A STATISTICAL ANALYSIS OF FRACTIONAL FACTORIAL DESIGN USING A FUZZY PROBABILISTIC APPROACH

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Abstract

In factorial experiments, treatment combinations increase as the number of factors increases. While handling a large number of factors, many difficulties are encountered. Moreover, mechanical errors like mistaken identification of plots, wrong labeling of treatments, etc., may creep in. To overcome these difficulties, only a fraction of treatment combinations can be tested. This technique is known as fractional replication. The design with fractional replication is known as fractional factorial design (FFD). In FFD, the choice of the fraction of treatment depends on what type of information is sacrificed. Usually, the interactions with higher-order are omitted, and all main effects and two-factor interactions are estimated without loss of information. The procedure for the layout of FFD is closely related to the concept of confounding. The analysis of fractional factorials is similar to the analysis of full factors. FFD is used to reduce treatment combinations by a fraction. FFD plays a significant role when the experiment is too large. When compared to classical designs, FFD yields a cost-benefit relationship. Fuzzy theory is used to deal with the imprecise observations in this design. This paper proposes the statistical analysis of fuzzy fractional factorial design with numerical illustration.

Keywords: Fuzzy Fractional Factorial Design, Fuzzy Sets, Trapezoidal Fuzzy Number, $\alpha-$ cut interval method.

1. Introduction

In a complete factorial, each treatment combination is applied to at least one of the experimental units. In some situations, the total number of treatment combinations is too large. Each factor involves two levels. If there are 8 factors, then there are $2^8 = 256$ plots needed for the experiment. As the number of factors increases, treatment combinations also increase. Sometimes, it is difficult to handle such a big experiment practically. Since the time, experimental material, cost, manpower,

etc., also increases, it is impossible to conduct a complete factorial experiment. Typically, the higher-order interactions are not much significant; also, it is difficult to interpret, and these can be used to estimate the error. The total degrees of freedom for 2^8 design is 255 with 8 main effects and 28 two-factor interactions. However, the error degrees of freedom are quite large (219). In handling such a big experiment, the non-experimental error may also lurk in. The higher-order interactions are ignored. The main effects and the lower order interactions information are obtained by a fraction of the complete factorial experiments. This type of experiment is known as the fractional factorial design (FFD). Sometimes, the observations that correspond to FFD will be imprecise. In this case, the fuzzy sets are used to calculate such a design. The fuzzy sets were developed by Lofti A. Zadeh [17] in 1965. Some of the authors who scrutinized to the relevant study are Holland, C.W., [4] outlined the fractional factorial design with its uses in marketing problems. Cotter, S.C., [3] describes the blocking in a fractional factorial experiment by using incomplete block designs without aliasing. Stolle, D.P. [14] suggests that fractional factorial design is the best alternative for factorial designs where the psycholegal researcher examines the main effects of a large number of factors. Ke, W., et.al., [6] propound an efficient method of selecting blocking two-level fractional factorial designs when some two-factor interactions are non-negligible. By screening the important drugs and drug interactions, the sequential usage of two-level and three-level fractional factorial designs was shown, and also Jaynes, J., et.al. [5] provoked the potential optimal drug dosages through contour plots. Parthiban S and Gajivaradhan P [10] have studied the 2^2 factorial experiment using fuzzy environments and compared the result with various tests. Using 3^{5-2} fractional factorial design with resolution III, Zaluski, D., et.al., [16] discern the effect of 5 cultivation factors at 3 levels of intensity under various weather conditions. Using a three-level three-factor factorial design, Anand, R, and Sridhar, V.G [1] focus FSW interlock lap joint of AA7475-T7 to study the correlation among process parameters. In addition to LiDAR range on the robot's navigation time, Mazen, A, et.al., [8] studies the effect of choice for 5 factors of forward and angular velocities using fractional factorial experiment with resolution V. Qamar, S, et.al., [11] determined the effectiveness of four main factors on the extraction of cannabinoids using scCO2 by half-fractional factorial design and identified the highest yield of cannabinoids provided by the extraction of scCO2at high pressure and temperature. In this study, statistical analysis of FFD using TrFNs with $\alpha-$ interval method was proposed through a numerical example.

2. Preliminaries

2.1 Factorial Design

The factorial experiment is defined as the experiment consisting of two or more factors, each with two or more levels. Each factor with the same number of levels is called symmetrical factorial; otherwise, it is called an asymmetrical factorial experiment.

2.2 Fractional Factorial Design (FFD)

When the number of factors is too large, even at two levels, the treatment combinations are also large. While planning such a big experiment, non-experimental types of errors may occur. So therefore, Finney resorted to fractional replicating the more extensive factorial experiments. The information on the main effects and lower order interactions are obtained only from a fraction of the complete factorial experiment called the Fractional Factorial Experiment or Fractional Replicated Design.

2.3 Features of FFD

The features of FFD are: (i) If the fraction of the factorial design s^n (n - number of factors, s -

levels) is of order $\frac{1}{s^m}$. Then, the fractional design is defined as $\frac{1}{s^m}s^n$, where, $m < n$. (ii) If the key block of the factorial experiments with block size s^{n-m-k}, then $\frac{s^{k-1}}{s-1}$ interactions are confounded (iii) The only one block obtained from the defining relation is taken as the defining relation are not estimable (iv) In FFD, confounding is necessary to reduce the block size, and aliases for interactions are confounded.

2.4 Trapezoidal Fuzzy Number (TrFN)

The TrFN is defined as, if a fuzzy set $\tilde{A} = (a_1, a_2, a_3, a_4)$, then its membership function is stated as;

$$\mu_{\tilde{A}}(x) = \begin{cases} 0; & x < a_1 \text{ or } x > a_4 \\ \dfrac{x - a_1}{a_2 - a_1}; & a_1 < x \le a_2 \\ 1; & a_2 < x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}; & a_3 < x \le a_4 \end{cases} \quad (1)$$

where, $a_1 \le a_2 \le a_3 \le a_4$. A TrFN becomes triangular fuzzy number if it satisfies $a_2 = a_3$. In terms of $\alpha -$ cut interval, TrFN is defined as follows:

$$\tilde{A} = [a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha]; \qquad 0 \le \alpha \le 1,$$

where, $a_1 \le a_2 \le a_3 \le a_4$.

3. Methodology

3.1 One-Half Fraction of the 2^5 Design

Consider five factors each at two levels, that is, $2^{5-1} = 32$ treatment combinations. The treatment seems quite large. This leads to a one-half fraction of 2^5 design. We select the 16 treatment combinations as a, b, c, d, e, abc, acd, abd, bcd, abe, ace, bce, ade, bde, cde, $abcde$ as one-half fraction. The signs in the Yates' table [13] is derived by writing down first the five main effects and then forming the interactions of those effects by using + and -. A further process gives the interaction between the five factors. From the Yates' sign table of 2^5, linear combinations for the estimate of the main effects and the interactions for one-half fraction of 2^{5-1} design are

$$l_A = \frac{1}{8}(abc + a - b - bcd - d + abd + acd - c - e + abe + ade - bce + ace - cde - bde + abcde);$$

$$l_B = \frac{1}{8}(b - a - c + abc - d + abd - acd + bcd - e + abe - ace + bde - ade + bce - cde + abcde);$$

Similarly, other effects can be calculated. The formal expressions of the interactions worth nothing. In terms of treatment combinations, the expressions given by the ordinary rules of algebra. Thus, $l_I = l_{ABCDE}$, $l_A = l_{BCDE}$, $l_B = l_{ACDE}$, $l_C = l_{ABDE}$, $l_D = l_{ABCE}$, $l_E = l_{ABCD}$, $l_{AB} = l_{CDE}$, $l_{AC} = l_{BDE}$, $l_{AD} = l_{BCE}$, $l_{AE} = l_{BCD}$, $l_{BC} = l_{ADE}$, $l_{BD} = l_{ACE}$, $l_{BE} = l_{ACD}$, $l_{CD} = l_{ABE}$, $l_{CE} = l_{ABD}$, $l_{DE} = l_{ABC}$, also it is impossible to differentiate A and $BCDE$, B and $ACDE$, and so on. In fact, by estimating main effects and two-factor interactions, actually estimates $A + BCDE$, $B + ACDE$, etc., Two are more effects that have this property are called aliases. The alias structure of this design can be determined

by defining the relation $I = ABCDE$. Multiplying any effect by using the defining relation yields the alias structure of that effect. The alias structure of A is $A.I = A.ABCDE = A^2BCDE$, the square of any column is just identity I, that is, $A + BCDE$. Similarly, the alias structure for other main effects and interaction effects are determined. In this one-half fraction, $I = ABCDE$ is called as the principal fraction. Suppose, considering the other one-half fraction with the minus sign in $ABCDE$ column. The defining relation is $I = -ABCDE$. By this type of fraction, the aliases $A - BCDE$, $B - ACDE$, and so on.

3.2 Statistical Analysis of 2^{5-1} FFD

Consider the Randomised Block Design (RBD) linear model for 2^{5-1} factorial design,

$$y_{ij} = \mu + \tau_i + b_j + e_{ij}; \qquad i = 1,...,t; j = 1,...,r \tag{2}$$

where μ is the general mean effect, τ_i is the fixed effect due to i^{th} treatment, b_j is the fixed effect due to j^{th} replicate, e_{ij} is the random error effect. From the Yates' table of 2^5 design (table 3.1),

$$A = (abc - b - c + a - d + acd + abd - bcd - e + ace + abe - bce + ade - cde - bde + abcde);$$

$$B = (b - c + abc - a - d + abd - acd + bcd - e + bce + abe - ace - ade - cde + bde + abcde);$$

Similarly, other effects can be calculated. Since, the higher order interactions are negligible. The sum of squares sare $SS_A = \frac{[A]^2}{N}$; $SS_B = \frac{[B]^2}{N}$; $SS_C = \frac{[C]^2}{N}$; $SS_D = \frac{[D]^2}{N}$; $SS_E = \frac{[E]^2}{N}$; $SS_{AB} = \frac{[AB]^2}{N}$; $SS_{AC} = \frac{[AC]^2}{N}$; $SS_{AD} = \frac{[AD]^2}{N}$; $SS_{AE} = \frac{[AE]^2}{N}$; $SS_{BC} = \frac{[BC]^2}{N}$; $SS_{BD} = \frac{[BD]^2}{N}$; $SS_{BE} = \frac{[BE]^2}{N}$; $SS_{CD} = \frac{[CD]^2}{N}$; $SS_{CE} = \frac{[CE]^2}{N}$ and $SS_{DE} = \frac{[DE]^2}{N}$ each with 1 degrees of freedom (*df*), where, $N = rt$.

The sum of squares for replications, total and errors are $SS_R = \frac{1}{t}\sum_{j=1}^{r} y_{.j}^{\ 2} - \frac{G^2}{N}$ with $(r-1)$ *df*, $SS_T = \sum\sum y_{ij}^{\ 2} - \frac{G^2}{N}$ with $(rt-1)$ *df* and $SS_{Er} = SS_T - [SS_R + SS_A + SS_B + SS_C + ... + SS_{DE}]$ respectively. All these values will be filled in the ANOVA table (table 3.1).

Table: 3.1 *ANOVA table for* 2^{5-1} *FFD*

SV	df	SS	MSS	F – Ratio
Replicates	$r-1$	SS_R	$\frac{SS_R}{r-1}$	$F_R = \frac{SS_R}{SS_{Er}}$
Treatments	$t-1$	-	-	-
Main effect A	1	SS_A	SS_A	$F_A = \frac{SS_A}{SS_{Er}}$
Main effect B	1	SS_B	SS_B	$F_B = \frac{SS_B}{SS_{Er}}$
Main effect C	1	SS_C	SS_C	$F_C = \frac{SS_C}{SS_{Er}}$
Main effect D	1	SS_D	SS_D	$F_D = \frac{SS_D}{SS_{Er}}$
Main effect E	1	SS_E	SS_E	$F_E = \frac{SS_E}{SS_{Er}}$
Interaction effect AB	1	SS_{AB}	SS_{AB}	$F_{AB} = \frac{SS_{AB}}{SS_{Er}}$
Interaction effect AC	1	SS_{AC}	SS_{AC}	$F_{AC} = \frac{SS_{AC}}{SS_{Er}}$

Interaction effect AD	1	SS_{AD}	SS_{AD}	$F_{AD}=\frac{SS_{AD}}{SS_{Er}}$
Interaction effect AE	1	SS_{AE}	SS_{AE}	$F_{AE}=\frac{SS_{AE}}{SS_{Er}}$
Interaction effect BC	1	SS_{BC}	SS_{BC}	$F_{BC}=\frac{SS_{BC}}{SS_{Er}}$
Interaction effect BD	1	SS_{BD}	SS_{BD}	$F_{BD}=\frac{SS_{BD}}{SS_{Er}}$
Interaction effect BE	1	SS_{BE}	SS_{BE}	$F_{BE}=\frac{SS_{BE}}{SS_{Er}}$
Interaction effect CD	1	SS_{CD}	SS_{CD}	$F_{CD}=\frac{SS_{CD}}{SS_{Er}}$
Interaction effect CE	1	SS_{CE}	SS_{CE}	$F_{CE}=\frac{SS_{CE}}{SS_{Er}}$
Interaction effect DE	1	SS_{DE}	SS_{DE}	$F_{DE}=\frac{SS_{DE}}{SS_{Er}}$
Error	$(r-1)(t-1)$	SS_{Er}	$\frac{SS_{Er}}{(r-1)(t-1)}$	-
Total	$rt-1$	SS_T	-	-

Note: Souces of Variation – SV and Degrees of Freedom – df, Sum of Squares – SS and Mean Sum of Squares - MSS.

Inference: If the calculated value is less than the table value, then there is no significant difference between the replications and factors.

3.3 Statistical Analysis of FFD with α – Interval Method

If the sample observations are in the form of TrFNs, that is, if the yield of a particular plot (cell) receives the value in the form of (a,b,c,d), then it is converted to interval model to analyze factorial model using TrFNs $\alpha-$ cut relation

$$\tilde{y}_{ij}=[a_{ij}+\alpha(b_{ij}-a_{ij}),d_{ij}-\alpha(d_{ij}-c_{ij})];\qquad i=1,...,t;j=1,...,r \tag{3}$$

where, $\tilde{y}_{ij}$ is the observation corresponding to i^{th} treatment and j^{th} replicate; $a_{ij}+\alpha(b_{ij}-a_{ij})$ is the lower level of the observed interval in i^{th} treatment and j^{th} replicate; $d_{ij}-\alpha(d_{ij}-c_{ij})$ is the upper level of the observed interval in i^{th} treatment and j^{th} replicate and now, split this expression into two levels (lower level and upper level) as $\tilde{y}_{ij}{}^{L}=a_{ij}+\alpha(b_{ij}-a_{ij})$ and $\tilde{y}_{ij}{}^{L}=a_{ij}+\alpha(b_{ij}-a_{ij}).$

Hypothesis: The null hypothesis $H_0:\mu_1=\mu_2=...=\mu_i$ against alternative hypothesis $H_1:\mu_1\neq\mu_2\neq...\neq\mu_i$. The crisp hypothesis is then converted into the fuzzy hypothesis for lower and upper-level models $H_0{}^{L},H_0{}^{U}:\mu_1{}^{L},\mu_1{}^{U}=\mu_2{}^{L},\mu_2{}^{U}=...=\mu_i{}^{L},\mu_i{}^{U}$ against $H_1{}^{L},H_1{}^{U}:\mu_1{}^{L},\mu_1{}^{U}\neq\mu_2{}^{L},\mu_2{}^{U}\neq...\neq\mu_i{}^{L},\mu_i{}^{U}$.

Lower Level Model (L.L.M): Let the sum of observations in the i^{th} treatment be $\tilde{y}_{i.}{}^{L}=[a_{i.}+\alpha(b_{i.}-a_{i.})]=T_i^{L}$; the sum of observations in the j^{th} block be $\tilde{y}_{.j}{}^{L}=[a_{.j}+\alpha(b_{.j}-a_{.j})]=R_j^{L}$ where, $i=1,...,t;j=1,...,r.$ Then, the grand total is $G^{L}=\sum\sum y_{ij}$. The sum of squares are $SS_A{}^{L}=\frac{[A]^2}{N}$; $SS_B{}^{L}=\frac{[B]^2}{N}$; and similarly other interactions

are also calculated. $SS_R{}^L = \frac{1}{t}\sum_{j=1}^{r}(y_{.j}{}^L)^2 - \frac{\left(G^L\right)^2}{N}$; $SS_T^L = \sum\sum y_{ij}{}^2 - \frac{\left(G^L\right)^2}{N}$ and

$SS_{Er}^L = SS_T^L - \left[SS_R^L + SS_A^L + SS_B^L + SS_C^L + SS_D^L + SS_E^L + SS_{AB}^L + SS_{AC}^L + SS_{AD}^L + SS_{AE}^L + SS_{BC}^L + SS_{BD}^L + SS_{BE}^L + SS_{CD}^L + SS_{CE}^L + SS_{DE}^L\right]$.

All the calculated values are presented in the ANOVA table as in Table: 3.1.

Upper Level Model (U.L.M): Let the sum of observations in the i^{th} treatment be $\tilde{y}_{i.}{}^U = [a_{i.} + \alpha(b_{i.} - a_{i.})] = T_i^U$; the sum of observations in the j^{th} block be $\tilde{y}_{.j}{}^U = [a_{.j} + \alpha(b_{.j} - a_{.j})] = R_j^U$ where, $i = 1,...,t; j = 1,...,r.$ Then, the grand total is $G^U = \sum_{i=1}^{t}\sum_{j=1}^{r} y_{ij}$. The sum of squares are $SS_A{}^U = \frac{[A]^2}{N}$; $SS_B{}^U = \frac{[B]^2}{N}$; and so on;

$SS_R{}^U = \frac{1}{t}\sum_{j=1}^{r}(y_{.j}{}^U)^2 - \frac{\left(G^U\right)^2}{N}$; $SS_T^U = \sum\sum y_{ij}{}^2 - \frac{\left(G^U\right)^2}{N}$ and

$SS_{Er}^U = SS_T^U - \left[SS_R^U + SS_A^U + SS_B^U + SS_C^U + SS_D^U + SS_E^U + SS_{AB}^U + SS_{AC}^U + SS_{AD}^U + SS_{AE}^U + SS_{BC}^U + SS_{BD}^U + SS_{BE}^U + SS_{CD}^U + SS_{CE}^U + SS_{DE}^U\right]$.

All the calculated values are presented in the ANOVA table as in Table: 3.1.

Decision Rule

Lower-Level Model (L.L.M)

If the calculated value is less than the *F* table value, then the null hypothesis is accepted. That is, the effects due to treatments are equal.

Upper-Level Model (U.L.M)

If the calculated value is less than the *F* table value, then the null hypothesis is accepted. That is, the effects due to treatments are equal.

The partial acceptance of the null hypothesis in lower and upper-level models will be considered as null hypothesis is accepted.

3.4. Advantages of Fuzzy Fractional Factorial Design

- It reduces cost and time when compared to other experimental designs.
- It is used to optimize yield with minimum defects.
- It also reduces the non-experimental type of errors when handling a big experiment.

4. Applications

Example 4.1

The following table 4.1 shows the yield of mustard seeds with five fertilizers by investigating a 2^{5-1} design to improve the yield. The five fertilizers were Farm Yard Manure (FYM) (17 Quintel, 25 Quintel), Nitrogen (120 Kg/Ha, 130 Kg/Ha), Phosphorus (40 Kg/Ha, 50 Kg/Ha), Potassium (60 Kg/Ha, 70 Kg/Ha) and Calcium (10 Kg/Ha, 15 Kg/Ha). Test whether there is a significant difference between the factors A - FYM, B – Nitrogen, C – Phosphorus, D – Potassium and E -Calcium or not?

Table: 4.1 *The yield of mustard seeds with five fertilizers*

Treatment Combination	Response 1	Response 2	Response 3
e	(7,9,11,13)	(8,10,11,13)	(8,9,12,14)
a	(9,11,12,14)	(9,11,13,14)	(8,10,12,13)
b	(31,33,35,36)	(30,33,34,35)	(30,32,33,35)
abe	(53,55,56,58)	(52,54,56,57)	(49,51,52,54)
c	(14,15,16,17)	(16,18,19,21)	(13,16,17,19)
ace	(20,22,23,26)	(24,26,27,29)	(21,23,24,25)

bce	(42,45,46,47)	(41,43,46,47)	(42,44,45,47)
abc	(58,60,61,63)	(54,57,58,60)	(55,58,59,62)
d	(7,8,9,10)	(9,12,13,14)	(5,9,10,12)
ade	(11,13,15,16)	(14,16,17,18)	(10,13,15,17)
bde	(30,31,32,34)	(28,29,30,32)	(31,33,35,36)
abd	(50,51,53,54)	(52,54,55,57)	(48,51,52,55)
cde	(14,16,18,20)	(13,16,17,19)	(16,18,19,21)
acd	(20,22,23,25)	(23,26,27,29)	(21,23,24,26)
bcd	(42,44,45,47)	(39,42,43,45)	(42,45,46,47)
abcde	(63,65,67,69)	(61,64,66,68)	(58,61,63,66)

First, the given trapezoidal fuzzy observations are converted into interval data using (3 and are given in the table 4.2.

Table: 4.2 *The interval observations of the TFN data*

Treatment Combination	Response	Treatment Combination	Response
e	$23+5\alpha, 40-6\alpha$	*d*	$21+8\alpha, 36-4\alpha$
a	$26+6\alpha, 41-4\alpha$	*ade*	$35+7\alpha, 51-4\alpha$
b	$91+7\alpha, 106-4\alpha$	*bde*	$89+4\alpha, 102-5\alpha$
abe	$154+6\alpha, 169-5\alpha$	*abd*	$150+6\alpha, 166-6\alpha$
c	$43+6\alpha, 57-5\alpha$	*cde*	$43+7\alpha, 60-5\alpha$
ace	$68+6\alpha, 80-6\alpha$	*acd*	$64+7\alpha, 80-6\alpha$
bce	$125+7\alpha, 141-4\alpha$	*bcd*	$123+9\alpha, 139-5\alpha$
abc	$167+8\alpha, 185-7\alpha$	*abcde*	$185+8\alpha, 203-7\alpha$

Hypothesis H_0^L, H_0^U : There is no significant difference between the factors A (FYM), B (Nitrogen), C (Phosphorus), D (Potassium) and E (Calcium).

Here, the lower-level and upper-level models are calculated separately per the methodology constructed.

Lower-Level Model (L.L.M.)

The effects and sum of squares for the main effects (A, B, C, D, E) and the two-factor interactions ($AB, AC, AD, AE, BC, BD, BE, CD, CE, DE$) of the L.L.M. is calculated and given in the table 4.3.

Table: 4.3 *Effects and sum of squares of the main effects and interactions L.L.M.*

Variable	Estimated Effect	Sum of Squares
A	$\frac{1}{8}\left[70756+532\alpha+\alpha^2\right]$	$\frac{1}{48}\left[70756+532\alpha+\alpha^2\right]$
B	$\frac{1}{8}\left[617796+4716\alpha+9\alpha^2\right]$	$\frac{1}{48}\left[617796+4716\alpha+9\alpha^2\right]$
C	$\frac{1}{8}\left[41616+3672\alpha+81\alpha^2\right]$	$\frac{1}{48}\left[41616+3672\alpha+81\alpha^2\right]$
D	$\frac{1}{8}\left[1444+380\alpha+25\alpha^2\right]$	$\frac{1}{48}\left[1444+380\alpha+25\alpha^2\right]$
E	$\frac{1}{8}\left[144-168\alpha+49\alpha^2\right]$	$\frac{1}{48}\left[144-168\alpha+49\alpha^2\right]$
AB	$\frac{1}{8}\left[36100+380\alpha+\alpha^2\right]$	$\frac{1}{48}\left[36100+380\alpha+\alpha^2\right]$

AC	$\frac{1}{8}\left[256+32\alpha+\alpha^2\right]$	$\frac{1}{48}\left[256+32\alpha+\alpha^2\right]$
AD	$\frac{1}{8}\left[2500-100\alpha+\alpha^2\right]$	$\frac{1}{48}\left[2500-100\alpha+\alpha^2\right]$
AE	$\frac{1}{8}\left[64+112\alpha+49\alpha^2\right]$	$\frac{1}{48}\left[64+112\alpha+49\alpha^2\right]$
BC	$\frac{1}{8}\left[784+504\alpha+81\alpha^2\right]$	$\frac{1}{48}\left[784+504\alpha+81\alpha^2\right]$
BD	$\frac{1}{8}\left[324+252\alpha+49\alpha^2\right]$	$\frac{1}{48}\left[324+252\alpha+49\alpha^2\right]$
BE	$\frac{1}{8}\left[1024-192\alpha+9\alpha^2\right]$	$\frac{1}{48}\left[1024-192\alpha+9\alpha^2\right]$
CD	$\frac{1}{8}\left[1296+216\alpha+9\alpha^2\right]$	$\frac{1}{48}\left[1296+216\alpha+9\alpha^2\right]$
CE	$\frac{1}{8}\left[196-84\alpha+9\alpha^2\right]$	$\frac{1}{48}\left[196-84\alpha+9\alpha^2\right]$
DE	$\frac{1}{8}\left[576+48\alpha+\alpha^2\right]$	$\frac{1}{48}\left[576+48\alpha+\alpha^2\right]$

By screening experiments, the factors with larger effects are considered and their sum of squares is presented in the ANOVA table (table 4.4).

Table: 4.4 *ANOVA for L.L.M.*

SV	df	SS	MSS	F – Ratio
Replications	2	$\frac{1}{48}[402-102\alpha+200\alpha^2]$	$\frac{1}{2(48)}[402-102\alpha+200\alpha^2]$	$\frac{41(402-102\alpha+200\alpha^2)}{2(-12287+32215866\alpha+883\alpha^2)}$ $<1\forall 0\le\alpha\le 1$
Main effect A	1	$\frac{1}{48}\left[70756+532\alpha+\alpha^2\right]$	$\frac{1}{48}\left[70756+532\alpha+\alpha^2\right]$	$\frac{41(70756+532\alpha+\alpha^2)}{(-12287+32215866\alpha+883\alpha^2)}$ $<1\forall 0\le\alpha\le 1$
Main effect B	1	$\frac{1}{48}\left[617796+4716\alpha+9\alpha^2\right]$	$\frac{1}{48}\left[617796+4716\alpha+9\alpha^2\right]$	$\frac{41(617796+4716\alpha+9\alpha^2)}{(-12287+32215866\alpha+883\alpha^2)}$ $<1\forall 0.8\le\alpha\le 1$
Main effect C	1	$\frac{1}{48}\left[41616+3672\alpha+81\alpha^2\right]$	$\frac{1}{48}\left[41616+3672\alpha+81\alpha^2\right]$	$\frac{41(9409+194\alpha+\alpha^2)}{(-12287+32215866\alpha+883\alpha^2)}$ $<1\forall 0\le\alpha\le 1$
Interaction effect AB	1	$\frac{1}{48}\left[36100+380\alpha+\alpha^2\right]$	$\frac{1}{48}\left[36100+380\alpha+\alpha^2\right]$	$\frac{41(36100+380\alpha+\alpha^2)}{(-12287+32215866\alpha+883\alpha^2)}$ $<1\forall 0\le\alpha\le 1$
Error	41	$\frac{1}{48}\left[-12287+32215866\alpha+883\alpha^2\right]$	$\frac{1}{41(48)}\left[-12287+32215866\alpha+883\alpha^2\right]$	-
Total	47	$\frac{1}{48}\left[754383-32827242\alpha+1175\alpha^2\right]$	-	-

Note: Souces of Variation – SV and Degrees of Freedom – df, Sum of Squares – SS and Mean Sum of Squares - MSS.

Inference: For replications, the table value is $F_t(2,41)=3.23$ and for the treatments, the table value is $F_t(1,41)=4.08$. When comparing the calculated values with these table values, it is less. Therefore, there is no significant difference between the factors A (FYM), B (Nitrogen), C (Phosphorus), D (Potassium) and E (Calcium).

Upper-Level Model (U.L.M.)

The effects and the sum of squares of the main effects and the two-factor interactions of the U.L.M. is given in table 4.5.

Table: 4.5 *Effects and the sum of squares of the main effects and interactions U.L.M.*

Variable	Estimated Effect	Sum of Squares
A	$\frac{1}{8}\left[85849-3516\alpha+36\alpha^2\right]$	$\frac{1}{48}\left[85849-3516\alpha+36\alpha^2\right]$
B	$\frac{1}{8}\left[585225-3060\alpha+4\alpha^2\right]$	$\frac{1}{48}\left[585225-3060\alpha+4\alpha^2\right]$
C	$\frac{1}{8}\left[54289-2796\alpha+36\alpha^2\right]$	$\frac{1}{48}\left[54289-2796\alpha+36\alpha^2\right]$
D	$\frac{1}{8}\left[361-76\alpha+4\alpha^2\right]$	$\frac{1}{48}\left[361-76\alpha+4\alpha^2\right]$
E	$\frac{1}{8}[1225]$	$\frac{1}{48}[1225]$
AB	$\frac{1}{8}\left[31329-2832\alpha+64\alpha^2\right]$	$\frac{1}{48}\left[31329-2832\alpha+64\alpha^2\right]$
AC	$\frac{1}{8}\left[81-144\alpha+64\alpha^2\right]$	$\frac{1}{48}\left[81-144\alpha+64\alpha^2\right]$
AD	$\frac{1}{8}[961]$	$\frac{1}{48}[961]$
AE	$\frac{1}{8}\left[729+108\alpha+4\alpha^2\right]$	$\frac{1}{48}\left[729+108\alpha+4\alpha^2\right]$
BC	$\frac{1}{8}[289]$	$\frac{1}{48}[289]$
BD	$\frac{1}{8}\left[1+8\alpha+16\alpha^2\right]$	$\frac{1}{48}\left[1+8\alpha+16\alpha^2\right]$
BE	$\frac{1}{8}\left[9+12\alpha+4\alpha^2\right]$	$\frac{1}{48}\left[9+12\alpha+4\alpha^2\right]$
CD	$\frac{1}{8}[14400]$	$\frac{1}{48}[14400]$
CE	$\frac{1}{8}\left[121+44\alpha+4\alpha^2\right]$	$\frac{1}{48}\left[121+44\alpha+4\alpha^2\right]$
DE	$\frac{1}{8}\left[2209-188\alpha+4\alpha^2\right]$	$\frac{1}{48}\left[2209-188\alpha+4\alpha^2\right]$

By screening experiments, the factors with more significant effects are considered, and their sum of squares is presented in the ANOVA table (table 4.6).

Table: 4.6 *ANOVA for U.L.M.*

SV	df	SS	MSS	F – Ratio
Replications	2	$\frac{1}{48}\left[146+108\alpha+24\alpha^2\right]$	$\frac{1}{2(48)}\left[146+108\alpha+24\alpha^2\right]$	$\frac{15(146+108\alpha+24\alpha^2)}{(4871-2820\alpha+652\alpha^2)}$ $<1\forall 0\le\alpha\le 0.6$
Main effect A	1	$\frac{1}{48}\left[85849-3516\alpha+36\alpha^2\right]$	$\frac{1}{48}\left[85849-3516\alpha+36\alpha^2\right]$	$\frac{30(10201-606\alpha+9\alpha^2)}{(4871-2820\alpha+652\alpha^2)}$ $0\le\alpha\le 1$
Main effect B	1	$\frac{1}{48}\left[585225-3060\alpha+4\alpha^2\right]$	$\frac{1}{48}\left[585225-3060\alpha+4\alpha^2\right]$	$\frac{30(585225-3060\alpha+4\alpha^2)}{(4871-2820\alpha+652\alpha^2)}$ $0\le\alpha\le 1$
Main effect C	1	$\frac{1}{48}\left[54289-2796\alpha+36\alpha^2\right]$	$\frac{1}{48}\left[54289-2796\alpha+36\alpha^2\right]$	$\frac{30(54289-2796\alpha+36\alpha^2)}{(4871-2820\alpha+652\alpha^2)}$ $0\le\alpha\le 1$
Interaction effect AB	1	$\frac{1}{48}\left[31329-2832\alpha+64\alpha^2\right]$	$\frac{1}{48}\left[31329-2832\alpha+64\alpha^2\right]$	$\frac{30(31329-2832\alpha+64\alpha^2)}{(4871-2820\alpha+652\alpha^2)}$ $0\le\alpha\le 1$
Interaction effect CD	1	$\frac{1}{48}[14400]$	$\frac{1}{48}[14400]$	$\frac{30(14400)}{(4871-2820\alpha+652\alpha^2)}$

$0 \le \alpha \le 1$

Error	30	$\frac{1}{48}\left[4871-2820\alpha+652\alpha^2\right]$	$\frac{1}{30(48)}\left[4871-2820\alpha+652\alpha^2\right]$	-
Total	47	$\frac{1}{48}\left[766367-12120\alpha+816\alpha^2\right]$	-	-

Note: Souces of Variation – SV and Degrees of Freedom – df, Sum of Squares – SS and Mean Sum of Squares - MSS.

Inference: For replications, the table value is $F_t(2,30)=3.32,$ and for the treatments, the table value is $F_t(1,30)=4.17.$ When comparing the calculated values with these table values, it is high. Therefore, there is a significant difference between the factors A (FYM), B (Nitrogen), C (Phosphorus), D (Potassium) and E (Calcium).

4. Conclusion

This paper proposes a new method of FFD using trapezoidal fuzzy numbers. This method is used to deal with imprecise observations. When compared to classical designs, fuzzy FFD yields a cost-benefit relationship. In the yield of mustard seeds with five-fertilizers FYM (17 Quintel, 25 Quintel), Nitrogen (120 Kg/Ha, 130 Kg/Ha), Phosphorus (40 Kg/Ha, 50 Kg/Ha), Potassium (60 Kg/Ha, 70 Kg/Ha) and Calcium (10 Kg/Ha, 15 Kg/Ha), the hypothesis for the L.L.M. is accepted and U.L.M. is rejected. But it is concluded that there is no significant difference between the factors A (FYM), B (Nitrogen), C (Phosphorus), D (Potassium) and E (Calcium), where the hypothesis is partially accepted. From the applications given, it is proved that the factors can be tested at different values of α. This method can be applied in the agricultural field, the engineering field, the medical field and so on. In the future, this work could be extended to one quarter fraction, asymmetrical factorial experiments and some special types of designs.

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