ESTIMATION OF FRECHET PARAMETERS WITH TIME-CENSORED DATA IN ACCELERATED LIFE TESTING UTILISING THE GEOMETRIC PROCESS

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Abstract

The geometric process (GP) has been applied to estimate constant stress accelerated life testing for the Frechet failure item with time-censored data. A geometric process (GP) is developed by the failure time of tested items when stress levels are constantly rising. The estimates of the various parameters are calculated using the maximum likelihood estimation procedure. The asymptotic variance of estimates is obtained using a Fisher information matrix. The asymptotic variance is then used to calculate the distribution parameter asymptotic interval values. The statistical properties and confidence intervals of the required parameters are then illustrated using a simulation technique.

Keywords: Frechet Distribution, Geometric Process, Maximum Liklihood Estimate, Asymptotic Confidence Interval Estimate, Simulation Study.

I. Introduction

The most common approach to product evaluation is accelerated life testing (ALT), which gives the necessary details on the product's life under normal usage. It's widely employed in the manufacturing sector for the purpose of enhancing product quality. In order to gather data swiftly than under normal conditions, it enables the researcher to enhance the stresses on the life distribution parameters. Because working under normal conditions would be time consuming, it is impractical to test items under greater stress than is normal in order to induce early failure. The life distribution of a product, as well as any related characteristics under normal stress, must be extrapolated from test data using accelerated life analysis. Comparing such a test to tests conducted under normal conditions, time and money are saved.

Making the decision as to what stress should be imposed and how is the most challenging job in Accelerated Life Testing. The ALT contains a variety of stress loading types, such as constant stress , progressive stress, step stress, random stress and cyclic stress. ALT has mainly two types of data that is complete (every failure time is available) and censored (some failure time is unavailable).
Numerous authors have given their perspective on accelerated life testing (for constant stress), references includes [1, 2, 3, 4]. Yang[5], introduced optimal design using a four-level ALT and compared it with three-level ALT for different censoring schemes. ALT utilised Lam's[6] GP concept to investigate the problem of repair replacement. ALT plans for Generalized Exponential

Distribution under GP was analyse by Lone[7]. A great number of literature on ALT under GP model are (see,[8, 11, 9, 10]). Zhou[12] showed ALT with progressive hybrid censoring under geometric process for Rayleigh distribution. And the same time Huang[13] preseneted GP for exponential failure model in respect of complete as well censored observation. Fan [14] explored the constant ALT design for the generalised gamma model. For life distributions like exponential and lognormal distributions, Chen[15] discovered Bayesian approximations of the parameters in a generalised linear model (GP). So many works has done on GP in ALT, see Lone SA[16], Kamal M[17], Lone SA[18], Kamal M[19], Zarrin S[20], Lone SA[22], Lone SA[23], Ismail[24], Lone SA[25], Aly H[26], Alam I[27]. Using Informative and Noninformative Priors, Sindhu[28] performed a Bayesian Study for Censored Shifted Gompertz Mixture Distributions. Nassr, SG[29] extended Weibull distribution under adaptive type II progressive hybrid censoring. Hemmati, F[30] provided the log-normal distribution under type-II progressive hybrid censoring . For the Modified Kies exponential distribution, Hussam E[31] investigated simple and multiple ramp-stres ALT design for type-II censored data and Binomial Removal.

II. Model Description and Testing Procedure

I. Geometric Process (GP)

A sequence of stochastic variable $\{X_n\}$, $n = 1, 2, 3...$ is referred to as a Geometric Process (GP) if $\{\lambda^{n-1}X_n, n = 1, 2, 3...\}$ established a renewal process. Where, ratio of GP $\lambda(> 0)$ is a real valued. It may be demonstrated that if $\{X_n\}$, $n = 1, 2, 3...$ develops a GP and there exist a random variable having pdf $f(x)$ with mean γ and variance σ^2 then the subsequent pdf of X_n will be given as $\lambda^{n-1}f(\lambda^{n-1}x)$ with mean $E(X_n) = \gamma/\lambda^{n-1}$ and variance $V(X_n) = \sigma^2/\lambda^{2(n-1)}$.

II. Frechet Failure Model

Probability density function of the Frechet variable is :

$$f(x) = \alpha\beta^{\alpha}x^{-\alpha-1}e^{-(\frac{x}{\beta})^{-\alpha}}, \qquad x > 0, \alpha > 0, \beta > 0 \tag{1}$$

where, α(shape) and β (scale) are parameters of the life distribution:

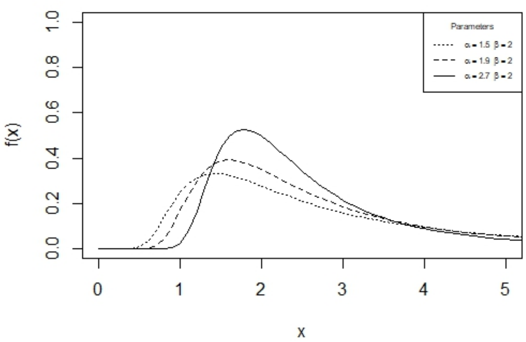

Figure 1: *PDF at different shapes and fixed scale ($\beta = 2$)*

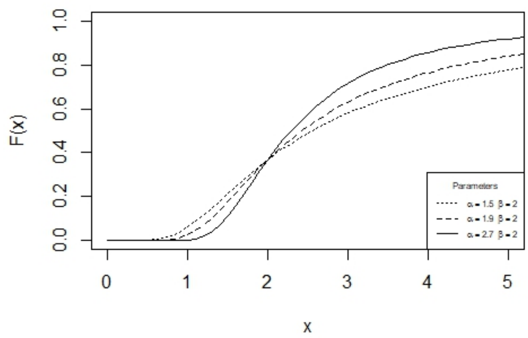

Figure 2: *CDF at different shapes and fixed scale ($\beta = 2$)*

CDF of Frechet variable takes the following expression:

$$F(x) = e^{-(\frac{x}{\beta})^{-\alpha}}, \qquad x > 0, \alpha > 0, \beta > 0 \tag{2}$$

Survival function for the Frechet variable is given as:

$$S(x) = 1 - e^{-(\frac{x}{\beta})^{-\alpha}}, \qquad x > 0 \tag{3}$$

The hazard function (HF) is

$$h(x) = \frac{\alpha\beta^{\alpha}x^{-\alpha-1}e^{-(\frac{x}{\beta})^{-\alpha}}}{1 - e^{-(\frac{x}{\beta})^{-\alpha}}} \tag{4}$$

III. Assumptions

1. The lifetime of failed items follows Frechet distribution at each stress.

2. Supoose a life test is organized with s number of stresses(increasing order). A random sample of n items are place on each stress and begin to operate simultaneously. Let the failure time of i^{th}(ranges from 1 to n) item in k^{th}(ranges from 1 to s) stress is denoted by x_{ki}. Now, failed items are removed and the test will run till the complete sample is exhausted at a predetermined t(censoring time)at each stress.

3. Stress is a log-linear function of the scale parameter β i.e., $log(\beta_k) = a + bS_k$, where a and b are unknown parameters, values dependind on the nature of products and test method.

4. Let say the lifespan of items on each stress is represented by random variables $X_0, X_1, X_2, ..., X_s$, where X_0 is the lifespan of the items under normal stress and sequence $X_k, k = 1, 2, ...s$ formulates a GP with ratio parameter $\lambda > 0$.

All of the preceding assumptions, except for the last one assumption (4^{th} assumption), are generally accepted in the ALT. Last one is based on the notion of a geometric procedure that is better than the traditional one without making computation more difficult. The following theorem, which presupposes a log linear function between life and stress is demonstrates as:

Theorem 1. If stresses in an ALT increases constantly, then the lifespan of products at each stress develops a GP. i.e., if the difference $(S_{k+1} - S_k) = \ (\Delta S\ )constant$, for $k = 1, 2, ..., s - 1$, then $\{X_k\}$, $k = 1, 2, ..., s$ develops a GP.

Proof. Last assumption (4^{th} assumption) states that,

$$\log\left(\frac{\beta_{k+1}}{\beta_k}\right) = b(S_{k+1} - S_k) = b\ (\Delta S\ ) \tag{5}$$

This demonstrates that the excessive stresses comprise an arithmetic series with a difference ΔS. (constant) Now, the previous expression can be rewritten as:

$$\frac{\beta_{k+1}}{\beta_k} = e^{b\Delta S} = \frac{1}{\lambda}, \tag{6}$$

from (6), we have

$$\beta_k = \frac{1}{\lambda}\beta_{k-1} = \frac{1}{\lambda^2}\beta_{k-2} = ... = \frac{1}{\lambda^k}\beta$$

The lifetime pdf of an object has the following structure at the kth stress level

$$f(x) = \alpha\beta^\alpha x^{-\alpha-1} e^{-(\frac{x}{\beta})^{-\alpha}}$$

$$f_{X_k}(x) = \lambda^{-\alpha k r_k}\ \alpha^{r_k}\beta^{\alpha r_k} x_{ki}\ e^{\left(-\frac{\lambda^k x_{ki}}{\beta}\right)} \tag{7}$$

And the cdf is written as

$$F_{X_k}(x) = e^{\left(-\frac{\lambda^k x_{ki}}{\beta}\right)} \tag{8}$$

This shows that

$$f_{X_k}(x) = \lambda^k f_{X_0}(\lambda^k x) \tag{9}$$

Hence, from the theory of GP and using the equation(9) it is obvious that if the pdf of lifespan of the X_0 (normal stress) is $f_{X_0}(x)$, then the pdf of the lifespan X_k (kth stress) is given by $\lambda^k f_{X_0}(\lambda^k x)$. Finally , it is evident that lifespan of a arithmetically increasing stresses results a GP. ■

IV. Maximum Likelihood Estimation

The most significant and frequently applied estimation technique is the maximum likelihood (ML) approach. The applicability of other methods is restricted, whereas it can be used with any probability distribution. ML estimation achievement in ALT is more complicated, and closed-form estimates of parameters are typically unavailable. Consequently, to calculate them, an arithmetic technique Newton Raphson (NR) method has been used.
Let's say the test is stopped at time t for each stress level, and only $x_{ki}(\leq t)$ failures are recorded. Suppose that $r_k \leq n$ failures at the k^{th} stress levels are obtained prior to suspending the test, and remaing $(n - r_k)$ items are survived till the entire test without any failure. The likelihood function of a particular stress is provided for time-censored Frechet failure data under GP with s number of stress:

$$L_k = \frac{n!}{(n-r_k)!}\left[\lambda^{-\alpha k r_k}\ \alpha^{r_k}\beta^{\alpha r_k}\prod_{i=1}^{r_k} x_{ki}\ e^{\left(-\frac{\lambda^k x_{ki}}{\beta}\right)}\right]\left[1 - e^{\left(-\frac{\lambda^k t}{\beta}\right)}\right]^{n-r_k} \tag{10}$$

Consequently, the likelihood function for overall stresses is

$$L_s(\alpha,\beta,\lambda) = \prod_{s=1}^{k} L_k = \prod_{k=1}^{s} \frac{n!}{(n-r_k)!}\left[\lambda^{-\alpha k r_k}\, \alpha^{r_k}\beta^{\alpha r_k}\prod_{i=1}^{r_k} x_{ki}\, e^{\left(-\frac{\lambda^k x_{ki}}{\beta}\right)}\right]\left[1-e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}}\right]^{n-r_k} \tag{11}$$

Taking log both sides and the likelihood function for the above equation is

$$l = \ln L_s(\alpha,\beta,\lambda) = \sum_{k=1}^{s}\left[\ln\left(\frac{n!}{(n-r_k)!}\right) + r_k(\ln\alpha + \ln\beta - \alpha k\ln\lambda) - (\alpha+1)\sum_{i=1}^{r_k}\ln x_{ki} - \left(\frac{\lambda^k}{\beta}\right)^{-\alpha}\sum_{i=1}^{r_k} x_{ki}^{-\alpha} + (n-r_k)\ln\left\{1-e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}}\right\}\right] \tag{12}$$

MLE's of different parameters α, β and λ has found after solving the these normal equations $\frac{\partial l}{\partial\alpha}$, $\frac{\partial l}{\partial\beta}$ and $\frac{\partial l}{\partial\lambda} = 0$.

$$\frac{\partial l}{\partial\alpha} = \sum_{k=1}^{s}\left[-k\,r_k\ln\lambda + \alpha^{-1}r_k - \sum_{i=1}^{r_k}\ln x_{ki} + \left(\frac{\lambda^k}{\beta}\right)^{-\alpha}\ln\left(\frac{\lambda^k}{\beta}\right)\sum_{i=1}^{r_k} x_{ki} - (n-r_k)A^{-1} \times\left\{e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}}\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}\ln\left(-\frac{\lambda^k t}{\beta}\right)\right\}\right] \tag{13}$$

$$\frac{\partial l}{\partial\beta} = \sum_{k=1}^{s}\left[\beta^{-1}r_k - \left(\frac{\lambda^k}{\beta}\right)^{-\alpha}\ln\beta\sum_{i=1}^{r_k} x_{ki} + (n-r_k)A^{-1}\left\{e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}}\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}\ln\beta\right\}\right] \tag{14}$$

$$\frac{\partial l}{\partial\lambda} = \sum_{k=1}^{s}\left[-\alpha\,k\,r_k\lambda^{-1} - \beta^{\alpha}\alpha\,k\,\lambda^{-(\alpha k+1)}\sum_{i=1}^{r_k} x_{ki} - (n-r_k)A^{-1}\left\{\alpha\,k\,\lambda^{-(\alpha k+1)}\left(\frac{t}{\beta}\right)^{-\alpha} e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}}\right\}\right] \tag{15}$$

where $A = 1 - e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}}$

As we can see,expressions(13), (14) and (15) are not linear. Consequently, it is challenging to find a closed-form answer. Therefore, the estimate of α, β and λ is obtained by concurrently solving these expressions using the NR method.

V. Asymptotic Confidence Interval

Under some specific regularity restrictions, large sample theory assures the consistency and normality of ML estimators. Because the estimate of parameters are not forming closed form, exact confidence intervals of the parameters cannot be determined . As a consequence, asymptotic confidence intervals rather than exact confidence intervals develop using the asymptotic property of MLE's.

Providing the The Fisher Information Matrix (FIM) as:

$$F = \begin{bmatrix} -\frac{\partial^2 l}{\partial \alpha^2} & -\frac{\partial^2 l}{\partial \alpha \partial \beta} & -\frac{\partial^2 l}{\partial \alpha \partial \lambda} \\ -\frac{\partial^2 l}{\partial \beta \partial \alpha} & -\frac{\partial^2 l}{\partial \beta^2} & -\frac{\partial^2 l}{\partial \beta \partial \lambda} \\ -\frac{\partial^2 l}{\partial \lambda \partial \alpha} & -\frac{\partial^2 l}{\partial \lambda \partial \beta} & -\frac{\partial^2 l}{\partial \lambda^2} \end{bmatrix}$$

The components of FIM can be obtained as follows:

$$\frac{\partial^2 l}{\partial \alpha^2} = \sum_{k=1}^{s} \left[-\alpha^{-1} r_k - \left(\frac{\lambda^k}{\beta}\right)^{-\alpha} \left\{ \ln\left(\frac{\lambda^k}{\beta}\right) \right\}^2 \sum_{i=1}^{r_k} x_{ki} - (n - r_k) A^{-2} \right. \\ \left. \times \left\{ e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}} \left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha} \ln\left(-\frac{\lambda^k t}{\beta}\right) \right\} \left\{ A \left(\left(\frac{\lambda^k t}{\beta}\right)^{-\alpha} - 1 \right) - e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}} \left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha} \right\} \right] \tag{16}$$

$$\frac{\partial^2 l}{\partial \beta^2} = \sum_{k=1}^{s} \left[\beta^{-2}\, r_k - \left(\frac{\lambda^k}{\beta}\right)^{-\alpha} (\ln\beta)^2 \sum_{i=1}^{r_k} x_{ki} + (n - r_k) A^{-2} \right. \\ \left. \times \left\{ e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}} \left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha} \ln\beta \left(A \left(\beta^{\alpha} \ln\beta - e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}} \left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha} \ln\beta \right) - 1 \right) \right\} \right] \tag{17}$$

$$\frac{\partial^2 l}{\partial \lambda^2} = \sum_{k=1}^{s} \left[-\alpha\, k\, r_k \lambda^{-2} + \beta^{\alpha}\, \alpha(\alpha+1)\, k\, \lambda^{-(\alpha k+2)} \sum_{i=1}^{r_k} x_{ki} + (n - r_k) A^{-2} \right. \\ \left. \times \left\{ \alpha\, k\, \lambda^{-(\alpha k+2)} \left(\frac{t}{\beta}\right)^{-\alpha} e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}} \left(A \left(\alpha\, k \lambda^{-(\alpha k+1)} \left(\frac{t}{\beta}\right)^{-\alpha} \right) + \left(\frac{t}{\beta}\right)^{-\alpha} e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}} \right) \right\} \right] \tag{18}$$

$$\frac{\partial^2 l}{\partial \alpha \partial \beta} = \sum_{k=1}^{s} \left[-\left(\frac{\lambda^k}{\beta}\right)^{-\alpha} \left(\frac{1}{\beta}\right) \left(1 + \alpha \ln\left(\frac{\lambda^k}{\beta}\right) \right) \sum_{i=1}^{r_k} x_{ki} + (n - r_k) A^{-2} \right. \\ \left. \times \left\{ e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}} \left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha} \left(\frac{1}{\beta} + \alpha \ln\left(\frac{\lambda^k}{\beta}\right) \right) + \left(\frac{1}{\beta}\right) \ln\beta - e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}} \left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha} \ln\beta \right\} \right] \tag{19}$$

$$\frac{\partial^2 l}{\partial \alpha \partial \lambda} = \sum_{k=1}^{s} \left[-\lambda^{-1} k\, r_k + \left(\frac{\lambda^k}{\beta}\right)^{-\alpha} \frac{k}{\lambda} \left(\frac{1}{\beta} + \alpha \ln\left(\frac{\lambda^k}{\beta}\right)^{-\alpha} \right) + (n - r_k) A^{-2} \right. \\ \left. \times \left\{ \alpha\, k\, \lambda^{-(\alpha k-1)} \left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha} e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}} \left(\alpha\, k \lambda^{-(\alpha k+1)} \left(\frac{t}{\beta}\right)^{-\alpha} \right) + \left(\frac{t}{\beta}\right)^{-\alpha} e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}} \right\} \right] \tag{20}$$

$$\frac{\partial^2 l}{\partial\beta\partial\lambda} = \sum_{k=1}^{s}\Bigg[-\lambda^{-1}k\alpha\left(\frac{\lambda^k}{\beta}\right)^{-\alpha}\ln\beta\sum_{i=1}^{r_k}x_{ki} - (n-r_k)A^{-2} \times \left\{e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}}\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}\ln\beta\left(\frac{1}{\beta}+\alpha\ln\left(\frac{\lambda^k}{\beta}\right)\right) - e^{\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}}\left(-\frac{\lambda^k t}{\beta}\right)^{-\alpha}\ln\beta\right\}\Bigg] \tag{21}$$

The var-covariance matrix is:

$$F^{-1} = \begin{bmatrix} -\frac{\partial^2 l}{\partial\alpha^2} & -\frac{\partial^2 l}{\partial\alpha\partial\beta} & -\frac{\partial^2 l}{\partial\alpha\partial\lambda} \\ -\frac{\partial^2 l}{\partial\beta\partial\alpha} & -\frac{\partial^2 l}{\partial\beta^2} & -\frac{\partial^2 l}{\partial\beta\partial\lambda} \\ -\frac{\partial^2 l}{\partial\lambda\partial\alpha} & -\frac{\partial^2 l}{\partial\lambda\partial\beta} & -\frac{\partial^2 l}{\partial\lambda^2} \end{bmatrix}^{-1}$$

The diagonal elements in the above matrix are representing variances terms, and the non-diagonal are indicating covariances .

Estimates of asymptotic confidence interval for the proposed parameters α, β and λ are written as:

$\left[\hat{\alpha} \pm Z_{1-\frac{\varphi}{2}}\left(SE(\hat{\alpha})\right)\right]$, $\left[\hat{\beta} \pm Z_{1-\frac{\varphi}{2}}\left(SE(\hat{\beta})\right)\right]$ and $\left[\hat{\lambda} \pm Z_{1-\frac{\varphi}{2}}\left(SE(\hat{\lambda})\right)\right]$ respectively.

III. Simulation study

Simulation study is a computational approach to analyze the behavior of the function. The uniform distribution has used to better understand the characteristics of the parameters. The proposed simulation method going through these steps.

1. Generating a pseudo random sample using the distribution $u[0,1]$.
2. Inverse-cdf method applied to transform the equation (8) in terms of u. Expression of $x_{ki}(\leq t)$ is:
$$x_{ki} = \frac{\beta}{\lambda^k\left(-\log u\right)^{1/\alpha}}$$
3. 5000 random samples of size 25, 50, 80, 120 and 150 have been produced from the Frechet Distribution.
4. Opted a fixed censoring time $t = 3.5$ at normal condition.
5. Taken values of number of failed items $r_k = (0.8 \times n)$, where n is sample size.
6. Chosen stress levels are $s = 4, 6$ and 8 along with parameter values $\alpha = 1.4$, $\lambda = 0.5$, $\beta = 1.1$.
7. Finally, optim () function in R-Programming Software has been used to calculate the ML estimates of mean along with various statistical measurements such as root mean squared error (RMSE), relative absolute bias (RAB), and lower and upper limit of 95% confidence intervals for different samples of different sizes.

Table 1: *Simulation findings for $\alpha = 1.4$, $\lambda = 0.5$, $\beta = 1.1$ and $s = 4$*

Sample size(n)	Failed items(r_k)	Estimates	Mean	RMSE	RAB	Lower limit	Upper limit
		α	1.3478	0.0721	0.0373	1.2065	1.4891
25	20	λ	0.4010	0.0523	0.1980	0.2985	0.5035
		β	1.0501	0.1230	0.0454	0.8090	1.2912
		α	1.3731	0.0515	0.0192	1.2722	1.4740
50	40	λ	0.3044	0.0415	0.3912	0.2231	0.3857
		β	1.0623	0.0871	0.0343	0.8916	1.2330
		α	1.3836	0.0409	0.0117	1.3034	1.4638
80	64	λ	0.7133	0.0328	0.4266	0.6490	0.7776
		β	1.0603	0.0688	0.0361	0.9255	1.1951
		α	1.3926	0.0335	0.0053	1.3269	1.4583
120	96	λ	0.5040	0.0456	0.0080	0.4146	0.5934
		β	1.0847	0.0561	0.0139	0.9747	1.1946
		α	1.3963	0.0300	0.0026	1.3375	1.4551
150	120	λ	0.4659	0.0431	0.0688	0.3814	0.5504
		β	1.0847	0.0501	0.0139	0.9865	1.1829

Table 2: *Simulation findings for $\alpha = 1.4$, $\lambda = 0.5$, $\beta = 1.1$ and $s = 6$*

Sample size(n)	Failed items(r_k)	Estimates	Mean	RMSE	RAB	Lower limit	Upper limit
		α	1.3218	0.0508	0.0558	1.2222	1.4214
25	20	λ	0.8050	0.0714	0.6100	0.6651	0.9449
		β	1.1030	0.1230	0.0027	0.8619	1.3441
		α	1.3429	0.0363	0.0407	1.2718	1.4140
50	50	λ	0.8024	0.0513	0.6048	0.7019	0.9029
		β	1.1260	0.0710	0.0236	0.9868	1.2652
		α	1.3571	0.0288	0.0306	1.3006	1.4135
80	64	λ	0.3097	0.0363	0.3806	0.2386	0.3808
		β	1.0979	0.0561	0.0019	0.9879	1.2079
		α	1.3648	0.0236	0.0251	1.3185	1.4110
120	96	λ	0.1058	0.0213	0.7884	0.0641	0.1475
		β	1.0921	0.0459	0.0072	1.0021	1.1821
		α	1.3683	0.0212	0.0226	1.3267	1.4098
150	120	λ	0.0504	0.0113	0.8992	0.0283	0.0725
		β	1.0900	0.0411	0.0091	1.0094	1.1706

Table 3: *Simulation findings for $\alpha = 1.4$, $\lambda = 0.5$, $\beta = 1.1$ and $s = 8$*

Sample size(n)	Failed items(r_k)	Estimates	Mean	RMSE	RAB	Lower limit	Upper limit
		α	1.3812	0.0454	0.0134	1.2922	1.4702
25	20	λ	0.2087	0.0583	0.5826	0.0944	0.3229
		β	1.1030	0.0977	0.0027	0.9115	1.2945
		α	1.3690	0.0285	0.0221	1.3131	1.4249
50	40	λ	0.8024	0.0367	0.6048	0.7305	0.8743
		β	1.1505	0.0614	0.0459	1.0301	1.2708
		α	1.3692	0.0226	0.0220	1.3249	1.4135
80	64	λ	0.7041	0.0513	0.4082	0.6035	0.8046
		β	1.1447	0.0484	0.0406	1.0498	1.2396
		α	1.3742	0.0184	0.0184	1.3381	1.4103
120	96	λ	0.7084	0.0273	0.4168	0.6549	0.7619
		β	1.1943	0.0395	0.0857	1.1169	1.2717
		α	1.3767	0.0165	0.0166	1.3443	1.4090
150	120	λ	0.6099	0.0127	0.2198	0.5850	0.6348
		β	1.1681	0.0355	0.0619	1.0985	1.2376

In this study, various measures such as average mean values, RMSE and RAB are calculated using 5000 replications of different samples to avoid randomness. The results presented in Table 1-3 are based on different sample sizes with parameter values $\alpha = 1.4$, $\lambda = 0.5$, $\beta = 1.1$ and stresses $s = 4, 6$ and 8 to analyse the performance of the MLEs of the Frechet parameters.
Table 1-3 shows that, nearly all of the parameter estimates in Table 3 result in lesser RMSEs and RABs compared with the estimates in Table 1-2. In every situation, the RMSEs of the MLEs of the parameters in Table 1-3 decrease as sample size increases.

IV. Conclusion

In the current study, a Frechet failure item accelerated life testing (ALT) design with time-censored data has been taken into consideration. The likelihood equation for the Frechet parameter is built using a geometric process, which is produced by the failure time of tested objects under constantly increasing stress leveles. Since the likelihood equation does not have the closed-form, the Newton-Raphson technique is used to calculate the mean, root mean square error (RMSE), and relative absolute bias (RAB) for the parameters. The results provided in Table 1-3 show that the estimates are reasonably near to their true values with low RMSEs. A larger sample number results in lower RMSE values and a narrower confidence interval. This work can be extended for various censoring schemes such progressive censoring.

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