q_{35}(t) \circledS \phi_5(t) + Q_{36}(t) \circledS \phi_6(t)$
$\phi_4(t) = Q_{40}(t) \circledS \phi_0(t) + Q_{47}(t) \circledS \phi_7(t) + Q_{48}(t) \circledS \phi_8(t)$
$\phi_5(t) = Q_{53}(t) \circledS \phi_3(t) + Q_{59}(t) \circledS \phi_9(t)$
$\phi_6(t) = Q_{64}(t) \circledS \phi_4(t) + Q_{610}(t) \circledS \phi_{10}(t)$
$\phi_7(t) = Q_{73}(t) \circledS \phi_4(t) + Q_{711}(t) \circledS \phi_{11}(t)$
$\phi_8(t) = Q_{84}(t) \circledS \phi_4(t) + Q_{812}(t) \circledS \phi_{12}(t)$
Taking Laplace Stieljes transform of above equation and solving for $\phi_0^{**}(s)$, by crammer rule, we get
$\phi_0^{**}(s) = \frac{M_1(s)}{T_1(s)}$
where,

$$M_1(s) = \begin{vmatrix} 1 & -Q_{01}^{**}(s) & -Q_{02}^{**}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -Q_{13}^{**}(s) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -Q_{24}^{**}(s) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -Q_{35}^{**}(s) & -Q_{36}^{**}(s) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -Q_{47}^{**}(s) & -Q_{48}^{**}(s) \\ -Q_{59}^{**}(s) & 0 & 0 & -Q_{53}^{**}(s) & 0 & 1 & 0 & 0 & 0 \\ -Q_{610}^{**}(s) & 0 & 0 & 0 & -Q_{64}^{**}(s) & 0 & 1 & 0 & 0 \\ -Q_{711}^{**}(s) & 0 & 0 & -Q_{73}^{**}(s) & 0 & 0 & 0 & 1 & 0 \\ -Q_{812}^{**}(s) & 0 & 0 & 0 & -Q_{84}^{**}(s) & 0 & 0 & 0 & 1 \end{vmatrix}$$

and

$$T_1(s) = \begin{vmatrix} 1 & -Q_{01}^{**}(s) & -Q_{02}^{**}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -Q_{13}^{**}(s) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -Q_{24}^{**}(s) & 0 & 0 & 0 & 0 \\ -Q_{30}^{**}(s) & 0 & 0 & 1 & 0 & -Q_{35}^{**}(s) & -Q_{36}^{**}(s) & 0 & 0 \\ -Q_{40}^{**}(s) & 0 & 0 & 0 & 1 & 0 & 0 & -Q_{47}^{**}(s) & -Q_{48}^{**}(s) \\ 0 & 0 & 0 & -Q_{53}^{**}(s) & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -Q_{64}^{**}(s) & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -Q_{73}^{**}(s) & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -Q_{84}^{**}(s) & 0 & 0 & 0 & 1 \end{vmatrix}$$

The mean time to system failure (MTSF) is given by:

$$MTSF = \lim_{s \to 0} \frac{1 - \phi_0^{**}(s)}{s}$$
$$= \lim_{s \to 0} \frac{1 - \frac{M_1(s)}{T_1(s)}}{s}$$
$$= \lim_{s \to 0} \frac{T_1(s) - M_1(s)}{sT_1(s)}$$

Apply L'Hospital Rule , We get:

$$MTSF = \lim_{s \to 0} \frac{T_1'(s) - M_1'(s)}{sT_1'(s) + T_1(s)} = \frac{T_1'(0) - M_1'(0)}{T_1(0)}$$

which is evaluated by solving determinants in $M_1(s), T_1(s)$ and then further required steps using MATLAB.

6.2. Availability Analysis

We can obtain the following expression for the availability by proceeding the same way as in earlier section 6.1,
$A_0^*(s) = \frac{M_2(s)}{T_2(s)}$
where,

$$M_2(s) = \begin{vmatrix} B_0^*(s) & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ B_1^*(s) & 1 & 0 & -q_{13}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ B_2^*(s) & 0 & 1 & 0 & -q_{24}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ B_3^*(s) & 0 & 0 & 1 & 0 & -q_{35}^*(s) & -q_{36}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 \\ B_4^*(s) & 0 & 0 & 0 & 1 & 0 & 0 & -q_{47}^*(s) & -q_{48}^*(s) & 0 & 0 & 0 & 0 \\ B_5^*(s) & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -q_{59}^*(s) & 0 & 0 & 0 \\ B_6^*(s) & 0 & 0 & 0 & -q_{64}^*(s) & 0 & 1 & 0 & 0 & 0 & -q_{6,10}^*(s) & 0 & 0 \\ B_7^*(s) & 0 & 0 & -q_{73}^*(s) & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -q_{7,11}^*(s) & 0 \\ B_8^*(s) & 0 & 0 & 0 & -q_{84}^*(s) & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -q_{8,12}^*(s) \\ 0 & 0 & 0 & 0 & 0 & -q_{95}^*(s) & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{10,7}^*(s) & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -q_{11,5}^*(s) & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{12,7}^*(s) & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$B_i^*(s) = L(B_i(t)) = \lim_{s \to 0} \int_0^\infty e^{-st} B_i(t) dt$

$$T_2(s) = \begin{vmatrix} 1 & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -q_{13}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -q_{24}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -q_{30}^*(s) & 0 & 0 & 1 & 0 & -q_{35}^*(s) & -q_{36}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 \\ -q_{40}^*(s) & 0 & 0 & 0 & 1 & 0 & 0 & -q_{47}^*(s) & -q_{48}^*(s) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -q_{59}^*(s) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -q_{64}^*(s) & 0 & 1 & 0 & 0 & 0 & -q_{6,10}^*(s) & 0 & 0 \\ 0 & 0 & 0 & -q_{73}^*(s) & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -q_{7,11}^*(s) & 0 \\ 0 & 0 & 0 & 0 & -q_{84}^*(s) & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -q_{8,12}^*(s) \\ 0 & 0 & 0 & 0 & 0 & -q_{95}^*(s) & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{10,7}^*(s) & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -q_{11,5}^*(s) & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{12,7}^*(s) & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

The steady-state availability is given by

$$A_0 = \lim_{s \to 0} sA_0^*(s) = \lim_{s \to 0} \frac{s.M_2(s)}{T_2(s)}$$

After being solved, this takes on an indeterminant form. Applying the L'Hospital Rule, we thus obtain:

$$A_0 = \lim_{s \to 0} \frac{sM_2'(s) + M_2(s)}{T_2'(s)}$$

$$= \lim_{s \to 0} \frac{M_2(s)}{T_2'(s)} = \frac{M_2(0)}{T_2{}'(0)}$$

Further calculations have been performed using MATLAB since there is a huge determinant and it's derivative to solve.

6.3. Busy Period Analysis for Major Repair

The average time for which repairman is busy for major repair of the system is given by $BJ_0^*(s) = \frac{M_3(s)}{T_2(s)}$
where

$$M_3(s) = \begin{vmatrix} 1 & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -q_{13}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -q_{24}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ W_3^*(s) & 0 & 0 & 1 & 0 & -q_{35}^*(s) & -q_{36}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -q_{47}^*(s) & -q_{48}^*(s) & 0 & 0 & 0 & 0 \\ W_5^*(s) & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -q_{59}^*(s) & 0 & 0 & 0 \\ W_6^*(s) & 0 & 0 & 0 & -q_{64}^*(s) & 0 & 1 & 0 & 0 & 0 & -q_{6,10}^*(s) & 0 & 0 \\ 0 & 0 & 0 & -q_{73}^*(s) & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -q_{7,11}^*(s) & 0 \\ 0 & 0 & 0 & 0 & -q_{84}^*(s) & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -q_{8,12}^*(s) \\ 0 & 0 & 0 & 0 & 0 & -q_{95}^*(s) & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ W_{10}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & -q_{10,7}^*(s) & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -q_{11,5}^*(s) & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{12,7}^*(s) & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

and $T_2(s)$ is same as mention in section 6.2
In steady state, the average time for which repairman is busy for major repair of the system is given by:

$$BJ_0 = \lim_{s \to 0} sBJ_0^*(s) = \lim_{s \to 0} \frac{s.M_3(s)}{T_2(s)}$$

After being solved, this takes on an indeterminant form. Applying the L'Hospital Rule, we thus obtain:

$$BJ_0 = \lim_{s \to 0} \frac{sM_3'(s) + M_3(s)}{T_2'(s)}$$

$$= \lim_{s \to 0} \frac{M_3(s)}{T_2'(s)} = \frac{M_3(0)}{T_2{}'(0)}$$

Further calculations have been performed using MATLAB since there is a huge determinant and it's derivative to solve.

6.4. Busy Period Analysis for Minor Repair

The average time for which repairman is busy for minor repair of the system is given by
$BM_0^*(s) = \frac{M_4(s)}{T_2(s)}$

where,

$$M_4(s) = \begin{vmatrix}
1 & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -q_{13}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -q_{24}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -q_{35}^*(s) & -q_{36}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & -q_{47}^*(s) & -q_{48}^*(s) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -q_{73}^*(s) & 0 & 1 & 0 & 0 & 0 & -q_{59}^*(s) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -q_{64}^*(s) & 0 & 1 & 0 & 0 & 0 & -q_{6,10}^*(s) & 0 & 0 \\
W_7^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -q_{7,11}^*(s) & 0 \\
W_8^*(s) & 0 & 0 & 0 & -q_{84}^*(s) & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -q_{8,12}^*(s) \\
0 & 0 & 0 & 0 & 0 & -q_{95}^*(s) & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{10,7}^*(s) & 0 & 0 & 1 & 0 & 0 \\
W_{11}^*(s) & 0 & 0 & 0 & 0 & -q_{11,5}^*(s) & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{12,7}^*(s) & 0 & 0 & 0 & 0 & 1
\end{vmatrix}$$

where,$T_2(s)$ is same as mention in section 6.2

In steady state, the average time for which repairman is busy for minor repair of the system is given by

$$BM_0 = \lim_{s\to 0} sBM_0^*(s) = \lim_{s\to 0} \frac{s.M_4(s)}{T_2(s)}$$

After being solved, this takes on an indeterminant form. Applying the L'Hospital Rule, we thus obtain:

$$BM_0 = \lim_{s\to 0} \frac{sM_4'(s) + M_4(s)}{T_2'(s)}$$

$$= \lim_{s\to 0} \frac{M_4(s)}{T_2'(s)} = \frac{M_4(0)}{T_2{}'(0)}$$

Further calculations have been performed using MATLAB since there is a huge determinant and it's derivative to solve.

6.5. Expected Number of the Visit of the Repairman

The expected number of server's visits obtained as:

$V_0^*(s) = \frac{M_5(s)}{T_2(s)}$

where,

$$M_5(s) = \begin{vmatrix}
-q_{01}^*(s) & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-q_{02}^*(s) & 1 & 0 & -q_{13}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -q_{24}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -q_{35}^*(s) & -q_{36}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & -q_{47}^*(s) & -q_{48}^*(s) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -q_{59}^*(s) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -q_{64}^*(s) & 0 & 1 & 0 & 0 & 0 & -q_{6,10}^*(s) & 0 & 0 \\
0 & 0 & 0 & -q_{73}^*(s) & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -q_{7,11}^*(s) & 0 \\
0 & 0 & 0 & 0 & -q_{84}^*(s) & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -q_{8,12}^*(s) \\
0 & 0 & 0 & 0 & 0 & -q_{95}^*(s) & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{10,7}^*(s) & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -q_{11,5}^*(s) & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{12,7}^*(s) & 0 & 0 & 0 & 0 & 1
\end{vmatrix}$$

and $T_2(s)$ is same as mention in section 6.2
In steady state, the expected number of server's visits obtained as

$$V_0 = \lim_{s\to 0} sV_0^*(s) = \lim_{s\to 0} \frac{s.M_5(s)}{T_2(s)}$$

After being solved, this takes on an indeterminant form. Applying the L'Hospital Rule, we thus obtain:

$$V_0 = \lim_{s\to 0} \frac{sM_5'(s) + M_5(s)}{T_2'(s)}$$
$$= \lim_{s\to 0} \frac{M_5(s)}{T_2'(s)} = \frac{M_5(0)}{T_2'(0)}$$

Further calculations have been performed using MATLAB since there is a huge determinant and it's derivative to solve.

7. Profit Analysis:

Profit is the difference between total value generated and total expenditure. Thus, in steady state, the expected profit is
Profit (P) = $C_0(A_0) - C_1(BJ_0) - C_2(BM_0) - C_3(V_0)$
where
C_0= Revenue per unit up time
C_1= Cost per unit up time for which the repairman is busy for major repair.
C_2= Cost per unit up time for which the repairman is busy for minor repair.
C_3= Cost per visit of the repairman

8. Numerical analysis

Various profit function graphs have been generated to determine the effect of various parameters, as illustrated. Assuming the hypothetical value of parameters as
$\lambda = 0.1, \lambda_2 = 0.2, \beta = 0.3, \alpha_1 = 0.4, \alpha_2 = 0.05, C_0 = 200, C_1 = 500, C_2 = 350, C_3 = 50$
The change in profit function (P) for different value of failure rate (λ)and revenue(C_O) shown in Figure 2.

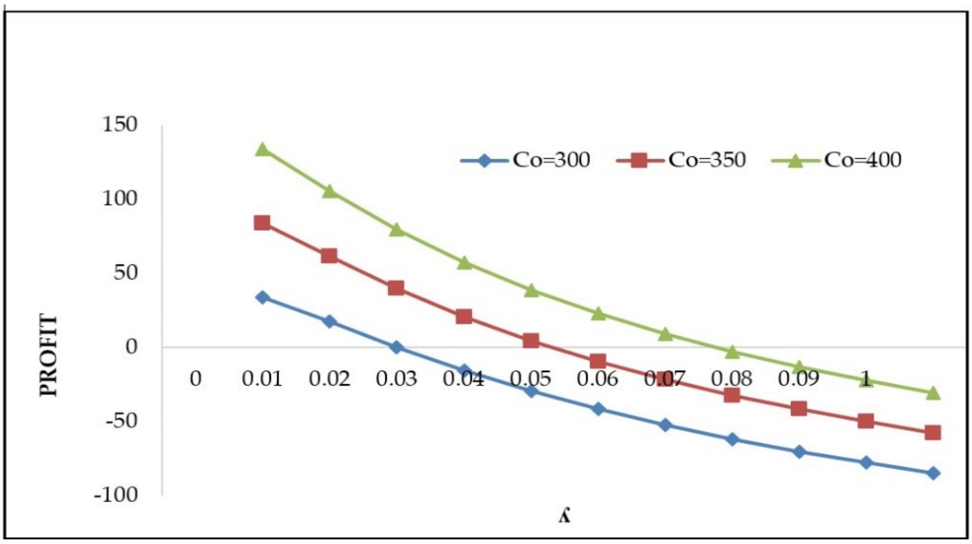

Figure 2: *Profit for varied λ and C_o*

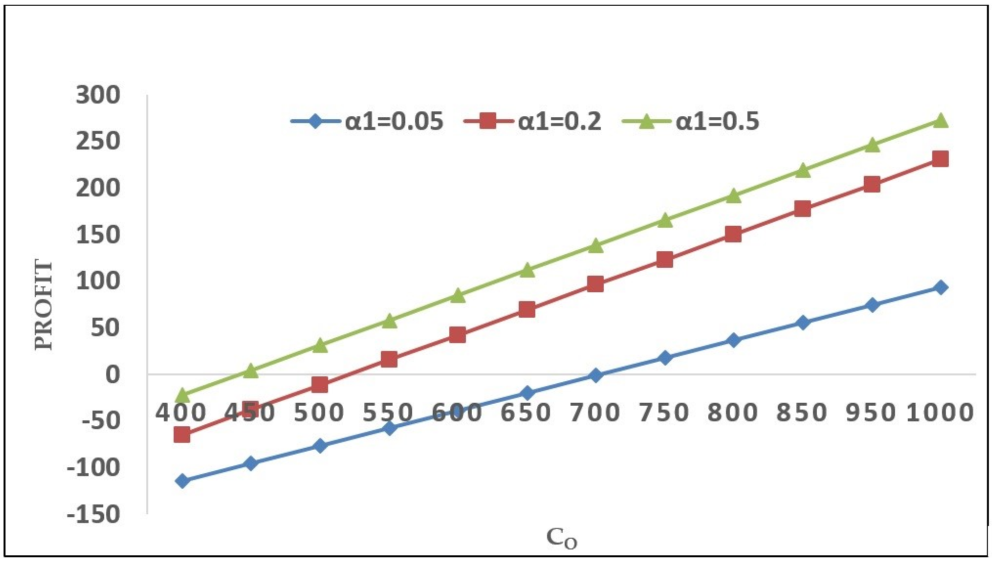

Figure 3: *Profit for varied C_0 and α_1*

It can be seen that, as the failure rate (λ) of operative unit increases, profit decreases and with increases in unit revanue (C_O), the profit increases.
Similarly, in figure 3, the effect of repair rate(α_1)on profit function has revealed . As the value of repair rate(α_1) higher, the profit also become higher. Moreover, the change in profit due to activation rate (β) and cost per visit of repairman (C_3) is shown in Figure 4.

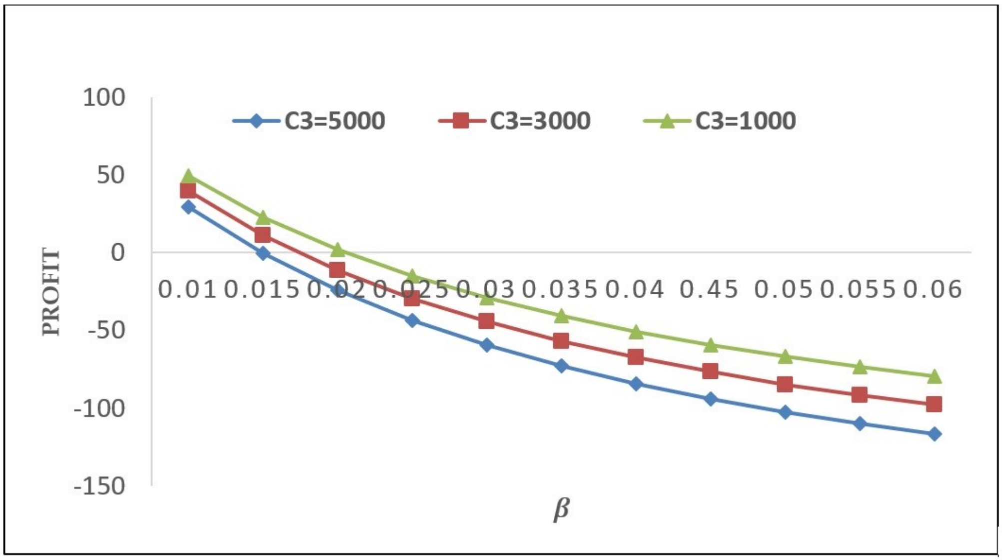

Figure 4: *Profit for varied β and C_3*

With increase in β and C_3 the profit decreases.Figure 5 reveals the change in profit w.r.t. C_2 and λ_1 respectively. With increase in C_2 and failure rate (λ_1) of warm standby unit, the profit decreases.

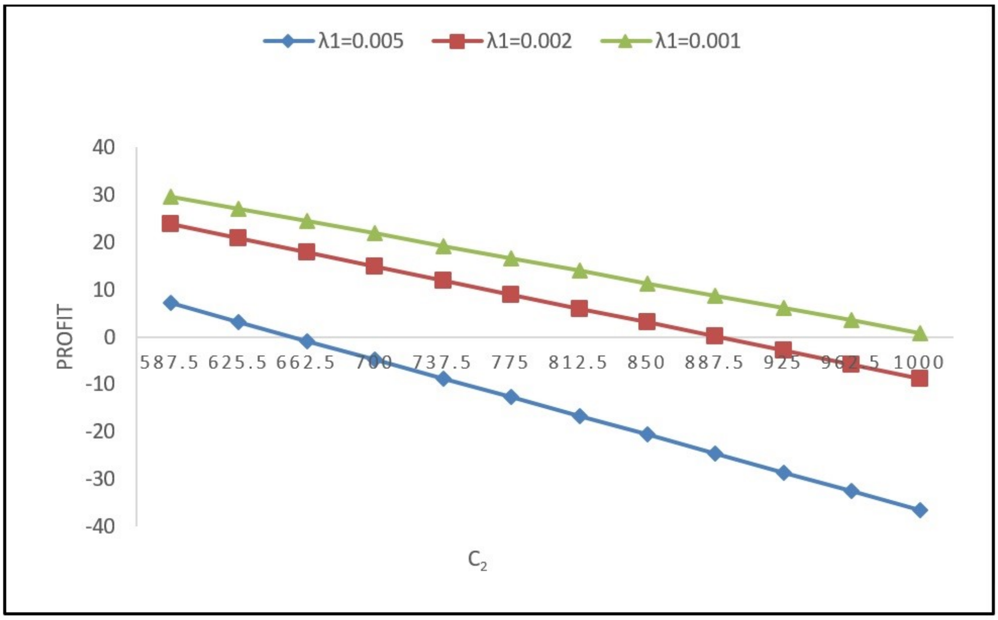

Figure 5: *Profit for varied C_2 and λ_1*

The bounds of λ,C_0,β and C_2 for profitable of system depicted in Table 1. It can be interpreted for $C_0 = 300$, failure rate should be less than equal to 0.003 for system to be profitable. Similarly, interpretation can be done for other value from previous figures 2, 3, 4 and located in Table 1.

Table 1: *Profit Analysis of the model*

Fig No.	Parameter value	Profit$\geq$ 0 if
2	$C_0 = 300$	$\lambda \leq 0.03$
	$C_0 = 350$	$\lambda \leq 0.05$
	$C_0 = 400$	$\lambda \leq 0.08$
3	$\alpha_1 = 0.05$	$C_0 \geq 450$
	$\alpha_1 = 0.02$	$C_0 \geq 550$
	$\alpha_1 = 0.5$	$C_0 \geq 700$
4	$C_3 = 5000$	$\beta \leq 0.015$
	$C_3 = 3000$	$\beta \leq 0.02$
	$C_3 = 1000$	$\beta \leq 0.025$
5	$\lambda = 0.005$	$C_2 \leq 662.5$
	$\lambda = 0.002$	$C_2 \leq 887.5$
	$\lambda = 0.001$	$C_2 \leq 1000$

9. Conclusion

A system reliability model that takes into account one operational, one cold standby, and one warm standby unit is examined. Various system measures have been drafted. The exponential case is studied numerically. Profit declines in accordance to the failure rate, activation rate, and cost per visit of the repairman. On the other hand, profit increases as revenue rises. Cut-off points for the system's cost and revenue per unit of time have also been determined in order to analyze the profitability element, which may aid them in making crucial judgments about the system's economics.

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