RELIABILITY TEST PLAN FOR THE POISSON-POWER LINDLEY DISTRIBUTION

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Abstract

In this article, we introduce a new member to Poisson-X family namely, the Poisson-power Lindley distribution. The statistical as well as the distributional properties of the new distribution are studied. The flexibility of the distribution is illustrated by means of real data sets. We also introduce a reliability test plan for acceptance or rejection of a lot of products submitted for inspection when lifetimes follow the new distribution. The minimum sample size using binomial and Poisson approximations, operating characteristic values and minimum ratios of the true value and the required value of the parameter with a given producer's risk are also developed with respect to the newly introduced sampling plans. A real data example is also given to illustrate the sampling plan developed.

Keywords: Poisson-X family, Poisson-power Lindley distribution, Reliability test plan, Producers risk, Operating characteristic function

1. Introduction

Recently researchers shows a special attention to Lindley distribution proposed by Lindley [16] by considering its importance in modelling complex lifetime data. Also, it is one of the well known distribution to analyze failure time data with different shapes of hazard rates. The probability density function and the cumulative distribution function of Lindley distribution are respectively given by,

$$g(t) = \frac{\beta^2}{\beta+1}(1+t)e^{-\beta t}, x > 0, \beta > 0 \tag{1}$$

and

$$G(t) = 1 - \frac{\beta+1+\beta t}{\beta+1}e^{-\beta t} x > 0, \beta > 0. \tag{2}$$

Ghitany et al. [14] studied the Lindley distribution and its applications in the contest of reliability studies and shows that its mathematical properties are more flexible than those of the exponential distribution. Ghitany et al. [13] introduced power Lindley distribution, a new generalization of Lindley distribution by considering the power transformation $X = T^{\frac{1}{\alpha}}$ in Lindley distribution. The pdf and cdf of power Lindley distribution are respectively

$$g(x) = \frac{\alpha\beta^2}{\beta+1}(1+x^{\alpha})x^{\alpha-1}e^{-\beta x^{\alpha}} \quad x > 0, \alpha \quad \beta > 0 \tag{3}$$

and

$$G(x) = 1 - (1 + \frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}} \quad x > 0, \alpha, \beta > 0. \tag{4}$$

Here, we introduce a new distribution based on power Lindley distribution called Poisson-power Lindley, a new member of Poisson-X family. Tahir et al. [21] developed Poisson-X family from T-X family of distributions introduced by Alzaatreh et al.[4] with the cumulative distribution function

$$F(x) = \int_{a}^{W(G(x))} r(t)dt \tag{5}$$

where W(G(x)) satisfies the conditions

- W(G(x)) $\in$ [a,b],
- W(G(x)) is differentiable and monotonically non decreasing and
- W(G(x)) $\to a$ as x $\to -\infty$ and $W(G(x)) \to b$ as x $\to \infty$.

Here r(t) is the pdf of the random variable T$\in$ [a,b] for -∞ < a < b < ∞.
For a Poisson-X family, the cdf and pdf of T are given respectively by Tahir et al. [21] as

$$R(t) = \frac{1 - e^{-\delta t^m}}{1 - e^{-\delta}} \tag{6}$$

and

$$r(t) = \frac{\delta m}{1 - e^{-\delta}} t^{m-1} e^{-\delta t^m} - \qquad 0 \leq t \leq 1. \tag{7}$$

Let G(x) be the baseline cdf. By substituting G(x) for the upper limit W(G(x)) and r(t) as (7) with δ=1 , we get the cdf and pdf of Poisson-X family respectively from (5) as

$$F(x; c; \xi) = \left(1 - e^{-1}\right)^{-1} \left[1 - e^{-[G(x)]^m}\right] \tag{8}$$

and

$$f(x; c; \xi) = \frac{m}{1 - e^{-1}} g(x; \xi) \left[G(x; \xi)\right]^{m-1} e^{-[G(x;\xi)]^m}. \tag{9}$$

Here after a random variable X with cdf (9) is denoted by $X \sim PX(m; \xi)$.

Lemma 1. If X have a density of the form (9), then the random variable T=G(x) has a pdf of the form (7) with δ=1. The converse is also true, that is, if T has density of the form (7) with δ=1 then the random variable $X = G^{-1}(T)$ has Poisson-X distribution with density (9).

The rest of the paper is unfolded as follows. In section 2, we introduce Poisson-power Lindley (PPL) distribution and study its statistical properties. We explore the feasibility of the new model by means of real data sets in section 3. In section 4, we developed a reliability test plan and the operating characteristic values for the PPL distribution. We illustrate the results with a real data example. Finally, we conclude the work by section 5.

2. Poisson-power Lindley (PPL) Distribution

For a Poisson-X family of distribution, if the parent distribution is power Lindley, we obtain Poisson-power Lindley distribution having cdf and pdf as

$$F(x) = \left(1 - e^{-1}\right)^{-1} \left[1 - e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^m}\right] \quad x > 0\ \alpha,\ \beta,\ m > 0 \tag{10}$$

and

$$\begin{aligned} f(x) \quad &= \quad \frac{m}{(1 - e^{-1})} \left[\frac{\alpha\beta^2}{\beta+1}(1 + x^{\alpha})x^{\alpha-1}e^{-\beta x^{\alpha}}\right] \\ &\left[e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^m}\right] \left[1 - (1 + \frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{m-1} \quad x > 0\ \alpha,\ \beta, m > 0. \end{aligned} \tag{11}$$

The PDF graphs of the PPL distribution for various values of the parameters are given in Figure 1

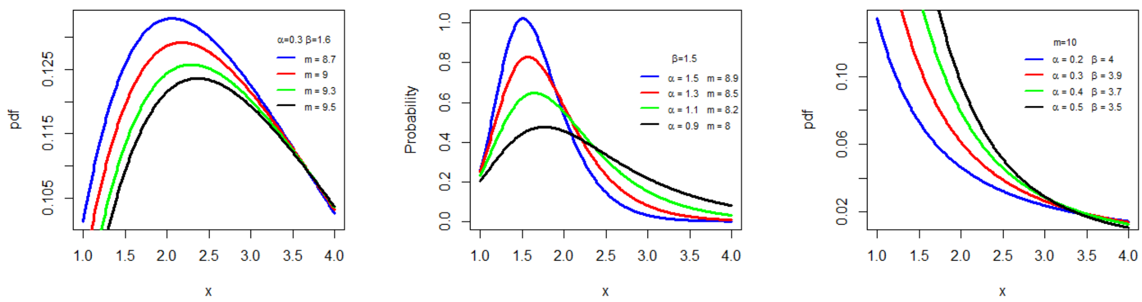

Figure 1: *pdf graphs of Poisson-power Lindley distribution for various values of parameters*

2.1. Properties of Poisson-power Lindley Distribution

The survival function (sf), hazard rate function (hrf), cumulative hazard rate function (chrf), Reversed hazard rate function of Poisson-power Lindley distribution are respectively given by

$$S(x) = 1 - \left(1 - e^{-1}\right)^{-1} \left[1 - e^{-[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}]^m}\right], \tag{12}$$

$$h(x) = \frac{m\frac{\alpha\beta^2}{\beta+1}(1+x^{\alpha})x^{\alpha-1}e^{-\beta x^{\alpha}}\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{m-1}}{1-e^{-(1-[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}]^m)}}, \tag{13}$$

$$H(x) = -log\left[\frac{e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^m} - e^{-1}}{1-e^{-1}}\right] \tag{14}$$

and

$$q(x) = \frac{m\frac{\alpha\beta^2}{\beta+1}(1+x^{\alpha})x^{\alpha-1}e^{-\beta x^{\alpha}}\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{m-1}}{1-e^{-[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}]^m)}}. \tag{15}$$

The residual life time of the PPL distribution at time t and the corresponding survival function are respectively given by

$$r_{x_t}(x) = \frac{m\frac{\alpha\beta^2}{\beta+1}(1+x^{\alpha})x^{\alpha-1}e^{-\beta x^{\alpha}}\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{m-1}}{1-e^{-(1-[1-(1+\frac{\beta}{\beta+1}t^{\alpha})e^{-\beta t^{\alpha}}]^m)}} \tag{16}$$

and

$$R_{x_t}(x) = \frac{e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^m} - e^{-1}}{e^{-\left[1-(1+\frac{\beta}{\beta+1}t^{\alpha})e^{-\beta t^{\alpha}}\right]^m} - e^{-1}}. \tag{17}$$

The past life time and corresponding distribution function of PPL distribution are

$$d_{t_x}(x) = \frac{m\frac{\alpha\beta^2}{\beta+1}(1+x^{\alpha})x^{\alpha-1}e^{-\beta x^{\alpha}}\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{m-1}}{1-e^{-[1-(1+\frac{\beta}{\beta+1}t^{\alpha})e^{-\beta t^{\alpha}}]^m)}} \tag{18}$$

and

$$D_{t_x}(x) = \frac{1-e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^m}}{1-e^{-\left[1-(1+\frac{\beta}{\beta+1}t^{\alpha})e^{-\beta t^{\alpha}}\right]^m}}. \tag{19}$$

The hrf graphs of the PPL distribution for different values of the parameters are given in Figure 2

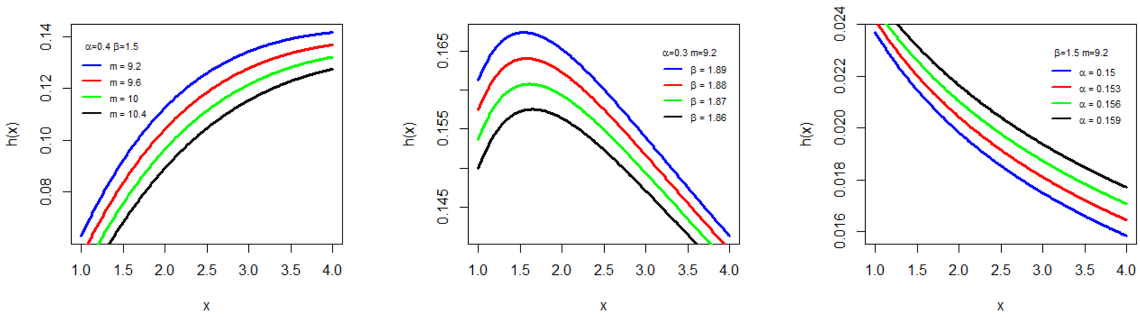

Figure 2: *hrf graphs of Poisson-power Lindley distribution for different values of the parameters*

For fixed values of α and β with different m values, we can see an increasing failure rate here in the hrf graph. Again, for a fixed values of β and m with different α values, we can see a decreasing failure rate. A reverse J-shaped curve can also be seen for fixed values of α and m with different β values.

2.2. Linear Representation

Here we derive some useful expansions for (10) and (11) using the concept of exponentiated distributions. A random variable with an arbitrary baseline cdf G(x) is said to have the exp-G distribution with parameter $\gamma > 0$, if its pdf and cdf are $l_\gamma = \gamma G^{\gamma-1}(x)g(x)$ and $L_\gamma(x) = G^\gamma(x)$, respectively. The exponential function of (10) can be expanded as

$$F(x;c,a,b) = \sum_{i=0}^{\infty} w_{i+1}\left(1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right)^{(i+1)^m} \tag{20}$$

where $w_{i+1} = \frac{((-1)^i)}{[(i+1)!(1-e^{-1})]}$ (for i$\geq$0) thereby the pdf is

$$\begin{aligned} f(x;c,a,b) \quad &= \quad \sum_{i=0}^{\infty} w_{i+1}\left((i+1)m\frac{\alpha\beta^2}{\beta+1}(1+x^\alpha)x^{\alpha-1}e^{-\beta x^\alpha}\right) \\ &\quad \left[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right]^{(i+1)m-1}. \end{aligned} \tag{21}$$

This can also be expressed as

$$f(x;c,a,b) = \sum_{i=0}^{\infty} w_{i+1} l_{(i+1)m}(x) \tag{22}$$

where $l_{(i+1)m}(x)$ is the density function of the exp-power Lindley distribution with power parameter $(i+1)m$.
(22) reveals that the Poisson-power Lindley density can be expressed as a linear representation of exp-power Lindley density.

Lemma 2. If $Y \sim$ power Lindley(α,β), then

$$X = \left[-1-\frac{1}{\beta}-\frac{1}{\beta}W_{-1}\left(\frac{\beta+1}{e^{\beta+1}}\left[1-\left\{-log\left(1-Y\left(1-e^{-1}\right)\right)\right\}^{\frac{1}{m}}\right]\right)\right]^{\frac{1}{\alpha}}$$

follows Poisson-power Lindley (m,α,β). Where $W_{-1}(.)$denotes the negative branch of the Lambert W function.

Proof. Consider the cdf of power Lindley(α,β),

$$F(x) = 1 - (1 + \frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}.$$

If T have the form (7) with $\delta = 1$, then from Lemma 1 we have,

$$\begin{aligned} X &= G^{-1}(T) \\ &= \left[-1 - \frac{1}{\beta} - \frac{1}{\beta}W_{-1}\left[\frac{\beta+1}{e^{\beta+1}}(T-1)\right]\right]^{\frac{1}{\alpha}} \end{aligned} \tag{23}$$

follows Poisson-power Lindley (m,α, β). Here for simulating T, we use R(T) given in (6) with $\delta = 1$, (by inverse probability integral transformation).
We have,

$$T = \left\{-log\left[1 - Y\left(1 - e^{-1}\right)\right]\right\}^{\frac{1}{m}}.$$

and by substituting the value of T in (23), we get

$$X = \left[-1 - \frac{1}{\beta} - \frac{1}{\beta}W_{-1}\left(-\frac{\beta+1}{e^{\beta+1}}\left[1 - \left\{-log\left(1 - Y\left(1 - e^{-1}\right)\right)\right\}^{\frac{1}{m}}\right]\right)\right]^{\frac{1}{\alpha}} \tag{24}$$

which follows Poisson-power Lindley(m,α,β). ∎

2.3. Quantile function and Median

The quantile function of Poisson-power Lindley distribution is given by

$$X = \left[-1 - \frac{1}{\beta} - \frac{1}{\beta}W_{-1}\left(-\frac{\beta+1}{e^{\beta+1}}\left[1 - \left\{-log\left(1 - u\left(1 - e^{-1}\right)\right)\right\}^{\frac{1}{m}}\right]\right)\right]^{\frac{1}{\alpha}} \tag{25}$$

Hence the median is,

$$X = \left[-1 - \frac{1}{\beta} - \frac{1}{\beta}W_{-1}\left(-\frac{\beta+1}{e^{\beta+1}}\left[1 - \{0.165\}^{\frac{1}{m}}\right]\right)\right]^{\frac{1}{\alpha}} \tag{26}$$

2.4. Moments

Lemma 3. If X is a random variable from a Poisson-power Lindley (m,α, β) distribution, then the k^{th} moment of X is

$$\mu'_k = E(X^k) = E_t\left[\left(-1 - \frac{1}{\beta} - \frac{1}{\beta}W_{-1}\left(-\frac{\beta+1}{e^{\beta+1}}(1-t)\right)\right)^{\frac{k}{\alpha}}\right]$$

where T follows distribution of the form (6) with $\delta = 1$

Proof.

$$\begin{aligned} E(X^k) &= \int_0^{\infty} \frac{c}{1-e^{-1}}\left[\frac{\alpha\beta^2}{\beta+1}(1+x^{\alpha})x^{\alpha-1}e^{-\beta x^{\alpha}}\right] \\ & \left[1 - (1 + \frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{m-1} e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{m}} \end{aligned}$$

By using Lemma 1). and the transformation

$$\begin{aligned} T &= G(x) \\ &= 1 - (1 + \frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}} \end{aligned}$$

We have

$$
\begin{aligned}
E(X^k) &= \int_0^\infty \left(-1-\frac{1}{\beta}-\frac{1}{\beta}W_{-1}\left(-\frac{\beta+1}{e^{\beta+1}}(1-t)\right)\right)^{\frac{k}{\alpha}} \frac{m}{1-e^{-1}} t^{m-1} e^{-t^m} dt \\
&= E_t\left[\left(-1-\frac{1}{\beta}-\frac{1}{\beta}W_{-1}\left(-\frac{\beta+1}{e^{\beta+1}}(1-t)\right)\right)^{\frac{k}{\alpha}}\right]
\end{aligned}
$$

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2.5. Order Statistics

Suppose $X_1, ...X_n$ is a random sample from the Poisson-power Lindley distribution. Let $X_{(r)}$ denote the rth order statistics. The pdf of $X_{(r)}$ of PPL distribution can be expressed as

$$
\begin{aligned}
f_r(x) \quad = \quad & \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{m\alpha\beta^2}{\beta+1}(1+x^\alpha)x^{\alpha-1}e^{-\beta x^\alpha}\right]\left[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right]^{m-1} \\
& e^{-\left[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right]^m}\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}(1-e^{-1})^{-(r+j)} \\
& \left[1-e^{-\left[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right]^m}\right]^{r+j-1}.
\end{aligned} \tag{27}
$$

Now the cdf, $F_n(x)$ of the largest order statistics $X_{(n)}$ is given by,

$$
F_n(x) \quad = \quad \left(1-e^{-1}\right)^{-n}\left[1-e^{-\left[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right]^m}\right]^n \tag{28}
$$

and the corresponding pdf is

$$
\begin{aligned}
f_n(x) \quad = \quad & nm\left(1-e^{-1}\right)^{-n}\left[1-e^{-\left[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right]^m}\right]^{n-1} \\
& \left[\frac{\alpha\beta^2}{\beta+1}(1+x^\alpha)x^{\alpha-1}e^{-\beta x^\alpha}\right]\left[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right]^{m-1} \\
& e^{-\left[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right]^m}.
\end{aligned} \tag{29}
$$

Again, the cdf $F_1(x)$ and pdf $f_1(x)$ of the smallest order statistic $X_{(1)}$ is given by,

$$
F_1(x) \quad = \quad 1-\left[1-\left(1-e^{-1}\right)^{-1}\left[1-e^{-\left[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right]^m}\right]\right]^n \tag{30}
$$

and the corresponding pdf is

$$
\begin{aligned}
f_1(x) \quad = \quad & \frac{nm}{(1-e^{-1})}\left[1-\left(1-e^{-1}\right)^{-1}\left[1-e^{-[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}]^m}\right]\right]^{n-1} \\
& \left[\frac{\alpha\beta^2}{\beta+1}(1+x^\alpha)x^{\alpha-1}e^{-\beta x^\alpha}\right]\left[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right]^{m-1} \\
& e^{-\left[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right]^m}.
\end{aligned} \tag{31}
$$

2.6. Parameter Estimation

Here, we use maximum likelihood method of estimation. Let $X_1, X_2...X_n$ be independent and identically distributed Poisson-power Lindley random variables. The log likelihood function is,

$$\begin{aligned} L(\alpha,\beta,m;x) &= nlog\left(\frac{m}{1-e^{-1}}\right) + nlog\left(\frac{\alpha\beta^2}{\beta+1}\right) + \sum_{i=1}^{n} log(1+x_i^{\alpha}) + (\alpha-1) \\ &\sum_{i=1}^{n} log(x_i) - \beta\sum_{i=1}^{n} x_i^{\alpha} - \sum_{i=1}^{n}\left[1-\left(1+\frac{\beta x_i^{\alpha}}{\beta+1}\right)e^{-\beta x_i^{\alpha}}\right] + (m-1) \\ &\sum_{i=1}^{n} log\left[1-\left(1+\frac{\beta x_i^{\alpha}}{\beta+1}\right)e^{-\beta x_i^{\alpha}}\right]. \end{aligned} \tag{32}$$

The computations were implemented using the nlm function of R software.

3. APPLICATIONS

In this section, we illustrate the flexibility of the Poisson-power Lidley distribution using two real data sets. The model parameters are estimated by the method of maximum likelihood using the nlm function of R software. The K-S statistics and the corresponding p values were also calculated for establishing its goodness of fit.

3.1. Dataset 1 Repair times data

We have considered a data set which represents the maintenance data with 46 observations reported on active repair times (hours) for an airborne communication transceiver given by [17]. The data set is
0.2, 0.3, 0.5, 0.5, 0.5, 0.5, 0.6, 0.6, 0.7, 0.7, 0.7, 0.8, 0.8, 1.0,1.0, 1.0, 1.0, 1.1 1.3, 1.5, 1.5, 1.5, 1.5, 2.0, 2.0, 2.2, 2.5, 2.7,3.0, 3.0, 3.3, 3.3, 4.0, 4.0, 4.5, 4.7, 5.0, 5.4, 5.4, 7.0, 7.5, 8.8,9.0, 10.3, 22.0, 24.5.
We plot the histogram of the observed data and the embedded pdf plot of PPL disribution. It is seen that the PPL distribution is a good fit for the observed data. For comparison study we include the embedded pdfs of power Lindley(PL), Poisson Weibull(PW) and Inverse power Lindly(IPL) distributions also in the same graph and it is given in Figure 3

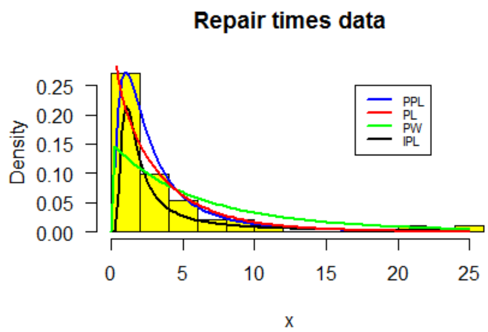

Figure 3: *Fitted pdf plot of Dataset 1*

The numerical values of statistics of the fitted models for Dataset 1 are presented in Table 1. We note that the PPL distribution has the lowest value of -logl, AIC, BIC, K-S and highest p value as compared to power Lindley(PL), Poisson Weibull(PW) and Inverse power Lindly(IPL) distributions. Therefore PPL distribution provides a better fit for the given Dataset 1 than the other considered distributions.

Table 1: *The MLE, -logl, AIC, BIC, K-S and p-value for the fitted models to the Dataset1*

Distribution	Parameters	-logl	AIC	BIC	K-S	p value
PPL	$\alpha = 0.4218$ $\beta = 1.9947$ m=8.2000	0.0635	6.1270	11.6130	0.28986	0.972
PL	$\alpha = 0.7581$ $\beta = 0.6757$	105.0133	214.0266	217.6839	0.76087	0.2171
PW	$\alpha = 0.4218$ $\beta = 0.1249$ $\gamma = 1.0004$	112.6021	231.2042	236.6901	0.63043	0.2129
IPL	$\alpha = 1.6899$ $\beta = 0.6222$	69.3208	142.6417	146.302	0.5	0.7241

3.2. Dataset 2 Carbon fibers data

The data provides the tensile strength of 69 carbon fibers, measured in GPa, tested under tension at gauge given by [13]
1.312, 1.314, 1.479, 1.552, 1.700, 1.803, 1.861, 1.865, 1.944, 1.958, 1.966, 1.997, 2.006, 2.021, 2.027, 2.055, 2.063, 2.098, 2.14, 2.179, 2.224, 2.240, 2.253, 2.270, 2.272, 2.274, 2.301, 2.301, 2.359, 2.382, 2.382, 2.426, 2.434, 2.435, 2.478, 2.490, 2.511, 2.514, 2.535, 2.554, 2.566, 2.57, 2.586, 2.629, 2.633, 2.642, 2.648, 2.684, 2.697, 2.726, 2.770, 2.773, 2.800, 2.809, 2.818, 2.821, 2.848, 2.88, 2.954, 3.012, 3.067, 3.084, 3.090, 3.096, 3.128, 3.233, 3.433, 3.585, 3.585.
The histogram and the embedded pdfs of Poisson-power Lindley (PPL), power Lindly (PL), Poisson Exponential (PE) and Exponentiated power Lindley (EPL) distributions for Dataset 2 are give in Figure 4.

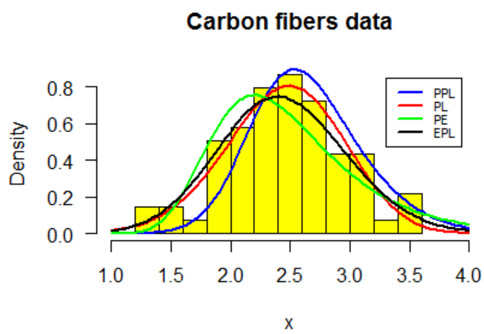

Figure 4: *Fitted pdf plot of Dataset 2*

The MLE, -logl, AIC, BIC, K-S and p-value for the fitted models are presented in Table 2. Here also PPL distribution seems to be the better fit as it has the lowest value of -logl, AIC, BIC, K-S and highest p value when compared to power Lindly (PL), Poisson Exponential (PE), Exponentiated power Lindley (EPL) distributions.

Table 2: *The MLE, -logl, AIC, BIC, K-S and p-value for the fitted models to the Dataset 2*

Distribution	Parameters	-logl	AIC	BIC	K-S	p value
PPL	$\alpha = 2.3845$ $\beta = 0.3095$ m=5.1124	46.67928	99.35855	106.0609	0.3333	0.9068
PL	$\alpha = 3.868$ $\beta = 0.050$	49.06274	102.1255	106.5937	0.5	0.7161
PE	$\theta = 91.2776$ $\lambda = 2.0476$	54.4334	112.8668	117.3351	0.5	0.7161
EPL	$\beta = 2.5554$ $\theta = 0.2487$ $\alpha = 2.2514$	117.7761	241.5522	248.2545	0.42029	0.6901

That is in the case of these two data sets, PPL distribution seems to be the better fit.

4. RELIABILITY TEST PLAN

In Reliability test plan, we want to decide whether to accept or not to accept the lot based on a sample of products taken from the lot. In a life testing experiment, the procedure is to terminate the test by a predetermined time t and note the number of failures. If the number of failures at the end of time t does not exceed a given number c, called acceptance number then we accept the lot with a given probability of at least p^*. But if the number of failures exceeds c before time t then the test is terminated and the lot is rejected. For such truncated life test and the associated decision rule, we are interested in obtaining the smallest sample size to arrive at a decision. Assume that the lifetime of a product follows Poisson-power Lindley distribution. If a scale parameter $\lambda > 0$ is introduced, the cdf of PPL is given by,

$$G(x;m;\xi) = \left(1-e^{-1}\right)^{-1}\left[1-e^{-\left[1-(1+\frac{\beta}{\beta+1}(\frac{x}{\lambda})^{\alpha})e^{-\beta(\frac{x}{\lambda})^{\alpha}}\right]^{m}}\right], x > 0, \alpha, \beta, m > 0. \tag{33}$$

The average life time depends only on λ if α,β and m are known. Let λ_0 be the required minimum average life time. Then

$$G(t,\alpha,\beta,m,\lambda) \leq G(t,\alpha,\beta,m,\lambda_0) \Leftrightarrow \lambda \geq \lambda_0$$

A sampling plan consists of the following quantities: (1) the number of units n on test; (2) the acceptance number c; (3) the maximum test duration t, and (4) the minimum average lifetime represented by λ_0. The probability of accepting a bad lot, that is the consumer's risk should not exceed the value $1-p^*$ where p^* is a lower bound for the probability that a lot of true value λ below λ_0 is rejected by the sampling plan. For fixed p^* the sampling plan is characterized by $(n, c, \frac{t}{\lambda_0})$. By sufficiently large lots we can apply binomial distribution to find acceptance probability. The problem is to determine the smallest positive integer n for given value of c and $\frac{t}{\lambda_0}$ such that

$$L(P_o) = \sum_{i=0}^{c}\binom{n}{i}p_0^i(1-p_0)^{n-i} \leq 1-p^* \tag{34}$$

where $p_0 = G(t,\alpha,\beta,m,\lambda_0)$. The function $L(p)$ is the operating characteristic function of the sampling plan i.e. the acceptance probability of the lot as a function of the failure probability $p(\lambda) = G(t,\alpha,\beta,m,\lambda)$. The operating characteristic function is an increasing function in λ as the average life time of the product is increasing with λ and thus the failure probability $p(\lambda)$ decreases. Table 3 gives the minimum values of n satisfying (34) for $\alpha = 1$, $\beta = 2$, $m = 2$, p^*=

0.75, 0.90, 0.95, 0.99 and $\frac{t}{\lambda_0}$= 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712.

$$L_1\left(p_0\right)=\sum_{i=0}^{c}\frac{\theta^i}{i!}e^{-\theta}\leq 1-p^*. \tag{35}$$

The minimum values of n satisfying (35) are obtained for the same combination of values of $\alpha, \beta, m, \frac{t}{\lambda_0}$ and p^* when the binomial probability approximated by Poisson probability with parameter $\theta = np_0$ in the case of $p_0 = G\left(t, \alpha, \beta, m, \lambda_0\right)$ is very small and n is large, are presented in Table 4. The operating characteristic function of the sampling plan $(n, c, \frac{t}{\lambda_0})$ gives the probability $L(p)$ of accepting the lot with

$$L(P)=\sum_{i=0}^{c}\binom{n}{i}p_0^i\left(1-p_0\right)^{n-i} \tag{36}$$

where $p = G(t, \lambda)$ is considered as a function of λ. The values of n and c are determined by means of operating characteristic function for given values of p^*, $\frac{t}{\lambda_0}$ and considering $p = G\left(\frac{t}{\lambda_0}/\frac{\lambda_0}{\lambda}\right)$ are displayed in Table 5. The producer's risk, probability of rejecting a lot having the quality $\lambda \geq \lambda_0$ is specified by a value say 0.05, it is interested to know that which values of $\frac{\lambda}{\lambda_0}$ will ensure a producer's risk less than or equal to 0.05 for a given sampling plan. The value of $\frac{\lambda}{\lambda_0}$ is the smallest positive number for which the following inequality

$$\sum_{i=0}^{c}\binom{n}{i}p_0^i\left(1-p_0\right)^{n-i}\geq 0.95 \tag{37}$$

holds. For some sampling plan $\left(n, c, \frac{t}{\lambda_0}/\frac{\lambda}{\lambda_0}\right)$ and values of p^*, the minimum values of $\frac{\lambda}{\lambda_0}$ satisfying (37) are given in Table 6 and Table 7 gives the values of L(p) for given values of $\frac{\lambda}{\lambda_0}$

Table 3: *Minimum sample sizes using Binomial probabilities*

		t/λ_0							
p^*	c	0.628	0.942	0.1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	3	2	1	1	1	1	1	1
	1	5	3	3	2	2	2	2	2
	2	8	5	4	4	3	3	3	3
	3	10	7	5	5	4	4	4	4
	4	13	8	7	6	5	5	5	5
	5	15	10	8	7	6	6	6	6
	6	17	11	9	8	7	7	7	7
	7	19	13	11	10	8	8	8	8
	8	22	15	12	11	9	9	9	9
	9	24	16	13	12	10	10	10	10
	10	26	18	14	13	12	11	11	11
0.90	0	4	2	2	2	1	1	1	1
	1	7	4	3	3	2	2	2	2
	2	10	6	5	4	3	3	3	3
	3	13	8	6	5	5	4	4	4
	4	15	10	8	7	6	5	5	5
	5	18	11	9	8	7	6	6	6
	6	20	13	10	9	8	7	7	7
	7	23	15	12	10	9	8	8	8
	8	25	16	13	12	10	9	10	10
	9	28	18	14	13	11	10	10	10
	10	30	20	16	14	12	11	11	11
0.95	0	5	3	2	2	1	1	1	1
	1	9	5	4	3	3	2	2	2
	2	11	7	5	5	4	3	3	3
	3	14	9	7	6	5	4	4	4
	4	17	11	8	7	6	5	5	5
	5	20	13	10	8	7	6	6	6
	6	22	15	11	10	8	7	7	7
	7	25	16	13	11	9	9	8	8
	8	27	18	14	12	10	10	9	9
	9	21	15	13	12	12	11	12	11
	10	32	21	17	15	12	12	11	11
0.99	0	8	4	3	3	2	1	1	1
	1	11	7	5	4	3	2	2	2
	2	15	9	7	6	4	3	3	3
	3	18	11	8	7	5	4	4	4
	4	21	13	10	8	7	5	5	5
	5	24	15	11	10	8	6	6	6
	6	26	17	13	11	9	7	7	7
	7	29	18	14	12	10	9	8	8
	8	32	20	16	13	11	10	9	9
	9	35	22	17	15	12	11	10	10
	10	37	24	19	16	13	12	11	11

Table 4: *Minimum sample sizes using Poisson probabilities*

		t/λ_0							
p^*	c	0.628	0.948	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	4	3	2	2	2	2	2	2
	1	6	4	4	4	3	3	3	3
	2	9	6	5	5	5	4	4	4
	3	11	8	7	6	6	6	6	6
	4	14	10	8	8	7	7	7	7
	5	16	11	10	9	8	8	8	8
	6	19	13	11	10	9	9	9	9
	7	21	15	12	11	11	10	10	10
	8	24	16	14	13	12	11	11	11
	9	19	15	14	13	13	13	13	13
	10	28	20	16	15	14	14	14	14
0.90	0	5	4	3	3	3	3	3	3
	1	9	6	5	5	4	4	4	4
	2	12	8	7	6	6	6	6	6
	3	15	10	9	8	7	7	7	7
	4	17	12	10	9	9	9	9	8
	5	20	14	12	11	10	10	10	10
	6	23	16	13	12	11	11	11	11
	7	25	18	15	14	13	12	12	12
	8	28	19	16	15	14	14	14	13
	9	31	21	18	16	15	15	15	15
	10	33	23	19	18	16	16	16	16
0.95	0	7	5	4	4	4	4	4	4
	1	11	8	7	6	6	6	5	5
	2	14	10	9	8	7	7	7	7
	3	15	12	10	10	9	9	9	8
	4	17	14	12	11	10	10	10	10
	5	20	16	14	13	12	12	11	11
	6	23	18	16	14	13	13	13	13
	7	25	20	17	16	14	14	14	14
	8	28	22	19	17	16	15	15	15
	9	31	24	20	19	17	17	17	17
	10	33	26	22	20	18	18	18	18
0.99	0	10	7	6	6	5	5	5	5
	1	15	10	9	8	7	7	7	7
	2	18	13	11	10	9	9	9	9
	3	22	15	13	12	11	11	11	11
	4	25	17	15	13	12	12	12	12
	5	28	20	17	15	14	14	14	14
	6	31	22	18	17	15	15	15	15
	7	24	22	20	18	17	17	17	17
	8	37	26	22	20	18	18	18	18
	9	40	28	24	22	20	19	19	19
	10	43	30	25	23	21	21	21	21

Table 5: *OC values for the plan $(n, c, t/\lambda_0)$ for given confidence level $p^\star$, $\alpha = 1$, $m = \beta = 2$*

$p^\star$	n	c	$\frac{t}{\lambda_0}$	λ/λ_0					
				2	4	6	8	10	12
0.75	8	2	0.628	0.8249	0.9913	0.9989	0.9997	0.9999	0.9999
	5	2	0.942	0.7877	0.9865	0.9982	0.9996	0.9998	0.9999
	4	2	1.257	0.7263	0.9774	0.9966	0.9992	0.9997	0.9999
	4	2	1.571	0.5416	0.9427	0.9902	0.9976	0.9992	0.9997
	3	2	2.356	0.5095	0.9145	0.9822	0.9951	0.9983	0.9993
	3	2	3.141	0.2885	0.7945	0.9438	0.9822	0.9934	0.9972
	3	2	3.927	0.1537	0.6502	0.8792	0.95589	0.9822	0.9920
	3	2	4.712	0.0793	0.5095	0.7945	0.9145	0.9623	0.9822
0.90	10	2	0.628	0.7168	0.9829	0.9978	0.9995	0.9998	0.9999
	6	2	0.942	0.6772	0.9754	0.9966	0.9992	0.9997	0.9999
	5	2	1.257	0.5498	0.9514	0.9923	0.9982	0.9994	0.9998
	4	2	1.571	0.5416	0.9427	0.9902	0.9976	0.9992	0.9997
	3	2	2.356	0.5095	0.9145	0.9822	0.9951	0.9983	0.9993
	3	2	3.141	0.2885	0.7945	0.9438	0.9822	0.9934	0.9972
	3	2	3.927	0.1537	0.6502	0.8792	0.95589	0.9822	0.9920
	3	2	4.712	0.0793	0.5095	0.7945	0.9145	0.9623	0.9822
0.95	11	2	0.628	0.6606	0.9775	0.9971	0.9993	0.9998	0.9999
	7	2	0.942	0.5669	0.9607	0.9943	0.9987	0.9996	0.9998
	5	2	1.257	0.5498	0.9514	0.9923	0.9982	0.9994	0.9998
	5	2	1.571	0.3345	0.8843	0.9780	0.9944	0.9982	0.9993
	4	2	2.356	0.1985	0.7711	0.9427	0.9831	0.9940	0.9997
	3	2	3.141	0.2885	0.7945	0.9438	0.9822	0.9934	0.9972
	3	2	3.927	0.1537	0.6502	0.8792	0.95589	0.9822	0.9920
	3	2	4.712	0.0793	0.5095	0.7945	0.9145	0.9623	0.9822
0.99	15	2	0.628	0.4485	0.9477	0.9926	0.9984	0.9995	0.9998
	7	2	0.942	0.3732	0.9211	0.9875	0.9971	0.9991	0.9996
	5	2	1.257	0.2725	0.8732	0.9768	0.9942	0.9982	0.9993
	6	2	1.571	0.1930	0.8125	0.9606	0.9895	0.9965	0.9986
	4	2	2.356	0.1985	0.7711	0.9427	0.9831	0.9940	0.9997
	4	2	3.141	0.0595	0.5419	0.8398	0.9427	0.9774	0.9902
	3	2	3.927	0.1537	0.6502	0.8792	0.95589	0.9822	0.9920
	3	2	4.712	0.0793	0.5095	0.7945	0.9145	0.9623	0.9822

Table 6: *Minimum ratio of true mean life to specified mean life for the acceptability of a lot with $\alpha = 0.05$*

		t/λ_0							
p^*	c	0.628	0.942	0.1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	0.31235	0.46621	0.62574	0.78206	1.1728	1.5636	1.9549	2.3456
	1	0.30953	0.46696	0.62487	0.77886	1.1680	1.5572	1.9469	2.3360
	2	0.30830	0.46375	0.61861	0.77314	1.1622	1.5495	1.9372	2.3245
	3	0.30689	0.45844	0.6152	0.7689	1.1508	1.5343	1.9182	2.3017
	4	0.3032	0.4573	0.6121	0.7651	1.1450	1.5266	1.9086	2.2901
	5	0.3029	0.4567	0.6076	0.7619	1.1374	1.5164	1.8959	2.2748
	6	0.2989	0.4534	0.6049	0.7535	1.0983	1.4642	1.8307	2.1966
	7	0.2971	0.4502	0.5996	0.7477	1.1234	1.4977	1.8725	2.2468
	8	0.2961	0.4484	0.5965	0.7442	1.1169	1.4891	1.8617	2.2339
	9	0.2945	0.4440	0.5916	0.7170	1.1098	1.4796	1.8499	2.2197
	10	0.2943	0.4412	0.5931	0.7443	1.1029	1.4743	1.8433	2.2117
0.90	0	0.31261	0.46768	0.62616	0.78188	1.17351	1.56451	1.95602	2.34702
	1	0.31178	0.46738	0.62537	0.78159	1.17103	1.56121	1.95188	2.34206
	2	0.30937	0.46401	0.62008	0.77561	1.16362	1.5133	1.93953	2.32724
	3	0.30850	0.46235	0.61821	0.77131	1.15673	1.54169	1.92748	2.31278
	4	0.30644	0.45863	0.61273	0.76771	1.15097	1.53054	1.91354	2.29605
	5	0.30465	0.45734	0.60887	0.76212	1.13909	1.52488	1.90647	2.28757
	6	0.30254	0.45466	0.60539	0.75557	1.13375	1.50856	1.88606	2.26308
	7	0.30015	0.45209	0.60226	0.75048	1.12580	1.50019	1.87559	2.25052
	8	0.29960	0.44887	0.59802	0.74859	1.12348	1.49781	1.87262	2.24696
	9	0.29697	0.44492	0.59328	0.74248	1.11331	1.48242	1.85338	2.22387
	10	0.29489	0.44304	0.59198	0.73818	1.10732	1.48019	1.85060	2.22053
0.95	0	0.31342	0.46930	0.62630	0.78275	1.17417	1.56540	1.95713	2.34835
	1	0.31301	0.46742	0.62571	0.78190	1.17260	1.56200	1.95288	2.34325
	2	0.31285	0.46638	0.62510	0.78125	1.17059	1.56107	1.95172	2.34186
	3	0.31249	0.46599	0.62489	0.77827	1.16723	1.55472	1.94378	2.33233
	4	0.31058	0.46438	0.62139	0.77449	1.16150	1.55442	1.94339	2.33187
	5	0.30982	0.46382	0.61888	0.77433	1.16073	1.54632	1.93327	2.31973
	6	0.30889	0.46296	0.61848	0.77169	1.15626	1.54544	1.93217	2.31841
	7	0.30835	0.46215	0.61767	0.77075	1.15323	1.53748	1.92512	2.30994
	8	0.30808	0.46042	0.61302	0.76404	1.14812	1.53067	1.91120	2.29324
	9	0.30635	0.45935	0.61247	0.76234	1.14329	1.52422	1.90252	2.28284
	10	0.30533	0.45755	0.61117	0.76219	1.14322	1.52414	1.89925	2.27890
0.99	0	0.31410	0.47158	0.62932	0.78653	1.17517	1.57161	1.96488	2.35766
	1	0.31349	0.46950	0.62815	0.78583	1.17293	1.56519	1.95686	2.3480
	2	0.31322	0.46912	0.62662	0.78525	1.17282	1.56216	1.95308	2.34349
	3	0.31264	0.46901	0.62634	0.78099	1.1697	1.55585	1.94518	2.33402
	4	0.31248	0.46875	0.62437	0.77818	1.16764	1.55522	1.94440	2.33308
	5	0.31131	0.46633	0.62203	0.77685	1.16330	1.55066	1.93870	2.32624
	6	0.31025	0.46618	0.61933	0.77598	1.16289	1.55015	1.93806	2.32547
	7	0.31009	0.46322	0.61857	0.77578	1.15937	1.54816	1.93782	2.32519
	8	0.30864	0.46104	0.616415	0.76888	1.15524	1.54185	1.92134	2.30541
	9	0.30646	0.46035	0.61599	0.76727	1.15464	1.53299	1.91697	2.30017
	10	0.30629	0.45845	0.61446	0.76669	1.14908	1.53029	1.91078	2.29275

4.1. Description of the Tables

Assume that the lifetime distribution is Poisson-power Lindley distribution with $\alpha = 1$, $\beta = 2$, $m = 2$, and that the experimenter irested in establishing that the true unknown average life is at least 1000 hours with confidence $p^* = 0.75$. It is desired to stop the experiment at t = 628 hours. Then, for an acceptance number $c = 2$, the required n in Table 3 is 8. If, during 628 hours, no more than 2 failures out of 8 are observed, then the experimenter can assert, with a confidence level of 0.75 that the average life is at least 1000 hours. For the same situation we obtained the value of n = 9 from Table 4when the Poisson approximation to binomial probability is used. Comparing with reliability test plan for the two parameter Quasi Lindley distribution[1] and three parameter Lindley distribution [2] the minimum sample size using binomial approximation for the sampling plan $c = 10$, $\frac{t}{\lambda_0} = 0.628$ with confidence level $p^* = 0.75$ are 29 and 125 respectively, whereas for the Poisson-power Lindley distribution it is 26. This indicate that the newly developed reliability test plan gives a propitious improvement in making optimal decisions as compared to the other two distributions. For the sampling plan $(n = 8, c = 2, \frac{t}{\lambda_0} = 0.628)$ and confidence level $p^* = 0.75$ under Poisson power Lindley distribution with $\alpha = 1$,$\beta = 2$,$m = 2$, the values of the operating characteristic function from Table 5 are as follows:

Table 7: *Values of L(p) for various values of* $\frac{\lambda}{\lambda_0}$

$\frac{\lambda}{\lambda_0}$	2	4	6	8	10	12
L(p)	0.8249	0.9913	0.9989	0.9997	0.9999	0.9999

From Table 7 we can find that if the true mean lifetime is twice the required mean lifetime ($\frac{\lambda}{\lambda_0} = 2$) the producer's risk is approximately 0.1751. We can get the values of the ratio $\frac{\lambda}{\lambda_0}$ for various choices of $(c, \frac{t}{\lambda_0})$ in order that the producer's risk may not exceed 0.05, for example if $p^* = 0.75$, $\frac{t}{\lambda_0} = 4.712$, $c = 2$, Table 6 gives a reading of 2.3245. This means that the product can have an average life of 2.32 times the required average lifetime in order that under the above acceptance sampling plan the product is accepted with probability of at least 0.95. The actual average lifetime necessary to accept 95 percent of the lots is provided in Table 6.

4.2. Real Data Example

Consider the data studied by Bjerkeda [10] which represents the survival times (in hours) of guinea pigs infected with virulent tubercle bacilli, after they were injected with a given dose of tubercule bacilli in a medical experiment. This data can be regarded as an ordered sample as given below:
43, 45, 53, 56,56,57, 58, 66, 67, 73, 74, 79, 80, 80, 81, 81, 81, 82, 83, 83, 84, 88, 89, 91, 91, 92, 92, 97, 99, 99, 100, 100, 101, 102, 102, 102, 103, 104, 107, 108, 109, 113, 114, 118, 121, 123, 126, 128, 137, 138, 139, 144, 145, 147, 156, 162, 174, 178, 179, 184, 191, 198, 211, 214, 243, 249, 329, 380, 403, 511, 522, 598
Let the experimenter is interested to establish the true unknown mean life time is 70 hours with confidence $p^* = 0.75$ and testing time be 44 hours, which gives the ratio $\frac{t}{\lambda_0} = \frac{44}{70} = 0.628$.. Thus, for an acceptance number c=8 and confidence level $p^* = 0.75$, the required sample size n from Table 3 is found to be 22. Thus the sampling plan for the above sample data is $\left(n = 22, c = 8, \frac{t}{\lambda_0} = 0.628\right)$. Based on the data, we have to decide whether to accept the drug or reject it. We accept the drug only if the number of survival before 44 hours is greater than or equal to 14 among the first 22 observations. However, the confidence level is assured by the sampling plan only if the given lifetimes following the Poisson-power Lindley distribution. Thus in order to confirm it, the goodness of fit test for these observations were done and it gives a p-value 0.6889 and K-S statistics as 0.41667. The corresponding fitted embedded graph is given in Figure 5. Since there is only 2 failures before the time 45, we can say that in a testing time of 44 hours, 20 were survived among 22 observations. So we can accept the effectiveness of tubercle bacilli in developing acquired resistance against tuberculosis in guinea pigs according to our

sampling plan.

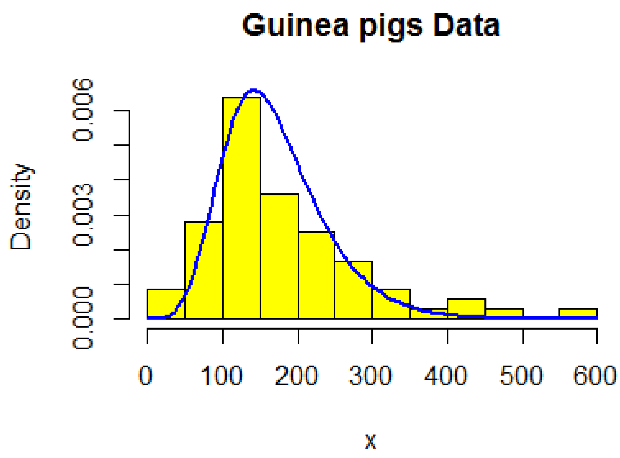

Figure 5: *fitted pdf plot for guinea pigs data*

5. CONCLUSION

In this paper, we introduce a new member of Poisson-X family called Poisson-power Lindley distribution. Some of its structural properties are investigated. The model parameters are estimated by the method of maximum likelihood. The flexibility of the new distribution is illustrated by means of real data sets. It is seen that the Poisson- power Lidley distribution provides a better fit than other compared distributions for these data sets. Also, a reliability test plan is developed when the life time follows the Poisson-power Lindley distribution. We have shown in general that under similar conditions, in order to ensure a specified mean life with a given confidence level, Poisson-power Lindley model results in smaller sample sizes than some other models used in acceptance sampling. In order to verify the applicability of the distribution in reliability test plan a real data analysis is also conducted and it found to be given as a better decision regarding the acceptability of the product.

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