EXPONENTIAL - POISSON DISTRIBUTION IN RELIABILITY ACCEPTANCE SAMPLING PLAN FOR LIFE TESTING

Dr. V.Kaviyarasu¹ and A.Nagarajan²

(1).Associate Professor, Department of Statistics, Bharathiar University
(2).Research Scholar, Department of Statistics, Bharathiar University
E.mail: <u>kaviyarasu@buc.edu.in</u>, <u>nagarajan.statistics@buc.edu.in</u>

Abstract

Statistical Quality Control is an important field in production and maintenance of quality product in manufacturing environments. Reliability sampling plans (RSP) were widely employed in the sectors of manufacturing to monitor the quality of products in order to safe guard the producer as well as the consumer also the experimental costs and time can be saved. This article is developed on the reliability sampling plan when the evaluating life of the product is set to be truncated at predetermined time follows Exponential-Poisson (EP) distribution. The probability of acceptance criteria for the single sampling is designed to achieve the lowest sample size for such proposed two parameter probability distribution with the corresponding decision rule. This study is conducted to design plan parameters on the basis of desired quality levels such as Acceptable Reliability Quality Level (ARQL), Indifference Reliability Quality Level (IRQL) and Rejectable Reliability Quality Level (RRQL). This study computes the median life for the specified producer's risk, its OC curve is provided along with the minimum ratio values. Furthermore, it determines the minimum size of the samples and the acceptance number. Table values have been obtained and provided for single sampling plan. Additionally, suitable examples are provided to conduct a study on a real time situations.

Keywords: Exponential – Poisson (EP) Distribution, Reliability, Median lifetime, Single Sampling Plan.

1. Introduction

Quality has become an inevitable term in the modern statistical society, especially in manufacturing sector. In such environment, every product must satisfy the required quality standards to achieve the goal. The act of employing statistical techniques to monitoring the quality and to maintaining the quality of the manufactured product in a systematic way is known as Statistical Quality Control (SQC). Due to technological advancements through mass production, it is an impossible to inspect every single product from a lot (i.e. 100% inspection is not feasible) and accepting a lot without inspection is also not acceptable hence both consumer and producer facing certain risks. So acceptance sampling is an important statistical technique to safeguard the consumer as well as the producer also. Here, the risks are termed as Producer risk (α) and Consumer risk (β) are the risk involved in the process of decision making.

Acceptance sampling is initially employed in the US military to test the quality of bullets from World War II and it acts as a vital tool in SQC, which focuses to make decisions about whether or not to accept a lot on the basis of the quality of randomly selected sample from a lot. This technique consists of the lot having size 'N' and 'n' is known to be the sample number of units and 'c' is the acceptance number. Reliability sampling plan is the one of the most important method in acceptance sampling which helps to assess the quality of product using time. Various techniques are employed to evaluate the quality of such a manufactured item to test the reliability of the item which is called as life test method here failure of an item follows a continuous probability distributions are adopted to model this methodology.

This paper is studied under the attribute sampling plan is studied to discover the life of an item to test the lower confidence limit on median life. According to Gupta and Groll (1961)[6], the median life constitutes a superior quality parameter than that of the average life for a skewed distribution. Our aim is to decrease the financial expenses and also the investment of time of the experimenter simultaneously truncated life test is studied to test the test termination time for the fixed time 't'. One can count the total number of failures occur during this process within the specified time, if no c failures occur prior to the scheduled time limit. If not, the experiment is terminated after the (c+1)th failure. Based on the values of the operating characteristics, the methodology for the smallest sample size is to be necessary for guaranteeing that the product's designated median life has been given along with the associated producer risk is presented here. An appropriate example have been discussed with suitable illustrations. The foremost objective of this article is to constitute a time truncated single sampling plan for median life under Exponential Poisson (EP) distribution.

2. Review of Literature

There has been extensive research about reliability sampling plans on the basis of truncated life tests done by various authors. Baklizi and El Masri (2004) [1] were studied acceptance sampling for Birnbaum–Saunders model, Barreto-Souva and Silva (2013) [2] pointed out that EP is better alternative to the gamma distribution, Cameron. J.M. (1952) [3] were studied about the construction of tables based on OC function of single sampling plans, Dodge. H.F and H.G. Romig (1959) [4] conducted a study on sampling inspection tables, Epstein (1954) [5] proposed a truncated test for the exponential case, Gupta and Groll (1961) [6] were conducted a study about acceptance sampling under Gamma distribution, Kaviyarasu and Fawaz (2017) [7] carried out a study on acceptance sampling on the modified weibull distribution, Kus (2007) [8] introduced a new life time distribution called as Exponential Poisson distribution(EP), Schilling and John (1980) [9] constructed a set of tables for various sampling plans, Sobel and Tischendrof (1959) [10] were studied about new life test objectives for acceptance sampling.

3. The Exponential-Poisson Distribution

The Exponential Poisson (EP) distribution is a two parameter continuous probability distribution that is used to model the time between events in a real time to test the life of an item. EP distribution is a compounded distribution under Exponential and zero truncated Poisson distribution. This distribution has several real time applications such as Network traffic modelling, manufacturing quality control, service queue management and stock price modelling etc.,.

According to Barreto-Souza and Silva (2013)[2], EP distribution is better alternative to the Gamma distribution. For a lifetime and reliability studies EP distribution performs a significant role in modelling the lifetime of the products.

The Cumulative distribution function of Exponential - Poisson distribution is

$$F(x;\lambda,\beta) = (e^{\lambda \exp(-\beta x)} - e^{\lambda})(1 - e^{\lambda})^{-1}$$
(1)

The Probability density function of Exponential - Poisson distribution is

$$f(x;\lambda,\beta) = \frac{\lambda\beta}{(1-e^{-\lambda})} e^{-\lambda - \beta x + \lambda \exp(-\beta x)}$$
(2)

Here $\lambda > 0$, $\beta > 0$ are the shape and scale parameter. Where λ is also known as Poisson parameter. When $\lambda \rightarrow 0$, the Exponential distribution is obtained by reducing the EP distribution.

The median function of EP distribution is

 $\log\{\log\left[2^{-1}(e^{\lambda}+1)\right]^{-1}\lambda\}\beta^{-1}$ (3)

1.e.
$$(1 \le \beta_0) = \beta$$

 $\beta_0 = -\beta^{-1} \log\{\log [2^{-1}(e^{\lambda} + 1)]^{-1}\lambda\}$
(4)

$$\eta = -\log\{\log\left[2^{-1}\left(e^{\lambda}+1\right)\right]^{-1}\lambda\}$$

$$\implies t = -\frac{\eta}{2}$$
(5)

$$\Rightarrow t_q = \gamma_\beta$$

$$\Rightarrow \beta = \eta_{\beta_0}$$

By substituting the scale parameter $\beta = \frac{\eta}{\beta_0}$, the CDF of EP distribution becomes

$$F(t) = (e^{\lambda \exp(-\frac{t}{\beta_0}*\eta)} - e^{\lambda})(1 - e^{\lambda})^{-1} \quad t > 0, \eta > 0$$

Let $\delta = \frac{t}{\beta_0}$
 $F(t; \delta) = (e^{\lambda \exp(\delta)} - e^{\lambda})(1 - e^{\lambda})^{-1} \quad , t > 0, \delta > 0$ (6)

4. Truncated Acceptance Sampling Plan

In acceptance sampling, a well-known simple plan is single sampling plan (SSP) and it has employed in many reliability studies. Here the product's lifetime (T) is assumed to follows the Exponential – Poisson (EP) distribution. The shape parameter λ is considered as a known parameter. The product's median life can be represented by m. In a truncated acceptance life test plans, the usual practice of testing the lifetime of the product is to terminating the experiment at a time (t) which has already determined. The decision of the acceptance criteria is purely based only the occurrence of defectives. If the total count of defectives is below the given acceptance number c, then it should be accepted. The main target of this experiment is to acquire a designated median life along with the help of probability P* (Consumer risk) and also to frame a lower confidence limit. For conducting a truncated life test experiment the following components should be considered.

- > The total number of sample units on the experiment (n);
- > The acceptance number (c); when the total count of defectives occurred is more than c at the final stage of pre-decided time then the inspected lot will be approved for the acceptance.
- > The ratio t/β_0 ; Here ' β_0 ' is known to be a described median life and the maximum amount of time for the experiment is known as 't'.

5. Minimum Sample Size

The chance cause of accidently accepting a lot without knowing that the chosen lot is a poor is known to be consumer risk (α) and it is fixed to not greater than 1-P*. Since the chance of accepting a poor lot with a median is at the minimum of P*, then it is evident that P* represents the confidence level. The lot size must be taken into consideration as being infinite and must be assumed to be sufficiently large, so in this case binomial distribution is employed to evaluate the lot acceptance. To find smallest sample size (n) such that

$$\sum_{x=0}^{n} \binom{n}{x} p^{x} (1-p)^{n-x} \le 1-p^{*}$$
(7)

In table 1, the minimum values (n) were presented that satisfies the above inequality, for $\frac{\tau}{\beta_0} = 0.3, 0.6, 0.9, 1.2, 1.5, 1.8, 2.1, 2.4, 2.7, 3$ and P* = 0.75, 0.90, 0.95, 0.99 and c = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

		t/β_0									
P*	Ν	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3
0.75	0	7	4	3	2	2	2	2	1	1	1
0.75	1	13	7	5	4	4	3	3	3	3	3
0.75	2	19	11	8	6	6	5	5	4	4	4
0.75	3	25	14	10	8	7	7	6	6	5	5
0.75	4	31	17	13	10	9	8	8	7	7	7
0.75	5	36	20	15	12	11	10	9	9	8	8
0.75	6	42	23	17	14	12	11	11	10	9	9
0.75	7	47	27	20	16	14	13	12	11	11	10
0.75	8	53	30	22	18	16	14	13	13	12	12
0.75	9	58	33	24	20	18	16	15	14	13	13
0.75	10	64	36	27	22	19	18	16	15	15	14
0.9	0	11	6	4	3	3	2	2	2	2	2
0.9	1	18	10	8	6	5	4	4	4	3	3
0.9	2	25	14	10	8	7	6	6	5	5	5
0.9	3	32	17	13	10	9	8	7	7	6	6
0.9	4	38	21	15	12	11	10	9	8	8	7
0.9	5	44	24	18	14	13	11	10	10	9	9
0.9	6	50	28	20	17	14	13	12	11	11	10
0.9	7	56	31	23	19	16	15	13	13	12	11
0.9	8	62	34	25	21	18	16	15	14	13	13
0.9	9	68	38	28	23	20	18	16	15	15	14
0.9	10	74	41	30	25	22	19	18	17	16	15
0.95	0	13	7	5	4	3	3	3	2	2	2
0.95	1	22	12	8	7	6	5	5	4	4	4
0.95	2	29	14	11	9	8	7	6	6	5	5
0.95	3	36	19	14	11	10	9	8	7	7	7
0.95	4	43	23	17	14	12	10	10	9	8	8
0.95	5	49	27	19	16	14	12	11	10	10	9
0.95	6	55	30	22	18	16	14	13	12	11	11
0.95	7	61	34	25	20	17	16	14	13	13	12
0.95	8	68	37	27	22	19	17	16	15	14	13
0.95	9	74	41	30	24	21	19	17	16	15	15
0.95	10	80	44	32	26	23	21	19	18	17	16
0.99	0	20	10	7	5	4	4	3	3	3	3
0.99	1	30	15	10	8	7	6	5	5	5	4
0.99	2	35	19	14	11	9	8	7	7	6	6
0.99	3	45	24	17	13	11	10	9	8	8	7
0.99	4	54	28	19	16	13	12	11	10	9	9
0.99	5	61	32	22	18	16	14	13	12	11	10
0.99	6	68	36	25	21	18	16	14	13	12	12
0.99	7	75	40	29	23	20	17	16	16	14	13
0.99	8	82	42	31	25	21	19	17	16	15	15
0.99	9	88	48	34	27	23	21	19	18	17	16
0.99	10	95	49	37	29	25	23	21	19	18	17

TABLE 1: Minimum sample sizes for the EP distribution.

6. Operating Characteristic Function

The Operating Characteristics (OC) function of the Acceptance sampling based on the Truncated Life Test (ASTLT) plan consists with the parameters of $(n, c, \frac{t}{\beta_0})$. For analysing the ASTLT, the probability is

$$L(p) = \sum_{x=0}^{c} {n \choose x} p^{x} (1-p)^{n-x}$$
(8)

Where $p = F(t; \theta)$ is a monotonically decreasing function of $\beta > \beta_0$. Based on the above inequality the operating characteristics (OC) values of $\frac{t}{\beta_0}$ were displayed in table 2.

			β/β_0								
P *	Ν	t/β_0	2	4	6	8	10	12	14	16	18
0.75	31	0.3	0.1515	0.6882	0.8939	0.9587	0.9816	0.9909	0.9951	0.9972	0.9983
0.75	17	0.6	0.1567	0.681	0.8883	0.9556	0.98	0.99	0.9946	0.9969	0.9981
0.75	10	1.2	0.1726	0.6737	0.8801	0.9507	0.9772	0.9884	0.9937	0.9963	0.9977
0.75	8	1.8	0.1542	0.6271	0.8507	0.935	0.9688	0.9837	0.9909	0.9946	0.9966
0.75	7	2.4	0.1419	0.5882	0.8228	0.9188	0.9596	0.9783	0.9876	0.9925	0.9953
0.75	7	3	0.0655	0.4278	0.707	0.8494	0.9188	0.9538	0.9724	0.9828	0.9888
0.9	38	0.3	0.0182	0.3597	0.69	0.8515	0.9247	0.9593	0.9767	0.9859	0.9911
0.9	21	0.6	0.0173	0.3358	0.6639	0.8338	0.9138	0.9526	0.9724	0.9832	0.9893
0.9	12	1.2	0.0235	0.3399	0.6563	0.8251	0.9072	0.948	0.9694	0.9811	0.9879
0.9	10	1.8	0.0115	0.2359	0.5407	0.7405	0.8515	0.912	0.9459	0.9655	0.9772
0.9	8	2.4	0.0224	0.284	0.5814	0.7657	0.8662	0.9207	0.9511	0.9688	0.9794
0.9	7	3	0.0303	0.301	0.5882	0.7662	0.8646	0.9188	0.9494	0.9674	0.9783
0.95	43	0.3	0.0041	0.214	0.5458	0.7562	0.8667	0.924	0.9546	0.9718	0.9818
0.95	23	0.6	0.0052	0.2158	0.5412	0.7503	0.8619	0.9205	0.9522	0.9701	0.9806
0.95	14	1.2	0.0036	0.163	0.4595	0.6814	0.8129	0.8873	0.9299	0.9549	0.9701
0.95	10	1.8	0.0082	0.2021	0.4986	0.7073	0.8285	0.8966	0.9355	0.9584	0.9724
0.95	9	2.4	0.0042	0.1376	0.3977	0.6162	0.7595	0.8474	0.9009	0.934	0.9549
0.95	8	3	0.0044	0.1294	0.3745	0.5889	0.7354	0.8282	0.8864	0.9232	0.9469
0.99	54	0.3	0.0002	0.0659	0.3056	0.5499	0.719	0.8236	0.8869	0.9256	0.9498
0.99	28	0.6	0.0004	0.0756	0.3209	0.562	0.7268	0.8284	0.8899	0.9275	0.951
0.99	16	1.2	0.0026	0.0684	0.292	0.5251	0.6936	0.8021	0.8701	0.9129	0.9403
0.99	12	1.8	0.0006	0.0634	0.269	0.4934	0.6633	0.7772	0.8507	0.8982	0.9292
0.99	10	2.4	0.0007	0.0599	0.2507	0.4662	0.6359	0.7536	0.8318	0.8835	0.9178
0.99	9	3	0.0005	0.0481	0.2132	0.4155	0.5862	0.711	0.7975	0.8566	0.897

 TABLE 2: OC values for (n, c =4, t/ β_0 = 0.60) for a given P* under EP distribution.

 β / β_0

7. Producer Risk

The chance of rejecting a lot without knowing the chosen lot is satisfying the quality requirements is known as producer risk (β), when $\beta > \beta_0$ it will be computed as

Prob(p) = Prob {Rejecting a lot} = 1 - Prob {Accepting a lot}

For a single sampling plan and the definite values of producer's risk, one may very curious in finding the value estimates of $\frac{\beta}{\beta_0}$ is going to guarantee the producer's risk which is not greater than or equal to 0.05 on the basis of the employed sampling plan. $\frac{\beta}{\beta_0}$ is having the values which are known to be the smallest non-negative integer for $p = F\left(\frac{t}{\beta_0}\frac{\beta_0}{\beta}\right)$ which satisfies the below mentioned inequality. $\sum_{i=1}^{c} \binom{n}{\gamma} p^i (1-p)^{n-i} \ge 0.95$ (9) The minimum values of $\frac{t}{\beta_0}$ satisfying the above inequality to the proposed sampling plan $(n, c, \frac{t}{\beta_0})$ at a specific confidence level P* were presented in table - 3.

		$\frac{1}{t/\beta_0}$									
P*	С	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3
0.75	0	31.171	35.869	40.333	35.642	44.484	53.61	62.603	35.236	39.857	44.375
0.75	1	24.053	24.977	25.813	26.566	33.198	27.825	32.642	37.306	41.727	46.363
0.75	2	15.17	16.605	17.279	16.251	20.314	19.187	22.383	18.473	20.779	23.087
0.75	3	11.794	12.401	12.452	12.512	13.017	15.62	14.602	16.689	13.883	15.507
0.75	4	10.091	10.286	11.176	10.579	11.468	11.568	13.496	12.468	14.026	15.604
0.75	5	8.764	8.9857	9.4756	9.4254	10.447	10.881	10.737	12.281	11.265	12.519
0.75	6	8.1086	8.1481	8.4278	8.6487	8.6371	9.0264	10.531	10.292	9.5259	10.57
0.75	7	7.4575	7.9256	8.1975	8.0653	8.3097	8.8619	9.0839	8.8853	9.9959	9.1862
0.75	8	7.1177	7.4086	7.54	7.6484	8.0468	7.8036	7.9908	9.1324	8.8559	9.8399
0.75	9	6.7202	7.0283	7.028	7.2891	7.8226	7.7769	8.1294	8.1754	7.9518	8.8354
0.75	10	6.5121	6.7127	6.9987	7.0168	7.051	7.7368	7.3757	7.4428	8.3901	8.0692
0.9	0	63.773	70.043	70.271	70.099	87.897	70.592	81.507	93.52	105.22	117.13
0.9	1	14.854	16.199	19.378	19.065	19.77	18.346	21.481	24.549	19.835	22.132
0.9	2	8.9139	9.7523	10.292	10.765	11.623	11.729	13.679	12.646	14.227	15.806
0.9	3	6.8377	7.0802	7.9756	7.9752	8.8518	9.3044	9.3117	10.642	9.9191	11.022
0.9	4	5.6044	6.0612	6.3422	6.6176	7.4869	8.0562	8.3495	8.2644	9.2975	8.7511
0.9	5	4.9038	5.2175	5.7527	5.8153	6.675	6.6329	6.9162	7.9003	7.8167	8.6849
0.9	6	4.426	4.846	5.074	5.6602	5.676	6.268	6.6585	6.8541	7.7109	7.6222
0.9	7	4.0913	4.4232	4.8269	5.2174	5.3801	5.9826	5.9171	6.7624	6.9	6.8725
0.9	8	3.841	4.1194	4.444	4.8948	5.1468	5.3923	5.8277	6.1435	6.3087	7.0097
0.9	9	3.6471	3.9867	4.3196	4.6404	4.9662	5.2773	5.3647	5.6783	6.3881	6.5131
0.9	10	3.4921	3.787	4.0623	4.443	4.8175	4.8834	5.3427	5.713	5.965	6.128
0.95	0	75.159	81.468	87.582	93.706	87.907	105.88	123.15	93.52	105.47	117.38
0.95	1	18.232	19.636	19.368	22.382	23.766	23.512	27.359	24.51	27.58	30.645
0.95	2	10.412	9.7842	11.4	12.251	13.44	13.916	13.683	15.66	14.232	15.813
0.95	3	7.7168	7.9544	8.6374	8.8752	9.9753	10.623	10.875	10.642	11.972	13.239
0.95	4	6.3752	6.6457	7.2535	7.8503	8.277	8.0892	9.4373	9.5422	9.2972	10.359
0.95	5	5.4765	5.8962	6.0749	6.7414	7.2681	7.3145	7.7384	7.9003	8.8878	8.6849
0.95	6	4.8859	5.1978	5.6126	6.0094	6.6146	6.8245	7.2994	7.6096	7.7102	8.5677
0.95	7	4.4704	4.882	5.2738	5.5147	5.7538	6.4561	6.4517	6.7624	7.6077	7.6667
0.95	8	4.2249	4.4986	4.8191	5.1542	5.4735	5.7862	6.2791	6.6603	6.9115	7.0097
0.95	9	3.9791	4.3181	4.6546	4.8692	5.2427	5.6116	5.7678	6.1244	6.3853	7.0948
0.95	10	3.7839	4.0732	4.3627	4.6362	5.055	5.4839	5.6991	6.111	6.4303	6.6407
0.99	0	116.71	114.93	121.94	117.08	115.67	138.58	121.54	140.99	158.8	176.56
0.99	1	24.936	24.705	24.367	25.823	27.987	28.519	27.211	31.093	34.966	30.645
0.99	2	12.575	13.457	14.693	15.215	15.315	16.127	16.23	18.622	17.617	19.575
0.99	3	9.6924	10.114	10.59	10.629	11.091	11.973	12.369	12.43	13.983	13.239
0.99	4	8.0504	8.172	8.1848	9.0434	9.0172	9.9322	10.493	10.74	10.735	11.924
0.99	5	6.8508	7.0482	7.1377	7.6714	8.4186	8.6986	9.367	9.7673	9.9505	9.8752
0.99	6	5.4395	5.5758	5.6231	6.0237	6.6008	6.8107	7.2994	7.6096	7.7102	7.6222
0.99	7	5.5184	5.7897	6.1898	6.4359	6.8933	6.9043	7.5359	8.6124	8.2951	8.4531
0.99	8	5.1131	5.141	5.6015	5.9245	6.1088	6.5678	6.7506	7.1953	7.4928	8.3253
0.99	9	4.7508	5.0853	5.3166	5.5287	5.8006	6.2968	6.5557	7.0449	7.4067	7.6639
0.99	10	4.5104	4.5646	5.0915	5.2251	5.5537	6.0677	6.3979	6.5112	6.8822	7.1316

TABLE 3: Minimum ratio of true mean life over β_0 at the producer's risk of 0.05

8. Numerical Illustration

In today's modern world, every region has its own food culture that is influenced by the environment, agriculture, whether, and so on. Nowadays, the beverage industry plays a significant role in the global food industry. Coca-Cola is the only beverage that is mass-produced and distributed globally. Coca-Cola was first sold in Atlanta in 1886. Despite the passage of many decades, the demand for Coca-Cola keeps on increasing on every single day. As a result, a beverage manufacturing company in the United States intends to increase Coca-Cola production. The exponential process is used to describe the increase in Coca-Cola production, and the Poisson process is used to describe the probability of manufacturing defects. Therefore, it is ensured that this production process is carried out using an exponential Poisson process.

Here it is considered that the Exponential-Poisson distribution is the appropriate distribution for evaluating life time of an item with the parameters λ =2. The quality inspector desires to investigate the median lifetime of an item has 1000 hours when the confidence level is P*= 0.75. The test was terminated after 600 hours. This leads to the ratio $t/\beta_0 = \frac{600}{1000} = 0.6$. The sampling plan which is used by the experimenter is (n=17, c=4, $t/\beta_0 = 0.60$).

	Tuble 1. OC curve for the pair (17, 1, 0.0) where Er for $p = 0.75$												
Р	2	4	6	8	10	12	14	16	18				
L(P)	0.15669	0.681	0.88834	0.95563	0.98003	0.99005	0.99462	0.9969	0.99811				

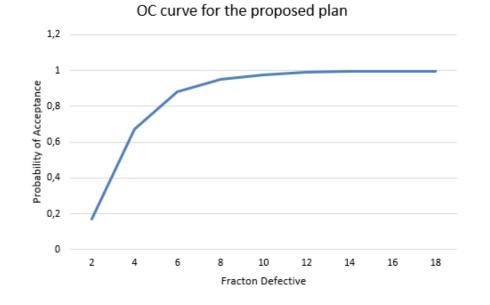


Table-4: OC curve for the plan (17, 4, 0.6) under EP for $p^* = 0.75$

Figure-1: OC curve for the plan (17, 4, 0.6) under EP for $p^* = 0.75$.

9. Construction of tables

- Step 1: Set the parameters λ =2 and the test termination ratio t/β_0 such as 0.3, 0.6, 0.9, 1.2, 1.5, 1.8, 2.1, 2.4, 2.7, 3 Step 2: To find the value of η , substitute the parameters in the equation (5)
- One can obtain η as 0.332832. Substitute $\beta = \frac{\eta}{\beta_0}$ in (1),

To find p, use the inequality $p = F(t, \delta)$

Step 3: By satisfying the inequality, determine the smallest sample size n.

$$\sum_{i=0}^{c} \binom{n}{i} p^{i} (1-p)^{n-i} \leq 1-p^{*}$$

Step 4: Utilize the inequality to determine the OC values.

$$L(p) = \sum_{i=0}^{c} {n \choose i} p^{i} (1-p)^{n-i}$$

Step 5: By satisfying the given inequality, determine the minimum mean ratio at Producer risk $\alpha = 0.05$

$$\sum_{i=0}^{c} \binom{n}{i} p^{i} (1-p)^{n-i} \geq 1-\alpha$$

10. Conclusion

This article is developed for time truncated single sampling plan which follows a pre-fixed time when lifetime of the products follows an Exponential-Poisson distribution. The required minimum sample size and OC values of Producer risk were displayed in the given tables in order to guarantee the determined median life along with a confidence level that is given. This study reveals that the EP distribution proves that the sample size is much smaller than other statistical distributions which are used in acceptance sampling. Further, the table values are explained with suitable illustration.

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