ANALYSIS OF ENCOURAGED ARRIVAL MULTIPLE WORKING VACATION QUEUING MODEL UNDER THE STEADY STATE CONDITION

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Abstract

Businesses typically entice customers with alluring offers and discounts. Encouraged arrivals is the name given to these curious clients. In certain situations, the service offered by queuing models, notably in transportation networks, enables the simultaneous serving of several consumers. In general, closed-form solutions to bulk service queuing models with idle servers are difficult to find. By coordinating the operations at each workstation using the Chapman-Kolmogorov research technique, the main objective of this study is to assess the performance of the car assembly line in order to reduce waiting times. The server is in a busy state, is idle, is regularly busy, and is in a busy state when it breaks down. Performance metrics are being tracked using a multiple working vacation approach. In this study, analysis of encouraged arrival multiple working vacation queuing model under the steady state condition. In this model, we included encouraged arrival. By resolving difference equations and Chapman Kolmogorov balancing equations, the steady state queue size problem is found. Additionally, the server is in the busy, idle state, regular, and breakdown busy states, and performance metrics are conducted. The server was sent for repair and is now completely repaired to avoid the crash at any time. After that, the server continues to offer the service. It is evidently identified that the efficiency level increased while the encouraged arrival is incorporated. The main contribution of this paper is to show the server is in the busy, idle state, regular, and breakdown busy states, and performance metrics efficient level increases. It is found that they offer more efficient results when compared with the Poisson process method

Keywords: Encouraged arrival, steady state, multiple working vacations, single server, queue size.

I. Introduction

There are times when the service provided in queuing models, particularly in transportation systems, allows for the simultaneous serving of many customers. In general, bulk service queuing models with idle servers are challenging to solve in closed form in [1]. A generic class of bulk queues with encouraged input was researched in [12].

A study on the examination of a GI/M/I- queue with several vacations is described in [2]. The Markovian M/ (q, **β**)/1 queuing model while taking several working vacations was also examined in [4]. An M/M/1 lines with working vacations (M/M/1/WV) model was investigated in [15]. A well-considered time-dependent bulk queuing service solution issues with queuing in [3].

Analyses the best management strategy for a heterogeneous M/M/1- queue with server downtime [5]. An analysis of the N-Policy and the M/M/1 queue with numerous working vacations is found in [10]. The queuing procedure using bulk services is examined in [19]. A Study on the Analysis of the M/G/1 Queue's Queue Length Distribution with Working Vacations [6].

Stochastic models with matrix geometric solutions were examined in [18]. GI/Geo/1 queue with several Vacations were investigated in [9]. Working vacation queue and matrix analysis a study was done in [7]. The optimal operation of a Markovian queuing system with a transportable and unreliable server was examined in [14].

The M/M/1-queue with a single working vacation was investigated in [21]. The M/M/1 queue with a single working holiday and setup times were investigated in [22]. A matrix analytical approach for the examination of the M/G/1 queue with exponential working vacations was looked at in [8 and 16]. Recent advancements in the queuing and bulk models were examined in [17]. An M/M/1/N queue system with encouraged arrivals was investigated in [23].

A finite and infinite M/H2/1 queuing system with a detachable, unreliable server was examined in [20]. The M/M(a,b)/1/MWV/Br Model [25] was investigated. Research on an M/G/1 queue with numerous working vacations may be found in [11]. Reducing wait times in an M/M/1/N encouraged arrival line by providing feedback, balking at unpaid customers, and sustaining those customers was investigated in [24]. A break-down-prone portable service station with M/Ek/1 queuing system optimization was examined in [13].

II. Model Recitation

In this model recitation provided by

- This method is encouraged with parameters λ^* (1+ϑ). In the manner of General-Bulk Service-Rule (G-B-S-R), the server handles the customers in batches.
- As a result of this rule, the server only begins to provide service when at least a "q" customer is present.
- The server serves the first β – customers, leaving the others in the line.
- Each batch of units must have a certain minimum and maximum number of "q" units to be used.
- The assumption is that the batch size α(q≤α≤β) will have an accelerated distribution of the parameter and will be an independent random variable with an identical distribution.
- When the server breaks down with the parameter $(1-e^{-bt})$.
- If the queue- size reaches minimum "q" imagine a situation where a server starts offering service while on vacation at a different rate from the standard one.
- The server was sent for repair and is now completely repaired to avoid the crash at any time. After that, the server continues to offer the service.

Let $K_A(t)$ represent the number of customers waiting in line at time t, and C(t) represent one of zero, one, two, or three depending on whether the server is idle, busy, in a typical busy state, or experiencing a breakdown respectively.

Let $M_k(t) = \text{Prob}\{K_A(t) = k, C(t) = 0\}; 0 \le k \le q-1,$

$A_k(t) = \text{Prob}\{K_A(t) = k, C(t) = 1\}; k \ge 0,$

$P_k(t) = \text{Prob}\{K_A(t) = k, C(t) = 2\}; k \ge 0,$

$\Upsilon_k(t) = \text{Prob}\{K_A(t) = k, C(t) = 3\}; k \ge 0,$

The queue-size and the system-size are equal for C(t) = 0.

$$M_k = \lim_{t.\to\infty} M_k(t); A_k = \lim_{t.\to\infty} Q_k(t); P_k = \lim_{t.\to\infty} P_k(t); \Upsilon_k = \lim_{t.\to\infty} \Upsilon_k(t);$$

As an outcome, the Chapman-Kolmogorov equations that satisfy the condition are as follows:

$$\lambda * (1+\vartheta)\ M_0 = \mu.P_0 + \mu_u\ A_0 \quad (1)$$

$$\lambda * (1+\vartheta)\ M_k = \lambda * (1+\vartheta)\ M_{k-1} + \mu\ P_k + \mu_u\ A_k; (1 \le k < q-1), \quad (2)$$

$$(\lambda * (1+\vartheta) + \chi + \mu_u)\ A_0 = \lambda * (1+\vartheta)\ M_{q-1} + \mu \sum_{k=q}^{\beta} A_k, \quad (3)$$

$$(\lambda * (1+\vartheta) + \chi + \mu_u)\ A_k = \lambda * (1+\vartheta)\ A_{k-1} + \mu_u\ A_{k+\beta}; (k \ge 1) \quad (4)$$

$$(\lambda * (1+\vartheta) + \mu + s)\ P_0 = \mu \sum_{k=q}^{\beta} P_k + \chi A_0 + b\Upsilon_0, \quad (5)$$

$$(\lambda * (1+\vartheta) + \mu + s) P_k = \lambda * (1+\vartheta) P_{k-1} + \mu P_{k+\beta} + \chi A_k + b\Upsilon_k; (k \ge 1), \quad (6)$$

$$(\lambda * (1+\vartheta) + b)\Upsilon_0 = sP_0, \quad (7)$$

$$(\lambda * (1+\vartheta) + b)\Upsilon_k = sP_k + \lambda * (1+\vartheta) \sum_{n=1}^{k} \Upsilon_{k-n}\, h_n; (k \ge 1). \quad (8)$$

The forward shifting operator Expectation on $P_k$and A_k are introduced as follows and will be used to solve the steady-state equations:

$Exp(P_k) = P_{k+1}; Exp(A_k) = A_{K+1}; Exp(\Upsilon_k) = \Upsilon_{k+1}; (k \geq 0)$.

Therefore, the homogeneous differential equation is given by equation (4).

$(\lambda *\ (1+\vartheta) + \chi + \mu_u)A_k = \lambda *\ (1+\vartheta)A_{k-1} + \mu_u A_{k+\beta}; (k \geq 1),$

$$\left[\mu_u Exp^{\beta+1} - Exp(\lambda *\ (1+\vartheta) + \chi + \mu_u) + \lambda *\ (1+\vartheta))\right]A_k = 0; (k \geq 0), \quad (9)$$

The difference characteristic equation is provided by

$g(o) = \left[\mu_u o^{\beta+1} - (\lambda *\ (1+\vartheta) + \chi + \mu_u)o + \lambda(1+\vartheta)\right] = 0$

by taking $e(o) = (\lambda *\ (1+\vartheta) + \chi + \mu_u)o; h(o) = \mu_u o^{\beta+1} + \lambda *\ (1+\vartheta)$

It is found that $|h(o)| < |e(o)|$ on $|o| = 1$.

The solution of the homogeneous differential equation is given by,

$$A_k = m_u^k A_0\ ; (k \geq 0) \quad (10)$$

Additionally, equation (6) will be expressed as,

$(\lambda *\ (1+\vartheta) + \mu + s)P_k = \lambda(1+\vartheta)p_{k-1} + \mu P_{k+\beta} + \chi A_k + b\Upsilon_k; (k \geq 1),$

$$\left[\mu Exp^{\beta+1} - Exp(\lambda *\ (1+\vartheta) + \mu + s) + \lambda *\ (1+\vartheta)\right]P_k = -\chi A_{k+1} - bExp(\Upsilon_k); (k \geq 0). \quad (11)$$

By applying, Rouche's theorem, we discover that the equation,

$\mu o^{\beta+1} - (\lambda *\ (1+\vartheta) + \mu + s)o + \lambda *\ (1+\vartheta) = 0$ has unique root with $|m| < 1$provided by $\frac{\lambda(1+\vartheta)}{\beta\mu}<1$.

Equation (8) will be expressed as follows:

$(\lambda *\ (1+\vartheta) + b)\Upsilon_k = sP_k + \lambda *\ (1+\vartheta)\Upsilon_{k-1}; (k \geq 1)$

$$\Upsilon_k = \frac{sExp(P_k)}{(\lambda *(1+\vartheta)+b)Exp - \lambda *(1+\vartheta)}. \quad (12)$$

By substituting (11) to (12),

We obtain,

$$\left[\mu Exp^{\beta+1} - Exp(\lambda *\ (1+\vartheta) + \mu + s) + \lambda *\ (1+\vartheta)\right]P_k = -\chi m_u^{k+1}A_0 - b\left[\frac{sExp^2(P_k)}{(\lambda *(1+\vartheta)+b)Exp - \lambda *(1+\vartheta)}\right]. \quad (13)$$

Consequently, the non-homogeneous difference equation (13) has the following solution:

$$P_k = \left[Qm^k - \frac{\chi m_u^{k+1}}{\mu m_u^{\beta+1} - (\lambda *\ (1+\vartheta) + \mu + s)m_u + \lambda *\ (1+\vartheta) + \frac{sbm_u^2}{(\lambda *\ (1+\vartheta) + b)m_u - \lambda *\ (1+\vartheta)}}\right]A_0$$

$$\text{(i.e)}P_k = (Q\ m^k + \Upsilon\ m_u^k). \quad (14)$$

The sequence $M_k (0 \leq k \leq q-1)$ for the condition equation (1) & (2) adding, we get

$\lambda *\ (1+\vartheta)\sum_{n=0}^{k} M_k = \lambda *\ (1+\vartheta)\sum_{n=0}^{k-1} M_N + \mu\sum_{N=0}^{k} P_N + \mu_u\sum_{N=0}^{k} A_n,$

$$\lambda *\ (1+\vartheta)\ M_k = \mu\sum_{n=0}^{k} P_N + \mu_u\sum_{N=0}^{k} A_N.$$

It is discovered by the interchange A_k and P_k in equations (9) and (14) that

$\lambda *\ (1+\vartheta)\ M_k = [\mu(Qm^k + \Upsilon m_u^k) + \mu_u(m_u^k)]A_0,$

$$M_k = \left[\frac{\mu}{\lambda *\ (1+\vartheta)}\left(\frac{Q(1-m^{k+1})}{(1-m)} + \frac{\Upsilon(1-m_u^{k+1})}{1-m_u}\right) + \frac{\mu_u}{\lambda *\ (1+\vartheta)}\left(\frac{1-m_u^{k+1}}{1-m_u}\right)\right]A_0.$$

Since Q and A_0 are unknowns, the steady-state queue size probabilities are also unknown.

To determine Q, we now take into account the equation (5),

$$(\lambda *\ (1+\vartheta) + \mu + s)P_0 = \mu\sum_{k=q}^{\beta} P_k + \chi A_0 + b\Upsilon_0$$

Equations (14) and (12)'s P_k will be substituted to determine that

$$(\lambda *\ (1+\vartheta) + \mu + s)(Q + \Upsilon)A_0 = \mu\left(Q\left[\frac{m^q - m^{\beta+1}}{1-m}\right] + \Upsilon\left[\frac{m_u^q - m_u^{\beta+1}}{1-m_u}\right]\right)A_0 + \chi A_0 + b\left(\frac{sP_1}{(\lambda *(1+\vartheta)+b)m_u - \lambda *(1+\vartheta))}\right).$$

It is expressed as,

$$Q\frac{\mu(1-m^q)}{(1-m)}=\frac{\chi}{(1-m_u)}-\frac{\Upsilon\mu(1-m_u^q)}{(1-m_u)}+b\left(\frac{sp_1}{(\lambda*\ (1+\vartheta)+b)(Exp-\lambda*\ (1+\vartheta))}\right)$$

$$Q=\left[\frac{\mu(1-m^q)}{(1-m)}-\frac{sbm}{\lambda*(1+\vartheta)(m-1)+bm}\right]^{-1}\left[\frac{\chi}{(1-m_u)}-\frac{\Upsilon\mu(1-m_u^q)}{(1-m_u)}+\frac{sb\Upsilon m_u}{\lambda*(1+\vartheta)(m_u-1)+bm_u}\right].$$

As an outcome, the steady-state queue-size probabilities A_0 and are provided by,

$$A_k=m_u^kA_0;(k\geq 0), \quad (15)$$

$$P_k=(Qm^k+\Upsilon m_u^k)A_0;(k\geq 0), \quad (16)$$

$$Q=\left[\frac{\mu(1-m^q)}{(1-m)}-\frac{sbm}{\lambda*(1+\vartheta)(m-1)+\Upsilon m}\right]^{-1}\left[\frac{\chi}{(1-m_u)}-\frac{\Upsilon\mu(1-m_u^q)}{(1-m_u)}+\frac{sb\Upsilon m_u}{\lambda*(1+\vartheta)(m_u-1)+bm_u}\right].$$

Where,

$$\Upsilon=\frac{\chi m_u(\lambda(1+\vartheta)\ (m_u-1)+bm_u)}{\left[\lambda*(1+\vartheta)\ (m_u-1)+\mu m_u\left(1-m_u^\beta\right)+sm_u\right][\lambda*(1+\vartheta)(m_u-1)+bm_u]-sbm_u^2}$$

$$M_k=\left[\frac{\mu}{\lambda*(1+\vartheta)}\left(\frac{Q(1-m^{k+1})}{(1-m)}+\frac{\Upsilon(1-m_u^{k+1})}{(1-m_u)}\right)+\frac{\mu_u}{\lambda*(1+\vartheta)}\left(\frac{1-m_u^{k+1}}{1-m_u}\right)\right]A;(0\leq k\leq q-1). \quad (17)$$

Using the normalizing condition, the expression for A_0 is determined as follows:

$$\sum_{k=0}^{\infty}A_k+\sum_{k=0}^{\infty}P_k+\sum_{k=0}^{q-1}M_k+\sum_{k=0}^{\infty}\Upsilon_k=1.$$

Which follows that,

$$A_0^{-1}=E(m_u,\mu_u)+QE(m,\mu)+\Upsilon E(m_u,\mu)+QS(m,s)+\Upsilon S(m_u,s).$$

Where $E(\pi,\delta)=\frac{1}{(1-\pi)}\left[1+\frac{\delta}{\lambda*(1+\vartheta)}\left(q-\frac{\pi(1-\pi^q)}{(1-\pi)}\right)\right]$ and $S(\pi,\delta)=\frac{1}{(1-\pi)}\left[\frac{\pi\delta}{\lambda*(1+\vartheta)(\pi-1)+b\pi}\right]$.

Thus, the value A_0^{-1} is evaluated.

III. Evaluating Performance

In this section, we have performance metrics of M/(q, **β**)/1/MWV/ϒm computed.

The expected queue- length (ι_a) is,

$$\iota_a=\sum_{k=1}^{\infty}k(A_k+P_k+\Upsilon_k)+\sum_{k=1}^{q-1}kM_k. \quad (18)$$

By Substituting the values of A_k , P_k, $\Upsilon_k$and M_k from (15) to (17), we get

$$\iota_a=\sum_{k=1}^{\infty}k\left(m_u^k+(Qm^k+\Upsilon m_u^k)\right)+\sum_{k=1}^{q-1}k\left[\frac{\mu}{\lambda*(1+\vartheta)}\left(\frac{Q(1-m^{k+1})}{(1-m)}+\frac{\Upsilon(1-m_u^{k+1})}{(1-m_u)}\right)+\frac{\mu_u}{(\lambda*(1+\vartheta)}\left(\frac{(1-m_u^{k+1})}{(1-m_u)}\right)\right]+$$
$$\sum_{k=1}^{\infty}k\left(\frac{smQm^k}{(\lambda*(1+\vartheta)+b)m-\lambda(1+\vartheta)}+\frac{sm_u\Upsilon m_u^k}{(\lambda*(1+\vartheta)+b)m_u-\lambda(1+\vartheta)}\right).$$

Moreover, ι_a can be simplified as,

$$\iota_a=QG(m,\mu)+\Upsilon G(m_u,\mu)+G(m_u,\mu_u)+QC(m,s)+\Upsilon C(m_u,s). \quad (19)$$

Where $G(\pi,\delta)=\frac{\pi}{(1-\pi)^2}+\frac{\delta}{\lambda(1+\vartheta)(1-\pi)}\left\{\frac{q(q-1)}{2}+\frac{q\pi^{q+1}(1-\pi)-\pi^2(1-\pi^2)}{(1-\pi)^2}\right\}$,

$$C(\pi,\delta)=\frac{\pi}{(1-\pi)^2}\left[\frac{\pi\delta}{\lambda(1+\vartheta)(\pi-1)+b\pi}\right].$$

Now P_u, $P_{by}(busy)$, $P_{ie}(idle)$ and $P_{\beta m}$ are given by,

$$P_u=\sum_{k=0}^{\infty}A_k=\frac{A_0}{(1-m_u)},$$

$$P_{by}=\sum_{k=0}^{\infty}P_k=\sum_{k=0}^{\infty}(Qm^k+\Upsilon m_u^k)A_0=\left[\frac{Q}{(1-m)}+\frac{\Upsilon}{(1-m_u)}\right]A_0,$$

$$P_{ie}=\sum_{k=0}^{q-1}M_k,$$

$$P_{\beta m}=\sum_{k=0}^{\infty}\Upsilon_k=\frac{Qsm}{(\lambda*\ (1+\vartheta)+b)m-\lambda*\ (1+\vartheta)}\left[\frac{m}{(1-m)^2}\right]+\frac{\Upsilon sm_u}{(\lambda*\ (1+\vartheta)+b)m_u-\lambda*\ (1+\vartheta)}\left[\frac{m_u}{(1-m_u)^2}\right].$$

IV. Conclusion

The analysis of encouraged arrival multiple working vacation queuing model under the steady state condition is discussed in this study in the context of the server being busy, idle state, regular, breakdown busy–state. Using Chapman Kolmogorov balancing equations, the total probability generating function was determined for this model. This model is more effective than the comparative Poisson arrival model [25].In the future to be included EASTA property for this encouraged arrival multiple vacation queuing model.

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