AN ATTRIBUTE CONTROL CHART FOR TIME TRUNCATED LIFE TESTS USING EXPONENTIATED INVERSE KUMARASWAMY DISTRIBUTION

B. Srinivasa Rao[1], M. Rami Reddy[2] and K. Rosaiah[3]

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[1]Department of Mathematics & Humanities, R.V.R& J.C College of Engineering, Chowdavaram, Guntur- 522 019, A.P, India
[2]Freshman Engineering Department, Lakireddy Bali Reddy College of Engineering, Mylavaram-521 230, A.P, India
[3]Department of Statistics, Acharya Nagarjuna University, Guntur, A.P, India[3]

[1]boyapatisrinu1@gmail.com, [2]ramireddy.munnangi@gmail.com, [3]rosaiah1959@gmail.com[3]

Abstract

In this article an attribute control chart is designed for the Exponentiated Inverse kumaraswamy distribution under a time truncated life test by assuming the life-time of the item follow the selected Exponentiated Inverse Kumaraswamy distribution with known parameters. In order to limit the cost of checking the quality of an item in any industrial process with time truncation, this process is much useful. By considering the average number of defective items from a specified lot that are failed before the time limit, the attribute control limits are constructed. The control chart is determined using Binomial distribution based on the Upper and Lower control limits. The functioning of the designed control chart is examined with the average run length (ARL) values. The control chart constants and limits are calculated for specific ARL values with assumed parameters at different sample sizes for an in-control process. These control chart constants are obtained by considering different combinations of parameters of the assumed distribution. With these in-control limits the ARL values are observed by shifting the parameter values. A simulation analysis is developed by taking a specific number of observations in each sample and the average number of failures from each sample is considered as a statistic to establish the execution of the control chart for a specified ARL at a particular shift in parameter. With that statistic of average number of failures from the samples the control chart is prepared. It is observed a specific change in defective number when there is shift in parameter values. The results are illustrated with an example.

Keywords: Exponentiated Inverse Kumaraswamy distribution; attribute control chart; time truncated life test; average run length; simulation.

1. Introduction

To examine the quality of an article in industrial production, control charts are much helpful. Simultaneously it is very important to maintain the standards or even to improve the quality of article to meet customer satisfaction levels. It requires a regular monitoring to assess the quality of the articles. To face the competition in the market it is very important to complete this screening process with less cost and within a shortest possible time. It is a common practice that the process is considered to be in control when the examined statistic values lies within the control limits known as Upper and Lower control limits (UCL and LCL) that are also believed as the extremities for specification of the product's quality. If the sample points exceeding the limits then the production procedure is treated as intemperate and these items are considered to be defective or imperfect. In terms of reducing the defective items or refining the quality of the items in a less span of time and with minimal cost, the control chart methods are much beneficial.

In general we have two kinds of control charts for variables and attributes. Variable control charts can be applied to any quality characteristic that is measurable. Whereas attribute control charts are useful in classifying defective and non-defective items in the production process.

There are several studies designed by various authors that the construction and implementation of different attribute and variable control charts. A few of them are Epprecht et al.[1] studied about the Adaptive control charts for attributes. Wu et al. [2] prepared an optimal np chart with curtailment. Ho and Quinino [3] discussed the monitoring process of variability through an attribute control chart. Wu and Wang [4] proposed np-control chart using double inspection. Further, some more attribute and variable controlcharts also established in Chiu and Kuo [5], A.D. Rodrigues et al. [6], Joekes and Barbosa [7], Arif et al. [8], and Shafqat et al. [9].

Generally most of the constructing processes of control charts are based on the supposition that the quality of items follow normality. While some circumstances where the its characteristic is unknown or doesn't follow the normal distribution. Various authors developed the procedure for the construction of control charts for non normal distributions, for example: Bai and choi [10], Chang and Bai, [11], Al-Oraini and Rahim [12], Aslam et al. [13] and Lin and Chou [14].

As it is essential for any industry to sustain in the competitive market by manufacturing more reliable products with less cost, with less manpower in a short period time it requires less time for inspection of defective products. To achieve this, it is necessary to have a time truncated life test based control chart. Hence, preparing a control chart for monitoring a non-normality characteristic product under the time truncated test is preferred to inspect the lifetime of the product.

Various authors proposed articles to expand the methodology of constructing control charts for various distributions under time truncated life test. A few references of such models are Aslam and Jun [15] developed a time truncated life test(TLT) based attribute control chart for Weibull distribution. Aslam et al. [16] designed a TLT based control chart for Pareto distribution of second kind. Similarly, Rao [17] proposed for exponentiated half-logistic distribution. Rosaiah et al. [18] considered for exponentiated Frechet distribution. Shruthi. G and O.S.Deepa [19] monitored ARL for Exponentiated distributions under TLT. Rao et al. [20] introduced TLT based chart for Dagum distribution. Adeoti and Ogundipe [21] developed for generalized exponential distribution under TLT. Rosaiah et al. [22] designed for type-II generalized log logistic distribution. Jafarian-Namin et al. [23] studied an efficient design of attribute control chart under TLT for weibull distribution. G. S .Rao and Al-Omari [24] designed for Length-Biased Weighted Lomax Distribution. Baklizi and Ghannam [25] proposed a TLT based attribute control chart for the inverse Weibull distribution.

Our interest is to develop an article of TLT based attribute control chart for monitoring the quality process when the lifetime of an article follow Exponentiated Inverse kumaraswamy distribution (EIKD). To monitor the functioning of a control chart ARL is general procedure which gives the average number of values that must be considered before an observation signals as out-of control. The ARL is determined as$ARL = \frac{1}{P}$; here P is the probability of any observation indicates out of control. The article is summarized in the following way: a concise introduction of the Exponentiated Inverse kumaraswamy distribution (EIKD) is provided in section-2, design and execution of control chart with calculations of ARL values when the parameter is shifted for EIKD is discussed with an application in Section-3. The process is evaluated with an analysis of simulation data in section-4. Few closing remarks are specified in Section-5.

2. Exponentiated Inverse Kumaraswamy distribution

The probability density function (pdf) of the Exponentiated inverse Kumaraswamy distribution (EIKD) is

$$f(x) = \alpha\beta\lambda(1+x)^{-(\alpha+1)}(1-(1+x)^{-\alpha})^{\beta\lambda-1} \;;0 < x < \infty,\ \ \alpha,\beta,\lambda > 0 \qquad (1)$$

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Its cumulative distribution function (cdf) is given by,

$$F(x) = [1-(1+x)^{-\alpha}]^{\beta\lambda}\,;\ 0<x<\infty,\ \ \alpha,\beta,\lambda>0 \tag{2}$$

Where α, β and λ are shape parameters

The mean of EIKD is $E(X) = \lambda\beta\, B\left(1-\frac{1}{\alpha}, \beta\lambda\right), \quad \alpha>1$

The Reliability function is

$$R(x) = 1-[1-(1+x)^{-\alpha}]^{\beta\lambda} \tag{3}$$

and the Hazard function is

$$H(x) = \frac{\alpha\lambda\beta(1+x)^{-(\alpha+1)}[1-(1+x)^{-\alpha}]^{\beta\lambda-1}}{1-[1-(1+x)^{-\alpha}]^{\lambda\beta}}\ ,0<x<\infty\ \ and\ \ \lambda,\alpha,\beta>0 \tag{4}$$

Graphs of the pdf, cdf, Reliability and hazard functions of EIKD for selected parameter values are plotted respectively.

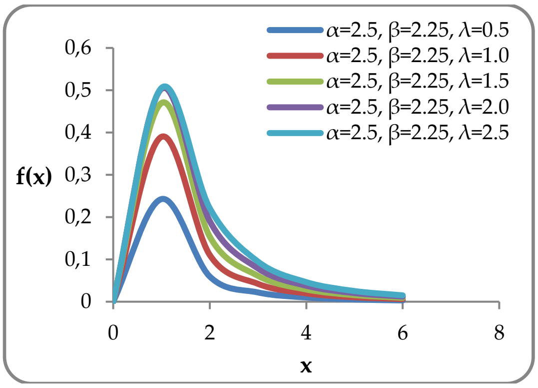

Figure 1: *The pdf plots of EIKD*

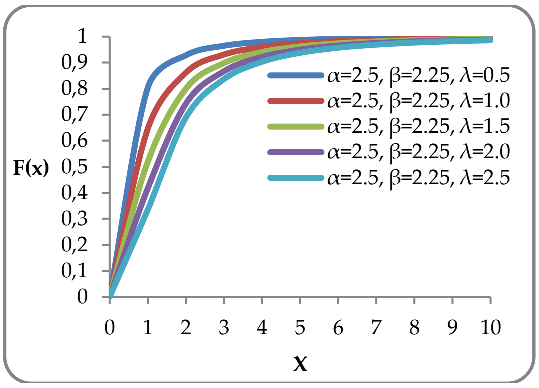

Figure 2: *The cdf plots of EIKD*

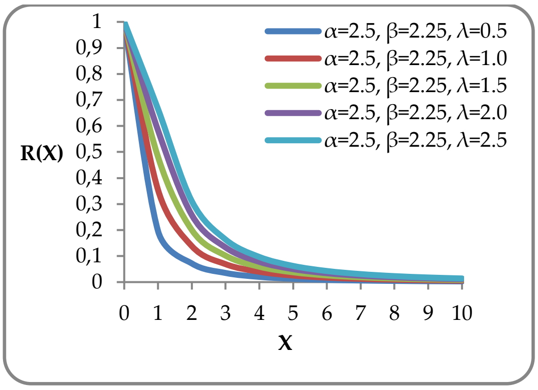

Figure 3: *Reliability function plots of EIKD*

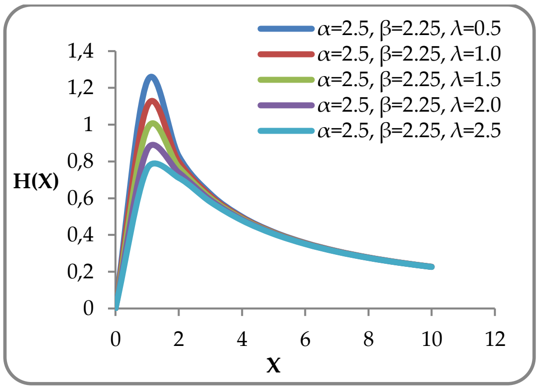

Figure 4: *Hazard function plots of EIKD*

3. Designing of the control chart

To construct the 'np' chart on the basis of defective articles in the production process the following methodology was implemented

Step1: A sample of 'n' articles is considered randomly from every subgroup lot and apply time truncated life test for these articles. Consider the number of articles (D) that are out of order (failure) within the termination time$t_0$stated as $t_0 = a\mu_0$, here $a$is constant related with target average life μ_0 when the process is supposed to be in-control.

Step2: Declare the process is under control if D rests in the limits of LCL and UCL, else, if D>UCL or D<LCL it is declared as not in control. Since D is a distinct count out of a sample of 'n' items, it can be considered as a "Binomial variate" as$n\ and\ p$ are parameters the control limits for in-control process are given as;

$$.UCL = np_0 + k\sqrt{np_0(1-p_0)} \tag{5a}$$

$$.LCL = MAX\left[0, np_0 - k\sqrt{np_0(1-p_0)}\right] \tag{5b}$$

Here'P_0'is the probability of an article is bungled earlier than$t_0$and it is determined from equation-(2) as"$p_0 = F(t_0)$"and k is the constant of the control chart. However, we state the process is in-control when$\mu = \mu_0$(or the parameters $\alpha = \alpha_0, \beta = \beta_0\ and\ \lambda = \lambda_0$). Then$p_0$ is obtained from equation (2) as

$$p_0 = F\left[t_0;\ \alpha_0, \beta_0, \lambda_0\right] = [1-(1+t_0)^{-\alpha_0}]^{\ \beta_0\lambda_0}$$

$$= \left[1-\left\{1+a\ \ \beta_0\lambda_0.B\left(1-\frac{1}{\alpha_0},\ \beta_0\lambda_0\right)\right\}^{-\alpha_0}\right]^{\ \beta_0\lambda_0} \tag{6}$$

In real time applications, the probability$p_0$is typically unknown; then the limits for such situations are

$$UCL = \bar{D} + k\sqrt{\bar{D}\left(1-\bar{D}/_n\right)} \tag{7a}$$

$$LCL = Max\left[0,\ \bar{D} - k\sqrt{\bar{D}\left(1-\bar{D}/_n\right)}\right] \tag{7b}$$

Here$\bar{D}$is mean count of failure articles in the samples.

The probability$P_{in}^0$of confirming in control process of the planned chart is specified as

$$P_{in}^0 = \sum_{d=LCL+1}^{UCL}\binom{n}{d}(P_0)^d\,(1-P_0)^{n-d} \tag{8}$$

$$P_{in}^0 = P[LCL \le D \le UCL/P_0]=$$

$$\sum_{d=LCL+1}^{UCL}\binom{n}{d}\left\{\left[1-\left[1+a\beta_0\lambda_0.B\left(1-\frac{1}{\alpha_0},\beta_0\lambda_0\right)\right]^{-\alpha_0}\right]^{\beta_0\lambda_0}\right\}^d\left\{\left[\left[1+a\beta_0\lambda_0.B\left(1-\frac{1}{\alpha_0},\beta_0\lambda_0\right)\right]^{-\alpha_0}\right]^{\beta_0\lambda_0}\right\}^{n-d} \tag{9}$$

The control charts efficacy can be examined by its "Average Run Length (ARL)", and is defined when the process is under control as

$$ARL_0 = \frac{1}{1-P_{in}^0} \tag{10}$$

3.1. ARLs with a shift in Parameter

To examine the performance of the control chart with a shift in one of the parameter (λ) as $\lambda_1 = c\lambda_0$, here c is specified as shift constant

The probability of an article is out of order prior to the experimental time$t_0$is consider asp_1, and is obtained as

$$p_1 = F\left[t_0;\ \alpha_0, \beta_0, \lambda_1\right] = [1-(1+t_0)^{-\alpha_0}]^{\ \beta_0\lambda_1}$$

$$= \left[1 - \left\{1 + a \;\; \beta_0 \lambda_0 . B\left(1 - \frac{1}{\alpha_0}, \;\; \beta_0 \lambda_0\right)\right\}^{-\alpha_0}\right]^{\beta_0 c \lambda_0} \tag{11}$$

The in-control probability of the process with the parameter shift as

$$P_{in}^1 = \sum_{d=LCL+1}^{UCL} \binom{n}{d} (P_1)^d (1 - P_1)^{n-d} \tag{12}$$

$P_{in}^1 = P[LCL \leq D \leq UCL/P_1]$=

$$\sum_{d=LCL+1}^{UCL} \binom{n}{d} \left\{\left[1 - \left[1 + a\beta_0\lambda_0 . B\left(1 - \frac{1}{\alpha_0}, \beta_0\lambda_0\right)\right]^{-\alpha_0}\right]^{\beta_0\lambda_1}\right\}^d \left\{\left[\left[1 + a\beta_0\lambda_0 . B\left(1 - \frac{1}{\alpha_0}, \beta_0\lambda_0\right)\right]^{-\alpha_0}\right]^{\beta_0\lambda_1}\right\}^{n-d} \tag{13}$$

The ARL for the process shift is given as

$$ARL_1 = \frac{1}{1 - P_{in}^1} \tag{14}$$

The approach for the calculations of intended chart is mentioned below

(1) Choose the ARL (sayr_0), parameter values (α_0, β_0 and λ_0), and the constant a.

(2) Control chart parameters to be determined for a specified sample size n, providedARL_0 which is specified in Eq-(10) very near to$r_0$that is$ARL_0 \geq r_0$.

(3) The parameters obtained in previous step are utilized to calculate ARL_1 as per the shift constant c using Eq-(14).

The control limits of the chart are determined for various parameter values and$r_0$values that are shown in the Tables 1 through 8.We have noticed a rapid reducing tend in ARL_1values with the decrement in shift value 'c'.

Table 1: $\boldsymbol{ARL_1}$ *Values for the designed chart with n=20;* $\boldsymbol{\lambda_0 = 1.5, \alpha_0 = 2.5}$ ***and*** $\boldsymbol{\beta_0 = 2.5}$

L C L	2	1	1	2
U C L	14	13	14	15
a	0.3224	0.2891	0.3246	0.3512
k	2.8561	2.9154	2.9628	2.9965
	ARLo=200	ARLo=250	ARLo=300	ARLo=370
C	ARL_1	ARL_1	ARL_1	ARL_1
1	200.088	250.258	300.372	370.897
0.9	118.620	120.348	119.549	178.751
0.8	49.315	46.023	46.680	71.215
0.7	20.178	18.085	19.162	28.765
0.6	8.803	7.716	8.452	12.287
0.5	4.212	3.674	4.090	5.664
0.4	2.273	2.014	2.231	2.888
0.3	1.434	1.319	1.420	1.686
0.2	1.095	1.058	1.091	1.178
0.1	1.005	1.002	1.004	1.014

Table 2: $\boldsymbol{ARL_1}$ *Values for the designed chart with n=20;* $\boldsymbol{\lambda_0 = 1.5, \alpha_0 = 2\ and\ \beta_0 = 2.5}$

LCL	5	1	0	1
UCL	17	13	12	14
a	0.4858	0.3015	0.2724	0.3328
k	2.8126	2.8682	2.9864	2.9962
	ARLo=200	ARLo=250	ARLo=300	ARLo=370
C	ARL_1	ARL_1	ARL_1	ARL_1
1	200.071	250.140	300.364	370.482
0.9	162.945	120.234	111.599	149.679
0.8	84.533	45.983	40.166	56.697
0.7	39.198	18.072	15.478	22.461
0.6	18.309	7.711	6.580	9.563
0.5	8.865	3.672	3.172	4.471
0.4	4.520	2.014	1.792	2.362
0.3	2.479	1.319	1.229	1.462
0.2	1.514	1.058	1.034	1.101
0.1	1.094	1.002	1.001	1.005

Table 3: $\boldsymbol{ARL_1}$ *Values for the designed chart with n=20;* $\boldsymbol{\lambda_0 = 2,\ \alpha_0 = 2\ and\ \beta_0 = 3}$

LCL	5	1	1	0
UCL	17	13	14	13
a	0.5421	0.351	0.3909	0.3498
k	2.6423	2.8645	2.9641	3.0125
	ARLo=200	ARLo=250	ARLo=300	ARLo=370
C	ARL_1	ARL_1	ARL_1	ARL_1
1	200.555	250.200	300.514	370.295
0.9	161.470	120.292	119.607	131.616
0.8	83.529	46.003	46.699	47.893
0.7	38.777	18.079	19.169	18.611
0.6	18.146	7.714	8.455	7.885
0.5	8.803	3.673	4.091	3.730
0.4	4.497	2.014	2.231	2.033
0.3	2.471	1.319	1.420	1.325
0.2	1.512	1.058	1.091	1.059
0.1	1.094	1.002	1.004	1.002

Table 4: $\boldsymbol{ARL_1}$ *Values for the designed chart with n=20;* $\boldsymbol{\lambda_0 = 2.5,\ \alpha_0 = 2.5\ and\ \beta_0 = 3}$

LCL	2	0	1	0
UCL	14	12	14	12
a	0.4198	0.3598	0.4221	0.3488
k	2.7562	2.8163	2.9564	2.9688
	ARLo=200	ARLo=250	ARLo=300	ARLo=370
C	ARL_1	ARL_1	ARL_1	ARL_1
1	200.081	250.221	300.745	370.192
0.9	118.599	92.574	119.700	141.897
0.8	49.306	34.284	46.731	49.380
0.7	20.175	13.640	19.179	18.264
0.6	8.802	5.984	8.458	7.455
0.5	4.211	2.972	4.092	3.456
0.4	2.273	1.726	2.231	1.885
0.3	1.434	1.210	1.420	1.256
0.2	1.095	1.031	1.091	1.039
0.1	1.005	1.001	1.004	1.001

Table 5: $\boldsymbol{ARL_1}$ *Values for the designed chart with n=30;* $\boldsymbol{\lambda_0 = 1.5,\ \alpha_0 = 2.5\ and\ \beta_0 = 2.25}$

LCL	9	3	6	8
UCL	24	18	22	24
a	0.4381	0.2897	0.2685	0.4246
k	2.7864	2.8636	2.9238	2.9817
	ARLo=200	ARLo=250	ARLo=300	ARLo=370
C	ARL_1	ARL_1	ARL_1	ARL_1
1	200.112	250.372	301.927	370.298
0.9	166.463	90.656	122.171	270.239
0.8	69.870	28.891	43.166	101.717
0.7	26.828	10.262	16.265	36.301
0.6	10.954	4.257	6.789	13.868
0.5	4.923	2.138	3.227	5.846
0.4	2.509	1.344	1.806	2.805
0.3	1.507	1.067	1.232	1.597
0.2	1.111	1.004	1.034	1.132
0.1	1.006	1.000	1.001	1.007

Table 6: $\boldsymbol{ARL_1}$ *Values for the designed chart with n=30;* $\boldsymbol{\lambda_0 = 1.5,\ \alpha_0 = 2\ and\ \beta_0 = 2.5}$

LCL	9	3	2	4
UCL	24	18	17	20
a	0.4657	0.3021	0.27939	0.3409
k	2.7951	2.8783	2.9438	2.9857
	ARLo=200	ARLo=250	ARLo=300	ARLo=370
C	ARL_1	ARL_1	ARL_1	ARL_1
1	200.157	250.489	300.090	370.029
0.9	166.398	90.720	96.226	136.724
0.8	69.836	28.908	29.187	43.416
0.7	26.817	10.267	10.039	15.051
0.6	10.951	4.258	4.087	5.950
0.5	4.922	2.138	2.045	2.774
0.4	2.509	1.344	1.301	1.584
0.3	1.507	1.067	1.053	1.141
0.2	1.111	1.004	1.003	1.014
0.1	1.006	1.000	1.000	1.000

Table 7: $\boldsymbol{ARL_1}$ *Values for the designed chart with n=30;* $\boldsymbol{\lambda_0 = 2,\ \alpha_0 = 2\ and\ \beta_0 = 3}$

LCL	9	2	1	5
UCL	24	17	16	21
a	0.5207	0.3327	0.31	0.415
k	2.7642	2.8645	2.9368	2.9864
	ARLo=200	ARLo=250	ARLo=300	ARLo=370
C	ARL_1	ARL_1	ARL_1	ARL_1
1	200.133	250.941	300.492	370.132
0.9	166.433	78.483	84.867	165.081
0.8	69.854	24.700	25.461	53.743
0.7	26.823	8.846	8.813	18.549
0.6	10.952	3.744	3.651	7.198
0.5	4.922	1.941	1.881	3.245
0.4	2.509	1.271	1.241	1.766
0.3	1.507	1.047	1.038	1.202
0.2	1.111	1.002	1.001	1.025
0.1	1.006	1.000	1.000	1.000

Table 8: ARL_1 *Values for the designed chart with n=30;* $\lambda_0 = 2.5,\ \alpha_0 = 2.5\ and\ \beta_0 = 3$

LCL	9	3	4	8
UCL	24	18	20	24
a	0.5421	0.3848	0.43039	0.5279
k	2.7123	2.8215	2.9452	2.9938
	ARLo=200	ARLo=250	ARLo=300	ARLo=370
C	ARL_1	ARL_1	ARL_1	ARL_1
1	200.041	250.287	300.702	370.128
0.9	166.565	90.609	103.844	270.558
0.8	69.926	28.878	34.402	101.855
0.7	26.846	10.259	12.545	36.343
0.6	10.960	4.256	5.211	13.880
0.5	4.925	2.137	2.545	5.849
0.4	2.510	1.344	1.513	2.806
0.3	1.507	1.067	1.122	1.598
0.2	1.111	1.004	1.012	1.132
0.1	1.006	1.000	1.000	1.007

3.2 Application of intended chart

To establish the applicability of the intended chart for the improvement of the quality of any manufactured article, we assume that the lifespan of the article follows the EIKD with parameters $\alpha_0 = 2.5, \beta_0 = 2.25\ and\ \lambda_0 = 1.5$. Consider the aimed mean life of the article is set as 1000 hrs with size of the sample n=20 of each group. If the target in control ARL value is fixed as $r_0 = 300$ for the control chart, as shown in Table 1, we obtain a= 0.3246, k=2.9628. From Equation-(6) we get $p_0 = 0.4221$. Using Equations-(5a & 5b) the LCL and UCL are determined as LCL =1 and UCL = 14. Then, the functioning of the prepared control chart is as follows:

Step 1: Take a sample of 20 articles from each subgroup and test their lifespan for period of 324.6 hours. Figure out the failure count of articles (D) through the test.

Step 2: We assert the process is in-control if 1 ≤ D ≤ 14, if not it is not in control.

4. Simulation study

A simulation study is presented to monitor the applicability of the designed control chart. It is executed using EIKD with specified parameters and the construction process is as follows:

The data is originated using EIKD with parameters $\alpha_0 = 2,\ \beta_0 = 2.5\ and\ \lambda_0 = 1.5$. A subgroup of 15 samples of size n=20 each are taken by considering the ARL as $r_0 = 300$. The process is affirmed as in control with these parameter values when $\mu_0 = 3.5483$. Another subgroup of 15 samples each of size n=20 are taken from EIKD with shift in parameter $\lambda_1 = c\lambda_0$ with shift value c =0.7. The control chart coefficient k=2.9864 is considered from Table 2, with ARL value as $r_0 = 300$ and n=20 when the process is in control.

The termination time of the life-test will be $t_0 = a\mu_0 = 0.2724 \times 3.5483 = 0.9665$. The number of articles that are failed before the termination time t_0 is considered as D which is calculated and presented in Table 9 for each sample. The number of average failures is $\bar{D} = 6$, the control limits are determined from equations (7a & 7b) are UCL=12 and LCL=0. The points of failure count (D) of each sample are exhibited in Figure 5. It is clearly observed that the planned chart indicated the shift at 18th sample (3rd sample after change in shift) while the corresponding

ARL value is 15. Thus the proposed chart effectively identifies the shift in this process.

5. Conclusion

In this article, we have designed a new ' np' control chart by assuming lifetime of the product follows EIKD to monitor the quality of manufactured articles under time truncation. The designed chart is then assessed by ARL's acquired from simulation study for different sample sizes; parameter values and objective in-control ARLs have been considered. The functioning of the intended chart is described with an explanatory example. For advance research, anyone can review applying the suggested control chart for any significant lifetime distribution. We have the feasibility to consider more quickened testing design to develop appropriate control charts for such circumstances.

Table 9: *The simulation Analysis*

	sample																				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	D
1	2.10	3.23	1.14	0.53	2.34	2.41	0.84	0.06	2.06	3.02	2.27	0.45	0.52	3.45	3.21	2.51	2.11	0.15	1.56	1.33	6
2	2.79	0.30	3.43	0.94	0.58	0.17	3.25	1.83	0.85	0.38	1.44	2.11	2.09	0.34	2.60	2.52	0.73	1.91	0.31	2.12	9
3	1.18	2.15	3.23	1.61	0.08	3.37	1.51	2.20	2.84	2.84	0.52	2.69	0.94	2.10	3.52	3.30	1.82	2.98	1.28	0.52	4
4	3.38	1.38	3.03	0.10	0.88	0.35	0.91	0.54	2.80	2.49	0.64	2.73	1.56	2.54	2.64	1.22	1.43	3.46	0.77	3.24	7
5	3.25	0.64	2.58	0.69	0.36	0.81	3.55	2.96	1.15	1.07	1.07	2.01	3.00	1.40	2.68	0.88	0.69	2.57	1.74	3.25	6
6	1.74	0.11	2.75	0.05	1.55	2.19	0.28	1.78	2.05	2.11	2.30	2.83	0.84	0.03	1.31	1.60	0.34	2.75	1.28	2.28	6
7	3.35	2.75	1.77	0.32	2.32	3.27	2.15	1.76	1.65	1.07	0.17	2.60	1.33	3.36	0.53	2.39	2.90	3.14	3.54	0.44	4
8	2.32	1.80	0.74	3.31	1.48	3.35	0.38	2.65	0.97	1.42	1.86	3.03	1.73	0.03	3.55	0.21	1.98	0.40	0.99	2.38	5
9	1.23	1.05	3.49	3.45	0.49	1.87	1.97	2.78	2.67	1.76	1.96	1.12	2.32	1.43	3.04	1.46	2.34	0.62	3.15	1.10	2
10	2.57	0.08	0.51	0.32	2.34	3.34	0.13	0.09	3.16	2.60	0.57	1.12	1.21	2.31	2.57	2.71	1.34	0.85	1.66	0.52	8
11	2.65	2.46	2.20	2.70	2.64	0.13	1.10	1.54	1.93	0.19	3.26	2.41	2.40	1.84	0.20	1.58	2.89	0.18	0.84	2.53	5
12	1.17	3.47	1.14	2.79	1.30	1.64	0.34	2.53	3.30	2.31	3.18	3.30	1.93	2.75	2.64	1.33	1.00	2.49	0.61	2.96	2
13	1.18	0.58	1.24	0.61	2.04	2.93	1.52	2.79	2.60	1.00	1.72	2.39	3.10	1.96	1.32	2.87	0.62	2.25	1.76	1.63	3
14	2.88	2.56	1.20	2.04	1.54	1.54	1.81	0.43	0.25	3.02	0.41	1.61	2.06	1.21	2.84	3.36	1.71	0.69	1.83	2.09	4
15	2.23	2.80	3.48	0.95	3.34	2.70	2.64	3.13	2.66	0.99	0.06	0.28	2.66	2.00	2.27	2.09	0.56	3.37	0.04	1.95	5
16	1.63	1.61	0.46	2.62	2.37	0.45	0.63	1.96	2.03	1.30	0.92	2.73	1.71	0.26	1.58	2.08	1.30	0.32	2.84	1.82	6
17	2.12	0.04	0.22	0.05	0.60	2.12	0.94	1.23	2.58	1.21	1.37	0.95	2.19	0.30	1.20	1.85	0.53	0.39	0.93	1.46	10
18	0.93	0.43	0.94	0.95	0.86	0.37	0.65	0.98	1.60	2.93	0.17	2.51	0.94	0.70	0.87	1.19	2.17	1.66	0.58	0.25	13
19	2.51	1.74	0.31	2.66	1.58	0.57	1.58	0.80	2.33	2.90	1.34	2.04	1.41	1.77	2.82	0.13	2.81	1.82	0.34	2.58	5
20	1.67	1.34	2.03	0.09	1.06	2.78	1.98	2.22	0.81	2.24	1.48	0.19	0.13	0.85	1.35	1.90	0.16	1.04	1.31	2.93	6
21	1.16	1.00	0.57	2.45	2.17	2.91	0.15	2.08	1.70	2.94	0.50	1.62	2.04	1.47	0.86	2.75	2.36	1.61	0.69	2.68	5
22	2.46	0.60	2.16	1.92	1.30	1.85	2.61	0.53	0.40	2.45	1.78	2.85	1.45	2.54	0.43	0.63	2.63	1.14	1.72	2.24	5
23	0.99	0.25	2.70	2.79	2.41	2.95	1.26	1.82	2.30	2.06	0.88	1.18	2.45	2.53	0.45	2.36	2.09	0.18	0.34	2.95	5
24	2.12	1.94	1.25	2.93	0.33	2.31	2.35	0.86	2.54	2.21	0.86	0.22	0.54	1.56	0.97	0.40	1.36	2.54	1.51	1.92	6
25	2.10	0.03	1.26	1.60	0.69	0.16	1.06	0.51	0.95	0.94	0.95	1.92	0.59	0.70	2.65	2.39	0.56	0.10	0.54	2.53	12
26	1.68	1.18	2.49	2.17	0.67	0.61	0.24	0.48	1.28	2.68	0.98	1.07	2.97	0.73	2.14	0.75	2.37	1.81	1.46	1.50	6
27	2.81	1.35	1.02	1.20	1.76	0.85	0.88	2.29	2.27	2.44	1.44	0.73	0.28	2.75	1.40	0.65	1.42	2.37	1.10	1.94	5
28	0.05	2.43	2.41	0.47	2.15	2.54	0.23	1.57	1.39	0.00	1.70	1.37	1.10	2.03	2.48	0.43	0.75	0.80	0.06	0.17	9
29	2.99	2.57	1.53	2.41	2.16	0.99	0.44	1.77	2.12	0.95	1.03	1.96	2.72	0.22	2.01	1.88	1.59	1.22	2.19	0.33	4
30	2.35	0.50	2.97	0.13	2.69	2.30	0.64	2.77	1.70	2.56	1.03	0.08	2.93	0.83	0.17	2.91	0.39	2.65	2.28	1.03	7

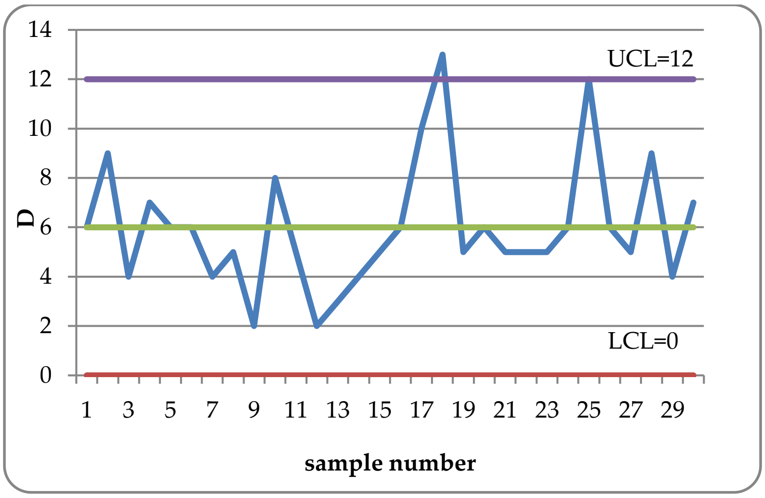

Figure 5: ***Control chart for simulation data.***

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