FAILURE RATE ESTIMATION BY WEIBULL DISTRIBUTION IN A STOCHASTIC ENVIRONMENT: APPLICATION TO THE HEMODIALYSIS MACHINE

Sofiene Fenina, Souheyl Jendoubi, Faker Bouchoucha,

Laboratoire de recherche LR-18ES45 Physique, Mathématique, Modélisation Quantique et Conception Mécanique, Institut Préparatoire aux Etudes d'Ingénieurs de Nabeul (IPEIN), université de Carthage, Tunisie, Campus Mrezgua, 8000 Nabeul.

E-mail address of the corresponding author: fakersbouchoucha@yahoo.fr

sofienefenina@yahoo.fr, jendoubi1souheyl@gmail.com

Abstract

This paper presented a study of the failure rate by introducing the effect of influencing variables. These variables have a random effect which depends on the external environment of the system. There are a multitude of variables and their modeling is difficult. The perturbation, to the failure rate, caused by external factors, has a direct impact on the time scale by the acceleration (or deceleration) of the degradation of the system. Therefore, the adopted methodology consists in introducing a perturbation on the Weibull parameters and studying its effect on the failure rate. Weibull parameters are considered random variables with a Gaussian distribution. The failure rate formulation in a random environment is offered through Weibull distribution. A case study of the hemodialysis machine is offered to illustrate the proposed approach and validate the results. The simulations presented show the failure rate statistics for different configurations of the Weibull distribution. The validation of the results was done using Monte Carlo simulations.

Keywords: Failure rate, Uncertain Factor, Gaussian perturbation, Weibull distribution

1. Introduction

Reliability is the probability that a system will perform a required function for a given period of time, under specified operating conditions [1, 2]. The conditions are all external constraints, whether mechanical, chemical, atmospheric, human, others. Dynamic reliability expands the static concept of failure by considering it dynamically over time [3]. This development may be due to variations in parameters influencing dysfunctional behavior (dynamic process, aging, etc.), modification of the system structure or environmental constraints [4].

The reliability and the functioning safety of the medical material are essential to ensure that a machine functions in accord with constructor instructions and assurances the patients and operators security. Damage of medical equipment may touch the healthcare services efficiency and causes severe harm to the patients and troubles the environment [5]. Scientific research plays an important role in today's society faced with major global challenges such as climate change, eradicating poverty and improving the quality of healthcare.

There are several methods and procedures employed to develop the reliability of the medical equipment. Dhillon [6] defined the maintenance of medical equipment as all actions necessary for retaining in, or returning to, a specified functioning condition. The aim of the study presented by Bahreini [7] is the extraction of the factors affecting the medical equipment reliability. The objective of the work presented by Khalaf [8] is the development of a mathematical approach that studies the influence of maintenance on the survival probability of medical equipment based on operating history and useful life.

Failure is a partial or total loss of the properties of an element which significantly decrease and leads to the total loss of its operating capacity. This failure may be due to its design, manufacture, installation, or even maintenance [9, 10, 11, 12]. Any production systems are subject to aging and wear [13]. These physical phenomena cause the failure, which has a significant impact on the cost of operating the system or on security.

In the literature, several authors have presented numerous classifications of failures. For example, Rausand [14] classified failures by cause, time, detectability, and degree. Deloux [15] categorized failures according to cause, on the one hand, and the impact on system performance, on the other. The classification of failures by cause differentiates between random and systematic failures. Classification of failures based on their impact on system performance distinguishes intermittent failures from extensive failures.

It is found that the failure rate in the different reliability databases vary significantly [16]. The causes of these discrepancies are numerous, for example, because of the characteristics of the equipment, the operating conditions, and the operating environment, the lack of precision in the information supplied and the increasing complexity of a reliable evaluation of equipment comparable to that of a component.

Stochastic degradation models are mathematical models that describe the degradation of the system over time. Degradation models were proposed as a tool to describe the state of a production system, to constitute a maintenance policy, to measure system availability and to obtain optimal maintenance periods [17].

The mechanisms of component degradation (operating conditions, fatigue, vibrations and other stresses, etc.) lead to time-dependent modeling of failure rates. Many studies show the impact that aging mechanical systems have on reliability as demonstrated [18]. Mechanical components are characterized by multiple, often complex degradation mechanisms of various origins (cracking, creep, wear, fatigue cracking, etc.) [19, 20]. These degradation modes include several parameters like material and dimensional characteristics, external stresses, etc. [21].

In some studies [22], two main types of models associated with the effects of aging factors are identified: physical models and empirical models. Booher [23] destined three models of degradation: shock model, wear model and hybrid model which combines the two processes (shock and wear). Degradation models may also be classified, according to Deloux [15], in two categories: discrete degradation models and continuous degradation models.

Influence factors are either internal or external factors that affect the reliability of the system. The influence can be positive by causing a reduction in the number of failures or, on the contrary, have negative effects on reliability. Depending on the type of system studied (human, electric/ electronic, mechanical), observed factors are generally different (human or organizational factors, system intrinsic or extrinsic factors). A classification of these factors, based on the life stages of the system under consideration, was proposed by Brissaud [24]: design factors, manufacturing factors, system installation factors, factors which influence system usage and maintenance factors. We can add to this list human and organizational factors that generally have a broader impact on the system [25].

The influence coefficients are calculated using physical relations, which are functions of many parameters like temperature, pressure, dimensions, properties of fluids and materials, etc. For many electrical and electronic components, failure rates, constant over time, are expressed analytically based on defined parameters. Couallier [26] developed a model adapted to the aeronautical maintenance data of on-board systems, some components of which are reliable according to the condition of other components. Without a priori knowledge of the physical relationships which link influencing factors to failure rates, statistical methods try to express correction coefficients. Ouakki [27] presented (figure 1) a cause-effect diagram (Ishikawa diagram) in order to better describe the different factors that cause the reliability varies.

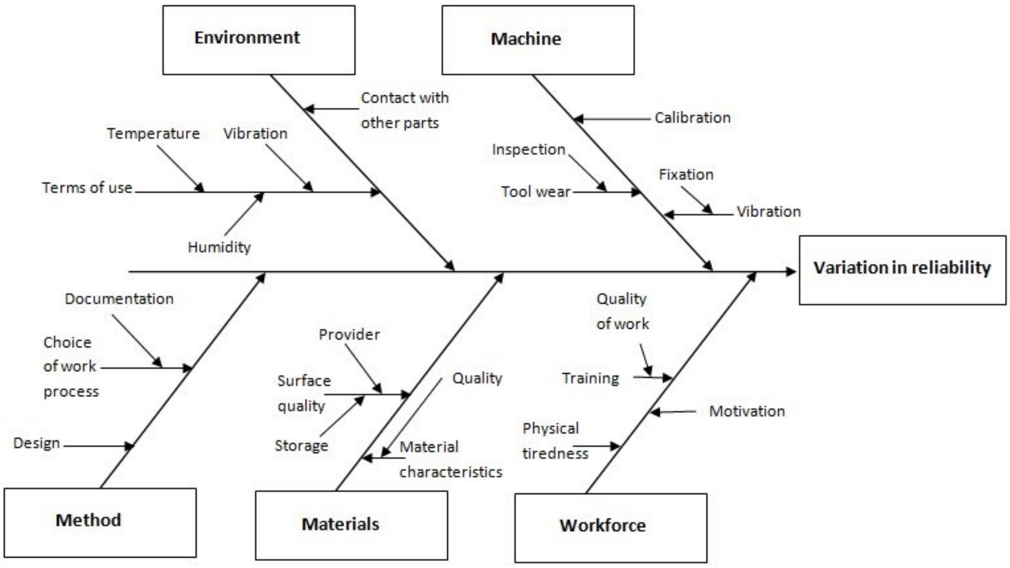

Figure 1: *Cause and effect diagram of the reliability*

The failure rate of mechanical systems is not constant and continually changes over time as a result of degradation phenomena such as wear, aging, etc. Any system is related to its external environment, the consideration of external influence variables allows a more robust modeling of the failure rate of the system. Several researchers proposed a failure rate model taking into account deterioration over time and influence factors where a Cox model is integrated to study the effect of the stress [24, 28, 29]. This is a semi-parametric model which describes the failure rate as a function of the basic failure rate of the system, which depends only on the time, and the influence function which depends only on the state of the influencing factors, it is independent of time. Several researchers have adopted the Cox model to estimate the influence function [24, 29, 30, 31, 32, 33]. The construction of this model requires the collection of input data, the preparation of a coding for the states of the influence factors, the determination of the parameters of the influence function, the determination of the parameters of the base failure rate and finally the synthesis of the results. It should be noted that in this model, the measures are always subject to a certain degree of uncertainty, such that the states of the influence factors can vary over time. The effectiveness of this model is closely associated with the quantity and quality of information available for the study. An analysis of the material is then required to select the influencing factors to consider.

In engineering studies, the distribution that best characterizes a set of data should be chosen [34]. In industry area, the Weibull distribution is one of the most used probability density functions. According to Lyonnet [35], the Weibull model is the best appropriate when carrying out reliability analysis for mechanical components. The main advantage of this distribution is its ability to account for small samples of failure data. Nevertheless the graphical method is recommended in case of little model size data in terms of the estimation precision and accuracy [36]. The flexibility in fitting different failure modes and in simulating many other statistical distributions is one of the important attractions of the mentioned distribution [37]. The Weibull probability analysis is widely employed for studying the life data and can be applied to several situations. Weibull distribution is used in various domains [38] such as aerospace, electronics, materials, automotive industries and civil aviation [39]. This statistical approach can be an important way to analysis the reliability of semi-conductors, ball bearings, engines, spot weldings, biological organisms… [40]. Ahsan [41] studied the reliability of gas turbine using three parameters Weibull distribution based on historical data.

This paper presented a study of failure rate in stochastic environment by introducing the effect of influential variables. These variables have a random effect that depends on the external environment of the system. The external factors, which can influence the reliability of a system, are much diversified and their modeling is difficult. An approach to characterize the failure rate is proposed, that takes into account the consequences of these factors without modeling them. Our methodology consists in introducing a perturbation on the Weibull parameters and studying its effect on the failure rate. This perturbation is the result of influence variables. Weibull parameters are considered random variables with a Gaussian distribution. The formulation of the failure rate in a stochastic environment is developed through the Weibull distribution. A case study is offered to illustrate the proposed approach and validate the results. The simulations presented show the failure rate statistics for different configurations of the Weibull distribution.

2. Failure rate estimation by Weibull distribution in stochastic environment

The two-parameter Weibull distribution is a continuous probability distribution widely used for analyzing reliability and lifetime data [42]. The Weibull distribution is characterized by two parameters β and η, where β is the shape parameter and η is the scale parameter.

The failure rate is expressed through the Weibull distribution by the following function:

$$\lambda(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1} \tag{1}$$

The Weibull distribution is characterized by two parameters (β, η):

β: is the shape parameter, (β >0). This parameter gives indications on the failure mode and on the evolution of the failure rate over time.

η: is the scale parameter, Which specifies the order of magnitude of the average lifespan.

The reliability analysis is based on a deterministic approach. In fact, all reliability parameters, which are uncertain, are described by unfavorable characteristic values. This often leads to unwarranted modeling and dimensioning. In that sense, behavioral prediction should preferably be in terms of probabilities. Uncertainties are related to variability in physical and geometric parameters, to fluctuations in load conditions, to stress boundary conditions and also to physical laws and simplifying assumptions used in the modeling process [21]. It is therefore the analysis of reliability by probabilistic approaches, taking into account the dispersion of the variables described by probabilistic distributions. Several factors contribute to the degradation of the component or entity. Consequently, the lifetimes must be explained by the different variables (degradation factors) contributing to the failure.

Several researchers [22, 29] have considered that the influencing variables mainly affect the weibull parameters. Their effect is to slow down or accelerate the degradation. The influence of these variables is random and hardly modelable. In this work, we will introduce a perturbation in the weibull parameters due to the influencing variables that contribute to the degradation and our objective is to estimate the effect of this perturbation on the failure rate. The shape and scale parameters are modeled by random variables which follow Gaussian distributions and written in the following form:

$$\tilde{\beta} = \beta + \sigma_\beta \varepsilon \quad (2)$$

$$\tilde{\eta} = \eta + \sigma_\eta \varepsilon \quad (3)$$

Where

- $\tilde{\beta}$, $\tilde{\eta}$: Random Weibull parameters following a Gaussian distribution,
- β, η: The mean of the random weibull parameters,
- σ_β, σ_η: The standard deviation of the weibull parameters (disturbance around the mean),
- ε: Reduced centered Gaussian variable,

The deterministic weibull parameters are represented by their means β and η. The perturbation caused by several external influencing factors is characterized by their standard deviation σ_β and σ_η. Hence, our study consists of estimating the effect of this perturbation on the failure rate using the Weibull distribution. The random failure rate is expressed as a random function following a Gaussian distribution as follow:

$$\tilde{\lambda}(t) = \lambda(t) + \sigma_\lambda(t)\varepsilon \quad (4)$$

Where λ and σ_λ are the mean and the standard deviation of the failure rate, respectively.

According to Weibull distribution, the random failure rate is offered by the following expression:

$$\tilde{\lambda}(t) = \frac{\tilde{\beta}}{\tilde{\eta}}\left(\frac{t}{\tilde{\eta}}\right)^{\tilde{\beta}-1} \quad (5)$$

In order to formulate the mean and the standard deviation of the failure rate, we can write:

$$log\left(\tilde{\lambda}(t)\right) = log(\tilde{\beta}) - log(\tilde{\eta}) + (\tilde{\beta} - 1)[log(t) - log(\tilde{\eta})] \quad (6)$$

$$log\left(\tilde{\lambda}(t)\right) = log(\tilde{\beta}) - \tilde{\beta}\, log(\tilde{\eta}) + (\tilde{\beta} - 1)\, log(t) \quad (7)$$

To linearize the failure rate equation, we will use the first-order Taylor series expansion of log ($\tilde{\lambda}$(t)), log ($\tilde{\beta}$) and log ($\tilde{\eta}$) in the vicinity of their mean, we obtain:

$$log\left(\tilde{\lambda}(t)\right) = log(\lambda(t)) + \frac{\sigma_\lambda(t)}{\lambda(t)}\varepsilon \quad (8)$$

$$log(\tilde{\beta}) = log(\beta) + \frac{\sigma_\beta}{\beta}\varepsilon \quad (9)$$

$$log(\tilde{\eta}) = log(\eta) + \frac{\sigma_\eta}{\eta}\varepsilon \quad (10)$$

Introducing equations (8), (9) and (10) in equation (7), we can write:

$$log(\lambda(t)) + \frac{\sigma_\lambda(t)}{\lambda(t)}\varepsilon = [log(\beta) - \beta\, log(\eta) + (\beta - 1)\, log(t)] + \left[\frac{\sigma_\beta}{\beta} - \frac{\beta}{\eta}\sigma_\eta - \sigma_\beta\, log(\eta) + \sigma_\beta\, log(t)\right]\varepsilon \quad (11)$$

The identification of the different terms in equation (11) leads to extract the mean and the standard deviation of the failure rate as:

$$log(\lambda(t)) = log(\beta) - \beta\, log(\eta) + (\beta - 1)\, log(t) \quad (12)$$

And

$$\frac{\sigma_\lambda(t)}{\lambda(t)} = \frac{\sigma_\beta}{\beta} - \frac{\beta}{\eta}\sigma_\eta - \sigma_\beta\, log(\eta) + \sigma_\beta\, log(t) \quad (13)$$

Finally, we obtain:

$$\lambda(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \quad (14)$$

$$\sigma_\lambda(t) = \lambda(t)\left[\frac{\sigma_\beta}{\beta} - \frac{\beta}{\eta}\sigma_\eta - \sigma_\beta\, log(\eta) + \sigma_\beta\, log(t)\right] \quad (15)$$

3. Numerical results and discussion

3.1. Case study: Weibull distribution analysis of the Hemodialysis machine

Hemodialysis machines are one of the important medical equipment which is directly responsible for the patient's life, used to treat kidney failure. The reliability of hemodialysis machines is very important for nephrologists to guarantee not only patient safety but also efficiency and continuity of treatment. Weibull distribution is very flexible and can, through an appropriate choice of parameters, model many types of failure rate behaviors.

In this paragraph, the failure rate of a group of 3 hemodialysis machines (M1, M2 and M3) will be studied. The data of the failure history of the 3 machines was collected during the period from 2013 to 2022. The data of the failure history of the 3 devices was collected and summarized in table 1.

Table 1: *The failure history (TBF) of hemodialysis machines.*

Failure number	M1		M2		M3	
	Date of failure	TBF	Date of failure	TBF	Date of failure	TBF
1	29/01/2014	1085	05/06/2015	2450	29/05/2013	413
2	04/01/2016	1904	29/05/2017	2040.5	12/07/2013	101.5
3	19/10/2016	609	07/08/2017	192.5	01/09/2014	1130.5
4	24/10/2016	10.5	28/02/2018	549.5	05/06/2015	763
5	29/05/2017	605.5	03/08/2018	311.5	09/11/2015	427
6	06/03/2020	2792	06/03/2020	584.5	07/08/2017	1750
7	12/11/2020	392	03/08/2020	280	31/10/2018	1228.5
8					09/08/2019	780.5
9					06/03/2020	560
10					29/07/2020	255.5
11					07/01/2022	1456

The shape and scale parameter will be extracted in order to characterize the failure rate behavior. The cumulative density function (CDF) is formulated using Weibull distribution in the following form:

$$F(t) = 1 - \exp\left(-\frac{t}{\eta}\right)^{\beta} \tag{16}$$

In order to extract the weibull parameters (β and η), we use the linearization of the cumulative density function given in equation (16) as follow:

$$F(t) - 1 = -\exp\left(-\frac{t}{\eta}\right)^{\beta} \tag{17}$$

$$\ln\big(1 - F(t)\big) = -\left(\frac{t}{\eta}\right)^{\beta} \tag{18}$$

$$\ln\big(-\ln\big(1 - F(t)\big)\big) = \ln\left(\frac{t}{\eta}\right)^{\beta} \tag{19}$$

$$\ln\,(\ln\left(\frac{1}{1-F(t)}\right) = \beta\ln(t) - \beta\ln\,(\eta) \tag{20}$$

The CFD equation can be written in the following linear form with a slop of β and an intercept of $\beta\ln(\eta)$:

$$y = \beta x - \beta\ln\,(\eta) \tag{21}$$

Where $y = \ln\,(\ln\left(\frac{1}{1-F(t)}\right)$ and $x = \ln(t)$

Failure times of Hemodialysis Machine were collected and arranged in ascending order to calculate the cumulative density function (CFD), denoted by $F(t_i)$, by using the Bernard's formula to assign median ranks to each failure given as:

$$F(t_i) = \frac{i - 0.3}{n + 0.4} \tag{22}$$

Where i is the order of failure and n is the total number of data

The necessary calculation steps performed to determine the Weibull parameters (β, η) are summarized in table 2.

The linear form of the cumulative density function given in equation (21) is presented in figure 2 for the 3 hemodialysis machines. The shape and scale parameters can be calculated from the linear equation offerred in figure 2. Table 3 shows the Weibull parameters β and η for the 3 studied devices (M1, M2 and M3).

Table 2: *The necessary calculation steps*

Failure number	M1		M2		M3	
	TBFᵢ	*F(ti)*	*TBFᵢ*	*F(ti)*	*TBFᵢ*	*F(ti)*
1	10.5	0,09459459	192,5	0,09459459	101,5	0,061403
2	392	0,22972973	280	0,22972973	255,5	0,149122
3	605.5	0,36486486	311,5	0,36486486	413	0,236842
4	609	0,5	549,5	0,5	427	0,324561
5	1085	0,63513514	584,5	0,63513514	560	0,412280
6	1904	0,77027027	2040,5	0,77027027	763	0,5
7	2792	0,90540541	2450	0,90540541	780,5	0,587719
8					1130,5	0,675438
9					1228,5	0,763157
10					1456	0,850877
11					1750	0,938596

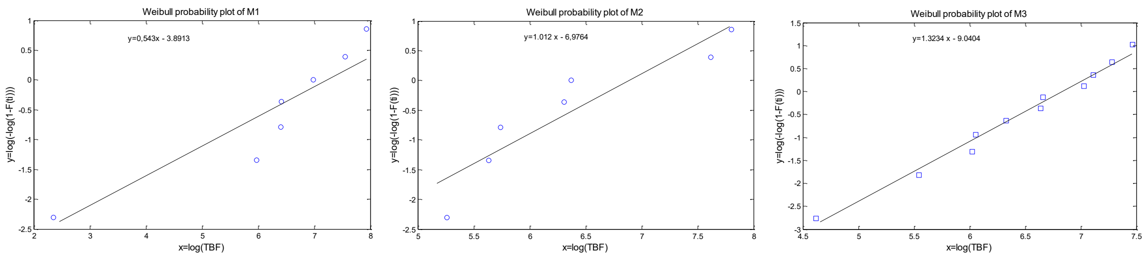

Figure 2: *Weibull probability plot of hemodialysis machines*

Table 3: *The shape and scale parameters of hemodialysis machines*

Machine	*β*	*η*
M1	0,543	1294
M2	1.012	986
M3	1,323	928

The uncertainties in the determination of the Weibull parameters lead to introduce a perturbation in the shape and scale parameters. In this part, the effect of this perturbation on the behavior of the failure rate will be studied. The mean and the standard deviation of the failure rate will be presented in many configurations. The shape and scale parameters, according to the equations (2) and (3), follow a Gaussian distribution as $\tilde{\beta} = \beta + \sigma_\beta \varepsilon$ and $\tilde{\eta} = \eta + \sigma_\eta \varepsilon$.

The simulations are performed for the 3 hemodialysis machines. Figure 3 presents the mean of the failure rate λ(t) expressed using the Weibull distribution by the equation (14). According to this study, the failure rate λ(t) of M3 increases significantly with time. M3 is in the aging life (β>1) and must be supervised continuously. The failure rate of M2 is constant during the time, the machine is in the useful life (β=1). The machine M1, which is in the early-life (β<1), have a decreasing failure rate.

Figure 4 presents the standard deviation of the failure rate for the 3 hemodialysis machines. The simulations were done, according to the equation (15), with a standard deviation introduced in the shape and scale parameters equivalent to: $\sigma_\beta = 10\%\beta$ and $\sigma_\eta = 10\%\eta$. The curves presented in figure 4 illustrate the evolution of the uncertainties in the failure rate for the different phases of the lifetime of the hemodialysis machine. The standard deviation for aging life (M3) is more significant and increases during this mature phase.

To highlight these interpretations, the correlation between the standard deviation and the mean of the failure rate for the 3 hemodialysis machines is studied. According to figure 5, the standard deviation evolves linearly according to the mean, which makes it possible to estimate the variation of the failure rate through the evaluation of the mean. In the aging phase, the effect of the perturbation introduced in the Weibull parameters is more serious and affect perilously the failure rate.

The uncertainty, introduced in the shape parameter, is related to the influencing factors which are ambiguous and described by unfavorable characteristic values. Many case studies are presented in order to illustrate the behavior of the failure rate following different level of uncertainty introduced in the shape parameter. Figure 6 shows the influence of the standard deviation of the shape parameter on the failure rate in the case of the aging life.

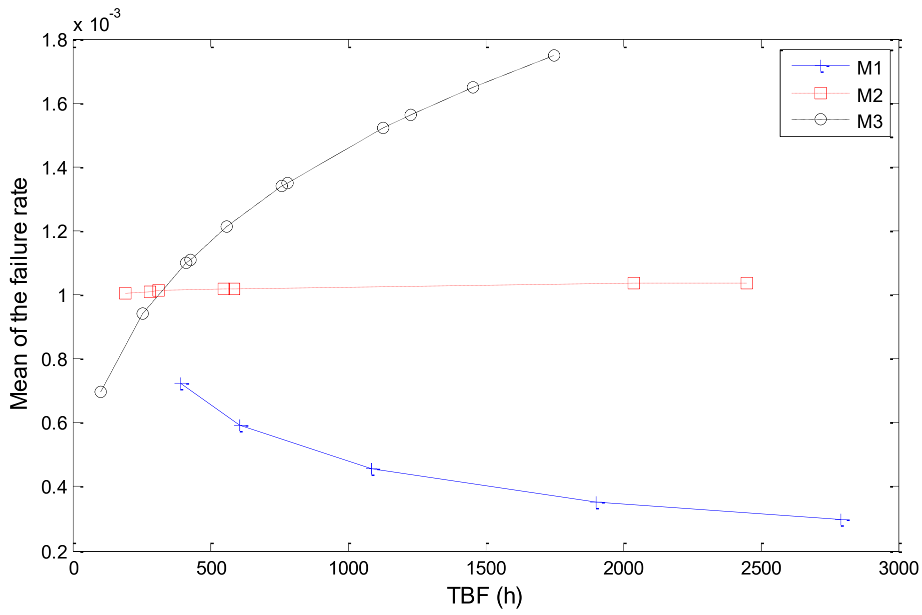

Figure 3: *The mean of the failure rate for the hemodialysis machines*

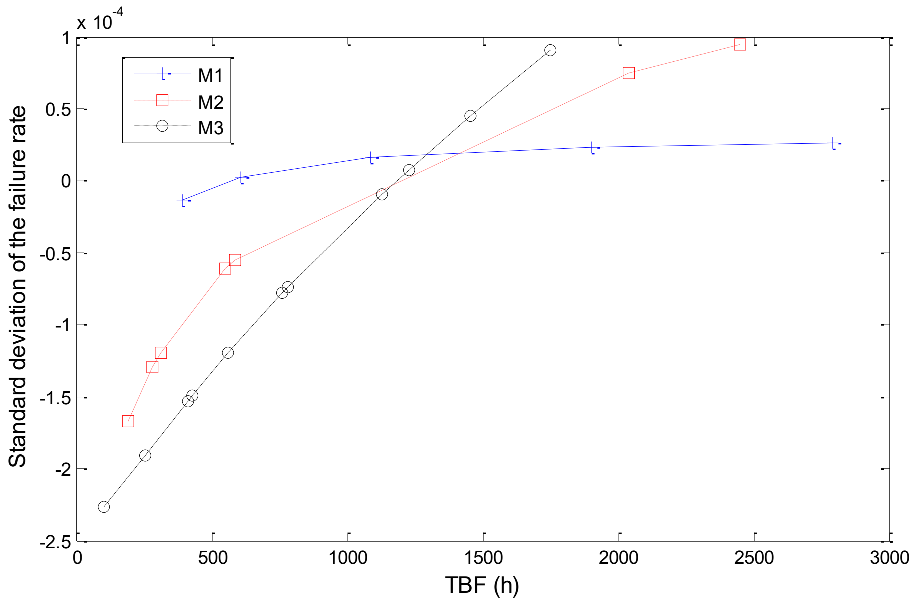

Figure 4: *The standard deviation of the failure rate for the hemodialysis machines*

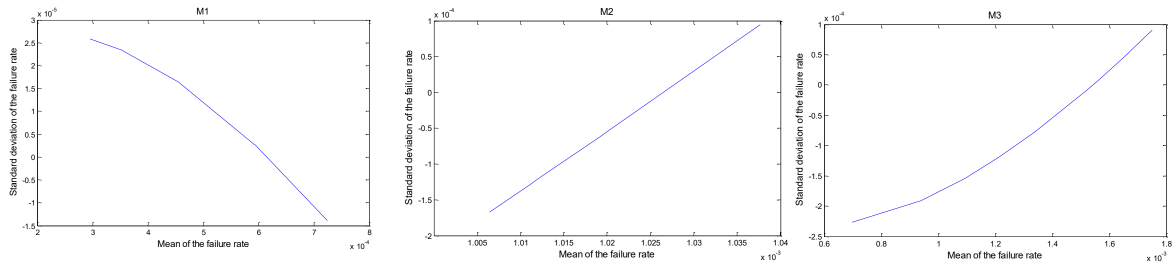

Figure 5: *Correlation between the standard deviation and the mean*

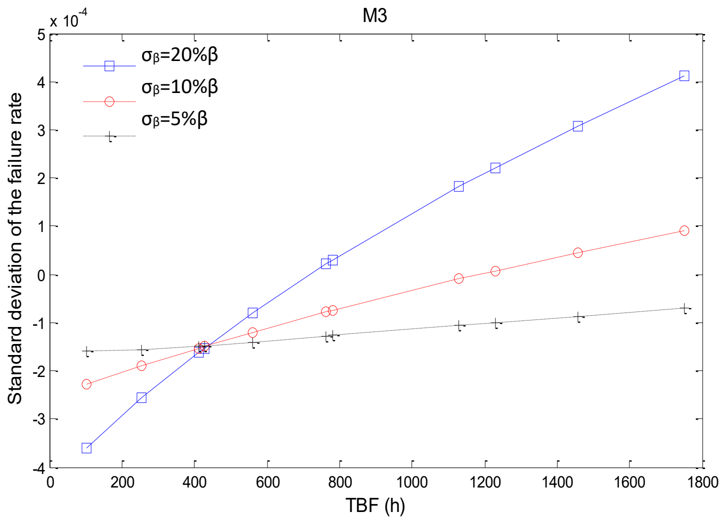

Figure 6: *Influence of the standard deviation of the shape parameter on the failure rate*

In conclusion, the results provide evidence of the propagation of errors that can affect a system and their significant influences on the failure rate. The presence of influencing factors can affect the determination of Weibull parameters and then the failure rate will be significantly affected, in particular, at aging phase. To avoid the inconvenient impact of the uncertainties in the failures on the dialysis system, it is recommended to upgrade the operation management. Moreover, the maintenance strategy must initially focus on the M3 and then at M2, which are on aging phase.

3.2. Validation of the results

The objective of this part is the validation of the results obtained by our proposed method in which the statistics of the failure rate were estimated using the Weibull distribution with uncertain parameters that follow a Gaussian distribution because of various external factors.

Validation will be ensured by comparing the obtained results with Monte Carlo simulations. The Monte Carlo (MC) method can be used as a reference for statistical methods. It consists of producing a Gaussian distribution of time and simulating several draws of the failure rate using the Weibull distribution and deducing the mean by:

$$\lambda(t) = \frac{1}{N}\sum_{i=1}^{N} \lambda_i(t) \quad (23)$$

Where $\lambda_i(t)$ is the failure rate for the i[th] draw at time (t) and N is the total number of draws.

The standard deviation of the failure rate is given by:

$$\sigma_\lambda(t) = \sqrt{\frac{\sum_{i=1}^{N}(\lambda_i(t)-\lambda(t))^2}{N}} \quad (24)$$

During the MC simulations, we performed N=10000 draws to ensure the convergence of the results.

Figure 7 and Figure 8 illustrate the validation of the mean and standard deviation of the failure rate by MC simulations. The simulations were prepared in the case of aging life which presents the most critical case.

To further our research, the probability function of the failure rate was estimated following a Gaussian perturbation introduced into the Weibull parameters. This study is accomplished for the aging life (M3) and at a time corresponding to 1500 hours. According to figure 9, the probability function of the failure rate follows a Gaussian distribution. This numerical result confirms the analytical formulations proposed in this work. The MC simulations show the validity of the results.

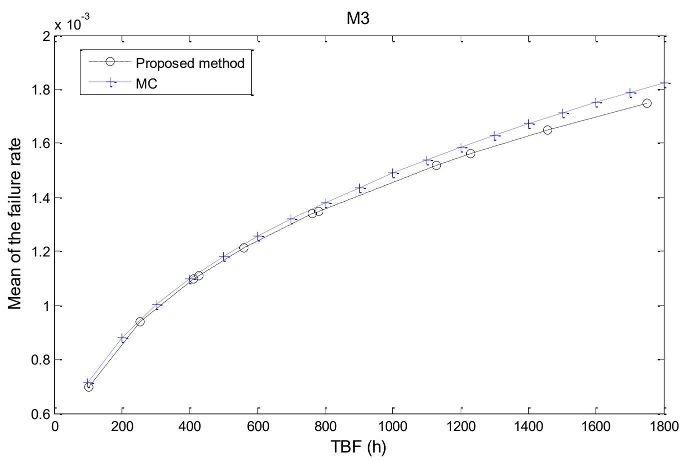

Figure 7: *Validation of the mean of the failure rate for the hemodialysis machine*

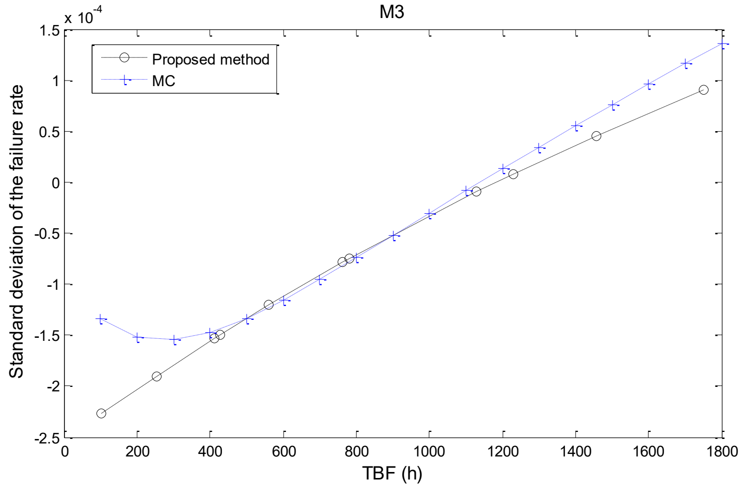

Figure 8: *Validation of the standard deviation of the failure rate for the hemodialysis machine*

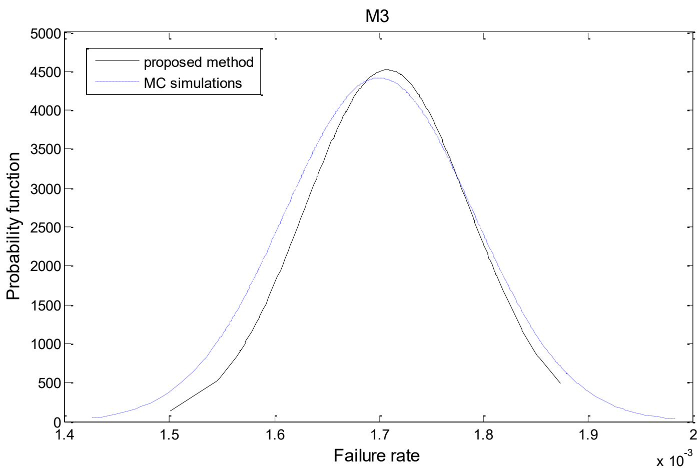

Figure 9: *Probability function of the failure rate*

4. Conclusion and perspectives

In this work, we studied the failure rate by introducing the effect of influencing variables. The effect of these variables is random and depends on the external environment of the system and directly affects the failure rate by accelerating (or decelerating) the degradation of the system.

The methodology adopted consists in introducing a perturbation on the Weibull parameters and studying its effect on the failure rate. Weibull parameters are considered random with a Gaussian distribution. The formulation of the failure rate in a stochastic environment is detailed using Weibull distribution. The simulations presented show the statistics of the failure rate for several configurations of the Weibull distribution. A case study is offered to illustrate the proposed approach and validate the results. The validation of the results was accomplished through Monte Carlo simulations.

This work can be extended along several lines of research such as the study of the reliability of a system in a stochastic environment taking into account external factors, as well as the study of availability in the presence of uncertain external influencing factors. The development of the failure rate following a second-order perturbation introduced in the Weibull parameters can be the subject of an in-depth and precise study.

Acknowledgments

The authors dedicate this work to the soul of Mr. Rabii Ben Kdissa that god took him for his participation in the realization of this article.

Statements and Declarations

The authors declare no conflict of interest, no funding was received.

References

[1] Dhillon, B. (2006). Maintainability and Reliability for Engineers. CRC Press, Taylor & Francis Group, ISBN-13: 978-0-8493-7243-8.

[2] Lyonnet, P. (1992). La maintenance. Mathématiques et méthodes. 3ème édition revue et augmentée, Tec Doc éditions, ISBN: 2-85206-745-5.

[3] Bertsche, B. (2008). Maintenance and Reliability, Reliability in Automotive and Mechanical Engineering. Springer, Berlin, Heidelberg, ISBN: 978-3-540-34282-3 https://doi.org/10.1007/978-3-540-34282-3_10

[4] Devooght, J., Lewins, J., & Becker, M. (2002). Dynamic Reliability Advances in Nuclear Science and Technology. 25 Springer, Boston, MA, https://doi.org/10.1007/0-306-47812-97

[5] Zamzam, A.H., Abdul Wahab, A.K., Azizan, M.M., Satapathy, S.C., Lai, K.W., & Hasikin, K. (2021). A Systematic Review of Medical Equipment Reliability Assessment in Improving the Quality of Healthcare Services. Front. Public Health, 9:753951. https://doi.org/10.3389/fpubh.2021.753951.

[6] Dhillon, B.S. (2011). Medical Equipment reliability: A review, analysis methods and improvement strategies. International Journal of Reliability, Quality and Safety Engineering, 18(4), 391–403, https://doi.org/10.1142/S0218539311004317

[7] Bahreini, R., Doshmangir, L., & Imani, A. (2018). Factors Affecting Medical Equipment Maintenance Management: A Systematic Review. Journal of Clinical and Diagnostic Research, 12 (4), IC01-IC07.

[8] Khalaf, A.B., & Hamam, Y. (2013). The effect of maintenance on the survival of medical equipment. Journal of Engineering, Design and Technology , 11(2), 142-157. https://doi.org/10.1108/JEDT-06-2011-0033

[9] Procaccia, H., & Piepszownik, L. (1992). Fiabilité des équipements et théorie de la décision statistique fréquentielle et bayésienne. Ed. Ayrolle, ISBN-13: 978-2212016314

[10] Arunraj, N.S., & Maiti, J. (2007). Risk-based maintenance-Techniques and applications. Journal of Hazardous Materials, 142(3), 653-661. ISSN 0304-3894, https://doi.org/10.1016/j.jhazmat.2006.06.069

[11] Monchy, F. (2010). Maintenance: Méthodes et organisations. 3ème édition, Dunod, Paris ISBN 978-2-10-055061-6

[12] Brissaud, F., Charpentier, D., Fouladirad, M., Barros, A., & Bérenguer, C. (2010). Failure rate evaluation with influencing factors. Journal of Loss Prevention in the Process Industries, 23(2), 187-193 ISSN 0950-4230, https://doi.org/10.1016/j.jlp.2009.07.013

[13] Ciriaco, V.F., & Richard, M.F. (1989). A survey of preventive maintenance models for stochastically deteriorating single-unit systems. Naval Research Logistics 36, 419-446 https://doi.org/10.1002/1520-6750(198908)36:4<419::AID-NAV3220360407>3.0.CO;2-5

[14] Rausand, M., & Hoyland, A. (2004). System Reliability Theory: Models, Statistical Methods, and Applications. Second édition, John Wilev & Sons. Inc.

[15] Deloux, E. (2008). Politiques de maintenance conditionnelle pour un système à dégradation continue soumis à un environnement stressant. Sciences de l'ingénieur [physics], Université de Nantes https://tel.archives-ouvertes.fr/tel-00348191

[16] Aupied, J. (1993). Retour d'expérience appliqué à la sûreté de fonctionnement des matériels en exploitation. Eyrolles ISBN-13:978-2-212-01638-3.

[17] Gölbaş, O., & Demirel, N. (2014). Stochastic Models in Preventive Maintenance Policies. Advanced Materials Research, 1016, 802–806, Trans Tech Publications, Ltd. https://doi.org/10.4028/www.scientific.net/amr.1016.802

[18] Alfares, H. (1999). A simulation model for determining inspection frequency. Computers & Industrial Engineering, 36(3), 685-696.

[19] Lalanne, C. (1999). Vibrations et Chocs Mecaniques T4: Dommage Par Fatigue. Hermes Sciences Publications, ISBN-13:978-2746200388.

[20] Habchi, G., & Barthod, C. (2016). An overall methodology for reliability prediction of mechatronic systems design with industrial application. Reliability Engineering & System Safety, 155, 236-254, https://doi.org/10.1016/j.ress.2016.06.013

[21] Tebbi, O. (2005). Estimation des lois de fiabilité en mécanique par les essaies accélérés. PhD thesis, Autre. Université d'Angers, Français. NNT: tel-00009407.

[22] Salameh, F. (2016). Méthodes de modélisation statistique de la durée de vie des composants en génie électrique. PhD thesis , Institut National Polytechnique de Toulouse, http://www.theses.fr/2016INPT0082/document

[23] Booher, T.B. (2006). Optimal Periodic Inspection of a Stochastically Degrading System. Theses and Dissertations 3291, https://scholar.afit.edu/etd/3291

[24] Brissaud, F., Lanternier, B., Charpentier, D., & Lyonnet, P. (2007). Modélisation des taux de défaillance en mécanique - Combinaison d'une loi de Weibull et d'un modèle de Cox pour la modélisation des taux de défaillance en fonction du temps et des facteurs d'influence. Performance et Nouvelles Technologies en Maintenance, PENTOM 2007, Mons Belgique. https://hal.archives-ouvertes.fr/hal-00196765

[25] Aven, T., Sklet, S., & Vinnem, J.E. (2006). Barrier and operational risk analysis of hydrocarbon releases (BORA-Release): Part I. Method description. Journal of Hazardous Materials, 137(2), 681-691. ISSN 0304-3894, https://doi.org/10.1016/j.jhazmat.2006.03.049

[26] Couallier, V., Denis, L., & Bayle, F. (2016). Calcul de MTBF des systèmes aéronautiques inspectés périodiquement soumis à rupture aléatoire de systèmes de régulation. 20ème Congrès de maîtrise des risques et de sûreté de fonctionnement, Saint-Malo, France, Institut pour la Maîtrise des risques https://doi.org/10.4267/2042/61795

[27] Ouakki, W. (2011). Prise en compte de la fiabilité en conception. Mémoire de maîtrise en génie mécanique, Université Laval, http://hdl.handle.net/20.500.11794/23131

[28] Lanternier, B., Charpentier, D., & Lyonnet, P. (2006). Modélisation de taux de défaillance en mécanique. Colloque de maîtrise des risques et sûreté de fonctionnement « Risques et performances », Lille, France, pp 4, ineris-00973237, https://hal-ineris.archives-ouvertes.fr/ineris-00973237

[29] Gouno, E., & Guérineau, L. (2015). Failure Rate Estimation in a Dynamic Environment. Economic Quality Control, 30(1), https://doi.org/10.1515/eqc-2015-6001

[30] Cox, D.R. (1972). Regression Models and Life-Tables. Journal of the Royal Statistical Society Series B (Methodological), 34(2), 187–220 [Royal Statistical Society, Wiley] http://www.jstor.org/stable/2985181.

[31] Ansell, J.I., & Phillips, M.J. (1994). Practical Methods for Reliability Data Analysis. Oxford Statistical Science Series, 14 Clarendon Press, 1st edition, December 8 ISBN-13: 978-0198536642.

[32] Carroll, K.J. (2003). On the use and utility of the Weibull model in the analysis of survival data. Control Clinical Trials, 24(6), 682-701. doi: 10.1016/s0197-2456(03)00072-2. PMID: 14662274

[33] Øien, K. (2001). A framework for the establishment of organizational risk indicators, Reliability Engineering & System Safety, 74(2), 147-167. ISSN 0951-8320, https://doi.org/10.1016/S0951-8320(01)00068-0

[34] O'Connor, D.T., & Kleyner, A. (2012). Practical reliability engineering. John Wiley and Sons, Chichester, UK, 5th edition.

[35] Lyonnet, P. (1991). Tools of Total Quality, An Introduction to statistical Process Control. Chapman and Hall, London.

[36] Yong, T. (2004). Extended weibull distributions in reliability engineering. PhD thesis, National University of Singapore.

[37] Lihou, D.A., & Spence, G.D. (1988). Proper use of data with the Weibull distribution. Journal of Loss Prevention in the Process Industries, 1(2), 110-113, ISSN 0950-4230, https://doi.org/10.1016/0950-4230(88)80021-4.

[38] Luko, S.N. (1999). A review of the Weibull Distribution and Selected Engineering Applications. SAE technical paper series. DOI: https://doi.org/10.4271/1999-01-2859

[39] Fidanoglu, M., Ungor, U., Ozkol, I., & Komurgoz, G. (2017). Application of Weibull Distribution Method for Aircraft Component Life Estimation in Civil Aviation Sector. Journal of Traffic and Logistics Engineering, 5(1).

[40] Rausand, M., Barros, A., & Hoyland, A. (2020). System Reliability Theory Models, Statistical Methods, and Applications. Third edition Wiley, ISBN: 9781119373957

[41] Ahsan, S., Lemma, T.A., & Gebremariam, M.A. (2019). Reliability analysis of gas turbine engine by means of bathtub-shaped failure rate distribution. Proc Safety Prog. e12115. https://doi.org/10.1002/prs.12115.

[42] Pasha, G.R., Khan, M.S., & Pasha, A.H. (2006). Empirical Analysis of The Weibull Distribution for Failure Data. Journal of Statistics, 13(1), 33-45.