RELIABILITY ANALYSES OF AN INDUSTRIAL SYSTEM BASED ON HERMITE POLYNOMIAL AND 2-PARAMETER WEIBULL

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Abstract

The current study aims to assess the structure's reliability using stochastic Hermite surface methodology. This approach uses series expansion of standard normal random variables to model uncertainty. (i.e., polynomial chaos expansion). The coefficients of the polynomial chaos expansion are found through stochastic collocation, which only requires a few performance function evaluations. After determining the order of the polynomial and its coefficients, first order methods calculates the reliability index. To demonstrate the applicability of the suggested based reliability analysis, failure rates were used and the duration for the evaluated were set to be 360 days. Numerical result are provided on monthly basics. On the other hand, to achieve our goal, we proposed the 2-parameter modified Weibull distribution. The simulation was performed using Maple software. The evaluation for each subsystem was displayed in the result and analyses section. The conclusion, however, draws a broad conclusion about the study.

Keywords: Reliability, Variables, Polynomials, Stochastic, Simulation, Distributions

I. Introduction

Derivatives are challenging to assess. For reliability analysis in this situation, Bucher and Bourgand [1] developed the response surface method (RSM). Near the failure region, a multidimensional quadratic polynomial in RSM is used to approximate the unknown limit surface. This method has recently been widely used by engineers and researchers for a variety of applications, including performance evaluation, crash simulation, and reliability-based design optimization. However, since the failure region is close to the polynomial approximation of the original limit state, it is frequently challenging to determine the ideal separation for limit states with multiple design points.Additionally, because this is a deterministic representation, it is unable to accurately depict the stochastic features of the initial limit state. The Askey-Wilson polynomials, which share the characteristics of more conventional orthogonal polynomials like the Legendre polynomials or the spherical harmonics, are the most general orthogonal polynomials [2].

A new dynamic reliability assessment method based on the decomposition method and the Hermite polynomials approximation is presented for the dynamic reliability analysis of stochastic structures under stationary random excitation. In this method, a multi-dimensional dynamic reliability response function is additively broken down into a one-dimensional function, and the one-dimensional dynamic reliability response function is then approximated using Hermite polynomials. Last but not least, the Maple software simulation yields the unconditional reliability of the explicit response function, and two expressions show the logic of the suggested approach. A systematic account of analytical calculus for distributions can be found in Potthoff-Yan [3]. Ordinary differentiation and integration can be thought of as being generalized to arbitrary order in the calculus of fractional order. The uncertainty of the rational order of the derivation gave rise to G.W [4]. Leibniz's question, which gave rise to the fractional calculus, in 1695 [5]. Recent years have seen a significant increase in interest in both theory and application for Jacobi polynomials. For the purpose of solving the multi-term fractional differential equations, the shifted Jacobi operational matrix of fractional derivatives and the spectral tau approach are combined. [6]. To calculate multi-term FDEs, a new explicit solution is created that is targeted for shifted Chebyshev polynomials with flexible degree and fractional order. The work in [7] can be used to solve the same linear problem. Laguerre polynomials have been used in a number of attempts to solve FDEs using various spectral methods within the realm of numerical methods [8]. The research done in [9] led to the creation of a novel time-dependent problem-targeting algorithm built on spectral Laguerre approximations. A novel tau method was proposed in a recent paper that examined the modified Laguerre functions [10]. Comparative analyses of Weibull distribution with Reliability, Availability, Maintainability, and Dependability analyses of an industrial system was conducted [14]. The reliability analyses of filtration system using Copula approach was conducted [15]. Complex reverse osmosis system was studied by [16] and [17] using RAMD analyses. And that of Photovoltaic using the same RAMD analyses was conducted by [18].

There is no comparison of the industrial system's performance using the Hermite polynomial and the two-parameter Weibull distribution in the literature currently available, this prompt our research. Additionally, notice how an industrial process innovation is developing.

II. Methods

Initial value conditions of multi-term fractional differential equations serve as a driving force behind many practical issues. The Hermite tau method is modified in this section to include the operational matrix in order to solve fractional differential equations. The following lists every step in the entire process.

2.2. The Properties of Hermite Polynomials

Let $\alpha = (-\infty, \infty)$ and $\gamma(l) = e^{-l^2}$be the reliability function on α. Hermite polynomials of degree m are defined in their analytical form [11]

$$Rm(l) = \sum_{d=0}^{[\frac{m}{2}]} \frac{(-1)^k m!(2l)^{m-2d}}{d!(m-2d)!} \quad (1)$$

where $R_0(l) = 1$ and $R_1(l) = 2l$

This recurrence relation is satisfied by hermite polynomials.

$$R_{m+1}(l) = 2lRm(l) + 2mR_{m-1}(l) \quad (2)$$

An orthogonal system is one in which the set of Hermite polynomials is an orthogonal polynomial

$$\int_{-\infty}^{\infty} Rm(l)R_j(l)\gamma(l)dx = h_j\delta_{mj} \quad (3)$$

where $\delta_{mj} = 0\ \forall\ m \neq j$ and $\delta_{mj} = m\ \forall\ m = j$ describes the role of Kronecker and $h_j = 2^j j!\sqrt{\pi}$

2.3. Fractional integration hermite operational reliability

We aim to derive a modified reliability equation of an industrial system for Hermite polynomials in this section.

Let $\lambda(l) \in L^2(\alpha)$, subsequently, $\lambda(l)$ can be defined as follows using Hermite polynomials

$$\lambda(l) = \sum_{j=0}^{\infty} a_j R_j(l) \quad (4)$$

Then, coefficient a_j can be written as

$$a_j = \frac{1}{2^j j!\sqrt{\pi}} \int_{-\infty}^{\infty} \lambda(l)R_j(l)\gamma(l)dl, \quad j = 0,1, \dots . \quad (5)$$

The initial $(N + 1)$ the only consideration is given to terms of Hermite polynomials, such that the modified failure rate of the industrial system is in the equation below:

$$\lambda_N(l) = \sum_{j=0}^{N} a_j R_j(l) = A^T \boldsymbol{\rho}(l) \quad (6)$$

where

$$A^T = [a_0 \quad a_1 \quad \dots \quad a_N] \text{ and } \phi(l) = [R_0(l) \quad R_1(l) \quad \dots \quad R_N(l)]^T \quad (7)$$

Integration of Hermite vector $\boldsymbol{\rho}(x)$ by $J^q\boldsymbol{\rho}(x)$ the system reliability in table 1 is obtained from the following equation.

$$J^q\phi(l) = \boldsymbol{P}^{(q)}\boldsymbol{\rho}(l) \quad (8)$$

where q indicates a fined integer value and $\boldsymbol{P}^{(q)}$ represents the actual operational reliability of integrated $\boldsymbol{\rho}(l)$.

2.4. For the reliability analyses using 2-parameter Weibull distribution

It is clearly stated by Quek and Ang [12]. If the lifetimes follow the Weibull distribution, the p.d.f.

$$f(t) = \beta\alpha^{-\beta}t^{\beta-1}e^{-(\frac{t}{\alpha})^{\beta}} \quad (9)$$

Where α and β are the scale and shape parameter of the Weibull distribution respectively.
The size of the units in which the random variable, t, is measured is reflected by the scale parameter, α. The distribution's form changes depending on the shape parameter,
β. We can create a diverse set of curves that reflect real lifetime failure distributions by modifying the value of β.
From (1), the Cumulative Distribution Function is given by:

$$F(t) = 1 - e^{-(\frac{t}{\alpha})^{\beta}} \quad (10)$$

From the relation,

$$R(t) = 1 - F(t) \quad (11)$$

We can substitute (2) into (3), and we have:

$$R(t) = e^{-(\frac{t}{\alpha})^{\beta}} \quad (12)$$

Where $R(t)$ is the reliability or survival function.
The failure rate function or the hazard function can therefore be derived from the following relation:

$$h(t) = \frac{f(t)}{R(t)} \tag{13}$$

Substituting (9) and (12) into (13) we have:

$$h(t) = \frac{\beta\alpha^{-\beta}t^{\beta-1}e^{-(\frac{t}{\alpha})^{\beta}}}{e^{-(\frac{t}{\alpha})^{\beta}}} \tag{14}$$

$$h(t) = \beta\alpha^{-\beta}t^{\beta-1} \tag{15}$$

If we consider M_0 to be the number of hours in a year (the size of the population). Out of which M_s units (the number of hours that the system is upstate) survive the test. While M_f fail, then reliability function $R(t)$ is given by:

$$R(t) = \frac{M_s}{M_0} = \frac{M_0 - M_f}{M_0} \tag{16}$$

Differentiating both sides of (16) and taking M_0 fixed, the following equation will result

$$\frac{dR(t)}{dt} = \frac{1}{M_0}\frac{dM_f}{dt} \tag{17}$$

The rate at which component fails can therefore be defined as:

$$\frac{dM_f}{dt} = -M_0\frac{dR(t)}{dt} \tag{18}$$

Dividing both sides of the above equation byM_s, we obtain the instantaneous probability $g(t)$ of failure, this is:

$$g(t) = \frac{1}{M_S}\frac{dM_f}{dt} = -\frac{M_0}{M_S}\frac{dR(t)}{dt} \tag{19}$$

Using equation (16) into (19) we get:

$$g(t) = -\frac{1}{R(t)}\frac{dR(t)}{dt} \tag{20}$$

Integrating both sides of the equation (20) we have:

$$\int g(t)dt = -\log R(t) \tag{21}$$

From the above equation, the $R(t)$ will be:

$$R(t) = e^{-\int_0^x g(t)dt} \tag{22}$$

Where x is variable.
The function $g(t)$ is called the hazard function or failure rate. Equation (22) can be considered as a generic expression of failure as it is applicable to both exponential and non-exponential failure distribution.

For our modified Weibull distribution, we compared (22) with (12)

$$(\frac{t}{\alpha})^{\beta} = \int_0^x g(t)dt \tag{23}$$

For,
$g(t) = -\frac{1}{R(t)}\frac{dR(t)}{dt}$, and the below facts:
The derivative of a definite integral of a function is the function itself only when the lower limit of the integral is a constant and the upper limit is the variable with respect to which we are differentiating. To summarize Ulrich et al. [13]:

- The derivative of an indefinite integral of a function is the function itself. i.e., $\frac{d}{dx}\int f(x)dx = f(x)$
- The derivative of a definite integral with constant limits is 0. i.e., $\frac{d}{dx}\int_a^b f(x)dx = 0$
- The derivative of a definite integral where the lower limit is a constant and the upper limit is a variable is a function itself in terms of the given variable (upper bound).
  i.e., $\frac{d}{dx}\int_a^x f(x)dx = f(x)$ where 'a' is a constant and 'x' is a variable.

$$(\frac{t}{\alpha})^{\beta} = -\frac{M_0}{M_S}(R(t)) \tag{24}$$

The NMW proposed for the present research follows;

$$R(t) = -\frac{M_S}{M_0}(\frac{t}{\alpha})^{\beta} \tag{25}$$

III. Results

The numerical values for the 2-parameter Weibull distribution and Hermite polynomial derived equations were presented in this section. For the two approaches, comparative analyses were conducted. 360 days were allotted for the analyses using the failure rate range of 0.75 to 0.95.

Table 1: *Hermite Polynomials reliability with time*

Time↓ X→	0.75	0.85	0.95	Exp(-m)
30	0.9483	0.9385	0.8383	0.7653
60	0.9245	0.9134	0.8134	0.7446
90	0.9188	0.9076	0.8075	0.7146
120	0.9176	0.8962	0.8056	0.7087
150	0.9171	0.8759	0.7946	0.6801
180	0.9148	0.8451	0.7676	0.6479
210	0.9117	0.8153	0.7373	0.6178
240	0.9092	0.8009	0.7187	0.5929
270	0.8977	0.7852	0.7089	0.5708
300	0.8832	0.7657	0.6806	0.5479
330	0.8633	0.7352	0.6735	0.5188
360	0.8558	0.7143	0.6566	0.5070

Table 2: *2-Parameter Weibull distribution's reliability with time*

Time↓ α→	0.75	0.85	0.95	Exp(-m)
30	0.9954	0.7854	0.5855	0.3667
60	0.9857	0.7789	0.5734	0.3584
90	0.9787	0.7607	0.5677	0.3499
120	0.9644	0.7525	0.5537	0.3312
150	0.9567	0.7457	0.5344	0.3290
180	0.9439	0.7346	0.5270	0.3043
210	0.9387	0.7277	0.5145	0.2932
240	0.9298	0.7190	0.5044	0.2823
270	0.9124	0.6932	0.4944	0.2732
300	0.9097	0.6854	0.4797	0.2617
330	0.8934	0.6724	0.4653	0.2476
360	0.8862	0.6656	0.4522	0.2332

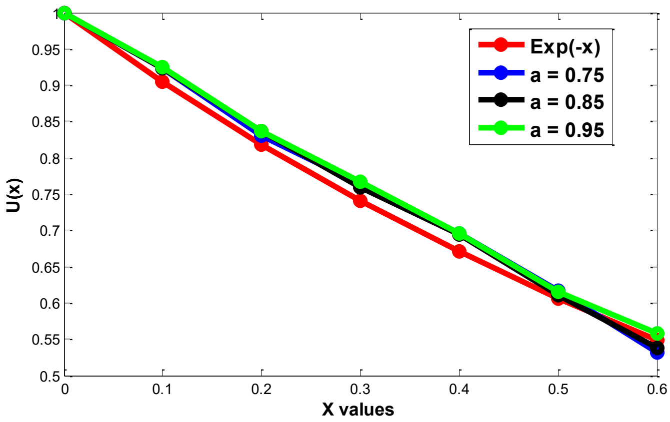

Figure 1: *Reliability analyses For failure against repair rate of a system using Hermite Polynomial*

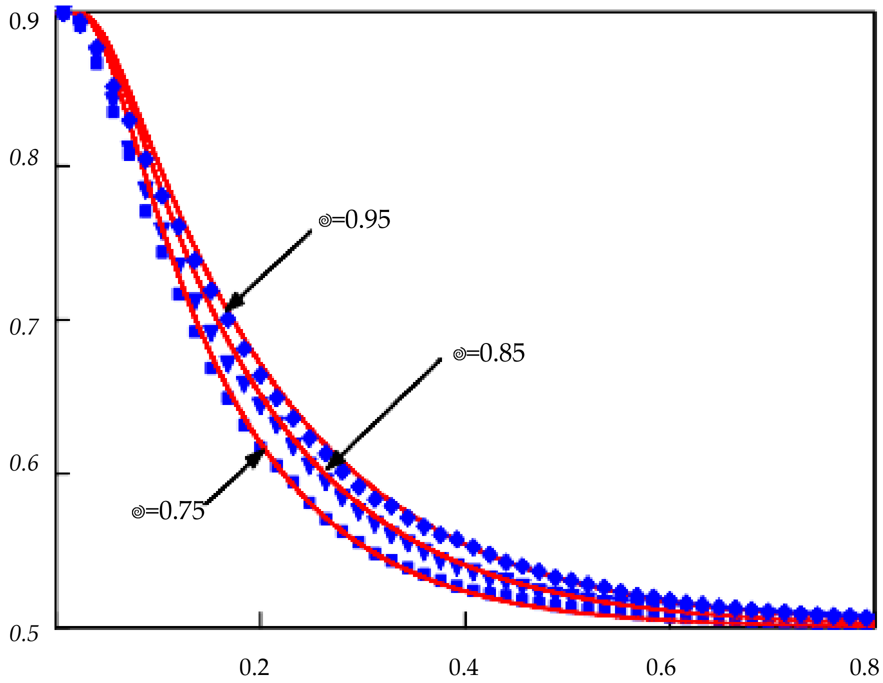

Figure 2: *Reliability analyses For failure against repair rate of a system using 2-parameter Weibull distribution*

IV. Discussion

Using the Hermite polynomial method, the result shown in table 1 that corresponds to Figure 1 demonstrated how the reliability of an industrial system is impacted by the rise in failure rate. The reliability of an industrial system was then shown to be affected as the failure rate was changed from 0.75 to 0.95 in table 2 that corresponds to Figure 2. When comparing the results in figures 1 and 2, it is obvious that the 2-parameter Weibull yields a more accurate result than the Hermite polynomials in terms of reliability estimation. Therefore, it is advised that future studies modify the Hermite polynomial to increase the dependability of industrial systems.

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