A Novel Three - Parameter Version of the Ailamujia Distribution

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Abstract

In this paper, a novel three - parameter continuous distribution is introduced. This novel distribution is an extented version of the Exponenentiated Ailamujia distribution. This extended version called as Exponentiated Generalized Ailamujia (EGA) distribution. The Exponentiated Generalized class is used to derive the proposed distribution by considering Ailamujia distribution as a baseline distribution. A special case of the EGA distribution called Generalized Ailamujia (GA) distribution is also derived. Properties of the proposed distribution such as moments, mean, variance, harmonic mean, moment generating function, survival function, hazard function, reverse hazard rate, Mills ratio and order statistics are derived. In addition, maximum likelihood approach is used to estimate the proposed distribution parameters. Finally, the proposed distribution is applied to two real datasets and compare with the Exponentiated Ailamujia and the Ailamujia Inverted Weibull distributions. Results reveal that the proposed distribution provides better estimate as compared to the said distributions for the given two real datasets.

Keywords: Ailamujia distribution, Exponentiated Generalized - G Class, Exponentiated Ailamujia distribution.

1. Introduction

Distributions with support on non-negative real numbers are important in modelling lifetime data. There are lifetime distributions which are popular in modelling lifetime data such as Weibull, log-logistic and lognormal distributions. These distributions are widely used in engineering and other related fields. Lv [8] proposed the Ailamujia distribution as an additional lifetime distribution and studied its properties such as mean, variance, median and maximum likelihood estimators. This distribution was studied further its properties such as interval estimation and hypothesis testing [9] and minimax estimation under non-informative prior using the loss functions [7].

There are different extensions of the Ailamujia distribution were considered in the literature. For example, Ahmad [1] introduced the Transmuted Ailamujia distribution and studied its several properties. Other identified extensions are the weighted analogue of Ailamujia distribution [14], the area biased distribution [6], the inverse analogue of Ailamujia distribution [2], the size biased Ailamujia distribution [11], the Power Ailamujia distribution [13] and the Power Ailamujia distribution [5].

Moreover, Rather [10] introduced the extended version of the Ailamujia distribution called Exponentiated Ailamujia distribution and explored some of its structural properties such as moments, reliability analysis and harmonic mean. They used the Exponentiated - G family of distributions in the derivation of the Exponentiated Ailamujia distribution. In addition, they fitted the Exponentiated Ailamujia distribution into two real datasets and they found that the Exponentiated Ailamujia distribution had a better fit compared to the Ailamujia and Lindley distributions.

In this paper, the main goals are the following: (i) to extend the exponentiated Ailamujia distribution using the exponentiated generalized class; (ii) to derive some properties of the proposed distribution such as moments, mean, variance, harmonic mean, moment generating function, survival function, hazard function, reverse hazard rate, Mills ratio, maximum likelihood estimates and order statistics; and (iii) to apply the proposed distribution into two real datasets and compare with the Exponentiated Ailamujia and Ailamujia Inverted Weibull distributions.

The rest of paper is organized as follows: Exponentiated Generalized Ailamujia distribution is introduced in section 2. In section 3, some statistical properties of the proposed distribution are derived. Order Statistics of the proposed distribution is presented in section 4 while the Maximum likelihood estimates of the proposed distribution parameters are discussed in section 5. In section 6, the application of the proposed distribution is illustrated. Some concluding remarks is presented in section 7.

2. Exponentiated Generalized Ailamujia distribution

This section presents the derivation of the Exponentiated Generalized Ailamujia (EGA) distribution. Let X be a random variable follows an Ailamujia distribution then the cumulative distribution function of the Ailamujia distribution is given by

$$G(x,\theta) = 1-(1+2\theta x)e^{-2\theta x}, x \geq 0, \theta > 0 \tag{1}$$

with corresponding probability density function given by

$$g(x,\theta) = 4\theta^2 x e^{-2\theta x}.$$

Cordeiro [4] introduced the exponentiated generalized class to extend any univariate continuous distribution into generalized distribution with additional two parameters. The cumulative distribution function of the exponentiated generalized class is given by

$$F(x) = (1-(1-G(x))^a)^b, a, b > 0, \tag{2}$$

where $G(x)$ is any baseline cumulative distribution function. The cumulative distribution function of the Exponentiated Generaized Ailamujia (EGA) distribution is obtained by inserting (1) into (2) and is

$$F(x,\theta,a,b) = (1-(1+2\theta x)^a e^{-2a\theta x})^b, x \geq 0, \theta, a, b > 0 \tag{3}$$

with corresponding probability density function given by

$$f(x,\theta,a,b) = 4ab\theta^2 x(1+2\theta x)^{a-1} e^{-2a\theta x}(1-(1+2\theta x)^a e^{-2a\theta x})^{b-1}. \tag{4}$$

Notethat if $a = 1$ then the EGA distribution reduces to Exponentiated Ailamujia distribution. Then it is Ailamujia distribution if $a = b = 1$. If $b = 1$ then the cumulative distribution function of the EGA distribution reduces to a cumulative distribution function of new special distribution that is given by

$$F(x,\theta,a) = 1-(1+2\theta x)^a e^{-2a\theta x} \tag{5}$$

with probability density function given by

$$f(x,\theta,a) = 4a\theta^2 x(1+2\theta x)^{a-1} e^{-2a\theta x}.$$

We name the cumulative distribution function (5) as the cumulative distribution function of the Generalized Ailamujia (GA) distribution.

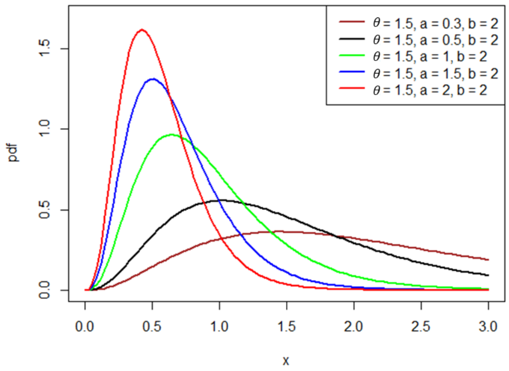

Figure 1: *pdf plots of EGA distribution for* $\theta = 1.5$, $b = 2$ *and varying values of* a

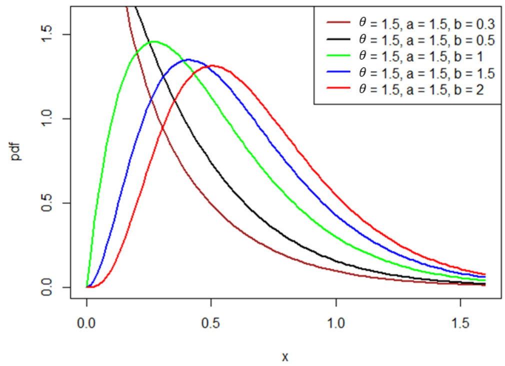

Figure 2: *pdf plots of EGA distribution for* $\theta = 1.5$, $a = 1.5$ *and varying values of* b

Figures 1 and 2 show some possible probability density shapes of the EGA distribution. It reveals that the probability density function of the EGA distribution can model a data with right tailed unimodal and exponential shapes.

3. Statistical Properties

In this section, we derive some properties of the EGA distribution such as moments, mean, variance, moment generating function, harmonic mean, survival fuction, hazard function, reverse hazard rate and Mills ratio.

3.1. Moments

Theorem 1. The rth raw moment of EGA with density (4) is

$$\mu_r' = \frac{b}{(2\theta)^r} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{l=0}^{\infty} \frac{(-1)^j \binom{a-1}{i} \binom{b-1}{j} \binom{aj}{l} \Gamma(r+l+i+2)}{a^{r+l+i+1} (j+1)^{r+l+i+2}}. \tag{6}$$

The mean μ_1' and variance σ^2 are respectively, given by

$$\mu_1' = \frac{b}{2\theta} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{l=0}^{\infty} \frac{(-1)^j \binom{a-1}{i} \binom{b-1}{j} \binom{aj}{l} \Gamma(l+i+3)}{a^{l+i+2} (j+1)^{l+i+3}}$$

and

$$\sigma^2 = \frac{b}{4\theta^2} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty} \frac{(-1)^j \binom{a-1}{i}\binom{b-1}{j}\binom{aj}{l}\Gamma(l+i+4)}{a^{l+i+3}(j+1)^{l+i+4}} - \frac{b^2}{4\theta^2}\left(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty} \frac{(-1)^j \binom{a-1}{i}\binom{b-1}{j}\binom{aj}{l}\Gamma(l+i+3)}{a^{l+i+2}(j+1)^{l+i+3}}\right)^2.$$

Proof. The rth raw moment is defined by

$$\begin{aligned}
\mu'_r &= E[X^r] \\
&= \int_{-\infty}^{\infty} x^r f(x) dx \\
&= \int_0^{\infty} x^r 4ab\theta^2 x(1+2\theta x)^{a-1} e^{-2a\theta x}(1-(1+2\theta x)^a e^{-2a\theta x})^{b-1} dx.
\end{aligned}$$

Using the binomial expansion for $(1+2\theta x)^{a-1}$, we have

$$(1+2\theta x)^{a-1} = \sum_{i=0}^{\infty} \binom{a-1}{i} 2^i \theta^i x^i.$$

Again, using the binomial expansion for $(1-(1+2\theta x)^a e^{-2a\theta x})^{b-1}$, we have

$$\begin{aligned}
(1-(1+2\theta x)^a e^{-2a\theta x})^{b-1} &= \sum_{j=0}^{\infty} (-1)^j \binom{b-1}{j} (1+2\theta x)^{aj} e^{-2aj\theta x} \\
&= \sum_{j=0}^{\infty} (-1)^j \binom{b-1}{j} e^{-2aj\theta x} \sum_{l=0}^{\infty} \binom{aj}{l} 2^l \theta^l x^l \\
&= \sum_{j=0}^{\infty}\sum_{l=0}^{\infty} (-1)^j \binom{b-1}{j}\binom{aj}{l} 2^l \theta^l x^l e^{-2aj\theta x}.
\end{aligned}$$

Now, μ'_r becomes

$$\mu'_r = 4ab\theta^2 \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty} (-1)^j B_i B_j B_l 2^{i+l}\theta^{i+l} \int_0^{\infty} x^{r+i+l+1} e^{-2a(j+1)\theta x} dx,$$

where $B_i = \binom{a-1}{i}$, $B_j = \binom{b-1}{j}$ and $B_l = \binom{aj}{l}$. Moreover,

$$\mu'_r = \frac{b}{(2\theta)^r} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty} \frac{(-1)^j B_i B_j B_l \Gamma(r+l+i+2)}{a^{r+l+i+1}(j+1)^{r+l+i+2}}.$$

The mean μ'_1 of EGA is obtained by setting $r=1$ in (6) and is

$$\mu'_1 = \frac{b}{2\theta} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty} \frac{(-1)^j \binom{a-1}{i}\binom{b-1}{j}\binom{aj}{l}\Gamma(l+i+3)}{a^{l+i+2}(j+1)^{l+i+3}}.$$

The μ'_2 is derived from (6) by setting $r=2$ and is

$$\mu'_2 = \frac{b}{4\theta^2} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty} \frac{(-1)^j \binom{a-1}{i}\binom{b-1}{j}\binom{aj}{l}\Gamma(l+i+4)}{a^{l+i+3}(j+1)^{l+i+4}}.$$

The variance σ^2 of EGA is obtained as

$$\begin{aligned}\sigma^2 =&\mu_2' - (\mu_1')^2\\ =&\frac{b}{4\theta^2}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j\binom{a-1}{i}\binom{b-1}{j}\binom{aj}{l}\Gamma(l+i+4)}{a^{l+i+3}(j+1)^{l+i+4}}\\ &-\left(\frac{b}{2\theta}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j\binom{a-1}{i}\binom{b-1}{j}\binom{aj}{l}\Gamma(l+i+3)}{a^{l+i+2}(j+1)^{l+i+3}}\right)^2\\ =&\frac{b}{4\theta^2}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j\binom{a-1}{i}\binom{b-1}{j}\binom{aj}{l}\Gamma(l+i+4)}{a^{l+i+3}(j+1)^{l+i+4}}\\ &-\frac{b^2}{4\theta^2}\left(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j\binom{a-1}{i}\binom{b-1}{j}\binom{aj}{l}\Gamma(l+i+3)}{a^{l+i+2}(j+1)^{l+i+3}}\right)^2.\end{aligned}$$

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3.2. Moment Generating Function

Theorem 2. Let X be a random variable follows EGA distribution then the moment generating function $M_X(t)$ is given by

$$M_x(t) = b\sum_{r=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty}\frac{t^r(-1)^j\binom{a-1}{i}\binom{b-1}{j}\binom{aj}{l}\Gamma(r+l+i+2)}{(2\theta)^r a^{r+l+i+1}(j+1)^{r+l+i+2}r!}, \tag{7}$$

where $t \in \mathbb{R}$.

Proof. By definition of moment generating function and equation (6), we have

$$M_X(t) = \mathbb{E}(e^{tX}) = \int_0^{\infty} e^{tx} f(x,\theta,a,b)dx.$$

Recall that $e^{tx} = \sum_{r=0}^{\infty}\frac{t^r}{r!}x^r$ then we have

$$M_X(t) = \int_0^{\infty}\sum_{r=0}^{\infty}\frac{t^r}{r!}x^r f(x,\theta,a,b)dx = \sum_{r=0}^{\infty}\frac{t^r}{r!}x^r\int_0^{\infty} f(x,\theta,a,b)dx = \sum_{r=0}^{\infty}\frac{t^r}{r!}\mu_r'.$$

Thus,

$$M_x(t) = b\sum_{r=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty}\frac{t^r(-1)^j\binom{a-1}{i}\binom{b-1}{j}\binom{aj}{l}\Gamma(r+l+i+2)}{(2\theta)^r a^{r+l+i+1}(j+1)^{r+l+i+2}r!},$$

where $t \in \mathbb{R}$. ■

3.3. Harmonic Mean

Theorem 3. Let X be a random variable follows EGA distribution then the harmonic mean of EGA is

$$H.M = 2\theta b\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j B_i B_j B_l\Gamma(l+i+1)}{a^{l+i}(j+1)^{l+i+1}}.$$

Proof. The harmonic mean is defined by

$$\begin{aligned} H.M &= \mathbb{E}\left[\frac{1}{X}\right] \\ &= \int_{-\infty}^{\infty} \frac{1}{x} f(x)dx \\ &= \int_{0}^{\infty} \frac{1}{x} 4ab\theta^2 x(1+2\theta x)^{a-1} e^{-2a\theta x}(1-(1+2\theta x)^a e^{-2a\theta x})^{b-1}dx \\ &= \int_{0}^{\infty} 4ab\theta^2 (1+2\theta x)^{a-1} e^{-2a\theta x}(1-(1+2\theta x)^a e^{-2a\theta x})^{b-1}dx. \end{aligned}$$

Using the binomial expansion for $(1+2\theta x)^{a-1}$, we have

$$(1+2\theta x)^{a-1} = \sum_{i=0}^{\infty} \binom{a-1}{i} 2^i \theta^i x^i.$$

Again, using the binomial expansion for $(1-(1+2\theta x)^a e^{-2a\theta x})^{b-1}$, we have

$$\begin{aligned} (1-(1+2\theta x)^a e^{-2a\theta x})^{b-1} &= \sum_{j=0}^{\infty} (-1)^j \binom{b-1}{j} (1+2\theta x)^{aj} e^{-2aj\theta x} \\ &= \sum_{j=0}^{\infty} (-1)^j \binom{b-1}{j} e^{-2aj\theta x} \sum_{l=0}^{\infty} \binom{aj}{l} 2^l \theta^l x^l \\ &= \sum_{j=0}^{\infty}\sum_{l=0}^{\infty} (-1)^j \binom{b-1}{j}\binom{aj}{l} 2^l \theta^l x^l e^{-2aj\theta x}. \end{aligned}$$

Now, $H.M$ becomes

$$H.M = 4ab\theta^2 \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty} (-1)^j B_i B_j B_l 2^{i+l}\theta^{i+l} \int_0^{\infty} x^{i+l} e^{-2a(j+1)\theta x} dx$$

where $B_i = \binom{a-1}{i}$, $B_j = \binom{b-1}{j}$ and $B_l = \binom{aj}{l}$. Thus,

$$H.M = 2\theta b \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty} \frac{(-1)^j B_i B_j B_l \Gamma(l+i+1)}{a^{l+i}(j+1)^{l+i+1}}.$$

■

3.4. Reliability Analysis

Let X be a random variable with cdf (3) and pdf (4) then the survival $S(x,\theta,a,b)$ and hazard $h(x,\theta,a,b)$ functions of the EGA distribution are respectively, given by

$$S(x,\theta,a,b) = 1-(1-(1+2\theta x)^a e^{-2a\theta x})^b, \; x \geq 0, \theta, a, b > 0$$

and

$$h(x,\theta,a,b) = \frac{4ab\theta^2 x(1+2\theta x)^{a-1} e^{-2a\theta x}(1-(1+2\theta x)^a e^{-2a\theta x})^{b-1}}{1-(1-(1+2\theta x)^a e^{-2a\theta x})^b}.$$

In addition, the reverse hazard rate $h_r(x,\theta,a,b)$ and the Mills ratio of the EGA distribution are respectively, given by

$$h_r(x,\theta,a,b) = \frac{4ab\theta^2 x(1+2\theta x)^{a-1} e^{-2a\theta x}}{1-(1+2\theta x)^a e^{-2a\theta x}}$$

and

$$MillsRatio = \frac{1-(1+2\theta x)^{a}e^{-2a\theta x}}{4ab\theta^{2}x(1+2\theta x)^{a-1}e^{-2a\theta x}}.$$

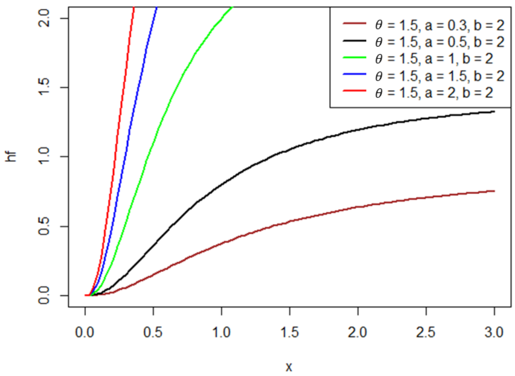

Figure 3: *hf plots of EGA distribution for $\theta = 1.5$, $b = 2$ and varying values of a*

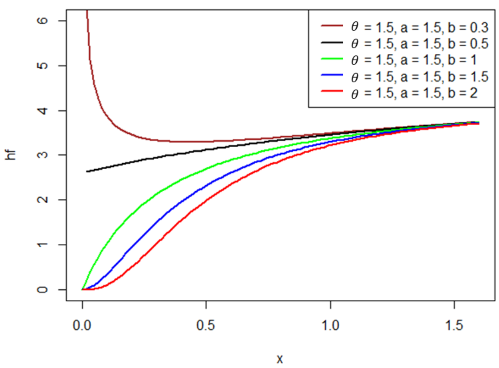

Figure 4: *hf plots of EGA distribution for $\theta = 1.5$, $a = 1.5$ and varying values of b*

Figures 3 and 4 present some possible shapes of the hazard function of EGA distribution. It reveals that the hazard function of the EGA distribution can model data with increasing or decreasing hazard rate behaviors.

4. Order Statistics

Let $X_{(1)}$, $X_{(2)}$,..., $X_{(n)}$ be the order statistics of a random sample X_1, X_2,..., X_n drawn from the continuous population with cumulative distribution function $F_X(x)$ and probability density function (pdf) $f_X(x)$, then the pdf of rth order statistics $X_{(r)}$ is given by

$$f_{X_{(r)}}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x)[F_X(x)]^{r-1}[1-F_X(x)]^{n-r}. \quad (8)$$

The pdf of rth order statistics $X_{(r)}$ of the EGA distribution is obtained by inserting (3) and (4) into (8) and is

$$f_{X_{(r)}}(x,\theta,a,b) = \frac{4ab\theta^{2}n!}{(r-1)!(n-r)!} xz^{a-1}e^{-2a\theta x}\left(1-z^{a}e^{-2a\theta x}\right)^{br-1}\left[1-\left(1-z^{a}e^{-2a\theta x}\right)^{b}\right]^{n-r}, \quad (9)$$

where $z = 1 + 2\theta x$. The pdf of the $1st$ or smallest order statistics of the EGA distribution is derived by setting $r = 1$ in equation (9) and is

$$f_{X_{(1)}}(x,\theta,a,b) = 4abn\theta^2 xz^{a-1}e^{-2a\theta x}\left(1 - z^a e^{-2a\theta x}\right)^{b-1}\left[1 - \left(1 - z^a e^{-2a\theta x}\right)^b\right]^{n-1}.$$

If $r = n$ then the pdf of the nth or largest order statistics of EGA distribution is given by

$$f_{X_{(n)}}(x,\theta,a,b) = 4ab\theta^2 nxz^{a-1}e^{-2a\theta x}\left(1 - z^a e^{-2a\theta x}\right)^{bn-1}.$$

5. Maximum Likelihood Estimation

This section is dedicated to maximum likelihood estimation as an estimation approach for EGA parameters.

Let X_1, X_2,..., X_n be a random sample of size n from EGA distribution then the likelihood function is given by

$$L = \prod_{i=1}^{n} 4ab\theta^2 x_i(1 + 2\theta x_i)^{a-1}e^{-2a\theta x_i}(1 - (1 + 2\theta x_i)^a e^{-2a\theta x_i})^{b-1}.$$

The log-likelihood is

$$l = nlog(4) + nlog(a) + nlog(b) + 2nlog(\theta) + \sum_{i=1}^{n} log(x_i) + a\sum_{i=1}^{n} log(1 + 2\theta x_i)$$
$$-\sum_{i=1}^{n} log(1 + 2\theta x_i) - 2a\theta\sum_{i=1}^{n} x_i + (b-1)\sum_{i=1}^{n} log(1 - (1 + 2\theta x_i)^a e^{-2a\theta x_i}). \quad (10)$$

Taking the the derivative of (10) with respect to parameters θ, a and b then we have the following equations:

$$\frac{\partial l}{\partial \theta} = \frac{2n}{\theta} + 2(a-1)\sum_{i=1}^{n}\frac{x_i}{z_i} - 2a\sum_{i=1}^{n} x_i + 4a\theta(b-1)\sum_{i=1}^{n}\frac{x_i^2 e^{-2a\theta x_i} z_i^{a-1}}{1 - z_i^a e^{-2a\theta x_i}}; \quad (11)$$

$$\frac{\partial l}{\partial a} = \frac{n}{a} + \sum_{i=1}^{n} log(z_i) - 2\theta\sum_{i=1}^{n} x_i + (b-1)\sum_{i=1}^{n}\frac{z_i^a e^{-2\theta a x_i}(2\theta x_i - log(z_i))}{1 - z_i^a e^{-2a\theta x_i}}; \quad (12)$$

$$\frac{\partial l}{\partial b} = \frac{n}{b} + \sum_{i=1}^{n} log(1 - z_i^a e^{-2a\theta x_i}), \quad (13)$$

where $z_i = 1 + 2\theta x_i$. The numerical maximum likelihood estimates of the EGA parameters can be computed by equating (11), (12) and (13) to 0, respectively.

6. Application

In this section, the EGA distribution is applied to two real datasets and compare with the following distributions:

- Exponentiated Ailamujia distribution (EA) [10]

$$f(x,\theta,b) = 4b\theta^2 xe^{-2\theta x}(1 - (1 + 2\theta x)e^{-2\theta x})^{b-1}, x \geq 0, \theta, b > 0.$$

- Ailamujia Inverted Weibull distribution (AIW) [12]

$$f(x,\theta,\alpha) = 4\alpha\theta^2 x^{-2\alpha-1}e^{-2\theta x^{-\alpha}}, x > 0, \theta, \alpha > 0.$$

In this application, we use two datasets from Badar[3]. These sets of data are presented as follow:

Data Set 1. This dataset is related to failure stresses (in GPa) and it is composed of 65 single carbon fibers of lengths 50 mm. The observations are given as follow: 1.339, 1.434, 1.549, 1.574, 1.589, 1.613, 1.746, 1.753, 1.764, 1.807, 1.812, 1.84, 1.852, 1.852, 1.862, 1.864, 1.931, 1.952, 1.974, 2.019, 2.051, 2.055, 2.058, 2.088, 2.125, 2.162, 2.171, 2.172, 2.18, 2.194, 2.211, 2.27, 2.272, 2.28, 2.299, 2.308, 2.335, 2.349, 2.356, 2.386, 2.39, 2.41, 2.43, 2.431, 2.458, 2.471, 2.497, 2.514, 2.558, 2.577, 2.593, 2.601, 2.604, 2.62, 2.633, 2.67, 2.682, 2.699, 2.705, 2.735, 2.785, 3.02, 3.042, 3.116 and 3.174.

Data Set 2. This dataset represents the strength data measured in GPa of 69 single carbon fibres tested under tension at gauge lengths of 20mm. The data is given as follows: 1.312, 1.314, 1.479, 1.552, 1.700, 1.803, 1.861, 1.865, 1.944, 1.958, 1.966, 1.997, 2.006, 2.021, 2.027, 2.055, 2.063, 2.098, 2.140, 2.179, 2.224, 2.240, 2.253, 2.270, 2.272, 2.274, 2.301, 2.301, 2.359, 2.382, 2.382, 2.426, 2.434, 2.435, 2.478, 2.490, 2.511, 2.514, 2.535, 2.554, 2.566, 2.570, 2.586, 2.629, 2.633, 2.642, 2.648, 2.684, 2.697, 2.726, 2.770, 2.773, 2.800, 2.809, 2.818, 2.821, 2.848, 2.880, 2.954, 3.012, 3.067, 3.084, 3.090, 3.096, 3.128, 3.233, 3.433, 3.585 and 3.585.

In this analysis, we use the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Kolmogorov - Smirnov (K-S), Anderson - Darling (A) and Cramer - von Mises (W*) statistics for comparison. In addition, a package "fitdistrplus" in R is used to fit the distributions into given datasets.

The ML estimates of the fitted models to both sets of data are presented in Tables 1 and 3. Furthermore, results are given in Table 2 for first set of data and in Table 4 for second set of data. Tables 2 and 4 indicate that the proposed distribution provides better estimate for two given datasets as compared to the EA and AIW distributions since it has a smallest values of AIC, BIC, K-S, A and W*. Moreover, same results are observed from figures 5 and 6.

Table 1: *MLEs of the fitted models for a first set of data.*

Distribution	$\hat{\theta}$	$\hat{\alpha}$	$\hat{a}$	$\hat{b}$
EGA	0.007532943		5404.596	12.49782
EA	1.473919			58.063764
AIW	12.894223	3.542585		

Table 2: *Numerical values of AIC, BIC, K-S, A and W* of the fitted models for a first set of data.*

Distribution	AIC	BIC	$K-S$	A	$W*$
EGA	77.65784	84.181	0.08196969	0.43719505	0.07281970
EA	79.85257	84.20135	0.09598235	0.75832969	0.12159989
AIW	85.65023	89.99900	0.1150352	1.2112933	0.1948092

Table 3: *MLEs of the fitted models for a second set of data.*

Distribution	$\hat{\theta}$	$\hat{\alpha}$	$\hat{a}$	$\hat{b}$
EGA	0.006973733		4720.681000000	9.017470000
EA	1.204724000			31.814881000
AIW	10.924163000	3.000044000		

Table 4: *Numerical values of AIC, BIC, K-S, A and W* of the fitted models for a second set of data.*

Distribution	AIC	BIC	$K-S$	A	$W*$
EGA	107.4429	114.1452	0.06677067	0.45740745	0.06486004
EA	111.7837	116.2520	0.09042448	0.99232483	0.14097939
AIW	122.5224	126.9907	0.1199293	1.9072115	0.2835300

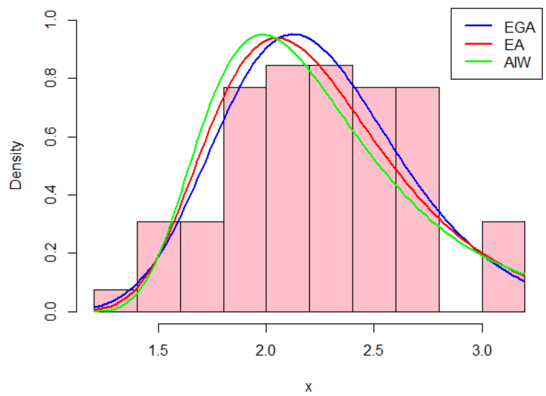

Figure 5: *Estimated pdf of the fitted models for the first set of data*

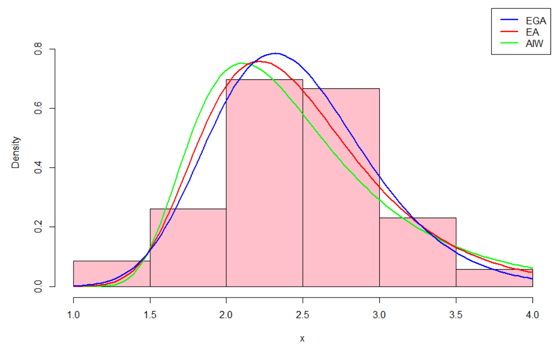

Figure 6: *Estimated pdf of the fitted models for the second set of data*

7. Concluding Remarks

In this paper, a novel generalized version of the Exponentiated Ailamujia distribution called Exponentiated Generalized Ailamujia distribution has been introduced. Some properties of the proposed distribution such as moments, mean, variance, harmonic mean, moment generating function, survival function, hazard function, reverse hazard rate, Mills ratio and order statistics were derived. Maximum likelihood approach was implemented to estimate the proposed distribution parameters. The applicability of the proposed distribution was evaluated by fitting on two real datasets and compared with the EA and AIW distributions. It was found that the proposed distribution provides better estimate for the given datasets compared to the said distributions.

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