Analysis of Retrial Queue with Two Way Communication, Working Breakdown and Collision

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Abstract

This study examines a two-way communication retrial queue with collision and working breakdown. Retrial incoming calls may interfere with service if the server is busy with primary incoming calls. Arriving primary incoming calls are sent to the orbit while the server is busy or enter for service if the server is discovered to be idle. The server places calls while it is idle. Incoming calls are given high priority, and outgoing calls are given low priority. During regular service, the system may fails at any time. The server will continue to provide service after a breakdown even though at a slower rate rather than shutting down completely. We assume that outgoing calls and service time distributions are modelled in terms of various PH distributions, while incoming calls are assumed to follow a Markovian arrival process (MAP). The matrix-analytic approach will be used to investigate the resulting QBD process in steady state. Some of the performance metrics' calculations have been figured out. At the end, the results are presented in both numerical and graphical form.

Keywords: Retrial queues, Priority queue, Two way communication, Working breakdown, Collision.

1. Introduction

This study examines a queueing model of the form $MAP, PH/PH, PH/1$ beneficial in the call centre service sector. If the server is idle and a call comes on connectivity issues, a change in the network, etc., the customer will receive service right away. If not, the server will enter orbit and the customer will have to attempt again afterward. The server may also call customers to inform them of the special offers during idle period.

Neuts [21] made a significant contribution to stochastic process theory.He created the adaptable Markovian point process, which made it feasible to create the Markovian arrival process (MAP) and batch Markovian arrival process ($BMAP$). The two arrival mechanisms were further extended by Lucantoni [19]. The most important MAP properties are those that allow for a matrix analytic solution to a stochastic model. [7] has also enhanced this useful tool to make it a more understandable in the encyclopaedia of operations research and management science. There are definitions for MAP's discrete and continuous situations. The parameters for MAP are D_0 and D_1 of dimension "m" in continuous time, where $D_0$is a non-singular stable matrix governing the transition corresponding to no arrival and D_1 regulates the transition corresponding to arrival. Take into consideration the generating matrix D, which is represented by the equation $D = D_0 + D_1$.

Retrial queues are given a lot of attention in recent years because they can be used to analyse the operation of many different systems, including telephone, call centres, computer networks, and communications systems. When customers who arrive but are unable to receive service enter the orbit and execute later, arbitrary time. The majority of queuing systems offer customers the option to leave the service area temporarily and join an orbit before returning to complete their request after a set amount of time. A customer's orbit is the time in between trials. The server solely serves incoming arrivals made by regular customers, according to the most of literature on retrials queues. Then there are real situations where servers can make outgoing phone calls while not conversing, simulating, for example, two-way communication.

Artalejo and Gomez-Correl [1] have analyzed the comparison of classical and retrial queues as well as advance retrial queues. Chakravarthy and Dudin [9] have examined a retrial queuing system in single server which are two types of customers arrives according to Markovian arrival process and service times follows exponential distribution. A retrial queues in finding a various kind of important problems has investigated by Falin and Templeton [12]. Ke and Chang [14] has examined the multi-server retrial queuing system with vacation and balking.

Artalejo and Phung-Duc [2] have examined single server $M/G/1$ retrial queues with two-way communication. During their analysis of the $M/G/1$ retrial queue, they found that the inward and outward service time distributions changed. An $M/G/1$-type mixed priority retrial queue with two-way communication, Bernoulli vacation, and discussions of collisions, working breakdown, negative arrival, repair, immediate feedback, and reneging was described by Ayyappan and Udayageetha [6]. The $MAP, PH_2^O/PH_1^I, PH_2^O/1$ retrial queue with vacation, feedback, two-way communication, and dissatisfied customers has been investigated by Ayyappan and Gowthami [4]. [20] . Krishnamoorthy et al. [16] has been investigated the multi server queueing system that has infinite capacity in which waiting customers generate into priority and non-preemptive service discipline.

The $M/M/1$ retrial queue with two-way communication and exponential service time distributions of ingoing and outgoing calls has been thoroughly examined by Artalejo and Phung-Duc [3], who also expanded their analysis to two-way communication for multi-server retrial queueing models. A multi-server queuing model with Markovian arrivals and multiple thresholds was researched by Chakravarthy [8]. A multi-server retrial queue with two types of customers arriving in accordance with MAP with type 1 customers having preemptive priority over type 2 customers was investigated by Kumar et al, [17]. A queueing model for automatic teller machines was created by Chakravarthy and Subramanian, [10]. These service systems are subject to failure because of catastrophic events. In this study, the effectiveness and accessibility of an ATM system during failures, repairs, and replacement were examined. Jeganathan [13] investigated a $M_1, M_2/M/1$ retrial inventory system with non-preemptive priority service.

Ayyappan and Thilagavathy [5] have investigated the $MAP/PH/1$ queueing model, consisting of setup, closedown, multiple vacations, standby servers, breakdown, repair, and reneging. They assume two different types of servers, especially main and standby server. For their model, standby server carries over the service at lower rates than main server. A queueing model with server breakdowns, repairs, vacations and backup server has been examined by Chakravarthy et al [11]. In their model, used two types of server, one is main server and another one is backup server, customers arrive as MAP. Suganya et al [22] explored a perishable inventory system with a limited waiting capacity. A multi-server retrial queue with batch Markovian arrival process and breakdowns have examined by Kim et al [15]. They considered the flow of breakdowns according to MAP and the repair period has PH type distributions.

2. Model Description

A single server retrial queueing model with two different categories of arrivals is explored. Some of the arrival as incoming calls that comes after a MAP which contains the parameters of matrices D_0 and D_1 in the order m_1. The server that places the outgoing calls follows the PH type distribution (α, R) in order m_2 while it is idle. At the arriving incoming calls get the service immediately, if the server is available. Whenever the server is busy, those calls are enter into the orbit. Both incoming and outgoing call service times are distributed corresponding to the PH type with (β, S) and (γ, T) in the positions n_1 and n_2, respectively. The server may struck with breakdown during the busy period with rate η. When the service is broken down, customers using the server will pay a lower rate (θ) for the service. The server will instantly begin the repair operation with the ξ parameter. If server is available, an arriving retrial customer receives the service immediately with probability π. When the server is busy, the arriving retrial customer which collides the current customer who are getting service and the customers enter into the orbit with probability p.

3. The Steady-State Analysis

The queueing model's steady-state analysis is discussed in this section. We need to start with a few notations. Let $N(t)$ denote number of customers in the orbit, $Y_1(t)$ denoted working mode of the server (if $Y_1(t) = 0$ normal working mode and if $Y(t) = 1$ slower working mode), $Y_2(t)$ denoted status of the server (if $Y_2(t) = 0$ server is idle, $Y_2(t) = 1$- server busy with incoming call, $Y_2(t) = 2$- server busy with outgoing call), $S_1(t)$ and $S_2(t)$ represented for the phases of service when the server was busy with incoming calls and outgoing calls, respectively. $A_1(t)$ and $A_2(t)$ represented for the phases of arrival of incoming calls and outgoing calls, respectively. A continuous-time Markov chain ($CTMC$) is applied in the process $\{N(t), Y_1(t), Y_2(t), S_1(t), S_2(t), A_1(t), A_2(t)\}$, where the state space is provided by

$$\begin{aligned}\Omega = &\{(i, j, 0, 0, 0, a_1, a_2) : i\epsilon\mathbb{Z}^+,\ j = 0 \text{ or } 1,\ 1 \le a_1 \le m_1,\ 1 \le a_2 \le m_2\}\\ &\bigcup\{(i, j, 1, s_1, 0, a_1, 0) : i\epsilon\mathbb{Z}^+,\ j = 0 \text{ or } 1,\ 1 \le s_1 \le n_1,\ 1 \le a_1 \le m_1\}\\ &\bigcup\{(i, j, 2, 0, s_2, a_1, 0) : i\epsilon\mathbb{Z}^+,\ j = 0 \text{ or } 1,\ 1 \le s_2 \le n_2,\ 1 \le a_1 \le m_1\}\end{aligned}$$

3.1. The Infinitesimal Generator Matrix

The infinitesimal generator matrix Q has the following structure of level dependent quasi birth-and-death ($LDQBD$).

$$Q = \begin{bmatrix} A_{00} & A_{01} & 0 & 0 & 0 & 0 & \dots \\ A_{10} & B_1 & B_0 & 0 & 0 & 0 & \dots \\ 0 & B_2 & B_1 & B_0 & 0 & 0 & \dots \\ 0 & 0 & B_2 & B_1 & B_0 & 0 & \dots \\ \vdots & \vdots & 0 & \ddots & \ddots & \ddots & \dots \end{bmatrix}$$

where the (block) matrices appearing in Q are as follows.

$$A_{00} = \begin{bmatrix} R\bigoplus D_0 & e_{m_2}\bigotimes\beta\bigotimes D_1 & R_0\bigotimes\gamma\bigotimes I & 0 & 0 & 0 \\ \alpha\bigotimes S_0\bigotimes I & (S\bigoplus D_0)-\eta I & 0 & 0 & \eta I & 0 \\ \alpha\bigotimes T_0\bigotimes I & 0 & (T\bigoplus D_0)-\eta I & 0 & 0 & \eta I \\ \zeta I & 0 & 0 & (R\bigoplus D_0)-\zeta I & e_{m_2}\bigotimes\beta\bigotimes D_1 & R_0\bigotimes\gamma\bigotimes I \\ 0 & \zeta I & 0 & \alpha\bigotimes(\theta S_0)\bigotimes I & \theta S\bigoplus D_0-\zeta I & 0 \\ 0 & 0 & \zeta I & \alpha\bigotimes\theta T_0\bigotimes I & 0 & \theta T\bigoplus D_0-\zeta I \end{bmatrix}$$

$$A_{01} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I\bigotimes D_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & I\bigotimes D_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I\bigotimes D_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & I\bigotimes D_1 \end{bmatrix}$$

$$A_{10} = \begin{bmatrix} 0 & (1-p)\pi(e_{m_2}\bigotimes\beta\bigotimes I) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-p)\pi(e_{m_2}\bigotimes\beta\bigotimes I) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} (R\bigoplus D_0)-(\pi I) & e_{m_2}\bigotimes\beta\bigotimes D_1 & R_0\bigotimes\gamma\bigotimes I & 0 & 0 & 0 \\ \alpha\bigotimes S_0\bigotimes I & (S\bigoplus D_0)-\eta I & 0 & 0 & \eta I & 0 \\ \alpha\bigotimes T_0\bigotimes I & 0 & (T\bigoplus D_0)-\eta I & 0 & 0 & \eta I \\ \zeta I & 0 & 0 & (R\bigoplus D_0)-(\zeta+\pi)I & e_{m_2}\bigotimes\beta\bigotimes D_1 & R_0\bigotimes\gamma\bigotimes I \\ 0 & \zeta I & 0 & \alpha\bigotimes(\theta S_0)\bigotimes I & \theta S\bigoplus D_0-\zeta I & 0 \\ 0 & 0 & \zeta I & \alpha\bigotimes\theta T_0\bigotimes I & 0 & \theta T\bigoplus D_0-\zeta I \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ p\pi(e_{n_1}\bigotimes\alpha\bigotimes I) & I\bigotimes D_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & I\bigotimes D_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p\pi(e_{n_1}\bigotimes\alpha\bigotimes I) & I\bigotimes D_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & I\bigotimes D_1 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & (1-p)\pi(e_{m_2}\bigotimes\beta\bigotimes I) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-p)\pi(e_{m_2}\bigotimes\beta\bigotimes I) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The boundary blocks B_1 and B_2 are of order $2mn$. A_0, A_1 and A_2 are square matrices of order mn.

4. System Analysis

4.1. Stability Condition

Using the definitions $B = B_0 + B_1 + B_2$ and δ as the steady-state probability vector of the irreducible matrix B, it can be confirmed that the vector δ satisfies

$$\delta B = 0, \quad \delta e = 1$$

The vector δ, partitioned as $delta = (\delta_0, \delta_1, \delta_2, \delta_3, \delta_4, \delta_5)$is evaluated with the assistance of the following equations:

$$\delta_0[(R \oplus D_0) - \pi I] + \delta_1[\alpha \otimes S_0 \otimes I] + \delta_2[\alpha \otimes T_0 \otimes I] + \delta_3[\zeta I] = 0$$
$$\delta_0[e_{m_2} \otimes \beta \times (D_1 + (1-p)\pi I)] + \delta_1[(S \oplus D) - \eta I] + \delta_4[\zeta I] = 0$$
$$\delta_0[R_0 \otimes \gamma \otimes I] + \delta_2[(T \oplus D) - \eta I] + \delta_5[\zeta I] = 0$$
$$\delta_3[(R \oplus D_0) - (\zeta + \pi)I] + \delta_4[(\alpha \otimes \theta S_0 \otimes I) + p\pi(e_{n_1} \otimes \alpha \otimes I)] + \delta_5[\alpha \otimes \theta T_0 \otimes I] = 0$$
$$\delta_1[\eta I] + \delta_3[e_{m_2} \otimes \beta \otimes (D + (1-p)\pi I)] + \delta_4[(\theta S \oplus D) - \zeta I] = 0$$
$$\delta_2[\eta I] + \delta_3[R_0 \otimes \gamma \times I] + \delta_5[(\theta T \oplus D) - \zeta I] = 0$$

subject to

$$\delta_0 e_{m_1 m_2} + \delta_1 e_{m_1 n_1} + \delta_2 e_{m_1 n_2} + \delta_3 e_{m_1 m_2} + \delta_4 e_{m_1 n_1} + \delta_5 e_{m_1 n_2} = 1$$

The condition $\delta B_0 e < \delta B_2 e$, require to maintain the queueing model's stability. i.e.,

$$\delta_1[e_{n_1} \otimes (p\pi\alpha e_{m_1} + D_1 e_{m_1})] + \delta_2[e_{n_2} \otimes e_{m_1} D_1] + \delta_4[e_{n_1} \otimes (p\pi\alpha e_{m_1} + D_1 e_{m_1})] + \delta_5[e_{n_2} \otimes e_{m_1} D_1] < \delta_0[(1-p)\pi e_{m_2} \otimes \beta e_{m_1}] + \delta_3[(1-p)\pi e_{m_2} \otimes \beta e_{m_1}]$$

4.2. The Invariant Probability Vector

Let x indicate the infinitesimal generator Q's transition probability vector. The dimensions of this probability vector's subdivisions are $x_i(i > 0) = 2(m_1 m_2 + m_1 n_1 + m_1 n_2)$, which can be written as $x = x_0, x_1, x_2$.

If x is a transition probability vector for Q, then it will also satisfy the following two requirements:

$$xQ = 0 \quad \text{and} \quad xe = 1$$

Once stability has been reached the following equations can be solved to determine the steady-state probability vector x.

$$x_{i+1} = x_1 R^i, \quad i \geq 1$$

When R is the least non-negative solution to the equation,

$$R^2 B_2 + R B_1 + B_0 = 0$$

Then the final two vectors, x_0 and x_1, can be obtained by resolving the following equations:
$x_0 A_{00} + x_1 A_{10} = 0$,
$x_0 A_{01} + x_1[B_1 + RB_2] = 0$
with the normalizing condition

$$x_0 e_{2(m_1 m_2 + m_1 n_1 + m_1 n_2)} + x_1 [I - R]^{-1} e_{2(m_1 m_2 + m_1 n_1 + m_1 n_2)} = 1$$

The "Logarithmic Reduction Algorithm" described by Latouche et al. [18] may be used to generate the rate matrix R.

5. Busy Period Analysis

The length of time between customers entering a system that is void and the first time the system becomes empty again is referred to as the busy period. As a result, it is the first instance where the QBD process has considered a transition from level i to level $i-1, i \geq 2$. It is necessary to discuss both $i = 0$ and $i = 1$ individually for the boundary states. There are $(m_1 m_2 + n_1 m_1 + n_2 m_1)$ states for every level i, where i is 1. Thus, the j^{th} state of level i may be denoted as (i, j) when the states are arranged in lexicographic order. Notations:

1. $G_{j,j'}(k, x)$ Given that it started in the state (i, j) at time $t = 0$, the conditional probability that starts in the state (i, j) at time $t = 0$ entered the level $(i-1)$ by making precisely k transitions to the left.

2. $\tilde{G}_{j,j'}(z,s) = \sum_{k=1}^{\infty} z^k \int_0^{\infty} e^{-sx} dG_{j,j'}(k,x); \qquad |z| \leq 1, \;\; Re(s) \geq 0$
3. The matrix $\tilde{G}(z,s) = \tilde{G}_{j,j'}(z,s)$
4. The matrix that concerns the initial flow of time without taking into consideration boundary states is $G = \tilde{G}(1,0)$.
5. $G_{i,j'}^{(1,0)}(k,x)$ - The conditional probability that, given that the QBD process began in level 1 at time $t = 0$, it will make precisely k transitions to the left to attain level 0.
6. $G_{j,j'}^{(0,0)}(k,x)$- conditional probability that was described for the initial return to level 0.
7. E_{1j} - Expected initial passage time from level i to level $i-1$, assuming a time $t = 0$ and the process in the state (i,j).
8. E_1 -column vector with E_{1j} as its entries.
9. E_{2j} -Number of customers who should have been served during the first passage time from level i to level $i-1$, assuming that the first passage time starts in the state (i,j).
10. E_2 -column vector with E_{2j} as its entries.
11. $E_1^{1,0}$ - The vector which indicates expected initial passage times between level 1 and level 0.
12. $E_2^{1,0}$ - The vector that indicates the expected amount of service completions during the initial passage from level 1 to level 0.
13. $E_1^{(0,0)}$ - The expected initial return to level 0 time.
14. $E_2^{(0,0)}$ - The expected amount of services to be completed during the first return time to level 0.

It can be easily seen that the matrix $\tilde{G}(z,s)$ satisfies the following equations

$$\tilde{G}(z,s) = z(sI - B_1)^{-1}B_2 + (sI - B_1)^{-1}B_0\tilde{G}^2(z,s)$$

once the rate matrix R is evaluated, we can easily find the matrix G by making the result

$$G = -(B_1 + RB_2)^{-1}B_2.$$

Another method for evaluating the matrix G is the logarithmic reduction algorithm. The following equations are satisfied by $\tilde{G}^{(1,0)}(z,s)$ and $\tilde{G}^{(0,0)}(z,s)$, respectively, when it comes to the boundary states, which are 0 and 1.

$$\tilde{G}^{(1,0)}(z,s) = Z(sI - B_1)^{-1}A_{10} + (sI - B_1)^{-1}B_0\tilde{G}(z,s)\tilde{G}^{(1,0)}(z,s)$$

$$\tilde{G}^{(0,0)}(z,s) = (sI - A_{00})^{-1}A_{01}\tilde{G}^{(1,0)}(z,s).$$

Therefore following the three matrices namely, G, $\tilde{G}^{(1,0)}(1,0)$ and $\tilde{G}^{(0,0)}(1,0)$ are stochastic, we may easily calculate the following moments

$$\tilde{E}_1 = -\frac{\partial \tilde{G}(z,s)}{\partial s}\Big|_{s=0,z=1} = -(B_1 + B_0(G+I))^{-1}e$$

$$\tilde{E}_2 = \frac{\partial \tilde{G}(z,s)}{\partial z}\Big|_{s=0,z=1} = -(B_1 + B_0(G+I))^{-1}e$$

$$\tilde{E}_1^{(1,0)} = -\frac{\partial \tilde{G}^{(1,0)}(z,s)}{\partial s}\Big|_{s=0,z=1} = -(B_1 + B_0 G)^{-1}(B_0 E_1 + e)$$

$$\tilde{E}_2^{(1,0)} = \frac{\partial \tilde{G}^{(1,0)}(z,s)}{\partial z}\Big|_{s=0,z=1} = -[B_1 + B_0 G]^{-1}[A_{10}e + B_0 E_2]$$

$$\tilde{E}_1^{(0,0)} = -\frac{\partial \tilde{G}^{(0,0)}(z,s)}{\partial s}\Big|_{s=0,z=1} = -A_{00}^{-1}(e + A_{01}E_1^{(1,0)})$$

$$\tilde{E}_2^{(0,0)} = \frac{\partial \tilde{G}^{(0,0)}(z,s)}{\partial z}\Big|_{s=0,z=1} = -A_{00}^{-1}A_{01}E_2^{(1,0)}$$

6. Performance Measure

- Probability that making outgoing calls by the server

$$P_{MOA} = \sum_{i=0}^{\infty}\sum_{a_2=1}^{m_2}\sum_{a_1=1}^{m_1} x_{i00a_2a_1}$$

- Probability that busy with incoming calls by the server in regular service rate

$$P_{BIS} = \sum_{i=0}^{\infty}\sum_{s_1=1}^{n_1}\sum_{a_1=1}^{m_1} x_{i01s_1a_1}$$

- Probability that server is busy with outgoing calls in regular service rate

$$P_{BOS} = \sum_{i=0}^{\infty}\sum_{s_2=1}^{n_2}\sum_{a_1=1}^{m_1} x_{i02s_2a_1}$$

- Probability that busy with incoming calls by the server in slower service rate

$$P_{WIS} = \sum_{i=0}^{\infty}\sum_{s_1=1}^{n_1}\sum_{a_1=1}^{m_1} x_{i11s_1a_1}$$

- Probability that server is busy with outgoing calls in slower service rate

$$P_{WOS} = \sum_{i=0}^{\infty}\sum_{s_2=1}^{n_2}\sum_{a_1=1}^{m_1} x_{i12s_2a_1}$$

- Expected number of customers in the orbit

$$E_N = \sum_{i=0}^{\infty}\sum_{j=0}^{1}\sum_{a_2=1}^{m_2}\sum_{a_1=1}^{m_1} ix_{ij0a_2a_1} + \sum_{i=0}^{\infty}\sum_{j=0}^{1}\sum_{s_1=1}^{n_1}\sum_{a_1=1}^{m_1} ix_{ij1s_1a_1} + \sum_{i=0}^{\infty}\sum_{j=0}^{1}\sum_{s_2=1}^{n_2}\sum_{a_1=0}^{m_1} ix_{ij2s_2a_1}$$

7. Particular case

We consider an exponential distribution of both arriving calls such as incoming and outgoing and service times. Consider the following

$$D_0 = [-\lambda],\ D_1 = [\lambda],\ \ \alpha = [1],\ R = [\delta],$$
$$\beta = [1],\ S = [\mu_1],\ \ \gamma = [1],\ T = [\mu_2].$$

From this assumption, the infinitesimal generator matrix becomes

$$Q = \begin{bmatrix} A_{00} & A_{01} & 0 & 0 & 0 & 0 & \dots \\ A_{01} & B_1 & B_0 & 0 & 0 & 0 & \dots \\ 0 & B_2 & B_1 & B_0 & 0 & 0 & \dots \\ 0 & 0 & B_2 & B_1 & B_0 & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \dots \end{bmatrix}$$

The entries of the Q matrix are defined by

$$A_{00} = \begin{bmatrix} -\lambda-\delta & \lambda & \delta & 0 & 0 & 0 \\ \mu_1 & -\mu_1-\lambda-\eta & 0 & 0 & \eta & 0 \\ \mu_2 & 0 & -\mu_2-\lambda-\eta & 0 & 0 & \eta \\ \zeta & 0 & 0 & -\lambda-\delta-\zeta & \lambda & \delta \\ 0 & \zeta & 0 & \theta\mu_1 & -\theta\mu_1-\lambda-\zeta & 0 \\ 0 & 0 & \zeta & \theta\mu_2 & 0 & \theta\mu_2-\lambda-\zeta \end{bmatrix}$$

$$A_{01} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

$$A_{10} = \begin{bmatrix} 0 & \pi(1-p) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \pi(1-p) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \pi p & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \pi p & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -\lambda-\delta-\pi(1-p) & \lambda & \delta & 0 & 0 & 0 \\ \mu_1 & -\mu_1-\lambda-\eta-\pi p & 0 & 0 & \eta & 0 \\ \mu_2 & 0 & -\mu_2-\lambda-\eta & 0 & 0 & \eta \\ \zeta & 0 & 0 & -\lambda-\delta-\zeta-\pi(1-p) & \lambda & \delta \\ 0 & \zeta & 0 & \theta\mu_1 & -\theta\mu_1-\lambda-\zeta-\pi p & 0 \\ 0 & 0 & \zeta & \theta\mu_2 & 0 & \theta\mu_2-\lambda-\zeta \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & \pi(1-p) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \pi(1-p) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In this sequel, the generator matrix B becomes

$$B = \begin{bmatrix} -\lambda-\delta-\pi(1-p) & \lambda-\pi(1-p) & \delta & 0 & 0 & 0 \\ \mu_1+\pi p & -\mu_1-\eta-\pi p & 0 & 0 & \eta & 0 \\ \mu_2 & 0 & -\mu_2-\eta & 0 & 0 & \eta \\ \zeta & 0 & 0 & -\lambda-\delta-\zeta-\pi(1-p) & \lambda-\pi(1-p) & \delta \\ 0 & \zeta & 0 & \theta\mu_1+\pi p & -\theta\mu_1-\zeta-\pi p & 0 \\ 0 & 0 & \zeta & \theta\mu_2 & 0 & \theta\mu_2-\zeta \end{bmatrix}$$

8. Numerical Result

In this section, we examine the outcome of our system with aid of numerical and graphical illustrations. For the arrival process, let us take four different MAP representations have same mean value, which is 1.

Arrival of incoming call in Erlang (ERL-A):

$$D_0 = \begin{bmatrix} -2 & 2 \\ 0 & -2 \end{bmatrix} \quad D_1 = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}$$

Arrival of incoming call in Exponential (EXP-A):

$$D_0 = [-1] \quad D_1 = [1]$$

Arrival of incoming call in Hyper exponential (HYP-A):

$$D_0 = \begin{bmatrix} -2.8 & 0 \\ 0 & -0.28 \end{bmatrix} \quad D_1 = \begin{bmatrix} 2.24 & 0.56 \\ 0.224 & 0.056 \end{bmatrix}$$

Arrival of incoming call in MAP-Negative Correlation (MAP-NC):

$$D_0 = \begin{bmatrix} -1.00222 & 1.00222 & 0 \\ 0 & -1.00222 & 0 \\ 0 & 0 & -225.75 \end{bmatrix} \quad D_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0.01002 & 0 & 0.99220 \\ 223.4925 & 0 & 2.2575 \end{bmatrix}$$

Arrival of outgoing call in Erlang (ERL-A):

$$\alpha = [1, 0] \quad R = \begin{bmatrix} -2 & 2 \\ 0 & -2 \end{bmatrix}$$

Arrival of outgoing call in Exponential (EXP-A):

$$\alpha = [1] \quad R = [-1]$$

Arrival of outgoing call in Hyper exponential (HYP-A):

$$\alpha = [0.8, 0.2] \quad R = \begin{bmatrix} -2.8 & 0 \\ 0 & -0.28 \end{bmatrix}$$

For service times let us consider phase type distribution as follows:

Service of incoming call in Erlang (ERL-S):

$$\beta = [1, 0] \quad T = \begin{bmatrix} -2 & 2 \\ 0 & -2 \end{bmatrix}$$

Service of incoming call in Exponential (EXP-S):

$$\beta = [1] \quad T = [-1]$$

Service of incoming call in Hyper exponential (HYP-S):

$$\beta = [0.8, 0.2] \quad T = \begin{bmatrix} -2.8 & 0 \\ 0 & -0.28 \end{bmatrix}$$

Service of outgoing call in Erlang (ERL-S):

$$\gamma = [1, 0] \quad S = \begin{bmatrix} -2 & 2 \\ 0 & -2 \end{bmatrix}$$

Service of outgoing call in Exponential (EXP-S):

$$\gamma = [1] \quad S = [-1]$$

Service of outgoing call in Hyper exponential (HYP-S):

$$\gamma = [0.8, 0.2] \quad S = \begin{bmatrix} -2.8 & 0 \\ 0 & -0.28 \end{bmatrix}$$

8.1. Illustrative Example 1

From Tables 1,2, 3 and 4 explore the effectiveness of the breakdown rate (η) on the traffic intensity (ρ) and expected queue size (EN). We fix $\lambda = 0.1$, $\delta = 0.01$, $\mu_1 = 2$, $\mu_2 = 1$, $\zeta = 2$, $\theta = 0.6$, $\pi = 0.9$ and $p = 0.4$. We observe from tables 1, 2, 3 and 4 as increasing breakdown rate (η), the traffic intensity increases and also mean queue length (EN) increases for various arrival and service distributions.

Table 1: *Breakdown rate (η) vs ρ and EN - $EXP - A$*

	$EXP-S$		$ERL-S$		$HYP-S$	
η	ρ	EN	ρ	EN	ρ	EN
1.1	0.3641	1.16678	0.33822	1.16428	0.49312	1.18627
1.2	0.3666	1.16913	0.34063	1.16661	0.49634	1.18903
1.3	0.3691	1.17142	0.34297	1.16889	0.49943	1.19170
1.4	0.3715	1.17364	0.34523	1.17110	0.50238	1.19431
1.5	0.3738	1.17581	0.34742	1.17325	0.50522	1.19684
1.6	0.3761	1.17791	0.34954	1.17535	0.50794	1.19930
1.7	0.3782	1.17996	0.35160	1.17739	0.51055	1.20170
1.8	0.3803	1.18196	0.35359	1.17938	0.51306	1.20403
1.9	0.3824	1.18391	0.35553	1.18132	0.51548	1.20631

Table 2: *Breakdown rate (η) vs ρ and EN - $ERL - A$*

	$EXP - S$		$ERL - S$		$HYP - S$	
η	ρ	EN	ρ	EN	ρ	EN
0.1	0.40496	2.60789	0.36928	2.61353	0.55800	2.60590
0.2	0.40742	2.61448	0.37161	2.62046	0.56098	2.61231
0.3	0.40978	2.62088	0.37385	2.62722	0.56382	2.61855
0.4	0.41207	2.62712	0.37603	2.63381	0.56655	2.62462
0.5	0.41427	2.63321	0.37813	2.64023	0.56915	2.63055
0.6	0.41640	2.63913	0.38017	2.64650	0.57165	2.63632
0.7	0.41846	2.64491	0.38215	2.65261	0.57405	2.64196
0.8	0.42044	2.65055	0.38407	2.65858	0.57635	2.64745
0.9	0.42237	2.65605	0.38593	2.66440	0.57856	2.65282

Table 3: *Breakdown rate (η) vs ρ and EN - $HYP - A$*

	$EXP - S$		$ERL - S$		$HYP - S$	
η	ρ	EN	ρ	EN	ρ	EN
0.1	0.26239	3.54972	0.25352	3.57397	0.31647	3.47495
0.2	0.26460	3.56408	0.25560	3.58893	0.31926	3.48861
0.3	0.26674	3.57805	0.25761	3.60351	0.32195	3.50192
0.4	0.26881	3.59165	0.25957	3.61772	0.32454	3.51488
0.5	0.27082	3.60490	0.26147	3.63157	0.32704	3.52751
0.6	0.27276	3.61780	0.26331	3.64507	0.32944	3.53982
0.7	0.27465	3.63038	0.26510	3.65824	0.33177	3.55182
0.8	0.27648	3.64264	0.26683	3.67109	0.33401	3.56354
0.9	0.27825	3.65460	0.26852	3.68363	0.33618	3.57497

Table 4: *Breakdown rate (η) vs ρ and EN - $NCM - A$*

	$EXP - S$		$ERL - S$		$HYP - S$	
η	ρ	EN	ρ	EN	ρ	EN
0.1	0.76028	3.06864	0.73819	3.08255	0.85856	3.09657
0.2	0.76097	3.07762	0.73880	3.09144	0.86000	3.10642
0.3	0.76165	3.08634	0.73940	3.10009	0.86139	3.11597
0.4	0.76230	3.09483	0.73998	3.10851	0.86271	3.12526
0.5	0.76293	3.10308	0.74054	3.11670	0.86398	3.13427
0.6	0.76354	3.11111	0.74109	3.12469	0.86519	3.14304
0.7	0.76413	3.11893	0.74162	3.13246	0.86635	3.15157
0.8	0.76471	3.12655	0.74214	3.14004	0.86746	3.15988
0.9	0.76526	3.13397	0.74264	3.14743	0.86853	3.16796

8.2. Illustrative Example 2

Two dimensional graphs are illustrated in the Figure 1. we fix $\lambda = 1$, $\delta = 0.1$, $\mu_1 = 2$, $\mu_2 = 1$, $\zeta = 3$, $\theta = 0.4$, $\pi = 0.6$ and $p = 0.5$. The illustration shows that the effect of the breakdown rate increase then the mean queue length is increase for various arrival and service distributions.

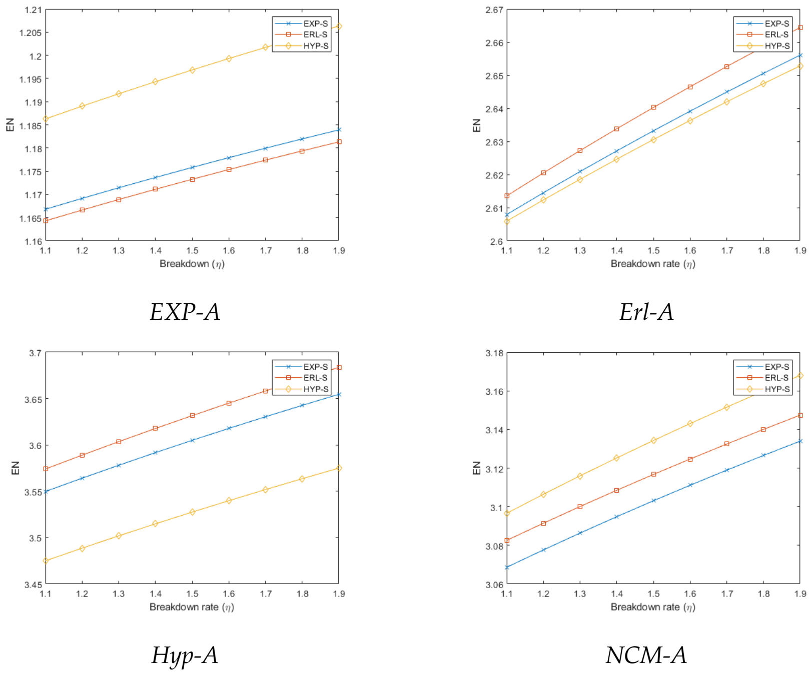

Figure 1: *Breakdown rate (η) vs Mean queue length (EN) with various arrival distributions*

8.3. Illustrative Example 3

Three dimensional graphs are illustrated in the Figure 2. We examine the influence of the breakdown rate (η) and collision probability (p) on the mean queue length. We fix $\lambda = 0.1$, $\delta = 0.01$, $\mu_1 = 2$, $\mu_2 = 1$, $\zeta = 6$, $\theta = 0.6$, and $\pi = 0.3$. we could observe that the surface denotes upward trend as expected to propagate the values of breakdown rate (η) and the collision probability (p) against the expected queue size (EN) for various arrival and service times.

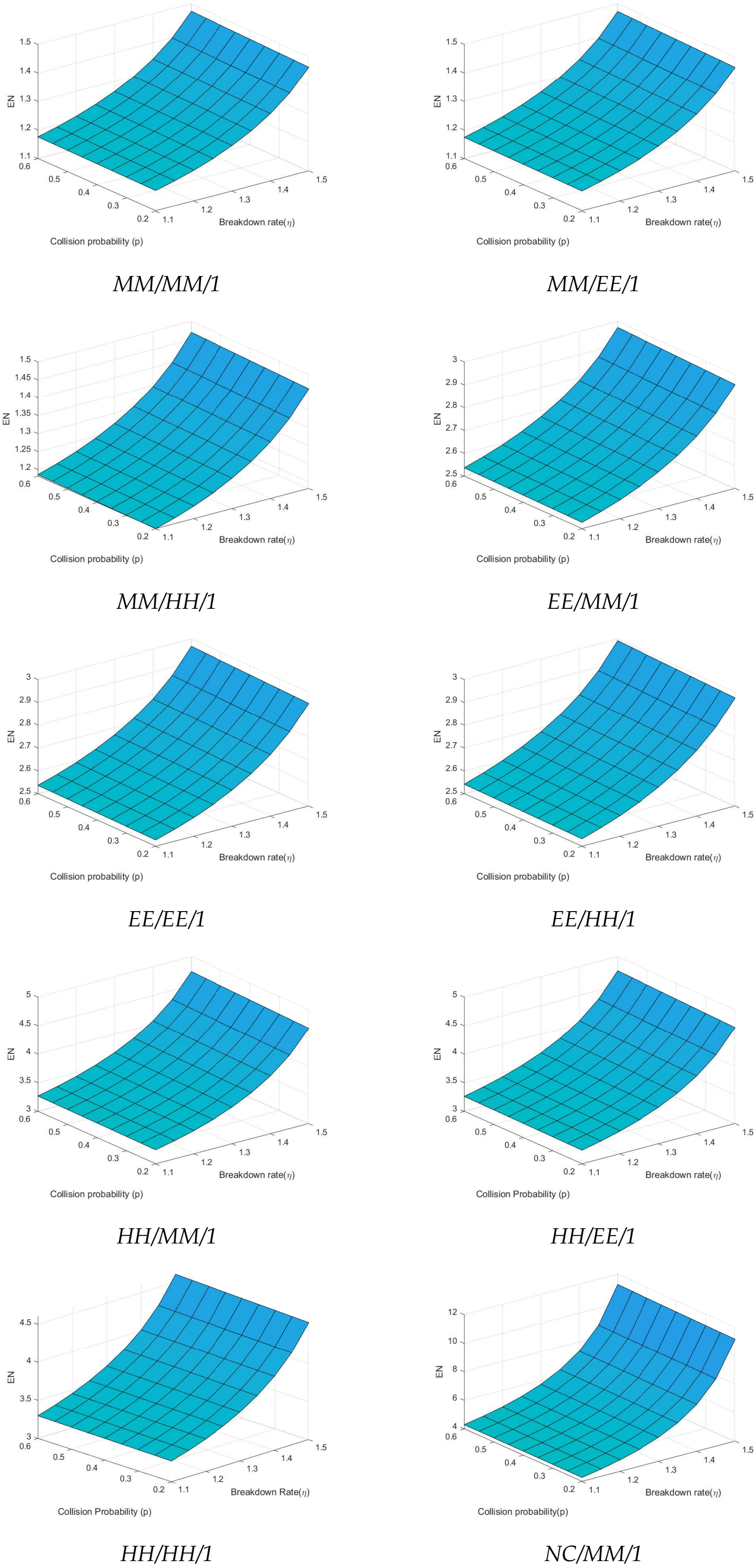

Figure 2: *Breakdown and collision vs mean queue length*

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