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A TWO NON-IDENTICAL UNIT PARALLEL SYSTEM WITH PRIORITY IN REPAIR

Alka Chaudhary, Shivali Sharma
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Principal, KLPG College, Meerut, 250002
Department of Statistics, Meerut College, Meerut, 250004
alkachaudhary527@gmail.com, sharma14shivali@gmail.com

Abstract

The paper deals with a system composed of two-non identical units (unit-1 and unit-2). Initially both the units are arranged in parallel configuration. Each unit has two possible modes- Normal (N) and Total Failure (F). The first unit gets priority in repair. System failure occurs when both the units stop functioning. A single repairman is always available with the system to repair a totally failed unit and repair discipline is first come, first served (FCFS).If during the repair of a failed unit the other unit also fails, then the later failed unit waits for repair until the repair of the earlier failed unit is completed. The repair times of both the units are exponential distribution with different parameters. Each repaired unit works as good as new. Using regenerative point technique, various important measures of system effectiveness have been obtained.

Keywords: Transition probabilities, mean sojourn time, reliability, MTSF, availability, expected busy period of repairman, net expected profit.

1. Introduction

Reliability is an important concept in the planning design and operation stages of various complex systems. Reliability is a significant area that is accepting awareness internationally and it is crucial for actual usage and care of any industrial system. It requires technical Knowledge for growing system effectiveness by decreasing the frequency of failure and reducing the worth of maintenance. Chaudhary and Tyagi [3] analyzed a two non-identical unit parallel system with two types of failure. Pundir et al. [7] analyzed a two non-identical unit parallel system with priority in repair. Chaudhary et al. [5] analyzed two non-identical unit warm standby repairable system with two types of failure. Saxena et al. [9] analyzed two unit parallel system with working and rest time of repairman.

A single repairman is always available with the system to repair a totally failed unit and repair discipline is first come, first served (FCFS). Chaudhary and Masih [1, 4] analyzed a two non-identical unit. Saini et al. [8], Chaudhary and Sharma [2] and Dabas et al. [6] analyzed a two non-identical unit system models assuming two modes- Normal mode and total failure mode of each unit and analyzed parallel system with priority in repair. System failure occurs when both the units stop functioning. The first unit gets priority in repair.

By using regenerative point technique, the following measures of system effectiveness are obtained-

i. Transition probabilities and mean sojourn times in various states.
ii. Reliability and mean time to system failure (MTSF).
iii. Point-wise and steady-state availabilities of the system as well as expected up time of the system during time interval (0, t).
iv. Expected busy period of repairman in the repair of unit-1 and unit-2 during time interval (0, t).
v. Net expected profit earned by the system in time interval (0, t).

2. System Description and Assumptions

1. The system comprises of two non-identical units (unit-1 and unit-2). Initially, both the units work in parallel configuration.
2. Each unit of the system has two possible modes-Normal (N) and total failure (F).
3. The first unit gets priority in repair.
4. System failure occurs when both the units stop functioning.
5. A single repairman is always available with the system to repair a totally failed unit and repair discipline is first come, first served (FCFS).
6. If during the repair of a failed unit the other unit also fails, then the later failed unit waits for repair until the repair of the earlier failed unit is completed.
7. The repair time of both the units is exponential distribution with different parameters. Each repaired unit works as good as new.

3. Notations and States of the System

We define the following symbols for generating the various states of the system-

N_{1O}, N_{2O}	:	Unit-1and Unit-2 is in N-mode and operative in parallel.
F_{1r}, F_{2r}	:	Unit-1 and Unit-2 is in F-mode and under repair.
F_{2w}	:	Unit-2 is in F-mode and under waiting for repair.

Considering the above symbols in view of assumptions stated in section-2, the possible states of the system are shown in the transition diagram represented by **Figure. 1**. It is to be noted that the epochs of transitions into the state S_1 from S_2 are non-regenerative, whereas all the other entrance epochs into the states of the systems are regenerative.

The other notations used are defined as follows:

E	:	Set of regenerative states.
α_1, α_2	:	Constant Failure rate of Unit-1 and Unit- 2.
β_1, β_2	:	Constant Repair rate of Unit-1 and Unit- 2.
$G(\bullet)$	:	CDF of time to repair and its repair is continued to state S1
$H(\bullet)$	:	General Distribution of Unit-2
*	:	Symbol for Laplace Transform i.e. $g_{ij}^*(s) = \int e^{-st} q_{ij}(t)dt$
~	:	Symbol for Laplace Stieltjes Transform i.e. $\widetilde{Q}_{ij}(s) = \int e^{-st} dQ_{ij}(t)t$
©	:	Symbol for ordinary convolution i.e. $A(t)©B(t)=\int_0^t A(u)B(t-u)du$

TRANSITION DIAGRAM

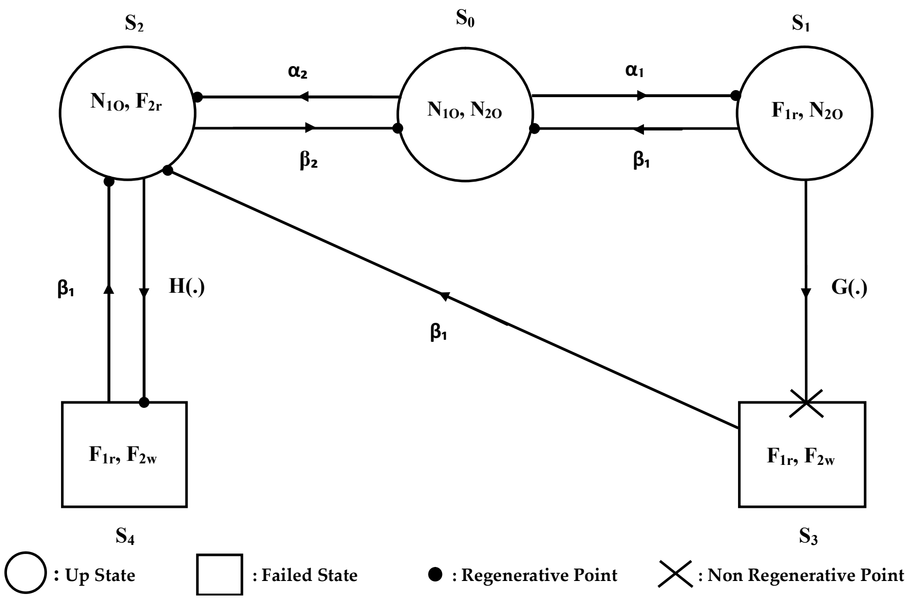

Figure 1: *Exponential Model*

4. Transition Probabilities and Sojourn Times

Let $X(t)$ be the state of the system at epoch t, then $\{X(t); t \geq 0\}$ constitutes a continuous parametric Markov-Chain with state space $E = \{S_0 to S_4\}$.The various measures of system effectiveness are obtained in terms of steady-state transition probabilities as follows:

$$p_{01} = \int \alpha_1 e^{-(\alpha_1+\alpha_2)t} dt = \frac{\alpha_1}{\alpha_1 + \alpha_2}$$

$$p_{02} = \int \alpha_2 e^{-(\alpha_1+\alpha_2)t} dt = \frac{\alpha_2}{\alpha_1 + \alpha_2}$$

$$p_{10} = \int \beta_1 e^{-\beta_1 t} \overline{G}(t) dt = 1 - \widetilde{G}(\beta_1)$$

$$p_{20} = \int \beta_2 e^{-\beta_2 t} \overline{H}(t) dt = 1 - \widetilde{H}(\beta_2)$$

$$p_{24} = \int e^{-\beta_2 t} dH(t) = \widetilde{H}(\beta_2) \qquad p_{42} = \int \beta_1 e^{-\beta_1 t} dt = 1 = p_{32}$$

The two step transition probability (Steady State) is given by

$$p_{12}^{(3)} = \left(1 - e^{-\beta_1 t}\right) \int_0^v e^{-\beta_1 u} dG(u) = \widetilde{G}(\beta_1)$$

It can be easily verified that,

$$p_{01} + p_{02} = 1, \tag{1}$$

$$p_{10} + p_{12}^{(3)} = 1, \tag{2}$$

$$p_{32} = p_{42} = 1, \tag{3}$$

$$p_{20} + p_{24} = 1 \tag{4}$$

†The limits of integration are 0 to ∞ whenever they are not mentioned.

5. Mean Sojourn Time

The mean sojourn time ψ_i in state S_i is defined as the expected time taken by the system in state S_i before transiting into any other state. If random variable U_i denotes the sojourn time in state S_i then,

$$\psi_i = \int P[U_i > t]dt$$

Therefore, its values for various regenerative states are as follows-

$$\psi_0 = \int e^{-(\alpha_1+\alpha_2)t}dt = \frac{1}{(\alpha_1+\alpha_2)} \quad (5)$$

So that,

$$\psi_1 = \int e^{-\beta_1 t}\overline{G}(t)\,dt \quad (6)$$

$$\psi_2 = \int e^{-\beta_2 t}\overline{H}(t)\,dt \quad (7)$$

$$\psi_4 = \int e^{-\beta_1 t}dt = \frac{1}{\beta_1} \quad (8)$$

6. Analysis of Characteristics

6.1. Reliability and MTSF

Let $R_i(t)$ be the probability that the system operates during (0, t) given that at t=0 system starts from $S_i \in E$. To obtain it we assume the failed states S_2 and S_4 as absorbing. By simple probabilistic arguments, the value of $R_0(t)$ in terms of its Laplace Transform (L.T.) is given by

$$R_0^*(s) = \frac{Z_0^* + q_{01}^* Z_1^* + q_{02}^* Z_2^*}{1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^*} \quad (9)$$

We have omitted the argument's from $q_{ij}^*(s)$ and $Z_i^*(s)$ for brevity. $Z_i^*(s)$; i = 0, 1, 2 are the L. T. of

$$Z_0(t) = e^{-(\alpha_1+\alpha_2)(1-r)t}, \qquad Z_1(t) = \int e^{-\beta_1 t}\overline{G}(t)dt, \qquad Z_2(t) = \int e^{-\beta_2 t}\overline{H}(t)dt$$

Taking the Inverse Laplace Transform of (9), one can get the reliability of the system when system initially starts from state S_0 .

The MTSF is given by,

$$E(T_0) = \int R_0(t) = \lim_{s\to 0} R_0^*(s) = \frac{\psi_0 + p_{01}\psi_1 + p_{02}\psi_2}{1 - p_{01}p_{10} - p_{02}p_{20}} \quad (10)$$

6.2. Availability Analysis

Let $A_i(t)$ be the probability that the system is up at epoch t, when initially it starts operation from state $S_i \in E$. Using the regenerative point technique and the tools of Laplace transform, one can obtain the value of $A_0(t)$ in terms of its Laplace transforms i.e. $A_0^*(s)$ given as follows-

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)} \quad (11)$$

Where,

$$N_1(s) = Z_0^*\left[1 - q_{24}^* q_{42}^*\right] + Z_1^* q_{01}^*\left[1 - q_{24}^* q_{42}^*\right] + Z_2^*\left[q_{01}^* q_{12}^{(3)*} + q_{02}^*\right]$$

and

$$D_1(s) = 1 - q_{24}^* q_{42}^* - q_{10}^* q_{01}^* \left(1 - q_{24}^* q_{42}^*\right) - q_{20}^* \left(q_{01}^* q_{12}^{(3)*} + q_{02}^*\right) \quad (12)$$

Where, $Z_i(t)$, i=0,1,2 are same as given in section 6.1.

The steady-state availability of the system is given by

$$A_0 = \lim_{t \to \infty} A_0(t) = \lim_{s \to 0} s\ A_0^*(s) \quad (13)$$

We observe that

$$D_1(0) = 0$$

Therefore, by using L. Hospital's rule the steady state availability is given by

$$A_0 = \lim_{s \to 0} \frac{N_1(s)}{D_1'(s)} = \frac{N_1}{D_1'} \quad (14)$$

Where,

$$N_1 = \psi_0 \left[1 - p_{24} p_{42}\right] + \psi_1 p_{01} \left[1 - p_{24} p_{42}\right] + \psi_2 \left[p_{01} p_{12}^{(3)} + p_{02}\right]$$

and

$$D_1' = \psi_0 p_{20} + \psi_1 p_{01} p_{20} + \psi_2 \left(1 - p_{10} p_{01}\right) + \psi_4 p_{24} \left(1 - p_{10} p_{01}\right) \quad (15)$$

The expected up time of the system in interval (0, t) is given by

$$\mu_{up}(t) = \int_0^t A_0(u)\,du$$

So that, $\mu_{up}^*(s) = \dfrac{A_0^*(s)}{s}$ (16)

6.3. Busy Period Analysis

Let $B_i^1(t)$ and $B_i^2(t)$ be the respective probabilities that the repairman is busy in the repair of unit-1 failed due to first repair with priority of unit-1 and unit-2 failed due to second repair at epoch t, when initially the system starts operation from state $S_i \in E$. Using the regenerative point technique and the tools of L. T., one can obtain the values of above two probabilities in terms of their L. T. i.e. $B_i^{1*}(s)$ and $B_i^{2*}(s)$ as follows-

$$B_i^{1*}(s) = \frac{N_2(s)}{D_1(s)}, \qquad B_i^{2*}(s) = \frac{N_3(s)}{D_1(s)} \quad (17\text{-}18)$$

Where,

$$N_2(s) = Z_1^* q_{01}^* \left(1 - q_{42}^* q_{24}^*\right) + Z_4^* \left(q_{01}^* q_{12}^{(3)*} q_{24}^* + q_{02}^* q_{24}^*\right)$$

and

$$N_3(s) = Z_2^* \left(q_{01}^* q_{12}^{(3)*} + q_{02}^*\right)$$

and $D_1(s)$ is same as defined by the expression (12) of section VI(II).

The steady state results for the above two probabilities are given by-

$$B_0^1 = \lim_{s \to 0} s\ B_0^{1*}(s) = N_2 \setminus D_1' \quad \text{and} \quad B_0^2 = \lim_{s \to 0} s\ B_0^{2*}(s) = N_3 \setminus D_1' \quad (19\text{-}20)$$

Where,

$$N_2(0) = \psi_1 p_{01} \left(1 - p_{24}\right) + \psi_4 \left(p_{01} p_{12}^{(3)} p_{24} + p_{02} p_{24}\right) \quad (21)$$

$$N_3(0) = \psi_2 \left(p_{01} p_{12}^{(3)} + p_{02}\right) \quad (22)$$

and D_1' is same as given in the expression (15) of section 6.2.

The expected busy period in repair of unit-1 failed due to first repair with priority of unit-1 and unit-2 failed due to second repair during time interval (0, t) are respectively given by-

$$\mu_b^1(t)=\int_0^t B_0^1(u)\,du \quad \text{and} \quad \mu_b^2(t)=\int_0^t B_0^2(u)\,du$$

So that,

$$\mu_b^{1*}(s)=\frac{B_0^{1*}(s)}{s} \quad \text{and} \quad \mu_b^{2*}(s)=\frac{B_0^{2*}(s)}{s} \tag{23-24}$$

6.4. Profit Function Analysis

The net expected total cost incurred in time interval (0, t) is given by

P (t) = Expected total revenue in (0, t) - Expected cost of repair in (0, t)

$$= K_0\mu_{up}(t)-K_1\mu_b^1(t)-K_2\mu_b^2(t) \tag{25}$$

Where, K_0 is the revenue per- unit up time by the system during its operation. K_1 and K_2 are the amounts paid to the repairman per-unit of time when the system is busy in repair of unit-1 failed due first repair with priority of unit-1 and unit-2 failed due to second repair respectively.

The expected total profit incurred in unit interval of time is $P = K_0A_0 - K_1B_0^1 - K_2B_0^2$

7. Particular Case

Let $G(t)=\lambda e^{-\lambda t}$, $H(t)=\mu e^{-\mu t}$

In view of above, the changed values of transition probabilities and mean sojourn times.

$$p_{10}=\frac{\beta_1}{\beta_1+\lambda}, \quad p_{20}=\frac{\beta_2}{\beta_2+\mu}, \quad p_{24}=\frac{\mu}{\beta_2+\mu}$$

$$p_{12}^{(3)}=\frac{\lambda}{\beta_1+\lambda}, \quad \psi_1=\frac{1}{\beta_1+\lambda}, \quad \psi_2=\frac{1}{\beta_2+\mu}$$

8. Graphical Study of Behaviour and Conclusions

For a more clear view of the behaviour of system characteristics with respect to the various parameters involved, we plot curves for MTSF and profit function in Fig. 2 and Fig. 3 w.r.t. α_1 for three different values of failure parameter α_2=0.1, 0.5, 0.9 and two different values of repair parameter β_1=0.01, 0.7 while the other parameters are β_2=0.99, μ= 0.01, λ= 0.06. It is clearly observed from Fig. 2 that MTSF increases uniformly as the value of α_2 and β_1 increase and it decrease with the increase in α_1. Further, to achieve MTSF at least 10 units we conclude for smooth curves that the values of α_1 must be less than 0.18, 0.29 and 0.49 respectively for α_2=0.1, 0.5, 0.9 when β_1=0.01. Whereas from dotted curves we conclude that the values of α_1 must be less than 0.15, 0.22 and 0.39 for α_2=0.1, 0.5, 0.9 when β_1=0.7.

Similarly, Fig.3 reveals the variations in profit (P) with respect to α_1 for three different values of α_2= 0.4, 0.6, 0.8 and two different values of β_1=0.03, 0.2, when the values of other parameters β_2=0.01, μ= 0.09, λ= 0.6, K_0=80, K_1=125 and K_2=175. Here also the same trends in respect of α_1, α_2 and β_1 are observed in case of MTSF. Moreover, we conclude from the smooth curves that the system is profitable only if α_1 is less than 0.20, 0.39 and 0.79 respectively for α_2= 0.4, 0.6, 0.8 when

β_1 =0.03. From dotted curves, we conclude that the system is profitable only if α_1 is less than 0.13, 0.27 and 0.58 respectively for α_2 = 0.4, 0.6, 0.8 when β_1 =0.2.

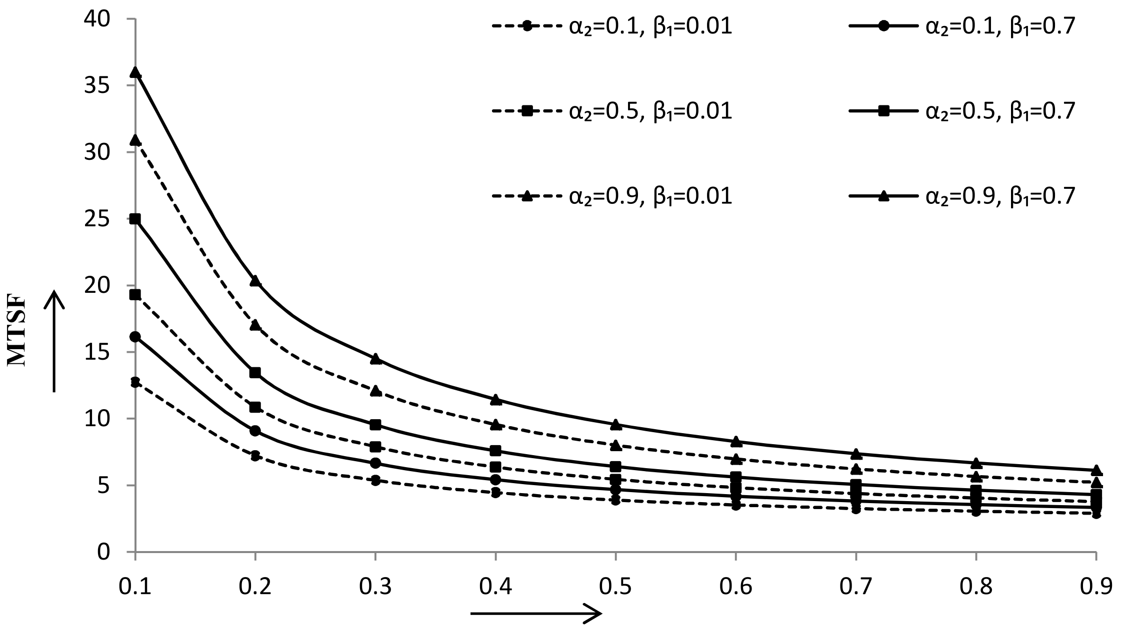

Figure 2: *Behaviour of MTSF w.r.t.* $\boldsymbol{\alpha_1}$ *for different values of* $\boldsymbol{\alpha_2}$ *and* $\boldsymbol{\beta_1}$

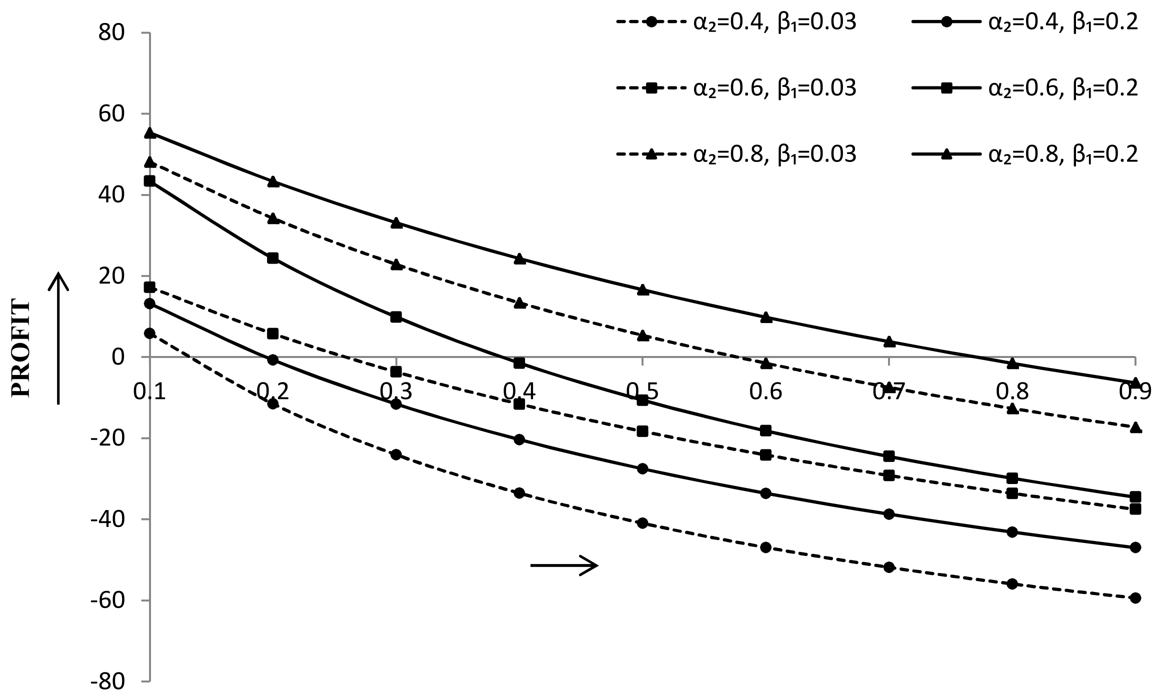

Figure 3: *Behaviour of* ***PROFIT (P)*** *w.r.t.* $\boldsymbol{\alpha_1}$ *for different values of* $\boldsymbol{\alpha_2}$ *and* $\boldsymbol{\beta_1}$

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