Reliability Acceptance Sampling Plan for One Parameter Polynomial Exponential Distribution

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Abstract

In this study, we construct a reliability acceptance sampling inspection plan to decide whether to accept or reject a lot of products where the One Parameter Polynomial Exponential (OPPE) family of distributions governs the lifetimes. The OPPE distribution has infinite support. To utilise finite support, it has transformed into its unit form, i.e. having the support (0, 1). The design of the plan, Operating characteristic curve, and Sampling procedure are discussed. Determination of the plan parameters using an algorithm is stated. The optimal sample size is determined to protect the consumer's confidence level. Two simplest particular choices of the OPPE family - the exponential and the Lindley are chosen as examples, and optimal plan parameters are tabulated and compared. The plan is executed with three real-life data sets.

Keywords: Consumer's risk, Operating characteristic function, Scale-invariant family of distributions, Truncated life test, unit-Lindley distribution.

1. Introduction

Quality control takes centre stage if one wants multiple copies of a product. The first question that arises throughout the repetitive process is what should the quality features of the product be for it to be satisfactory? The same prototype of products can be seen with the naked eye in the early days. As a result, making items identical is one of the most crucial aspects. Variation in the products is inevitable. In 1924, Walter A. Shewhart introduced many statistical approaches to assess product quality variation.

The sampling inspection plan aims to sentence a lot of products and make decisions to reject or accept that lot. In practice, two types of sampling plans are attribute sampling and variable sampling. In attribute sampling, products of a lot are judged as defective or non-defective, whereas the variable sampling plan measures the product's actual quantitative information. The main advantage of a variable sampling plan is that it requires fewer samples than an attribute plan with the same protection.

Acceptance sampling is traditionally used to decide on acceptance or rejection of the lot, not to determine the quality of the product by using estimation methods. As a result, most acceptance sampling plans are not adequately designed and worrying truth because buying a lot without knowing its quality seems risky. Therefore, to make an appropriate decision, it is better to develop a procedure for evaluating the value of the fraction defective of the lot.

In a sampling inspection plan, if sample observations represent the lifetime of products in the test, one may be interested in the hypothesis that the population average surpasses a specified average. Suppose the population average represents the average lifetime, defined as μ. If μ_0 is the specified minimum value, then one would like to test the hypothesis $\mu_{\geq}\mu_0$, which means the population average surpasses the specified average. In the test, n samples are placed for testing over a period of time t. The lot cannot be accepted if the observed failures exceed acceptance number (c). This sampling inspection plan is called Reliability Acceptance Sampling Plan (RASP) and it is characterised by the triplet $(n,\ c,\ t)$.

Many authors chalked out the RASP for quality characteristics following different parametric distributions, like exponential [22] , Weibull [9] , Gamma [11], Normal and Log-Normal [10], half Logistics [12], Log-Logistic [17], Birnbaum-Saunders [14] , exponential Frechet [1], three-parameter kappa [2] , generalised inverse exponential [21], generalised Weibull[8], Ishita [3] , transmuted generalised inverse Weibull [4] , generalised Pareto [20] , Quasi Shanker [5], etc.

The popularity of exponential distribution is well known in the context of life testing because of its simplicity in analysis. The constancy properties of failure rate and residual mean life limit the distribution in the present industrial scenario. The Lindley distribution is an excellent alternative to an exponential distribution, but it is overlooked for life-testing purposes. It is a mixture of two parametric distributions, exponential and gamma. The Lindley distribution is more flexible than the exponential distribution because of its increasing, decreasing and upside-down bathtub failure rate for the parameter at different values. [7] has proposed a new and unified approach in generalizing the Lindley's distribution. They investigated some structural properties like moments, skewness, kurtosis, median, mean deviations, Lorenz curve, entropies and limiting distribution of extreme order statistics; reliability properties like reliability function, hazard rate, stress-strength reliability, stochastic ordering; and estimation methods like the method of moment and maximum likelihood. We call the distribution as the one-parameter polynomial exponential (OPPE) family of distributions. The proposed RASP will be constructed assuming only OPPE distribution considering the rejectable quality level. The RASP for the Lindley distribution will be discussed in detail as particular example and compare it with that of the exponential distribution.

Few works have been done on acceptance sampling inspection plans assuming the Lindley distribution (see [15]; [**?**]); [6] and [23] worked on an acceptance sampling plan under a truncated life test assuming two-parameter Lindley distribution. Plan parameters are estimated based on two-point approaches on Operating characteristic (OC) curve-acceptable and rejectable quality levels.

Almost all works on RASP are done for scale-invariant distributions. Minimum sample size n and acceptance number (c) are determined for different times per mean $(\frac{t}{\mu_0})$. However, the OPPE distribution does not belong to a scale-invariant family of distributions. Therefore, the utilisation of time per mean is beyond our scope. We may directly chalk the plan with plan parameters $(n,\ c,\ t)$. Since the OPPE distribution has support (0,∞), we may utilise it with the finite support by transforming into its unit form, i.e. having the support (0,1) with the transformation $V = e^{-T}$. We make tables using the unit-Lindley form by choosing V and the mean μ_v in the interval (0,1). Utilising this benefit, we choose optimal $(n,\ c)$ and then revert to plan parameter t from the relation of the transformation. So, in a nutshell, our objective is to develop a RASP for the OPPE distributed quality characteristic. Based on the time-truncated life test, the plan has the advantage of saving the organisation's time and cost while also being very helpful in determining whether to accept or reject a lot. The OC is derived for choosing the optimal plan based on the consumer's confidence level. Tables of minimum sample sizes are examples for easy understanding and execution of the proposed plan. It is put into practice for real-life experimental data, and the OC surface is depicted to provide a clear picture of the plan.

The following is the arrangement of the rest of the paper. The OPPE and unit-OPPE distributions are described in section 2. In section 3, we describe the sampling design, operating characteristics function, and operating procedure. In section 4, an algorithm for calculating the minimal sample size of the proposed RASP is stated for the OPPE distribution, and examples for the Lindley and exponential distributions, in particular, are presented in tabular form. In section 5, we use the said sampling plan to work on real-world data. Section 6 concludes.

2. The One Parameter Polynomial Exponential Distribution and its unit version

The probability density function (PDF) of a random variable T of the OPPE distribution can be written as

$$f_T(t,\theta) = h(\theta)p(t)e^{-\theta t},\ t,\ \theta > 0, \tag{1}$$

where, $h(\theta) = \frac{1}{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}}}$, $p(t) = \sum_{k=0}^{r} a_k t^k$, a_k's are known non-negative constants and r is known non-negative integer.

The distribution can also be written as

$$\begin{aligned} f_T(t,\theta) &= h(\theta)\sum_{k=0}^{r} a_k t^k e^{-\theta t} \\ &= \frac{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}} f_{GA}(t;k+1,\theta)}{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}}}, \end{aligned} \tag{2}$$

where $f_{GA}(t;k+1,\theta)$ is the PDF of a gamma distribution with shape parameter $(k+1)$ and scale parameter θ. The distribution is a finite mixture of $(r+1)$ gamma distributions.

The cumulative density function (CDF) is given by

$$F_T(t,\theta) = 1 - \left(\frac{\sum_{k=0}^{r} \frac{a_k \Gamma(k+1)\Gamma(k+1,\theta t)}{\theta^{k+1}}}{\sum_{k=0}^{r} a_k \frac{k!}{\theta^{k+1}}} \right),\ t,\ \theta > 0, \tag{3}$$

where $\Gamma(m,t) = \frac{1}{\Gamma(m)} \int_t^{\infty} e^{-u} u^{m-1} du$.
The s-th order raw moment of OPPE is given by

$$\begin{aligned} \mu'_s &= E(T^s) \\ &= \frac{\sum_{k=0}^{r} a_k \frac{\Gamma(k+s+1)}{\theta^{k+s+1}}}{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}}}. \end{aligned} \tag{4}$$

Now, if we take a transformation $V = e^{-T}$, then the OPPE turns into unit-OPPE in range of (0,1). The PDF and CDF of unit-OPPE is given by ,

$$\begin{aligned} f_V(v,\theta) &= h(\theta)\sum_{k=0}^{r} a_k (-\ln v)^k v^{\theta-1} \\ &= \frac{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}} f_{UGA}(v;k+1,\theta)}{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}}},\ 0 < v < 1, \end{aligned} \tag{5}$$

where $f_{UGA}(v;k+1,\theta) = \frac{\theta^{k+1}}{\Gamma(k+1)}(-\ln v)^{k+1} v^{\theta-1}$ is the PDF of a unit-gamma distribution with shape parameter $(k+1)$ and scale parameter θ, and

$$F_V(v,\theta) = 1 - \left(\frac{\sum_{k=0}^{r} \frac{a_k \Gamma(k+1)\Gamma(k+1,-\theta \ln v)}{\theta^{k+1}}}{\sum_{k=0}^{r} a_k \frac{k!}{\theta^{k+1}}} \right),\ 0 < v < 1,\ \theta > 0, \tag{6}$$

respectively.
The s-th order raw moment of unit-OPPE is given by

$$\mu'_s = E(V^s) = \frac{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{(s+\theta)^{k+1}}}{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}}}. \quad (7)$$

The Lindley distribution (for $r = 1,\ a_0 = a_1 = 1$), introduced by Lindley (1958) to analyse failure time data has the PDF, CDF and hazard rate function (HRF) as

$$f_T(t;\theta) = \frac{\theta^2}{\theta+1}(1+t)e^{-\theta t} \qquad t > 0, \theta > 0, \quad (8)$$

$$F_T(t;\theta) = 1 - \frac{1+\theta+\theta t}{\theta+1}e^{-\theta t} \quad (9)$$

and

$$h_T(t;\theta) = \frac{\theta^2(1+t)}{1+\theta+\theta t} \qquad t > 0, \theta > 0, \quad (10)$$

respectively.

The mean of the random variable T is

$$\mu = \frac{\theta+2}{\theta(1+\theta)}. \quad (11)$$

The unit-Lindley distribution with parameter θ has the PDF , CDF and HRF respectively , as follows:

$$f(v;\theta) = \frac{\theta^2}{1+\theta}(1-log(v))\left(v^{\theta-1}\right) \qquad 0 < v < 1, \theta > 0, \quad (12)$$

$$F(v;\theta) = \frac{v^{\theta}(1+\theta(1-log(v)))}{1+\theta} \qquad 0 < v \le 1, \theta > 0, \quad (13)$$

$$h(v;\theta) = \frac{\theta^2(1-log(v))}{v(\theta log(v) - (1+\theta)(1-v^{-\theta}))} \qquad 0 < v < 1, \theta > 0. \quad (14)$$

The shapes of the PDF, CDF and HRF of unit-Lindley distribution for different θ are shown in Figure 1. Notably, the HRF is an increasing function. So, the distribution is capable of modeling life time data. Furthermore, the first moment about the origin of unit-Lindley distribution can be obtained as $m'_1 = \frac{\theta^2(2+\theta)}{(1+\theta)^3} = \mu_v$.
For comparison purpose, we will choose the exponential distribution ($r = 0,\ a_0 = 1$) and its corresponding unit version. The unit version of the exponential distribution with parameter θ has the PDF, CDF and HRF as

$$f(v,\theta) = \theta v^{\theta-1}, \qquad 0 < v < 1, \theta > 0, \quad (15)$$

$$F(v,\theta) = v^{\theta}, \qquad 0 < v \le 1, \theta > 0, \quad (16)$$

$$h(v,\theta) = \frac{\theta v^{\theta-1}}{1-v^{\theta}}, \qquad 0 < v < 1, \theta > 0, \quad (17)$$

respectively. In this case, $\mu_v = \frac{\theta}{1+\theta}$ implies $\theta = \frac{\mu_v}{1-\mu_v}$.

3. Reliability Acceptance Sampling Plan for OPPE distribution

According to the product's mean life, a product lot is labelled as good or bad in this sampling plan. The RASP has the plan parameters $n,\ c,$ and t. For proper implementation of the plan, engineers and practitioners use the tabulated value or algorithm. Tables are presented for fixed t and c, the optimal value of n. Since the value of $t \in (0,\ \infty)$, fixing t is tedious, whereas choosing $v \in (0,\ 1)$ is easy and comprehensive.

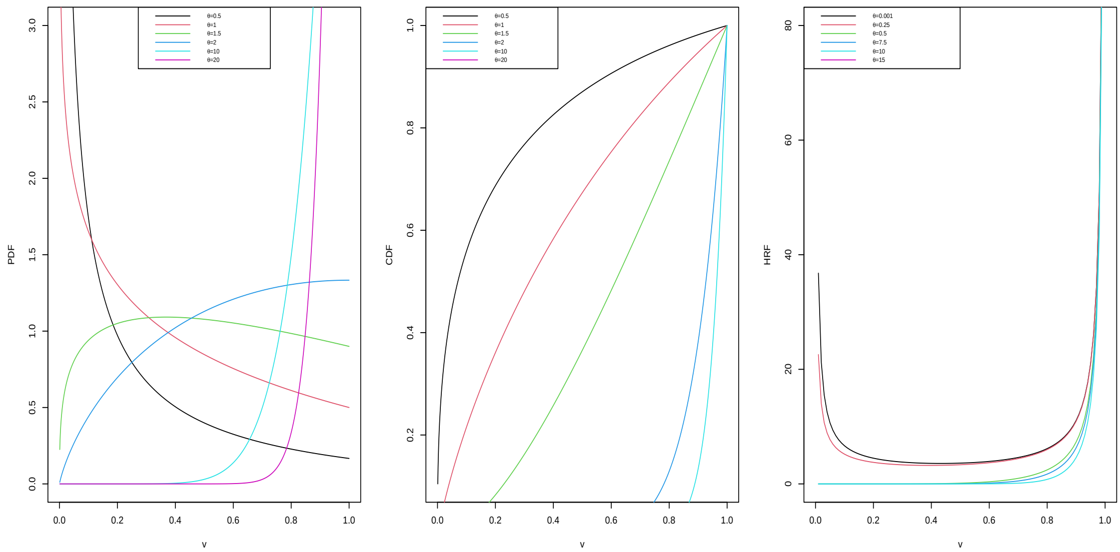

Figure 1: *PDF,CDF and HRF of unit-Lindley distribution for different θ*

3.1. Design of the sampling plan

A product is defective if it fails before truncation time v. The fraction defective i.e. the probability that a product is defective is

$$p(v) = F(v;\theta) = 1 - \left(\frac{\sum_{k=0}^{r} \frac{a_k \Gamma(k+1)\Gamma(k+1, v^{\theta})}{\theta^{k+1}}}{\sum_{k=0}^{r} a_k \frac{k!}{\theta^{k+1}}} \right), \; 0 < v < 1, \; \theta > 0. \tag{18}$$

In particular, for the Lindley distribution,

$$p(v) = F(v;\theta) = \frac{v^{\theta}(1+\theta(1-log(v)))}{1+\theta} \qquad 0 < v < 1, \theta > 0. \tag{19}$$

In this equation, we replace the shape parameter θ by product's mean life (μ). We can say that θ = g(μ), and we get the value of θ by solving the equation by numerical method and hence we have $p(v) = F(v, \mu_v)$.

The Operating Characteristic (OC) function plays a vital role in product control techniques. It gives the probability of acceptance of an individual lot from finite production. For some fixed p, our sampling plan characterized by $(n,\ c,\ t)$ or equivalently by $(n,\ c,\ v)$, where $v = e^{-t}$. For sufficient large lots, the binomial distribution can be applied. The OC function can be formulated as

$$\pi(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} = 1 - B_p(c+1,\ n-c)$$

with $p = F(v;\mu_v)$, and $B_p(c+1,\ n-c)$ is the incomplete beta function.
For determining small positive integer n for given c, v and μ_v^0 , we use

$$\pi(p_0) = \sum_{i=0}^{c} \binom{n}{i} p_0^i (1-p_0)^{n-i} \leq 1 - P^*, \tag{20}$$

where, P^* is the level of confidence. If p is very small and n is very large, and $\beta = np$ is finite, then the binomial distribution can be approximated by the Poisson distribution. Then, the OC function becomes

$$\pi(p) = \sum_{i=0}^{c} e^{-\beta} \frac{\beta^i}{i!} = 1 - \Gamma(c+1,\ \beta),$$

with $\Gamma(k,\ w) = \frac{1}{\Gamma(k)} \int_0^w x^{k-1} e^{-x} dx$, the incomplete gamma function.

3.2. Sampling Procedure

The RASP is conducted as follows.

1. Put n items on test.
2. Choose the acceptance number c and specify the maximum test time t.
3. Perform the experiment and count the number of failures.
4. Accept the lot if the number of failures is at most c by the experiment time t.
5. Terminate the experiment as soon as (c+1)th failure occurs and reject the lot.

4. Estimation of the plan parameters

A sampling scheme for unit-OPPE distribution with parameter $(n,\ c,\ v)$ satisfies the consumer's risk given in the previous section. To determine the smallest integer of sample size n for given $(c,\ v)$ is

$$Min_{(n|c,\mu_v^0,P^*)} n$$
subject to
$$\sum_{i=0}^{c} \binom{n}{i} p_0^i (1-p_0)^{n-i} \leq 1 - P^*, \tag{21}$$

where $p_0 = 1 - \left(\frac{\sum_{k=0}^{r} \frac{a_k \Gamma(k+1)\Gamma(k+1, v g(\mu_v^0))}{g(\mu_v^0)^{k+1}}}{\sum_{k=0}^{r} a_k \frac{k!}{g(\mu_v^0)^{k+1}}} \right)$, $0 < v < 1,\ g(\mu_v^0) > 0$.

Algorithm for determination of the plan parameters is as follows.

1. Specify the confidence level P^*.
2. Choose t and μ_0.
3. Calculate $v = e^{-t}$ and $\mu_v^0 = \frac{\sum_{k=0}^{r} a_k \frac{\Gamma(k+s+1)}{\theta^{k+s+1}}}{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}}}$, θ_0 is obtained by solving $\mu_0 = \frac{\sum_{k=0}^{r} a_k \frac{\Gamma(k+s+1)}{\theta^{k+s+1}}}{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}}}$ numerically.
4. Choose a value of c.
5. For given (c,t,μ_v^0), choose minimize n such that (21) satisfies.

The minimum values of n and c satisfying the inequality are obtained and shown in Table 1- 2 for the Lindley and that are in Tables 3-4 for the exponential distribution for $P^* = 0.95, 0.99$. The representative tables are shown for an easy and comprehensive understanding of the proposed plan. The contents of each table are described as follows. The first row is reserved for specifying confidence level (P^*). The first column represents the pre-specified mean value, μ_0^v of unit-Lindley/unit-exponential distribution and the corresponding mean value, μ_0 of Lindley/exponential distribution in the parenthesis. The second row represents the truncation time, v, of unit-Lindley/unit-exponential distribution and the corresponding time, t, of Lindley/exponential distribution in the parenthesis. For a combination of (μ_0^v, v) or (μ_0, t), in the cell, the optimal choices of n, the sample size and c, the acceptance number are presented.

Table 2: *Determination of optimal sample size for Lindley set up*

	$P^* = 0.99$					
$\mu_v^0(\mu_0)$v(t)	$6.74 \times 10^{-3}(5)$		$4.54 \times 10^{-5}(10)$		$3.05 \times 10^{-8}(15)$	
	c	n	c	n	c	n
0.1162(5)	0	3	0	5	0	8
	1	5	1	8	1	12
	2	6	2	10	2	16
0.0447(10)	0	2	0	2	0	3
	1	3	1	4	1	5
	2	4	2	5	2	6
0.0236(15)	0	1	0	2	0	2
	1	3	1	3	1	3
	2	4	2	4	2	5

Table 1: *Determination of optimal sample size for Lindley set up*

	$P^* = 0.95$					
$\mu_v^0(\mu_0)$v(t)	$6.74 \times 10^{-3}(5)$		$4.54 \times 10^{-5}(10)$		$3.05 \times 10^{-8}(15)$	
	c	n	c	n	c	n
0.1162(5)	0	2	0	3	0	6
	1	4	1	6	1	9
	2	5	2	8	2	12
0.0447(10)	0	1	0	2	0	2
	1	3	1	3	1	4
	2	4	2	4	2	5
0.0236(15)	0	1	0	1	0	2
	1	2	1	3	1	3
	2	3	2	4	2	4

Table 3: *Determination of optimal sample size for Exponential set up*

	$P^* = 0.95$					
$\mu_v^0(\mu_0)$v(t)	$6.74 \times 10^{-3}(5)$		$4.54 \times 10^{-5}(10)$		$3.05 \times 10^{-8}(15)$	
	c	n	c	n	c	n
0.1662(5)	0	7	0	21	0	94
	1	11	1	34	1	150
	2	15	2	45	2	197
0.0909(10)	0	4	0	7	0	16
	1	6	1	11	1	25
	2	8	2	15	2	34
0.0625(15)	0	3	0	5	0	8
	1	5	1	8	1	14
	2	7	2	10	2	18

A few observations from the Tables are noted below.
(i) With the confidence level (P^*) increase, the minimum sample size increases.
(ii) The optimal sample size for the Lindley distribution is smaller than that for the exponential distribution.
(iii) The sample size increases with the increase of truncation time (t) or mean (μ) or both.

Table 4: *Determination of optimal sample size for Exponential set up*

	$P^* = 0.99$					
$\mu_v^0(\mu_0)$v(t)	$6.74 \times 10^{-3}(5)$		$4.54 \times 10^{-5}(10)$		$3.05 \times 10^{-8}(15)$	
	c	n	c	n	c	n
0.1662(5)	0	10	0	32	0	72
	1	15	1	46	1	122
	2	20	2	60	2	166
0.0909(10)	0	5	0	11	0	12
	1	8	1	16	1	21
	2	11	2	20	2	27
0.0625(15)	0	4	0	7	0	7
	1	6	1	10	1	11
	2	8	2	13	2	16

5. Real life examples

Data Set 1 : The data is from [16] and arose in test on the cycle at which the yarn failed. The data are the number of cycles until failure of the yarn(100 units):
15, 20, 20, 38, 38, 40, 40, 42, 55, 55, 61, 61, 65, 71, 76, 81, 86, 88, 90, 93, 98, 105, 121, 124, 124, 131, 135, 135, 137, 143, 146, 149, 151, 157, 166, 169, 175, 176, 180, 180, 180, 182, 185, 185, 186, 188, 188, 193, 194, 195, 196, 198, 198, 203, 203, 211, 220, 224, 229, 229, 236, 239, 244, 246, 246, 249, 250, 251, 262, 264, 264, 264, 277, 279, 282, 284, 286, 290, 292, 315, 321, 325, 337, 338, 341, 350, 353, 364, 393, 396, 398, 400, 400, 423, 497, 568, 571, 597, 653, 829.
First, we have checked whether the considered data set is well fitted with the exponential or Lindley distribution by goodness-of-fit test. For this purpose, we have used the Akaike Information Criterion [AIC=-2log(likelihood)+2k, k is the parameter number] to verify which data fit better. The model that best fits the data could be the one with the lowest AIC value.

Table 5: *Comparison of Exponential and Lindley Distribution for Data Set 1*

Distribution	Estimate of θ	Negative Log-likelihood	AIC
Exponential	0.00450	640.2587	1282.517
Lindley	0.0089687	625.6708	1253.3410

Table 5 shows that the Lindley distribution gives a better fit as the AIC value is less than the exponential distribution. The histogram with fitted distributions and P-P plots is shown in Figure 2.
Suppose the truncation time of the testing number of cycles until failure of the yarn is 150, 200, 250 and 300. We construct the decision table for specified mean life as 150, 200, 150, and 300. The estimated average life of the number of cycles is 221.98. The sample size n(=100) is fixed, so Tables 6 and 7 are constructed for (c,v) or (c,t) values, and the decision regarding acceptance or rejection of the lot is made accordingly for the Lindley and the exponential distributions.
Based on the observations, we must make a sentence for the lot, whether it will be accepted or rejected. In this example, let us assume that truncation time, t is 200, that after transformation of t ($v = e^{-t}$), v is 0.0497, and that we take the specified mean of the Lindley distribution to be 150, with the corresponding unit-Lindley mean to be 0.3001. We accept the lot only if the number of failures before the specified mean,150, is less than or equal to acceptance number 60 (see Table 6). In this case, the decision is to accept the lot. We arrive at the same decision for exponential assumption, but the acceptance number, c(=94), is larger than that for the Lindley distribution. So, there may be a chance to come to a wrong decision if Lindley is a better fit, which will be

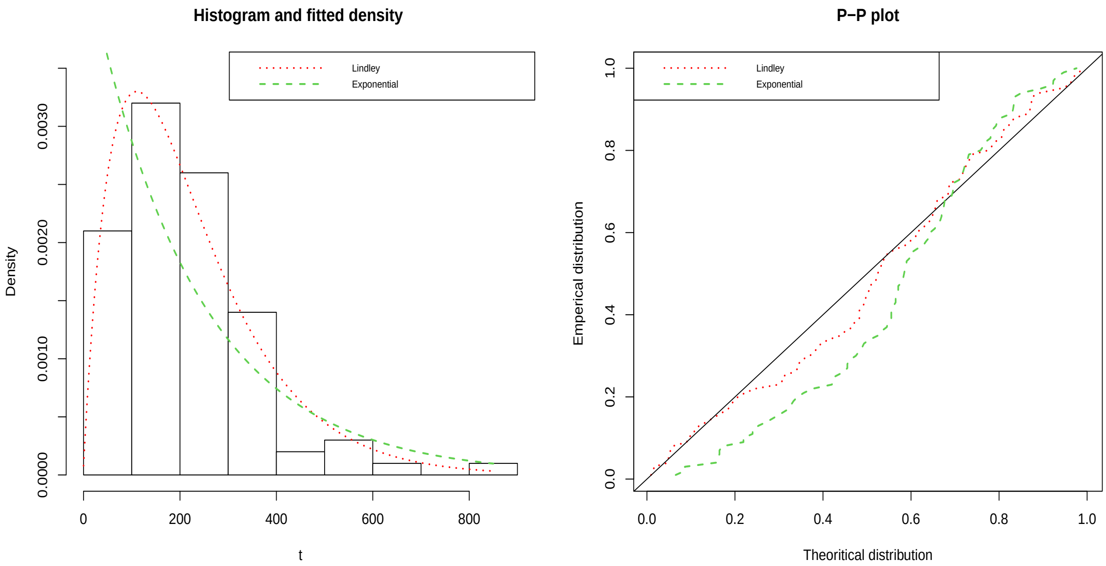

Figure 2: *Histogram with fitted distributions and P-P plot for Data Set 1*

noticed in the case of Data Set 2. The OC surface of the plan for $n = 100$ and $c = 72$ has been shown in Figure 3.

Table 6: *RASP for Lindley distribution for Data Set 1*

	$P^* = 0.95$											
$\mu_v^0(\mu_0)$v(t)	0.1353 (150)			0.0497 (200)			0.0183 (250)			0.0067 (300)		
	c	n	decision	c	n	decision	c	n	decision	c	n	decision
0.3001(150)	72	100	Accept	60	100	Accept	49	100	Accept	39	100	Accept
0.2065(200)	82	100	Accept	74	100	Accept	65	100	Accept	57	100	Accept
0.1515(250)	88	100	Accept	82	100	Accept	76	100	Accept	70	100	Accept
0.1162(300)	91	100	Accept	87	100	Accept	83	100	Accept	78	100	Accept

Table 7: *RASP for Exponential distribution for Data set 1*

	$P^* = 0.95$											
$\mu_v^0(\mu_0)$v(t)	0.1353 (150)			0.0497 (200)			0.0183 (250)			0.0067 (300)		
	c	n	decision	c	n	decision	c	n	decision	c	n	decision
0.0067(150)	96	100	Accept	94	100	Accept	93	100	Accept	93	100	Accept
0.0049(200)	96	100	Accept	95	100	Accept	95	100	Accept	94	100	Accept
0.0039(250)	97	100	Accept	96	100	Accept	95	100	Accept	95	100	Accept
0.0033(300)	97	100	Accept	96	100	Accept	96	100	Accept	95	100	Accept

Data Set 2:

This data represents the failure times in minutes for a sample of 15 electronic component in accelerated life test (see, [13]), also used by [19] and [18] :
1.4, 5.1, 6.3, 10.8, 12.1, 18.5, 19.7, 22.2, 23, 30.6, 37.3, 46.3, 53.9, 59.8, 66.2.

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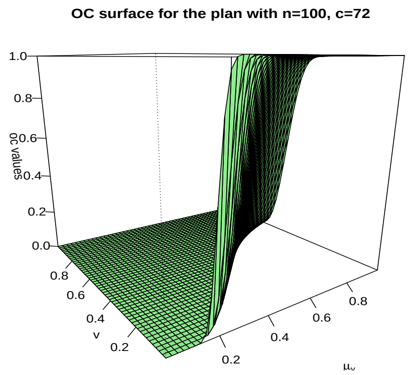

Figure 3: *OC Surface of the proposed plan for Data Set 1*

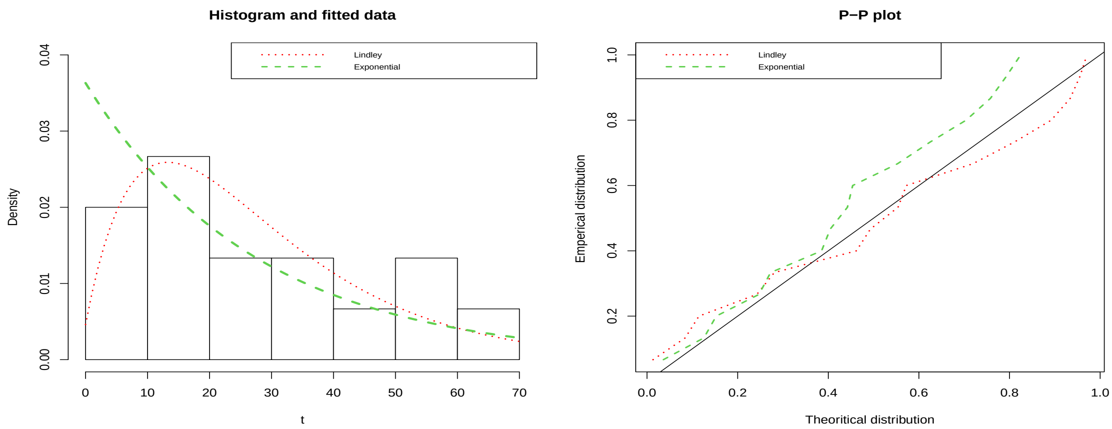

Figure 4: *Histogram with fitted distributions and P-P plot for Data Set 2*

Table 8: *Comparison of Exponential and Lindley Distributions for Data Set 2:*

Distribution	Estimate of θ	Negative Log-likelihood	AIC
Exponential	0.03630	64.7386	131.4764
Lindley	0.07025	64.40554	130.8110

Table 8 shows that the Lindley distribution gives a better fit as the AIC value is less than that for the exponential distribution. The histogram with fitted distributions and the P-P plot shown in Figure 4 substantiates the claim.

The estimated mean life is 27.54 for the fitted Lindley distribution. We have selected the testing time for failure times as 20, 30, 35 and 40. The corresponding hypothesis for testing is $\mu \geq \mu_0$ against $\mu < \mu_0$, where μ_0 is the specified mean, and μ is the average lifetime. We construct Tables 9 and 10 with sample size n =15 for the Lindley and exponential distributions, respectively.

For example, let the truncation time is 30, and the specified mean is 35. The decision regarding the lot is to reject under the Lindley assumption and accept under the exponential assumption. Hence the exponential assumption leads to a wrong conclusion as the data fits better with the Lindley distribution. Figure 5 shows the OC surface of the plan for $n = 15$ and $c = 9$.

Table 9: *RASP for the Lindley distribution for Data set 2*

	$P^* = 0.95$											
$\mu_v^0(\mu_0)$v(t)	0.3679 (20)			0.1353 (30)			0.0820 (35)			0.0498 (40)		
	c	n	decision	c	n	decision	c	n	decision	c	n	decision
0.4853(20)	9	15	Accept	5	15	Accept	4	15	Reject	3	15	Reject
0.3002(30)	11	15	Accept	8	15	Reject	7	15	Reject	6	15	Reject
0.2465(35)	11	15	Accept	9	15	Reject	8	15	Reject	8	15	Reject
0.2065(40)	12	15	Accept	10	15	Reject	9	15	Reject	9	15	Reject

Table 10: *RASP for Exponential distribution for Data set 2*

	$P^* = 0.95$											
$\mu_v^0(\mu_0)$v(t)	0.3679 (20)			0.1353 (30)			0.0820 (35)			0.0498 (40)		
	c	n	decision	c	n	decision	c	n	decision	c	n	decision
0.0476(20)	12	15	Accept	11	15	Accept	10	15	Accept	10	15	Accept
0.0323(30)	12	15	Accept	11	15	Accept	11	15	Accept	11	15	Accept
0.0278(35)	12	15	Accept	12	15	Accept	11	15	Accept	11	15	Accept
0.0244(40)	12	15	Accept	12	15	Accept	11	15	Accept	11	15	Accept

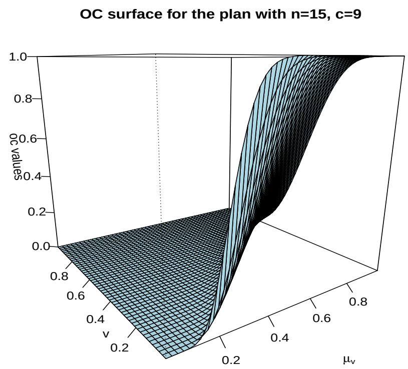

Figure 5: *OC Surface of the proposed plan for Data Set 2*

Data set 3: This data set shows that the cycle-to-failure numbers for 25 (100-cm) specimens of yarn tested at a particular strain level (see, [13]) are:
15, 20, 38, 42, 61, 76, 86, 98, 121, 146, 149, 157, 175, 176, 180, 180, 198, 220, 224, 251, 264, 282, 321, 325, 653.
Table 11 shows that the OPPE with $a_0 = 9, a_1 = 4, a_2 = 0.005$ distribution is a better fit as the AIC value is less than that for the exponential distribution. The histogram with fitted distributions and the P-P plot in Figure 6 justifies the claim. Table 12 shows different plans and their decisions for the fitted OPPE model. Table 13 shows that for the exponential model. The OPPE assumption shows its superiority. The OC surface of the plan for $n = 25$ and $c = 18$ with the fitted OPPE is at Figure 7.

Table 11: *Comparison of Exponential and OPPE(9,4,0.005) for Data Set 3*

Distribution	Estimate of θ	Negative Log-likelihood	AIC
Exponential	0.0056	154.5078	311.179
OPPE(9,4,0.005)	0.01115	152.5078	307.0156

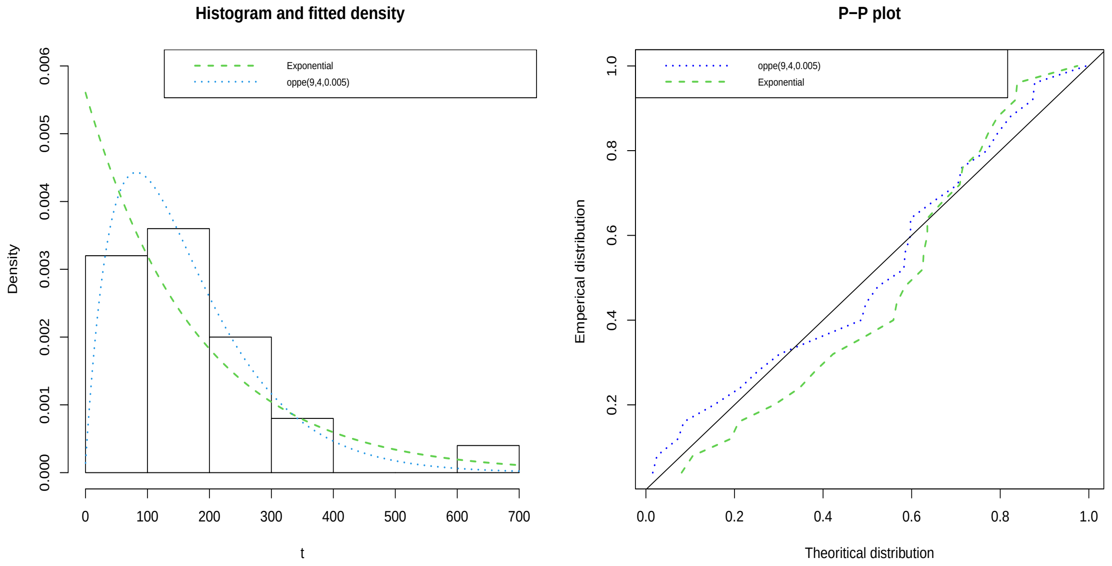

Figure 6: *Histogram with fitted distributions and P-P plot for Data Set 3*

Table 12: *RASP for the OPPE(9,4,0.005) distribution for Data set 3*

	$P^* = 0.95$											
$\mu_v^0(\mu_0)$v(t)	0.4065 (100)			0.2466 (150)			0.1469 (200)			0.0907 (250)		
	c	n	decision	c	n	decision	c	n	decision	c	n	decision
0.5208(100)	18	25	Accept	11	25	Accept	8	25	Accept	6	25	Reject
0.4054(150)	16	25	Accept	14	25	Accept	11	25	Accept	9	25	Reject
0.3282(200)	18	25	Accept	16	25	Reject	13	25	Reject	12	25	Reject
0.2730(250)	19	25	Accept	17	25	Reject	15	25	Reject	14	25	Reject

Table 13: *RASP for the Exponential distribution for Data set 3*

	$P^* = 0.95$											
$\mu_v^0(\mu_0)$v(t)	0.4065 (100)			0.2466 (150)			0.1469 (200)			0.0907 (250)		
	c	n	decision	c	n	decision	c	n	decision	c	n	decision
0.0099(100)	23	25	Accept	23	25	Accept	22	25	Accept	22	25	Accept
0.0066(150)	23	25	Accept	23	25	Accept	23	25	Accept	22	25	Accept
0.0049(200)	23	25	Accept	23	25	Reject	23	25	Accept	23	25	Accept
0.0039(250)	23	25	Accept	23	25	Accept	23	25	Accept	23	25	Accept

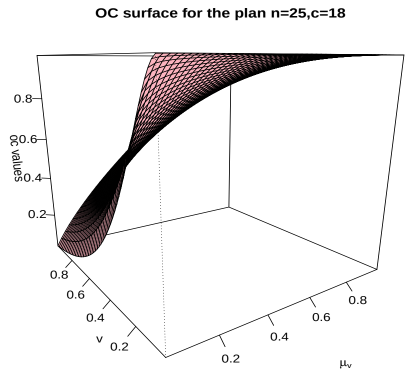

Figure 7: *OC Surface of the proposed plan for Data Set 3*

6. Concluding remarks

A reliability acceptance sampling plan is formulated for the OPPE distributed quality characteristic. The OPPE family of distributions does not belong to the scale-invariant family, whereas most of the RASP chalked out for the scale-invariant family in the literature. The optimal plan parameters are estimated by transforming the OPPE distribution into its unit form to utilize the advantage of finite range (in this case, (0,1)). A few examples are presented for finding optimal sample sizes for the proposed plan for the Lindley distribution, a particular choice of the OPPE family, which will be helpful to scientists and quality practitioners for implementation. Three data sets are analyzed for implementing the proposed plan. The approach may be adopted to construct RASP for other lifetime quality characteristic distributions that do not belong to the scale-invariant family.

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Declarations

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Ethical Approval: This article does not contain any studies with human participants performed by any of the authors.

Code availability: Codes are available on request.

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