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ALTERNATE QUADRA SUB - MERGING POLAR FUZZY SOFT GRAPH AND ITS APPLICATION

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Abstract

Fuzzy graph and Fuzzy soft graph are indispensable computing modules for presenting membership and non - membership values in the world of uncertain situations and incidents. In this research article, we introduce the new module of Alternate Quadra Submerging Polar Fuzzy Soft Graph with four co – ordinates with membership and non – membership values. The aim of this new fuzzy soft graph is to find the single output from different uncertain parametric sets of subjects and events, between the range [-1, 1]. The submerge level of fixed four co ordinates is a tool to find the precise and reliable membership degree values from uncertain problems and outcomes. In this artifact, we also investigate the different types of Alternate Quadra Submerging Polar Fuzzy Soft Graphs, corresponding parametric fuzzy values and submerge membership and non – membership values. We discussed Strong, Complete, Complement and μ complement properties of Alternate Quadra Submerging Polar Fuzzy Soft Graphs. We use this fuzzy soft graph in the Analysis of water related diseases to find the result of most and least affected diseases with the symptoms among the hostel students in the same locality. We find the maximum and minimum membership and non - membership value of the water related diseases in an unique way by using this Alternate Quadra Submerging Polar Fuzzy Soft Graph score function values.

Keywords: AQSP fuzzy graph , AQSP Fuzzy Soft Graph, Strong and Complete AQSP fuzzy soft graph, Complement and μ – Complement proprties of AQSP fuzzy soft graphs.

1. Introduction

The future is parametric uncertain universal set, but this uncertainty is at the very heart of human creativity. Mathematicians and Scientists have a lot of experience with ignorance, doubt, and uncertainty. In 1965 Prof.Lotfi.A.Zadeh[20], invented Fuzzy set theory with membership values to solve uncertain subjects and events. The concept of fuzzy graph was first introduced by Rosenfeld[16]. Kaufmann's[10] initial definition of a fuzzy graph was based on Zadeh's fuzzy relations. Bhattacharya[6] gave some remarks on fuzzy graphs. In 1994, Moderson[14] and Peng introduced several notations on fuzzy graphs and the concept of complement of fuzzy graphs. In 1999, Molodtsov[12] introduced the concept of soft set theory to deal with uncertainties. It has been applied in the field of Applied Mathematics, Artificial Computation intelligent, Engineering, Smoothness of functions, Medical Science and Environment. Since the research on soft fuzzy sets has been very active and received much attention from researchers in worldwide.

In this current computing era, a few research studies contributed into fuzzification of soft set theory. Feng et al, combined soft sets with rough sets and fuzzy sets, obtaining three types of hybrid models, rough, soft sets, soft, rough sets, and soft – rough fuzzy sets. In 2001, Maji et al, initiated the concept of fuzzy soft sets which is a combination of soft sets. In 2002, M.S.Sunitha [19] and Vijayakumar gave a modified definition of Complement of fuzzy graph.

In 2006, Nagooorgani[8] and Chandrasekaran defined. μ – Complement of fuzzy graph, which is different from the definition of M.S.Sunitha's Complement of fuzzy graph.In 2015 , Samanta and Mohinta[13] investigated the notions of fuzzy soft graphs, Operation of union,intersection of two fuzzy soft graphs with properties related to this fuzzy soft graph module. Akram[1],[2] and Nawaz introduced the notions of fuzzy soft graph, strong, complete fuzzy soft graph and regular fuzzy soft graph with properties are investigated.

In this paper, we introduce certain types of Alternate Quadra Sub – merging Polar Fuzzy soft graphs, μ – Complement of AQSP fuzzy soft graphs and some properties of μ – Complement of AQSP fuzzy soft graphs. And we explore some results of strong and complete AQSP fuzzy soft graphs and isolated AQSP fuzzy soft graphs with theorems, examples, and applications. Using submerging level of fixation method in four quadrant membership and non - membership values, [- 0.5, 0] ⊂ [-1, 0], [- 0.5, 0.5] ⊂ [-1, 1], [0, 0.5] ⊂ [0,1] and [0.5, 0.5] ⊂ [1,1] will provide the solution from uncertain membership values. It is the module of medical and psychological studies to interpret a particular type of Uncertainty with parametric set.

2. Preliminaries

2.1. Fuzzy Graph[16].

Let V be a nonempty finite set and $\sigma : V \longrightarrow [0,1]$. And, let $\mu : V \times V \longrightarrow [0,1]$ such that $\mu(x,y) \leq \sigma(x) \wedge \sigma(y), \forall (x,y) \in V \times V$. Then the ordered pair $G = (\sigma, \mu)$ is called a fuzzy graph over the set V, where σ and μ are fuzzy vertex and edge of fuzzy graph $G = (\sigma, \mu)$.

2.2. Fuzzy Soft Set[13].

Let X be an initial universe set and E be the set of parameters. Let $A \subset E$. A pair (F,A) is called fuzzy soft set over X, where F is a mapping given by $F : A \longrightarrow I^X$ and I^X denotes the collection of all fuzzy subsets of X.

2.3. Complete Fuzzy Graph [14]

A Complete fuzzy graph is a pair of functions $G : (\sigma, \mu)$, where σ is a fuzzy subset of X and μ is a symmetric fuzzy relation on σ. Here $\sigma : X \rightarrow [0,1]$ and $\mu : X \times X \rightarrow [0,1]$ such that $\mu(x,y) = \wedge(\sigma(x), \sigma(y)) \ \forall \ x,y \ \in \sigma^*$.

2.4. Strong Fuzzy Graph [14]

A strong fuzzy graph is a pair of functions $G : (\sigma, \mu)$ where σ is a fuzzy subset of X and μ is a symmetric fuzzy relation on σ . Here $\sigma : X \rightarrow [0,1]$ and $\mu : X \times X \rightarrow [0,1]$ such that $\mu(x,y) = \wedge(\sigma(x), \sigma(y)) \ \forall \ x,y \ \in \mu^*$

2.5. Complement of Fuzzy Graph [14]

Let $G : (\sigma, \mu)$ be a fuzzy graph. The complement of G is defined as $\bar{G} = (\sigma, \bar{\mu})$, where $\bar{\mu}(x,y) = \sigma(x) \wedge \sigma(y) - \mu(x,y) \forall x,y \in V$. When G is a fuzzy graph, $\overline{G} = (\sigma, \overline{\mu})$ is complement of fuzzy graph.

2.6. μ- Complement of Fuzzy Graph [19]

Let $G : (\sigma, \mu)$ be a fuzzy graph. The μ- complement of G is defined as $G^\mu = (\sigma, \bar{\mu^\mu})$, where $\bar{\mu^\mu}(x,y) = \sigma(x) \wedge \sigma(y) - \mu(x,y),\ if\ \mu(x,y) > 0,$ and $\mu^\mu(x,y),\ if \mu(x,y) = 0.$

3. Method

The essential definition of AQSP fuzzy soft graph method is deliberated with an examples.

3.1. Alternate Quadra Sub - merging Polar(AQSP) Fuzzy Graph

An Alternate Quadra - Submerging Polar (AQSP) Fuzzy Graph $G = (\sigma_{AQSP}, \mu_{AQSP})$ is a fuzzy graph with crisp graph $G^* = (\sigma^*_{AQSP}, \mu^*_{AQSP})$ is given as $V = (\sigma^P(x), \sigma^N(x), \rho^P(x), \rho^N(x))$ which is the membership value of vertices along with the uncertain membership value of edges is given as, $E = V \times V = (\mu^P(x,y), \mu^N(x,y), \gamma^P(x,y), \gamma^N(x,y))$. Here the vertex set V is defined with the given condition in a unique method which is an alternate contrast submerging polarized uncertain transformation.Here $\sigma^P = V \to [0,1]$, $\sigma^N = V \to [-1,0]$, $\rho^P = d\ |\ 0.5, \sigma^P\ (x)\ |$ and $\rho^N = -d\ |-0.5,\ \sigma^N\ (x)|$. Here (-0.5, 0.5) is the fixation of uncertain alternate contrast polarized submerging transformation into certain consistent preferable position. And the edge set E satisfies the following sufficient conditions.

$$(i)\quad \mu^P(x,y) \leq \min(\sigma^P(x), \sigma^P(y)), \quad (ii)\ \mu^N(x,y) \geq \max(\sigma^N(x), \sigma^N(y))$$

$$(iii)\ \gamma^P(x,y) \leq \min(\rho^P(x), \rho^P(y)) \quad (iv)\ \gamma^N(x,y) \geq \max(\rho^N(x), \rho^N(y)),$$

$\forall (x,y) \in E$. By definition, $\mu^P = V \times V \to [0,1] \times [1,0]$, $\mu^N = V \times V \to [-1,0] \times [0,-1]$ and the submerging mappings, $\gamma^P = V \times V \to [0,0.5] \times [0.5,0]$,
$\gamma^N = V \times V \to [-0.5,0] \times [0,-0.5]$, which denotes the impact of the alternate quadrant polarized fuzzy mapping. The maximum of submerging presumption to be at the level of confidence $[0,0.5] \subseteq [0,1]$ and the minimum of submerging presumption level of confidence is $[-0.5,0] \subseteq [-1,0]$ extension of the graph with its membership and non - membership values portrait the unique level of submerging destination in an AQSP fuzzy graph.

Also it must satisfy the condition, $-1 \leq \sigma^P\ (x) + \sigma^N\ (x) \leq 1$ *and* $|\rho^P\ (x) + \rho^N\ (x)| \leq 1$ with constrains $0 \leq \sigma^P\ (x) + \sigma^N\ (x) + |\rho^P\ (x) + \rho^N\ (x)| \leq 2$ such that the uncertain status of submerging presumption, transform into its precise consistent level with fixation mid - value 0.5, which implies that level of confidence 0.5 in an AQSP as the valuable membership of its position which is real and valid in the fuzzification. The example of AQSP fuzzy graph is given in Fig.1.

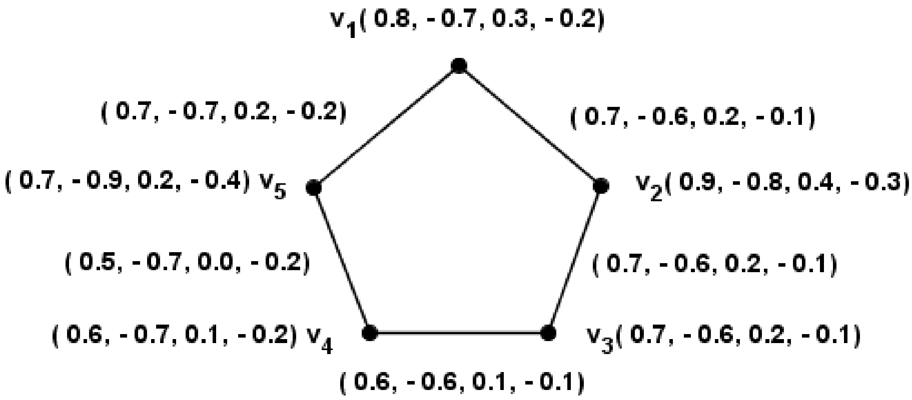

Figure 1: *AQSP Fuzzy Graph* $G = (\sigma_{AQSP}, \mu_{AQSP})$

3.2. AQSP Fuzzy Soft Graph

Let $V = ((\sigma_1^P(x), \sigma_1^N(x), \rho_1^P(x), \rho_1^N(x)), (\sigma_2^P(x), \sigma_2^N(x), \rho_2^P(x), \rho_2^N(x))...(\sigma_n^P(x), \sigma_n^N(x), \rho_n^P(x), \rho_n^N(x)))$ be a nonempty AQSP fuzzy set. E (Parameters set) and $A_{AQSP} \subset E$. Also let,

(i) $\sigma^P : A_{AQSP} \longrightarrow F_{AQSP}(V)$(Collection of all AQSP fuzzy subsets in V), $e \longmapsto \sigma_e^P$, and $\sigma_e^P : V \longrightarrow [0,1]$, $v_i \longmapsto \sigma_e^P$ then (A_{AQSP}, σ^P) : AQSP fuzzy soft vertex set.
(ii) $\sigma^N : A_{AQSP} \longrightarrow F_{AQSP}(V)$(Collection of all AQSP fuzzy subsets in V), $e \longmapsto \sigma_e^N$, and $\sigma_e^N : V \longrightarrow [-1,0]$, $v_i \longmapsto \sigma_e^N$ then (A_{AQSP}, σ^N) : AQSP fuzzy soft vertex set.

(iii) $\rho^P : A_{AQSP} \longrightarrow F_{AQSP}(V)$(Collection of all AQSP fuzzy submerge subsets in V), $e \longmapsto \rho_e^P$, and $\rho_e^P : V \longrightarrow [0,0.5]$, $v_i \longmapsto \rho_e^P$ then (A_{AQSP}, ρ^P) : AQSP fuzzy soft vertex set.
(iv) $\rho^N : A_{AQSP} \longrightarrow F_{AQSP}(V)$(Collection of all fuzzy submerge subsets in V), $e \longmapsto \rho_e^N$, and $\rho_e^N : V \longrightarrow [-0.5,0]$, $v_i \longmapsto \rho_e^N$ then (A_{AQSP}, ρ^N) : AQSP fuzzy soft vertex set.

(v) $\mu^P : A_{AQSP} \longrightarrow F_{AQSP}(V \times V)$(Collection of all AQSPfuzzy subsets in $V \times V$), $e \longmapsto \mu_e^P$, $\mu_e^P : V \times V \longrightarrow [0,1]$, $(v_i, v_j) \longmapsto \mu_e^P(v_i, v_j)$ then (A_{AQSP}, μ^P) : AQSP fuzzy soft membership edge set.

(vi) $\mu^N : A_{AQSP} \longrightarrow F_{AQSP}(V \times V)$(Collection of all AQSPfuzzy subsets in $V \times V$), $e \longmapsto \mu_e^N$, and $\mu_e^N : V \times V \longrightarrow [-1,0]$, $(v_i, v_j) \longmapsto \mu_e^N(v_i, v_j)$ then (A_{AQSP}, μ^N) : AQSP fuzzy soft non - membership edge set.

(vii) $\gamma^P : A_{AQSP} \longrightarrow F_{AQSP}(V \times V)$(Collection of all AQSPfuzzy subsets in $V \times V$), $e \longmapsto \gamma_e^P$, and $\gamma_e^P : V \times V \longrightarrow [0,0.5]$, $(v_i, v_j) \longmapsto \gamma_e^P(v_i, v_j)$ then (A_{AQSP}, γ^P) : AQSP fuzzy soft submerge membership edge set.
(viii) $\gamma^N : A_{AQSP} \longrightarrow F_{AQSP}(V \times V)$(Collection of all AQSPfuzzy subsets in $V \times V$), $e \longmapsto \gamma_e^N$, and $\gamma_e^N : V \times V \longrightarrow [-0.5,0]$, $(v_i, v_j) \longmapsto \gamma_e^N(v_i, v_j)$ then (A_{AQSP}, γ^N) : AQSP fuzzy soft submerge membership edge set. Then the AQSP fuzzy soft graph is, $((A_{AQSP}), (\sigma^P, \sigma^N, \rho^P, \rho^N)), ((A_{AQSP}), (\mu^P, \mu^N, \gamma^P, \gamma^N))$ if the conditions are satisfied

(a) $\mu_e^P(x,y) \leq \sigma_e^P(x) \wedge \sigma_e^P(y)$, (b) $\mu_e^N(x,y) \geq \sigma_e^N(x) \vee \sigma_e^N(y)$,

(c) $\gamma_e^P(x,y) \leq \rho_e^P(x) \wedge \rho_e^P(y)$, (d) $\gamma_e^N(x,y) \geq \rho_e^N(x) \vee \rho_e^N(y)$, for all $e \in A_{AQSP}$ and for all values of $x, y = 1,2,3,...,n$ and this AQSP fuzzy soft graph is denoted as $G_{AQSP}(A, V)$.

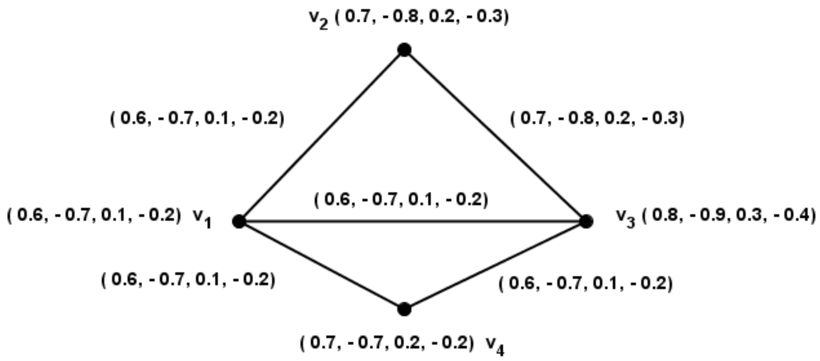

Figure 2: *$G_{AQSP}(A, V)$ - Corresponding to the parameter e_1*

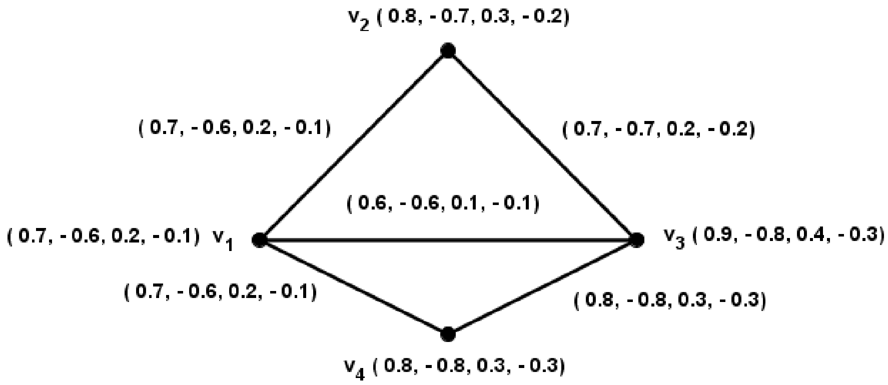

Figure 3: $G_{AQSP}(A, V)$ *- Corresponding to the parameter* e_2

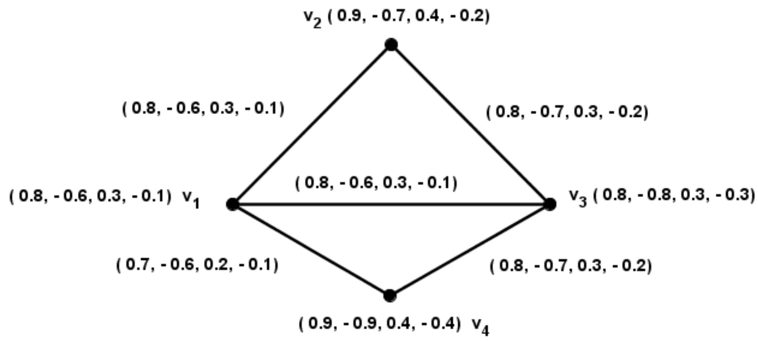

Figure 4: $G_{AQSP}(A, V)$ *- Corresponding to the parameter* e_3

3.3. Example of AQSP Fuzzy Soft Graph

Consider an AQSP fuzzy soft graph $G_{AQSP}(A, V)$, where $V = (v_1, v_2, v_3, v_4)$ and $E = (e_1, e_2, e_3)$. Here $G_{AQSP}(A, V)$ is described in Table.1. and $\mu_e(v_i, v_j) = 0$, $\forall (v_i, v_j) \in V \times V$ $\{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_1, v_4), (v_1, v_3)\}$ for all $e \in E$.

Table 1: *Tabular representation of AQSP Fuzzy Soft Graph parameter vertex set.*

(σ, ρ)	v_1	v_2	v_3	v_4
e_1	(0.6, - 0.7, 0.1,- 0.2)	(0.7, - 0.8, 0.2, -0.3)	(0.8, - 0.9, 0.3, - 0.4)	(0.6, - 0.7, 0.1,- 0.2)
e_2	(0.7, - 0.6, 0.2,- 0.1)	(0.8, - 0.7, 0.3, -0.2)	(0.9, - 0.8, 0.4, - 0.3)	(0.8, - 0.8, 0.3,- 0.3)
e_3	(0.8, - 0.6, 0.3,- 0.1)	(0.9, - 0.7, 0.4, -0.2)	(0.8, - 0.8, 0.3, - 0.3)	(0.9, - 0.9, 0.4,- 0.4)

Table 2: *Tabular representation of AQSP Fuzzy Soft Graph parameter edge set.*

(μ,γ)	v_1,v_2	v,v_3	v_3,v_4	v_4,v_1	v_1,v_3
e_1	(0.6, - 0.7, 0.1,- 0.2)	(0.7, - 0.8, 0.2, -0.3)	(0.6, -0.7, 0.1, - 0.2)	(0.6, - 0.7, 0.1,- 0.2)	(0.6, - 0.7, 0.1,- 0.2)
e_2	(0.7, - 0.6, 0.2,- 0.1)	(0.7, - 0.7, 0.2, -0.2)	(0.8, - 0.8, 0.3, - 0.3)	(0.7, - 0.6, 0.2,- 0.1)	(0.6, - 0.6, 0.1,- 0.1)
e_3	(0.8, - 0.6, 0.3,- 0.1)	(0.8, - 0.7, 0.3, -0.2)	(0.8, - 0.7, 0.3, - 0.2)	(0.7, - 0.6, 0.2,- 0.1)	(0.8, - 0.6, 0.3,- 0.1)

Table. 2. represents the AQSP fuzzy graph with parametric membership and non - membership with submerge values.

4. Results of Complete and μ - Complement of AQSP Fuzzy Soft Graph

4.1. Crisp graph of AQSP Fuzzy Soft Graph

Let $G_{AQSP}(A,V) = ((A_{AQSP}),\ (\sigma^P,\sigma^N,\rho^P,\rho^N)),\ ((A_{AQSP}),(\mu^P,\mu^N,\gamma^P,\gamma^N))$ be an AQSP fuzzy soft graph with underlying crisp graph is, $G^* = (\sigma^*,\mu^*)$, where
$\sigma^* = (v_i \in V : \sigma_e^P(v_i) > 0, \sigma_e^N(v_i) < 0, \rho_e^P(v_i) > 0, \rho_e^N(v_i) < 0)$
for some $e \in E$. $\mu^* = (v_i, v_j \in V \times V : \mu_e^P(v_i,v_j) > 0, \mu_e^N((v_i,v_j)) < 0,$
$\gamma_e^P((v_i,v_j)) > 0, \gamma_e^N((v_i,v_j)) < 0)$, $e \in E$.

4.2. Strong and Complete AQSP Fuzzy Soft Graph

Let $G_{AQSP}(A,V) = ((A_{AQSP},\ (\sigma^P,\sigma^N,\rho^P,\rho^N)),\ ((A_{AQSP},(\mu^P,\mu^N,\gamma^P,\gamma^N))$ is called as strong and complete AQSP fuzzy soft graph if,

(i) $\mu_e^P(x,y) = \sigma_e^P(x) \wedge \sigma_e^P(y)$, (ii) $\mu_e^N(x,y) = \sigma_e^N(x) \vee \sigma_e^N(y)$,
(iii) $\gamma_e^P(x,y) = \rho_e^P(x) \wedge \rho_e^P(y)$, (iv) $\gamma_e^N(x,y) = \rho_e^N(x) \vee \rho_e^N(y)$, for all $e \in A_{AQSP}$
and for all values of $x,y \in \mu^*$ is for strong AQSP fuzzy soft graph and for complete AQSP fuzzy soft graph is for all values of $x,y \in \sigma^*$.

4.3. Complement and μ- Complement of AQSP Fuzzy Soft Graph

Let $G_{AQSP}(A,V) = ((A_{AQSP}),\ (\sigma^P,\sigma^N,\rho^P,\rho^N)),\ ((A_{AQSP}),(\mu^P,\mu^N,\gamma^P,\gamma^N))$
be the AQSP fuzzy soft graph. The complement of AQSP fuzzy soft graph $G_{AQSP}(A,V)$ is defined as, $\overline{G}_{AQSP}(A,V) = ((A_{AQSP}),\ (\sigma^P,\sigma^N,\rho^P,\rho^N)),\ (A_{AQSP}),(\mu^P,\mu^N,\gamma^P,\gamma^N))$,
with the following sufficient conditions,
(i) $\overline{\mu}_e^P(x,y) = \sigma_e^P(x) \wedge \sigma_e^P(y) - \mu_e^P(x,y)$, (ii) $\overline{\mu}_e^N(x,y) = \sigma_e^N(x) \vee \sigma_e^N(y) \vee - \mu_e^N(x,y)$,
(iii) $\overline{\gamma}_e^P(x,y) = \rho_e^P(x) \wedge \rho_e^P(y) - \gamma_e^P(x,y)$, (iv) $\overline{\gamma}_e^N(x,y) = \rho_e^N(x) \vee \rho_e^N(y) - \gamma_e^N(x,y)$,
for all $e \in A_{AQSP}$ and for all values of $x,y \in V, e \in A_{AQSP}$.

4.4. Example of μ- Complement of AQSP Fuzzy Soft Graph

Consider an AQSP fuzzy soft graph $G_{AQSP}(A,V)$, where $V = (v_1,v_2,v_3,v_4)$ and

$E = (e_1,e_2,e_3)$. Here $G_{AQSP}(A,V)$ is described in Table.5. and $\mu_e(v_i,v_j) = 0$,
$\forall (v_i,v_j) \in V \times V\ \{(v_1,v_2),(v_2,v_3),(v_3,v_4),(v_2,v_4),(v_1,v_3)\}$ for all $e \in E$.

Table 3: *Tabular representation of AQSP Fuzzy Soft Graph parameter vertex set.*

(σ, ρ)	v_1	v_2	v_3	v_4
e_1	(0.7, - 0.8, 0.2,- 0.3)	(0.8, - 0.8, 0.3, -0.3)	(0.9, - 0.9, 0.4, - 0.4)	(0.9, - 0.6, 0.4,- 0.1)
e_2	(0.6, - 0.6, 0.1,- 0.1)	(0.7, - 0.7, 0.2, -0.2)	(0.8, - 0.8, 0.3, - 0.3)	(0.7, - 0.9, 0.2,- 0.4)
e_3	(0.8, - 0.8, 0.3,- 0.3)	(0.6, - 0.6, 0.1, -0.1)	(0.7, - 0.7, 0.2, - 0.2)	(0.9, - 0.9, 0.4,- 0.4)

Table 4: *Tabular representation of AQSP Fuzzy Soft Graph parameter edge set.*

(μ, γ)	v_1, v_2	v_2, v_3	v_3, v_4	v_4, v_2	v_1, v_3
e_1	(0.6, - 0.7, 0.1,- 0.2)	(0.8, - 0.7, 0.3, -0.2)	(0.8, -0.5, 0.3, 0.0)	(0.7, - 0.6, 0.2,- 0.1)	(0.7, - 0.7, 0.2,- 0.2)
e_2	(0.5, - 0.6, 0.0,- 0.1)	(0.6, - 0.6, 0.1, -0.1)	(0.6, - 0.7, 0.1, - 0.2)	(0.5, - 0.6, 0.0,- 0.1)	(0.5, - 0.5, 0.0, 0.0)
e_3	(0.6, - 0.6, 0.1,- 0.1)	(0.5, - 0.6, 0.0, -0.1)	(0.6, - 0.6, 0.1, - 0.1)	(0.5, - 0.5, 0.0, 0.0)	(0.6, - 0.6, 0.1,- 0.1)

Table. 5. represents the AQSP fuzzy graph with parametric membership and non - membership with submerge values.

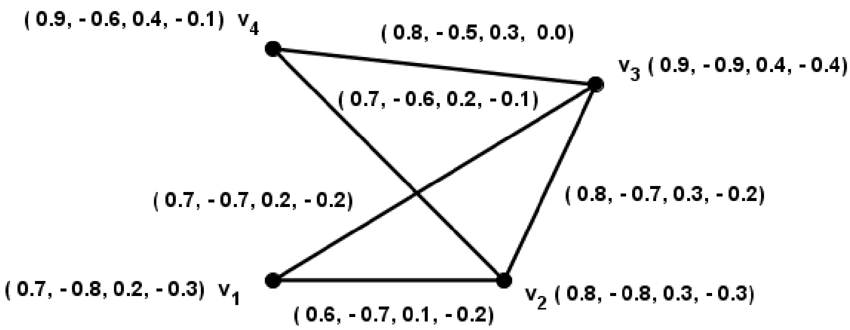

Figure 5: $G_{AQSP}(A, V)$ *- Corresponding to the parameter* e_1

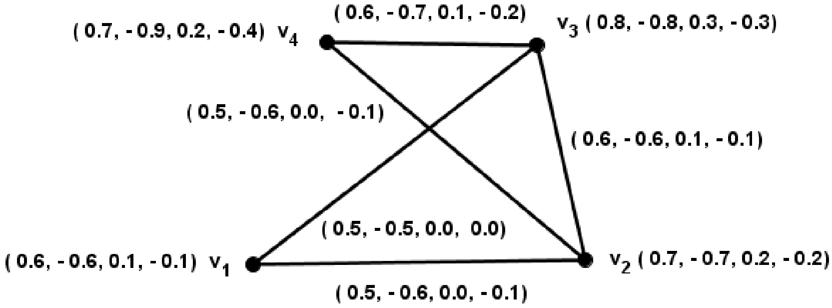

Figure 6: $G_{AQSP}(A, V)$ *- Corresponding to the parameter* e_2

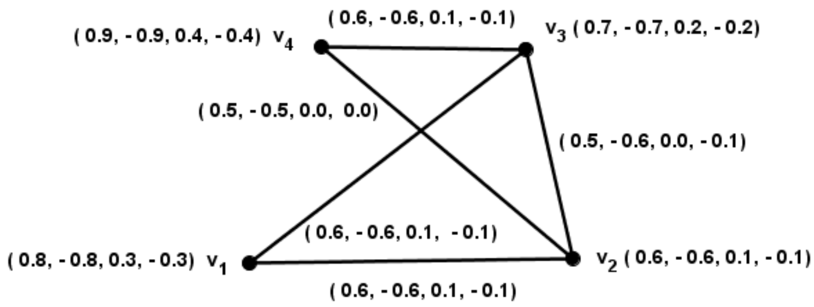

Figure 7: $G_{AQSP}(A,V)$ - *Corresponding to the parameter* e_3

4.5. Complement of AQSP Fuzzy Soft Graph

Consider an AQSP fuzzy soft graph $G_{AQSP}(A,V)$, where $V=(v_1,v_2,v_3,v_4)$ and $E=(e_1,e_2,e_3)$. Here $G_{AQSP}(A,V)$ is described in Figure 8,9,10 and we prove the Complement of AQSP fuzzy soft graph. $\mu_e(v_i,v_j)=0$, $\forall(v_i,v_j)\in V\times V\ \{(v_1,v_2),(v_2,v_3),(v_3,v_4),(v_2,v_4),(v_1,v_3)\}$ for all $e\in E$.

Theorem 1. Sum of the Size of $(\overline{G}_{AQSP})$ and the size of $(G_{AQSP}(A,V))$ is equal to the result to twice the sum of its minimum and maximum membership and non - membership submerging AQSP fuzzy soft graph values.Then we prove the following result such as,

(i) $S(\overline{G}_{AQSP})+S(G_{AQSP}(A,V))\leq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\sigma_e^P(x)\wedge\sigma_e^P(y))$,
(ii)$S(\overline{G}_{AQSP})+S(G_{AQSP}(A,V))\geq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\sigma_e^N(x)\vee\sigma_e^N(y))$,
(iii) $S(\overline{G}_{AQSP})+S(G_{AQSP}(A,V))\leq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\rho_e^P(x)\wedge\rho_e^P(y))$,
(iv)$S(\overline{G}_{AQSP})+S(G_{AQSP}(A,V))\geq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\rho_e^N(x)\vee\rho_e^N(y))$,
$\forall x,y\in V, e\in A_{AQSP}$.

Proof. The order of the complement of AQSP fuzzy soft graph of $S(\overline{G}_{AQSP})$ is equal to the order of the AQSP fuzzy soft graph $G_{AQSP}(A,V)$ is obvious.

$$\mu_e^P(x,y)\leq\sigma_e^P(x)\wedge\sigma_e^P(y)\ \forall x,y\in V,e\in A_{AQSP} \tag{1}$$

$$\overline{\mu}_e^P(x,y)=\sigma_e^P(x)\wedge\sigma_e^P(y)-\mu_e^P(x,y)\ \forall x,y\in V,e\in A_{AQSP}$$

$$\mu_e^P(x,y)\leq\sigma_e^P(x)\wedge\sigma_e^P(y)\ \forall x,y\in V,e\in A_{AQSP} \tag{2}$$

From (1) and (2), we get $\mu_e^P(x,y)\leq\sigma_e^P(x)\wedge\sigma_e^P(y)\ \forall x,y\in V,e\in A_{AQSP}$.

(i) $\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\mu_e^P(x,y)+\overline{\mu}_e^P(x,y))\leq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\sigma_e^P(x)\wedge\sigma_e^P(y))$
$\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\mu_e^P(x,y)+\sum_{e\in A_{AQSP}}\sum_{x\neq y}\overline{\mu}_e^P(x,y))\leq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\sigma_e^P(x)\wedge\sigma_e^P(y))$.

Hence, $S(\overline{G}_{AQSP})+S(G_{AQSP}(A,V))\leq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\sigma_e^P(x)\wedge\sigma_e^P(y))$,

$$\mu_e^N(x,y)\geq\sigma_e^N(x)\vee\sigma_e^N(y)\ \forall x,y\in V,e\in A_{AQSP} \tag{3}$$

$$\overline{\mu}_e^N(x,y)=\sigma_e^N(x)\vee\sigma_e^N(y)-\mu_e^N(x,y)\ \forall x,y\in V,e\in A_{AQSP}$$

$$\mu_e^P(x,y)\geq\sigma_e^N(x)\wedge\sigma_e^N(y)\ \forall x,y\in V,e\in A_{AQSP} \tag{4}$$

From (3) and (4), we get $\mu_e^N(x,y)\geq\sigma_e^N(x)\vee\sigma_e^P(y)\ \forall x,y\in V,e\in A_{AQSP}$.

(ii) $\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\mu_e^N(x,y)+\overline{\mu}_e^N(x,y))\geq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\sigma_e^(x)\wedge\sigma_e^N(y))$
$\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\mu_e^N(x,y)+\sum_{e\in A_{AQSP}}\sum_{x\neq y}\overline{\mu}_e^N(x,y))\geq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\sigma_e^N(x)\wedge\sigma_e^N(y))$.
Hence, $S(\overline{G}_{AQSP})+S(G_{AQSP}(A,V))\geq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\sigma_e^N(x)\vee\sigma_e^N(y))$,

Similarly we get the result for submerging membership and non - membership values.
(iii) $\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\gamma_e^P(x,y)+\overline{\gamma}_e^P(x,y))\leq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\rho_e^P(x)\wedge\rho_e^P(y))$
$\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\gamma_e^P(x,y)+\sum_{e\in A_{AQSP}}$
$\sum_{x\neq y}\overline{\gamma}_e^P(x,y))\leq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\rho_e^P(x)\wedge\rho_e^P(y))$.
Hence, $S(\overline{G}_{AQSP})+S(G_{AQSP}(A,V))\leq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\rho_e^P(x)\wedge\rho_e^P(y))$,

(iv) $\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\gamma_e^N(x,y)+\overline{\gamma}_e^N(x,y))\leq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\rho_e^N(x)\vee\rho_e^N(y))$
$\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\gamma_e^N(x,y)+\sum_{e\in A_{AQSP}}\sum_{x\neq y}\overline{\gamma}_e^N(x,y))\leq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\rho_e^N(x)\vee\rho_e^N(y))$.
Hence, $S(\overline{G}_{AQSP})+S(G_{AQSP}(A,V))\geq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\rho_e^N(x)\vee\rho_e^N(y))$,

4.6. Example of Complement of AQSP fuzzy graph

Consider the AQSP fuzzy soft graph $(G_{AQSP}(A,V))$ in Figure. 5,6,7. and its complement of AQSP fuzzy soft graph $S(\overline{G}_{AQSP})$ Figure. 8,9,10. we get,the order of the complement of AQSP fuzzy soft graph, $S(\overline{G}_{AQSP})$ is equal to the order of AQSP fuzzy soft graph $S(G_{AQSP}(A,V))$.i.e. $O(\overline{G}_{AQSP})=O(G_{AQSP}(A,V))=(9.1,-9.1,3.1,-3.1)$

And, $S(\overline{G}_{AQSP})=(0.4,-0.6,0.4,-0.4), S(G_{AQSP}(A,V))=(6.1,-8.6,1.6,-1.8)$
$(\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\sigma_e^P(x)\wedge\sigma_e^P(y)=2(10.2)),\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\sigma_e^N(x)\vee\sigma_e^N(y)=2(-9.2))$,
$\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\rho_e^P(x)\wedge\rho_e^P(y)=2(2.7)),\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\rho_e^N(x)\vee\rho_e^N(y)=2(2.7))$.

Then we have 2(10.2, - 9.2, 2.7, - 2.7) = (20.4, - 18.4, 5.4, - 5.4). Therefore,

(i) $S(\overline{G}_{AQSP})+S(G_{AQSP}(A,V))\leq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\sigma_e^P(x)\wedge\sigma_e^P(y))$,
(ii)$S(\overline{G}_{AQSP})+S(G_{AQSP}(A,V))\geq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\sigma_e^N(x)\vee\sigma_e^N(y))$,
(iii) $S(\overline{G}_{AQSP})+S(G_{AQSP}(A,V))\leq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\rho_e^P(x)\wedge\rho_e^P(y))$,
(iv)$S(\overline{G}_{AQSP})+S(G_{AQSP}(A,V))\geq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\rho_e^N(x)\vee\rho_e^N(y))\ \forall x,y\in V, e\in A_{AQSP}$.

4.7. Complement of AQSP fuzzy soft graph

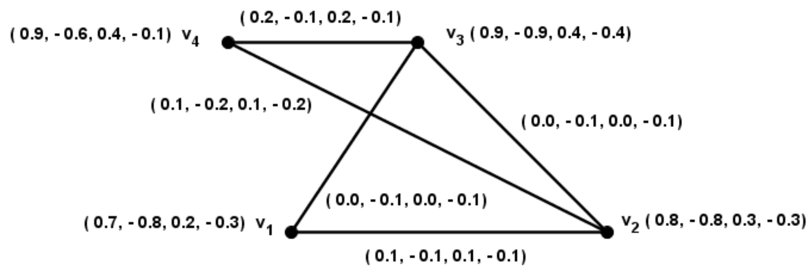

Figure 8: *Complement of $G_{AQSP}(A,V)$ - Corresponding to the parameter e_1*

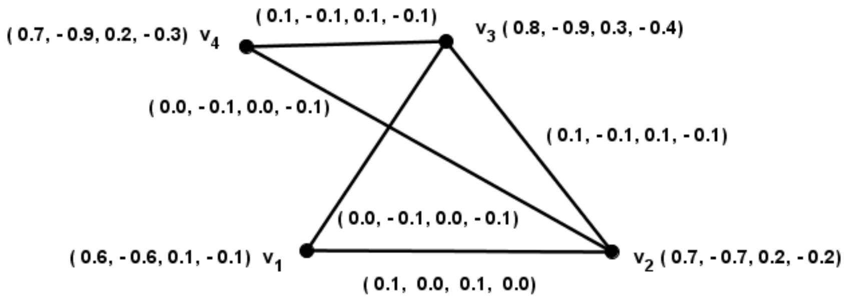

Figure 9: *Complement of $G_{AQSP}(A, V)$ - Corresponding to the parameter e_2*

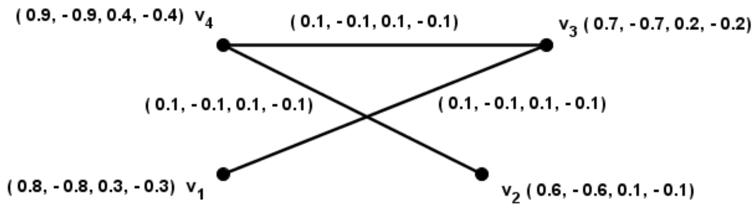

Figure 10: *Complement of $G_{AQSP}(A, V)$ - Corresponding to the parameter e_3*

4.8. Remark

The order of the complement of AQSP fuzzy soft graph $\overline{G}_{AQSP}(A, V)$ is equal to the order of the AQSP fuzzy soft graph $G_{AQSP}(A, V)$.

5. Properties of μ - Complement of AQSP Fuzzy Soft Graph

(i) The order of the complement AQSP fuzzy soft graph, $O(\overline{G}_{AQSP})$ is equal to the order of AQSP fuzzy soft graph $O(G_{AQSP}(A, V))$. And, $O(\overline{G^{\mu}}_{AQSP}) = O(G_{AQSP}(A, V))$ is presented in the Example.4.7. of AQSP fuzzy soft graph.

(ii) Vertex set of $(\overline{G^{\mu}}_{AQSP}) = (G_{AQSP}(A, V)$

(iii) The number of elements to the edge set of $\overline{G^{\mu}}$ is less than the number of elements in the node set of $(G_{AQSP}(A, V)$.

(iv) $\mu_e^{\mu^P}(x, y) > 0$ if $(x, y) \in \mu^*$, *otherwise* $\mu_e^{\mu^P}(x, y) = 0$

(v) $\mu_e^{\mu^N}(x, y) < 0$ if $(x, y) \in \mu^*$, *otherwise* $\mu_e^{\mu^N}(x, y) = 0$

(vi) $\mu_e^{\gamma^P}(x, y) > 0$ if $(x, y) \in \gamma^*$, *otherwise* $\gamma_e^{\gamma^P}(x, y) = 0$

(vii) $\mu_e^{\gamma^N}(x, y) < 0$ if $(x, y) \in \gamma^*$, *otherwise* $\gamma_e^{\gamma^N}(x, y) = 0$

For the size of $G_{AQSP}(A, V)$, μ complement AQSP fuzzy soft graph membership value is,
(viii) $S(G_{AQSP}(A, V) = \sum_{e \in A_{AQSP}} \sum_{x \neq y} (\mu_e \mu^P(x, y))$
= $\sum_{e \in A_{AQSP}} (\sum_{x,y \in \mu^*} (\sigma_e^P(x) \wedge \sigma_e^P(y) - \mu_e^P(x, y)\ \forall x, y \in V, e \in A_{AQSP}))$
= $\sum_{e \in A_{AQSP}} (\sum_{x,y \in \mu^*} (\sigma_e^P(x) \wedge \sigma_e^P(y) - \sum_{e \in A_{AQSP}} (\sum_{x,y \in \mu^*}\ x, y \in \mu_e^P(x, y)$

= $\sum_{e\in A_{AQSP}}(\sum_{x,y\in\mu^*}(\sigma_e^P(x)\wedge\sigma_e^P(y) - S(G_{AQSP}(A,V))$
i.e. $S(\overline{G}_{AQSP}) + S(G_{AQSP}(A,V)) \leq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\sigma_e^P(x)\wedge\sigma_e^P(y))$.

For the $G_{AQSP}(A,V)$, μ complement AQSP fuzzy soft graph non - membership value is,

(ix) $S(G_{AQSP}(A,V) = \sum_{e\in A_{AQSP}}\sum_{x\neq y}(\mu_e\mu^N(x,y))$
= $\sum_{e\in A_{AQSP}}(\sum_{x,y\in\mu^*}(\sigma_e^N(x)\vee\sigma_e^N(y) - \mu_e^N(x,y)\ \forall x,y\in V, e\in A_{AQSP}))$
= $\sum_{e\in A_{AQSP}}(\sum_{x,y\in\mu^*}(\sigma_e^N(x)\vee\sigma_e^N(y) - \sum_{e\in A_{AQSP}}(\sum_{x,y\in\mu^*} x,y\in\mu_e^N(x,y)$
= $\sum_{e\in A_{AQSP}}(\sum_{x,y\in\mu^*}(\sigma_e^N(x)\vee\sigma_e^N(y) - S(G_{AQSP}(A,V))$
i.e. $S(\overline{G}_{AQSP}) + S(G_{AQSP}(A,V)) \leq 2\sum_{e\in A_{AQSP}}\sum_{x\neq y}(\sigma_e^N(x)\vee\sigma_e^N(y))$

For the size of $G_{AQSP}(A,V)$, μ complement AQSP fuzzy soft graph membership value is,

(x) $S(G_{AQSP}(A,V) = \sum_{e\in A_{AQSP}}\sum_{x\neq y}(\gamma_e\gamma^P(x,y))$
= $\sum_{e\in A_{AQSP}}(\sum_{x,y\in\gamma^*}(\rho_e^P(x)\wedge\rho_e^P(y) - \gamma_e^P(x,y)\ \forall x,y\in V, e\in A_{AQSP}))$
= $\sum_{e\in A_{AQSP}}(\sum_{x,y\in\gamma^*}(\rho_e^P(x)\wedge\rho_e^P(y) - \sum_{e\in A_{AQSP}}(\sum_{x,y\in\mu^*}\ x,y\in\gamma_e^P(x,y)$
= $\sum_{e\in A_{AQSP}}(\sum_{x,y\in\gamma^*}(\rho_e^P(x)\wedge\rho_e^P(y) - S(G_{AQSP}(A,V))$.

For $G_{AQSP}(A,V)$, μ complement AQSP fuzzy soft graph submerging value is

(xi) $S(G_{AQSP}(A,V) = \sum_{e\in A_{AQSP}}\sum_{x\neq y}(\gamma_e\gamma^N(x,y))$
= $\sum_{e\in A_{AQSP}}(\sum_{x,y\in\gamma^*}(\rho_e^N(x)\vee\rho_e^N(y) - \gamma_e^N(x,y)\ \forall x,y\in V, e\in A_{AQSP}))$
= $\sum_{e\in A_{AQSP}}(\sum_{x,y\in\gamma^*}(\rho_e^N(x)\vee\rho_e^N(y) - \sum_{e\in A_{AQSP}}(\sum_{x,y\in\mu^*}\ x,y\in\gamma_e^N(x,y)$
= $\sum_{e\in A_{AQSP}}(\sum_{x,y\in\gamma^*}(\rho_e^N(x)\vee\rho_e^N(y) - S(G_{AQSP}(A,V))$

Theorem 2. The complement of a strong AQSP fuzzy soft graph $\overline{G}_{AQSP}(A,V)$ is also strong AQSP fuzzy soft graph $G_{AQSP}(A,V)$.

Proof. Let $G_{AQSP}(A,V)$ be an strong AQSP fuzzy soft graph by definition 4.3.of the complement of a strong AQSP fuzzy soft graph for the membership values,

$\overline{\mu}^P(x,y) = \sigma_e^P(x)\wedge\sigma_e^P(y) - \mu_e^P(x,y)\ \forall x,y\in V\times V, e\in A_{AQSP}$,
$= \sigma_e^P(x)\wedge\sigma_e^P(y) - (\sigma_e^P(x)\wedge\sigma_e^P(y)), \mu_e^P(x,y) > 0$,
$\sigma_e^P(x)\wedge\sigma_e^P(y),\qquad \mu_e^P(x,y) = 0$,
$= 0,\qquad \mu_e^P(x,y) > 0$,
$\sigma_e^P(x)\wedge\sigma_e^P(y),\qquad \mu_e^P(x,y) = 0$,
$= 0,\qquad \overline{\mu}_e^P(x,y) = 0$,
$\sigma_e^P(x)\wedge\sigma_e^P(y),\qquad \overline{\mu}_e^P(x,y) > 0$.
$\overline{\mu}^P(x,y) = \sigma_e^P(x)\wedge\sigma_e^P(y), \overline{\mu}_e^P(x,y) = 0\ \forall x,y\in V\times V$,
where (x,y) is the edge $\forall,\ (x,y)\in\overline{\mu}$.

The complement of a strong AQSP fuzzy soft graph for the non - membership values,
$\overline{\mu}^N(x,y) = \sigma_e^N(x)\vee\sigma_e^N(y) - \mu_e^N(x,y)\ \forall x,y\in V\times V, e\in A_{AQSP}$,
$= \sigma_e^N(x)\vee\sigma_e^N(y) - (\sigma_e^N(x)\vee\sigma_e^N(y)), \mu_e^N(x,y) < 0$,
$\sigma_e^N(x)\vee\sigma_e^N(y),\qquad \mu_e^N(x,y) = 0$,
$= 0,\qquad \mu_e^N(x,y) < 0$,
$\sigma_e^N(x)\vee\sigma_e^N(y),\qquad \mu_e^N(x,y) = 0$,
$= 0,\qquad \overline{\mu}_e^N(x,y) = 0$,
$\sigma_e^N(x)\vee\sigma_e^N(y),\qquad \overline{\mu}_e^N(x,y) < 0$.
$\overline{\mu}^N(x,y) = \sigma_e^N(x)\vee\sigma_e^N(y), \overline{\mu}_e^N(x,y) = 0\ \forall x,y\in V\times V$,
where (x,y) is the edge $\forall,\ (x,y)\in\overline{\mu}$.

Similarly the complement of the submerging AQSP fuzzy soft graph membership values are,

$$\begin{aligned}\overline{\gamma}^P(x,y) &= \rho_e^P(x) \wedge \rho_e^P(y) - \gamma_e^P(x,y)\ \forall x,y \in V \times V, e \in A_{AQSP},\\ &= \rho_e^P(x) \wedge \rho_e^P(y) - (\rho_e^P(x) \wedge \rho_e^P(y)), \gamma_e^P(x,y) > 0,\\ \overline{\gamma}^P(x,y) &= \rho_e^P(x) \wedge \rho_e^P(y), \overline{\gamma}_e^P(x,y) = 0\ \forall x,y \in V \times V,\end{aligned}$$

where (x,y) is the edge $\forall,\ (x,y) \in \overline{\gamma}$.

For the complement of the submerging AQSP fuzzy soft graph non - membership values are,

$$\begin{aligned}\overline{\gamma}^N(x,y) &= \rho_e^N(x) \wedge \rho_e^N(y) - \gamma_e^N(x,y)\ \forall x,y \in V \times V, e \in A_{AQSP}\ ,\\ &= \rho_e^N(x) \wedge \rho_e^N(y) - (\rho_e^N(x) \wedge \rho_e^N(y)), \gamma_e^N(x,y) > 0,\\ \overline{\gamma}^N(x,y) &= \rho_e^N(x) \wedge \rho_e^N(y), \overline{\gamma}_e^N(x,y) = 0\ \forall x,y \in V \times V,\end{aligned}$$

where (x,y) is the edge $\forall,\ (x,y) \in \overline{\gamma}$. Hence,the theorem is completed.

Theorem 3. The complement of a complete AQSP fuzzy soft graph $\overline{G}_{AQSP}(A,V)$ is also complete AQSP fuzzy soft graph $G_{AQSP}(A,V)$.

Proof. Let $G_{AQSP}(A,V) = ((A_{AQSP}),\ (\sigma^P,\sigma^N,\rho^P,\rho^N)),\ ((A_{AQSP}),(\mu^P,\mu^N,\gamma^P,\gamma^N))$ be the complete AQSP fuzzy soft graph by definition 4.3.of the complement of a complete AQSP fuzzy soft graph for the membership values,

$$\begin{aligned}\overline{\mu}^P(x,y) &= \sigma_e^P(x) \wedge \sigma_e^P(y) - \mu_e^P(x,y)\ \forall x,y \in V, e \in A_{AQSP}\ ,\\ &= \sigma_e^P(x) \wedge \sigma_e^P(y) - (\sigma_e^P(x) \wedge \sigma_e^P(y)), \mu_e^P(x,y) > 0,\\ &\sigma_e^P(x) \wedge \sigma_e^P(y), \qquad \mu_e^P(x,y) = 0,\\ &= 0, \qquad \mu_e^P(x,y) > 0,\\ &\sigma_e^P(x) \wedge \sigma_e^P(y), \qquad \mu_e^P(x,y) = 0,\\ &= 0, \qquad \overline{\mu}_e^P(x,y) = 0,\\ &\sigma_e^P(x) \wedge \sigma_e^P(y), \qquad \overline{\mu}_e^P(x,y) > 0.\\ \overline{\mu}^P(x,y) &= \sigma_e^P(x) \wedge \sigma_e^P(y), \overline{\mu}_e^P(x,y) = 0\ \forall x,y \in V, \forall\ (x,y) \in \overline{\sigma^*}.\end{aligned}$$

The complement of a Complete AQSP fuzzy soft graph for the non - membership values,

$$\begin{aligned}\overline{\mu}^N(x,y) &= \sigma_e^N(x) \vee \sigma_e^N(y) - \mu_e^N(x,y)\ \forall x,y \in V, e \in A_{AQSP}\ ,\\ &= \sigma_e^N(x) \vee \sigma_e^N(y) - (\sigma_e^N(x) \vee \sigma_e^N(y)), \mu_e^N(x,y) < 0,\\ &\sigma_e^N(x) \vee \sigma_e^N(y), \qquad \mu_e^N(x,y) = 0,\\ &= 0, \qquad \mu_e^N(x,y) < 0,\\ &\sigma_e^N(x) \vee \sigma_e^N(y), \qquad \mu_e^N(x,y) = 0,\\ &= 0, \qquad \overline{\mu}_e^N(x,y) = 0,\\ &\sigma_e^N(x) \vee \sigma_e^N(y), \qquad \overline{\mu}_e^N(x,y) < 0,\\ \overline{\mu}^N(x,y) &= \sigma_e^N(x) \vee \sigma_e^N(y), \overline{\mu}_e^N(x,y) = 0\ \forall x,y \in V, \forall\ (x,y) \in \overline{\sigma^*}.\end{aligned}$$

For the complement of the submerging Complete AQSP fuzzy soft graph membership values ,

$$\begin{aligned}\overline{\gamma}^P(x,y) &= \rho_e^P(x) \wedge \rho_e^P(y) - \gamma_e^P(x,y)\ \forall x,y \in V, e \in A_{AQSP}\ ,\\ &= \rho_e^P(x) \wedge \rho_e^P(y) - (\rho_e^P(x) \wedge \rho_e^P(y)), \gamma_e^P(x,y) > 0\\ \overline{\gamma}^P(x,y) &= \rho_e^P(x) \wedge \rho_e^P(y), \overline{\gamma}_e^P(x,y) = 0\ \forall x,y \in V, \forall\ (x,y) \in \overline{\sigma^*}.\end{aligned}$$

Similarly we get result for the complement of the submerging Complete AQSP fuzzy soft graph non - membership values ,$\overline{\gamma}^N(x,y) = \rho_e^N(x) \wedge \rho_e^N(y), \overline{\gamma}_e^N(x,y) = 0\ \forall x,y \in V$, where (x,y) denoted the vertices for all $(x,y) \in \overline{\rho^*}$. Hence the proof.

Theorem 4. Let the AQSP fuzzy soft graph be $G_{AQSP}(A,V)$. Then $G_{AQSP}(A,V)$ is an isolated AQSP fuzzy soft graph if and only if $\overline{G}_{AQSP}(A,V)$ is a complete AQSP fuzzy soft graph.

Proof. $G_{AQSP}(A,V) = ((A_{AQSP}),\ (\sigma^P,\sigma^N,\rho^P,\rho^N)),\ ((A_{AQSP}),(\mu^P,\mu^N,\gamma^P,\gamma^N))$ be the AQSP fuzzy soft graph. Then $G_{AQSP}(A,V)$.Let $\overline{G}_{AQSP}(A,V)$ be its complement of AQSP fuzzy soft graph. Then the given isolated AQSP fuzzy soft graph is $G_{AQSP}(A,V)$. Then,

$\mu_e^P(x,y) = 0, \forall (x,y) \in V \times V, e \in A_{AQSP}$.

Since,$\overline{\mu}^P(x,y) = \sigma_e^P(x) \wedge \sigma_e^P(y) - \mu_e^P(x,y)\ \forall x,y \in V \times V, e \in A_{AQSP}$,

$\overline{\mu}^P(x,y) = \sigma_e^P(x) \wedge \sigma_e^P(y), \forall\ x,y \in V \times V, e \in A_{AQSP}$,

Hence , $\overline{G}_{AQSP}(A,V)$ is complete AQSP fuzzy soft graph. Conversely, Given $\overline{G}_{AQSP}(A,V)$ to be a complete AQSP fuzzy soft graph. $\overline{\mu}^P(x,y) = \sigma_e^P(x) \wedge \sigma_e^P(y), \forall x,y \in V \times V, e \in A_{AQSP}$,

Since,$\mu^P(x,y) = \sigma_e^P(x) \wedge \sigma_e^P(y) - \overline{\mu}_e^P(x,y)\ \forall x,y \in V \times V, e \in A_{AQSP}$,

$\overline{\mu}^P(x,y) - \overline{\mu}^P(x,y), \forall\ x,y \in V \times V, e \in A_{AQSP}$,

$= 0$, $\forall\ x,y \in V \times V, e \in A_{AQSP}$,

$\mu^P(x,y) = 0, \forall\ x,y \in V \times V, e \in A_{AQSP}$.

Similarly, we get the result for non - membership values and submerge values such as,

$\overline{\mu}^N(x,y) = \sigma_e^N(x) \vee \sigma_e^N(y) - \mu_e^N(x,y)\ \forall x,y \in V \times V, e \in A_{AQSP}$,

$\overline{\mu}^N(x,y) = \sigma_e^N(x) \vee \sigma_e^N(y), \forall\ x,y \in V \times V, e \in A_{AQSP}$.

Hence $\overline{G}_{AQSP}(A,V)$ is complete AQSP fuzzy soft graph.

6. Application of AQSP Fuzzy Soft Graph

6.1. Analysis of AQSP fuzzy soft graph in Water - related diseases.

The analysis of the Water related diseases is done for different hostel students in the same locality. This kind of diseases occur by drinking polluted water . Children make up the majority of harmed diseases by contaminated water. This leads to a number of common ailments such as Diarrehea, Dysentery, Cholera, and Typhoid fever. We use the AQSP fuzzy soft graph module to find the most common diseases that the students are affected. And the corresponding parameteric symptoms of diesases is presented in AQSP fuzzy soft edges. The following descriptions will pave the way to find the cause of this sicknesess to precise the correct medicine .

6.2. Description of the Analysis

1. Let us consider the AQSP fuzzy soft sets such as, $((A_{AQSP}),\ (\sigma^P,\sigma^N,\rho^P,\rho^N)),\ ((A_{AQSP}),$ $(\mu^P,\mu^N,\gamma^P,\gamma^N))$. Which is the parametric set taken as the different symptoms of Water diseases .

2. Specify the vertex and edge sets of AQSP fuzzy soft graphs $G_{AQSP}(A,V)$, which corresponds to the symptoms of Water related diseases of the students in the hostel .

3. Measure the most common symptoms of this sickness by taking AQSP fuzzy soft graph membership and non - membership values with submerging level.

4. Calculate the score values of the $((A_{AQSP}),\ (\sigma^P,\sigma^N,\rho^P,\rho^N)),\ ((A_{AQSP}),(\mu^P,\mu^N,\gamma^P,\gamma^N))$ by using the score function, $\frac{1}{2}\ \ (\frac{1}{S_{AQSP}^P}\ \sum \theta_x^P\ -\ \frac{1}{S_{AQSP}^N}\ \sum \theta_x^N\ \ )$.

5. The maximum score membership value in AQSP fuzzy soft graph $G_{AQSP}(A,V)$, is the most common symptoms of Water related diseases.

6. Consider AQSP fuzzy soft vertex set, $v_1 =$ Typhoid fever, $v_2 =$ Diarrehea, $v_3 =$ Dysentery, and $v_4 =$ Cholera.

6.3. Discussion of the New AQSP fuzzy soft graph

We consider AQSP fuzzy soft graph corresponding to the parameter e_1 as,

v_1, v_2 = Fever, v_2, v_3 = Diarrehea, v_3, v_4 = Muscles aches, v_1, v_4 = Sweating, v_1, v_3 = Fatigue.

The AQSP fuzzy soft graph Corresponding to the parameter e_2, is

v_1, v_2 = Vomiting, v_2, v_3 = Muscles Cramps, v_3, v_4 = Nausea,
v_1, v_4 = Diarrehea, v_1, v_3 = Head ache.

The AQSP fuzzy soft graph Corresponding to the parameter e_3 is, v_1, v_2 = Cramps and Bloating, v_2, v_3 = Weightloss,

v_3, v_4 = Nausea, v_1, v_4 = Diarrehea, v_1, v_3 = Abdomend pain. The following Table.5. presents the membership and non - membership submerging values .

Table 5: *Tabular representation of AQSP Fuzzy Soft Graph parameter vertex set.*

(σ, ρ)	v_1	v_2	v_3	v_4
e_3	(0.8, - 0.8, 0.3,- 0.3)	(1.0, - 1.0, 0.5, -0.5)	(0.8, - 0.8, 0.3, - 0.3)	(0.9, - 0.9, 0.4,- 0.4)
e_1	(0.7, - 0.8, 0.2,- 0.3)	(0.8, - 0.8, 0.3, -0.3)	(0.9, - 0.9, 0.4, - 0.4)	(0.9, - 0.6, 0.4,- 0.1)
e_2	(0.6, - 0.6, 0.1,- 0.1)	(0.7, - 0.7, 0.2, -0.2)	(0.8, - 0.8, 0.3, - 0.3)	(0.7, - 0.9, 0.2,- 0.4)

Table 6: *Tabular representation of AQSP Fuzzy Soft Graph parameter edge set.*

(μ, γ)	v_1, v_2	v_2, v_3	v_3, v_4	v_4, v_1	v_1, v_3
e_1	(0.6, - 0.7, 0.1,- 0.2)	(0.8, - 0.7, 0.3, -0.2)	(0.8, -0.5, 0.3, 0.0)	(0.7, - 0.6, 0.2,- 0.1)	(0.7, - 0.7, 0.2,- 0.2)
e_2	(0.5, - 0.6, 0.0,- 0.1)	(0.6, - 0.6, 0.1, -0.1)	(0.6, - 0.7, 0.1, - 0.2)	(0.5, - 0.6, 0.0,- 0.1)	(0.5, - 0.5, 0.0, 0.0)
e_3	(0.6, - 0.6, 0.1,- 0.1)	(0.5, - 0.6, 0.0, -0.1)	(0.6, - 0.6, 0.1, - 0.1)	(0.5, - 0.5, 0.0, 0.0)	(0.6, - 0.6, 0.1,- 0.1)

Table 7: ***D****ifferent Hostel students affected by Water related diseases Score values.*

Hostel - 1	**Hostel - 2**	**Hostel -3**	**Hostel - 4**
($\sigma, \rho)v_1$ **)** Score	($\sigma, \rho)v_2$) Score	($\sigma, \rho)v_3$ Score	($\sigma, \rho)v_4$ Score
0.576	1.000	0.543	0.990
0.502	0.476	0.499	0.456
0.476	0.733	0.654	0.630

(i) The most affected common diseases from different water - related disease is

v_2=Diarrehea, which is the main symptoms of the students in different hostels in the same locality. The score value of the disease Diarrehea is, v_2 = 1.000 .

(ii) Corresponding to the parameter e_1 score value of (v_2, v_3) = 0.648.

Table 8: *Different Hostel students affected by Water related diseases Score values.*

Hostel - 1 ($\mu, \gamma)v_1, v_2$ Score	**Hostel - 2** ($\mu, \gamma)v_2, v_3$ Score	**Hostel -3** ($\mu, \gamma)v_3, v_4$ Score	**Hostel - 4** ($\mu, \gamma)v_1, v_4$ Score
0.489	0.648	0.509	0.634
0.478	0.500	0.487	0.466
0.485	0.646	0.500	0.633

Many students are affected by these sicknesess such as, v_1, v_2 = Fever, v_2, v_3 = Diarrehea, v_3, v_4 = Muscles aches, v_1, v_4 = Sweating, v_1, v_3 = Fatigue.

(iii)The least affected common diseases from different water - related disease is

v_4= Cholera, which is the main symptoms of the students in different hostels in the same locality. The score value of the disease Diarrehea is, v_4 = 0.456 .

(iv) Corresponding to the parameter e_1 score value of (v_1, v_4) = 0.466.

7. Conclusion

In this artifact , AQSP fuzzy soft graph definitions, complement of AQSP fuzzy soft graphs are introduced with theorems and examples. Some results about the strong AQSP fuzzy soft graph, complete AQSP fuzzy soft graph with μ - complement AQSP fuzzy soft graph and isolated AQSP fuzzy soft graph with complements is constructed.The analysis of water - related deseases result is the invention of AQSP fuzzy soft graph module.

Declarations

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