THE EFFICACY OF TRAPEZOIDAL FUZZY NUMBERS AND ITS APPLICATION

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Abstract

Numerous fields, including engineering, agriculture, and management sciences, have been using trapezoidal fuzzy numbers. In this study, we first develop Trapezoidal Fuzzy Number (TFN) and then attempt to formulate a model to handle element uncertainty in order to solve a linear programming problem. Making good decisions will only require this type of approximation.

Keywords: Trapezoidal Fuzzy Number, linear programming problem, initial problem and membership function.

1. Introduction

Optimization problem is a one of the most important operation research techniques, and it is used in many areas in agriculture planning, science, Technology and engineering which may be important in both economic and social point of view. We formulate the problem mathematically which may arise in our daily life our aim is to minimize cost and maximize the profit, with certain constrains or restrictions are to be considered. In order to get the best possible result of those problems faced by Agriculturalists to allocate the optimum number of vegetation in their farmhouses. To increase the area under cultivation there are numerous ways to achieving high productivity. If we utilize the resources in a proper way which may be helpful to increase the crop production. Many operation research techniques have been used in planning agriculture activity one of the technique is linear programming. In 1947 George Danzig was given the concept of Linear Programming problem. If we have a limited number of resources, we use linear programming method to optimize the problem. The Zimmermann [1] presented the concept of formulation of fuzzy linear programming problem. Orlovsky [2] made several attempts to investigate the potential of fuzzy set theory as a valuable tool for comprehensive mathematical analysis of practical problems. To address the many kinds of FLP problems, numerous authors utilize various techniques. In almost all areas of decision-making problems, fuzzy approaches have been developed. Particularly Tamiz [3], and Ross [4]. Delgado and Verdegay [5] construct a broad model of fuzzy linear programming within the fuzzy and fuzzy right side of technical coefficients and also demonstrates that it is possible to solve the dual problem using the same programme. Fung and Hu [6] introduced the fuzzy number-based approach coefficients. Kumar and Rajendra [7] solved a fuzzy linear programming problem with fuzzy variables in parametric form. By utilizing a ranking function and defining a crisp model, Verdegray [8] and Maleki [9] ranking function can be identified in comparisons between fuzzy numbers. In order to determine a workable and ideal solution, we study the linear programming issue in this essay in its conventional form. To solve the linear fuzzy linear programming problem, we utilize the algorithm by trapezoidal fuzzy number is considered.

2. Model Formulation

$$\begin{aligned} &Maxmize\ Z = CY \\ &AY \leq B \\ &Y \geq 0 \end{aligned} \tag{1}$$

Where C is vector component, A is coefficient Matrix, B is crisp parameters and Y is decision variable.

2.1 Generalized Trapezoidal Fuzzy Number (TFN)

The generalized Fuzzy Trapezoidal number $\widetilde{T} = (t_1, t_2, t_3, t_4, w)$ is a fuzzy subset of real line R, whose membership function $\mu_{\widetilde{T}}$ satisfies the following postulates:

- $t_1 \leq x \leq t_2$, is a continuous mapping from R to the closed interval [0, 1]
- $\mu_{\widetilde{T}}(x) = 0, -\infty < x \leq t_1$
- $\mu_{\widetilde{T}}(x)$ is strictly increasing with constant rate on $t_1 \leq x \leq t_2$
- $\mu_{\widetilde{T}}(x)$ is strictly decreasing with constant rate on $t_3 \leq x \leq t_4$
- $\mu_{\widetilde{T}}(x) = 0, t_4 \leq x < \infty$

Membership function is given by

$$\mu_{\widetilde{T}}(y) = \begin{cases} w(\frac{Y - t_1}{t_2 - t_1}), & t_1 \leq Y \leq t_2 \\ w, & t_2 \leq Y \leq t_3 \\ w(\frac{t_4 - Y}{t_4 - t_3}), & t_3 \leq Y \leq t_4 \\ 0, & elesewhere \end{cases} \tag{2}$$

Where, $t_1 < t_2 < t_3 < t_4\ and\ w \in (0,1]$

If w=1, the generalized TFN can be written as

$\widetilde{T} = (t_1, t_2, t_3, t_4)$ and the membership function is given by

$$\mu_{\tilde{T}}(y) = \begin{cases} w(\frac{Y-t_1}{t_2-t_1}), & t_1 \le Y \le t_2 \\ 1, & t_2 \le Y \le t_3 \\ w(\frac{t_4-Y}{t_4-t_3}), & t_3 \le Y \le t_4 \\ 0, & elesewhere \end{cases} \tag{3}$$

Now we can take ordered pair of parametric of fuzzy numbers with left-hand alpha- cut and right-hand alpha-cut, which are bounded left non-decreasing and bounded right non-increasing functions over [0,1],

i.e. $\widetilde{T} = \{(t_1 + \alpha(t_2 - t_1), t_4 + \alpha(t_4 - t_3)\}$

The above mathematical model can be formulated as given below

$$\begin{aligned} &Maximize && Z = C_1(y_{L1}, y_{R1}) + C_2(y_{L2}, y_{R2}).........+ C_n(y_{Ln}, y_{Rn}) \\ &Subject\ to && \\ & && a_{h1}(y_{L1}, y_{R1}) + a_{h2}(y_{L2}, y_{R2}) + ...+ a_{hn}(y_{Ln}, y_{Rn}) \le (b_{Ln}, b_{Rn}) \\ & && y_{Lj}, y_{Rj} \ge 0, \quad for\,all\ h = 1,2,3,......m \ \ and \ \ j = 1,2,3,....n. \end{aligned} \tag{4}$$

3. Applications

A 20 hectares of land is under cultivation of three different crops such as wheat, corn and pulses with certain requirement for capital (in euros) and working hours as shown below:

Table 1: *20 hectares of land under cultivation*

Crops per acre	Capital (€)	Workers (hours)
Wheat	50	10
Corn	33	8
Pulses	27	4

In this problem the profit of the above three different crops are wheat €38/ acre, Corn €32/ acre and pulses €28/ acre acres of land. The amount and working hours are respectively €460 and around 52 hours. Now, we decide how many hectares of land are required for each crop in order to maximize the profit. Let y_1 be the cultivated area with wheat, y_2 be the cultivated area with corn and y_3 be the cultivated area with pulses. We can have characterized the rough data by a trapezoidal fuzzy number as: 22 hectares = (18, 22, 24, 25) about €460 = (380, 410, 440, 450); around 55 hours = (46, 48, 50, 55). The problem can be written as:

$$\begin{aligned} &\text{Maximize} && Z = 38y_1 + 32y_2 + 28y_3 \\ &\text{Subject to} && \\ & && 50y_1 + 33y_2 + 27y_3 \le (380, 410, 440, 450) \\ & && 10y_1 + 8y_2 + 4y_3 \le (46, 48, 50, 55) \\ & && y_1 + y_2 + y_3 \le (18, 22, 24, 25) \end{aligned} \tag{5}$$

The crisp model of the above problem

$$\begin{aligned} &\text{Maximize} && Z_1 = 38y_{L1} + 32y_{L2} + 28y_{L3} \ \ and \\ & && Z_2 = 38y_{R1} + 32y_{R2} + 28y_{R3} \\ &\text{Subject to} && \end{aligned}$$

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\begin{aligned}
50y_{L1} + 33y_{L2} + 27y_{L3} &\leq 380 + 30\alpha \\
50y_{R1} + 33y_{R2} + 27y_{R3} &\leq 450 + 10\alpha \\
10y_{L1} + 8y_{L2} + 4y_{L3} &\leq 46 + 2\alpha \\
10y_{R1} + 8y_{R2} + 4y_{R3} &\leq 55 + 5\alpha \\
y_{L1} + y_{L2} + y_{L3} &\leq 18 + 4\alpha \\
y_{R1} + y_{R2} + y_{R3} &\leq 25 + \alpha
\end{aligned} \tag{6}
$$

LINGO 12.0 [10] can be used to acquire the results for the various values of presented in table (2) below. The ideal response to the initial problem is y_{L1} = (0, 0,0,0), y_{L2} = (0, 0, 0, 0), and y_{L3} = (11.62, 11.75, 11.87, 12.0) and is the ideal value Z_1 = (325.50,329,323.50,336).

Table 2: *Cropping combination provides best overall solution*

α	0.25	0.50	0.75	1.0
y_{L1}	0	0	0	0
y_{L2}	0	0	0	0
y_{L3}	11.62	11.75	11.87	12
y_{R1}	0	0	0	0
y_{R2}	0	0	0	0
y_{R3}	14.06	14.37	14.68	15
Z_1	325.50	329	323.50	336
Z_2	393.75	402.50	411.25	420

4. Conclusion

The application of fuzzy linear programming to resolve a production planning problem in agriculture has been covered in this study. The paper finishes by explaining how FLPP is transformed into clear multi-objective linear programming problems and how the farmer achieves the best possible outcomes while using constrained resources. Only the trapezoidal membership function is taken into account in this paper.

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