COMPLETELY RANDOMIZED DESIGN IN FUZZY OBSERVATIONS

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Abstract

The real world is vague, unclear and full of ambiguity, and are inevitable. The classical statistics disregards the extreme, aberrant, uncertain values, and hence a new appropriate tool had to surface. The Analysis of Variance (ANOVA) method is used to compare the response variable's means between several groups that are specified by the factor variable. Another method of data analysis offered by ANOVA is one that is based on statistics and is experimental design-driven, or Design of Experiment (DOE). In DOE, there are single and two-factor experimental designs depending on, observing the effect of number of factor(s) on output variable as a primary interest. Among all the single factor experimental designs, Completely Randomized Design (CRD) is the simplest and flexible design. In this design, treatments are randomly allocated to the experimental units over the entire experimental material. Each treatment is repeated to increase the efficiency of the design. CRD is more appropriate to use when the data is homogenous. The objective that deals with the preparation and analysis of experiments is experimental design. The treatments are apportioned to the exploratory units at random in the fully randomized experimental design. When the observed data are fuzzy observations rather than precise numerical values, the CRD is expanded in this study. In this paper, an innovative Triangular Fuzzy Number (TFN) in the fuzzy Completely Randomized Design (FCRD) analysis statistical method for evaluating CRD model hypotheses on fuzzy data is presented. To convert the fuzzy totally randomized design model into two crisps CRD models using the suggested way, and then convert to lower and upper models are used in fuzzy hypothesis. Determine the fuzzy hypothesis for the fuzzy CRD model based on the hypotheses of the two crisp CRD models using the decision rules. The fuzzy test appears to be a competitive tool in circumstances with ambiguous data, particularly linguistic ambiguity because it is more adaptable than the conventional test of significance. This paper presents and illustrates a novel fuzzy triangular number-based approach to fuzzy CRD analysis. This paper also explores how flexible a CRD may be when handling uncertain elements. This study provides an example of a new method for fuzzy CRD analysis employing TFN.

Keywords: Fuzzy Set, CRD, Fuzzy CRD, Triangular Fuzzy Number, Decision Rules

1. Introduction

The Analysis of Variance (ANOVA) was introduced in the 1920s by Prof. R.A. Fisher. This method can be used to solve the problems of variations, especially in the agricultural sector. The ANOVA has many independent demographic variables and it is a most powerful tool of the test of significance. The significance test in terms of t-distribution is the only adequate procedure to test the significance of the difference between the two-sample means. In such a situation, when three or more sample means are considered simultaneously, an alternative procedure is needed to test the hypothesis that all samples are taken from the same population. This is called ANOVA. The foundation for experimental designs was laid in 1935 by Prof. R.A. Fisher. The term design of tests is said to be the logical construction of tests, in which the degree of uncertainty can be well defined. The basic principles of experimental designs are randomization, replication, and local control. The local control is the method of increasing efficiency in test designs. A Completely Randomized Design (CRD) means that treatments are assigned to a completely randomized group so that each test unit has the same chance of receiving any one treatment. Since the principle of local control is not used, the CRD is considered simple and the experimental material is observed, but it is seen that the experimental material is not completely homogeneous. It is specifically designed to address mathematical uncertainty and inadequate specification and provide a systematic tool for dealing with the inherent fuzzy of many problems. The word fuzzy means ambiguity (vagueness). Fuzziness occurs when the boundary of information is not clearly cut. In 1965s Lotfi .A. Zadeh introduced fuzzy sets as an extension of the set with classical notations. The classical set theory allows membership of elements in a set of binary terms to be inside. Fuzzy sets theory allows the estimated membership function in intervals [0,1]. Sometimes, agricultural data is not recorded for natural calamities.

Therefore, fuzzy synthesis is the most inevitable. In both cases, the observed variable of the fuzziness often occurs. In the first case, due to technical problems, the response variable cannot be measured properly. So, in this case the data cannot be clearly recorded with the exact numbers and the measurement errors are computed linguistically to justify the required tolerance. The second phenomenon is that the response variable is presented in terms of linguistic forms such as a special linguistic report or variance report. As for his products, they are not counted. In both of the above cases the data can be represented by the concept of fuzzy sets for analyzing the test (Zadeh [23]). An example is cited by H.C. Wu [21], to illustrate in this situation. There are many real-life populations in which imprecise values can be assigned to their experimental outcomes. Some practical reasons may not be accurate for the agricultural observations so that fuzzy sets used and the fuzzy was introduced by Zadeh [22], to represent manipulate data and in order to non-statistical uncertainties. D. Dubois et al. Brett [6], defined any fuzzy numbers as a fuzzy subset of the real line. The symmetric triangular fuzzy approximation was presented by M. Ma et al. [15]. S. Chanas [2], presented a formula for determining approximations of intervals under humming distance. S. Chandrasekaran et al. Tamilmani [3], proposed the arithmetic operations of fuzzy numbers using the alpha-cut interval method. The Triangular Fuzzy Numbers (TFNs) result from addition or subtraction between TFNs results. Therefore, addition and subtraction between fuzzy numbers become a TFNs. Such areas include approximate reasoning, decision making, optimization, control, and so on. R.R. Hocking [11], has been the traditional statistical testing, the sample observations are crisp and a statistical experiment leads to a binary conclusion.

Applying fuzzy set theory to Statistics. K.G. Manton et al. [16], proposed a fuzzy test for testing hypotheses with fuzzy data and fuzzy testing created the acceptance of null and alternative hypotheses. Statistical hypothesis testing for ambiguous data by presenting the notions of pessimism and pessimism by H.C. Wu [20]. We provide decision rules that can be used to accept or reject ambiguous null and alternative hypotheses. The observed values of the classical random variable can be considered an ambiguous number, while the model for the observed values in the linear model. Note that Filsmoser and Viertl [7] and Viertl [17], use a similar idea. The proposed technique, ambiguous data as well, given the vague assumptions of the tests were imprecise data, along with two hypothesis test, replacing CRD models crisp data, that is the lower-level model and the upper-level model, then each CRD hypothesis, testing the crisp data, models and results, getting after using the results obtained in terms of the provisions of the proposed decision of the population receive the decision. In the decision rules of the proposed test technique, we did not use the confidence, distrust, and h-level set used in the H.C. Wu [21]. In this way, fuzzy numbers are appropriate models for formalizing and manipulating these populations (Gill et al. [9]). According to Kumari et al. [13] the methodology was expanded by introducing a fuzzy regression approach to randomized block designs that takes into account qualitative predictor factors in multiple linear regression. The concept from this study was a helpful thread for developing thorough connectedness between regression and randomized block designs. According to the researchers, fuzzy MLR can predict far more accurately than MLR alone. By comparing the RMSEs from various forecasting techniques, Koul et al. [12] suggested a study to identify the variance analysis experimental model approach between stock exchange trends. In this paper, we introduce a new technique using triangular fuzzy numbers in the fuzzy CRD analysis with an example.

2. Preliminaries

2.1 Completely Randomized Design (CRD)

CRD is the basic single factor design. In this design the treatments are assigned completely at random so that each experimental unit has the same chance of receiving any one treatment. But CRD is appropriate only when the experimental material is homogeneous. As there is generally large variation among experimental plots due to many factors CRD is not preferred in field experiments. In laboratory experiments and greenhouse studies it is easy to achieve homogeneity of experimental materials and therefore CRD is most useful in such experiments.

2.2 Triangular Fuzzy Number

The triangular fuzzy number membership function is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a} & ; \ a \le x \le b \\ \dfrac{x-c}{b-c} & ; \ b \le x \le c \end{cases}$$

Where a is indicate lower point, b is indicate centre point and c is indicate upper point.
A Triangular fuzzy number can be represented as an interval number form as follows.

$$\tilde{A} = \left[\left\{ (b-a)r + a \right\}^{L} ; \left\{ -(d-c)h + c \right\}^{U} \right]; \ 0 \le r, h \le 1.$$

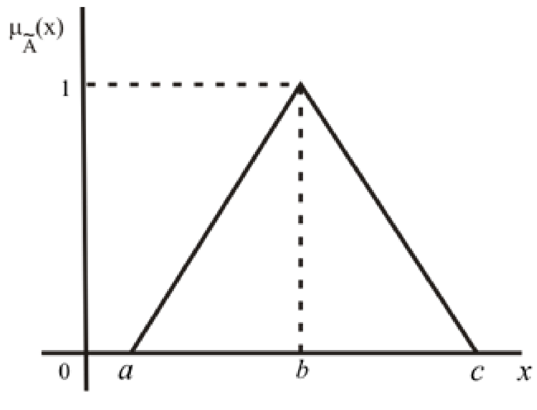

Figure 1: *Figure Triangular Fuzzy Numbers*

Note that r is the level of pessimistic value and h is the level of optimistic value of the fuzzy number $\tilde{A} = (a,b,c)$.

3. Statistical Analysis of CRD

The *CRD* is the one in which all the experimental units are taken in a single group which are homogeneous as far as possible. Suppose there are t treatments in an experiment. Let i^{th} treatment be replicates n_i times then, the total number of experimental units in the design is $\sum_{i=1}^{t} n_i = N$. Then, the treatment is allocated at random to entire experimental area. In this design provides a one-way classified data with different levels of a single factor is called treatments. For instance, y_{ij} can be the productivity of the j^{th} week in the i^{th} varieties, or the paddy seedling of the j^{th} week grown of the j^{th} type of shelf display. Since the number of cases or trials for the i^{th} factor level is denoted by N, so, $j = 1, 2, ..., n_i$. Now, the Statistical analysis of *CRD* is analogue to *ANOVA* one-way classified data, linear model becomes

$$y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \; ; i = 1, 2, ..., t \; ; j = 1, 2, ..., n_i \quad (1)$$

In which, y_{ij}'s is the j^{th} observations of the i^{th} treatment; μ is the general mean effect which is fixed; α_i is the fixed effect due to the i^{th} treatment and ε_{ij} is the random error effect which distributed as normal $N \sim (0, \sigma^2)$; $i = 1, 2, ..., t$ and $j = 1, 2, ..., n_i$.

The grand total of *n* observations of *CRD* is $\sum_{i=1}^{t}\sum_{j=1}^{n_i} y_{ij} = y_{..} = G$; the correction factor is $cf = \frac{y_{..}^2}{N}$ and the i^{th} treatment total taken is $\sum_{i=1}^{t}\sum_{j=1}^{n_i} y_{ij} = y_{i.} = T_i$.

Apply the *ANOVA* for one way classify and compute the total sum of squares (sst), the treatment sum of squares $(sstr)$ and the error sum of squares (sse) are given below;

$$Q_{sst} = \sum_{i=1}^{r}\sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{..})^2 = \sum_{i=1}^{r}\sum_{j=1}^{n_i} y_{ij}^2 - \frac{y_{..}^2}{N} \quad (2)$$

$$Q_{sstr} = \sum_{i=1}^{r} n_i (\bar{y}_{i.} - \bar{y}_{..})^2 = \sum_{i=1}^{r} \frac{y_{i.}^2}{n_i} - \frac{y_{..}^2}{N} \quad (3)$$

and

$$Q_{sse} = \sum_{i=1}^{r}\sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.})^2 = \sum_{i=1}^{r}\sum_{j=1}^{n_i} y_{ij}^2 - \sum_{i=1}^{r} \frac{y_{i.}^2}{n_i} \quad (4)$$

Where, sst, $sstr$ and sse which has $(N-1)$, $(t-1)$ and $(N-t)$ degrees of freedom (df), respectively. The mean sum squares are obtained as follows:

$$msstr = \frac{sstr}{t-1} \text{ and } msse = \frac{sse}{N-t} \quad (5)$$

Where, $msstr$ and $msse$ stands for treatment mean square and error mean square.

In order to test whether or not the factor level means μ are equal, the following classical testing hypotheses are considered.

$$H_0 : \mu_1 = \mu_2 = ... = \mu_t \text{ Vs } H_1 : \mu_1 \neq \mu_2 \neq ... \neq \mu_t$$

The test statistic to be used is

$$F = \frac{msstr}{msse} \sim F_{(t-1),(N-1)} \quad (6)$$

When the null hypothesis H_0 holds true, it is known that F is distributed as with degrees of freedom $(t-1)$ and $(N-t)$ that is $F_{(t-1),(N-t)}$.

All these values are referring in the *ANOVA* table and inference is drawn.

Table 1: *ANOVA table for CRD*

sv	*df*	*ss*	*mss*	$\tilde{F}-ratio$
Between Treatmen	$(t-1)$	Q_{sstr}	$msstr$	$F=\dfrac{msstr}{msse}$
Within Treatmen	$(N-t)$	Q_{sse}	$msse$	
Total	$(N-1)$	Q_{sst}		

Decision Rule:

The decision rules in the F test to accept or reject the null hypothesis and alternative hypothesis are the level of significance α is given by

(i) If $msstr > msse$ and $\tilde{F}_c = \dfrac{msstr}{msse} < \tilde{F}_t$ where $\tilde{F}_t$ and $\tilde{F}_c$ is the tabulated and calculated values of $\tilde{F}$ with $(t-1)\,(N-t)$, degrees of freedom at α level of significance, then we accept the null hypothesis $\tilde{H}_0$, otherwise the alternative hypothesis $\tilde{H}_1$ is accepted.

(ii) If $msstr < msse$ and $\tilde{F}_C = \dfrac{msse}{msstr} < \tilde{F}_T$ where $\tilde{F}_t$ and $\tilde{F}_c$ is the tabulated and calculated values of $\tilde{F}$ with $(N-t)\,(t-1)$, degrees of α level of significance, then we accept then null hypothesis $\tilde{H}_0$, otherwise the alternative hypothesis $\tilde{H}_1$ is accepted.

3.1 Statistical Analysis of Fuzzy CRD

In this real-world, sometimes agricultural data cannot be accurately recorded. For example, the growth of seeds grown in a field due to fluctuation cannot be exactly measured. Therefore, the fuzzy set theory provides an appropriate tool for processing naturally imprecise data. Under this consideration, the more appropriate way to describe the paddy seedlings level is to say that the initial stage paddy seedlings are around 10 centimeters. The phrase about 10 centimeters should be considered an ambiguous number, which is realized by the fuzzy set theory. Therefore, our aim is the statistical analysis of fuzzy *CRD* using the *TFNs* method. In this case, observations and recorded data are treated as *TFNs*. Statistical hypotheses and populations parameter are crisp and hence the linear model is considered as $\tilde{y}_{ij} = \mu + \alpha_i + \varepsilon_{ij}$; in which $\tilde{y}_{ij}$'s is the j^{th} observations of the i^{th} treatment; μ is the general mean effect which is fixed; α_i is the fixed effect due to the i^{th} treatment and ε_{ij} is the random error effect which distributed as normal $N \sim (0,\sigma^2)$; $i = 1,2,...,t$ and $j = 1,2,...,n_i$.

Statistical hypotheses are considered as classical ones:

$$\tilde{H}_0 = \tilde{\mu}_1 = \tilde{\mu}_2 = ... = \tilde{\mu}_t \text{ Vs } \tilde{H}_1 : \tilde{\mu}_1 \neq \tilde{\mu}_2 \neq ... \neq \tilde{\mu}_t.$$

But one point that deviates from the classical *ANOVA* assumptions in the linear model is that the sample observations did not change anything else in the *CRD* model before collecting *TFNs* and data rather than actual numbers. Regarding the fuzzy arithmetic of *TFNs* described in the observed values of statistics for simplicity of calculations, Zadeh's [20] fuzzy extension principle can be explained lower level and upper-level model as follows:

$$\tilde{Q}^{L}_{ssro} = \sum_{i=1}^{t}\sum_{j=1}^{n_i}\tilde{y}^2_{ij}\,;\tilde{Q}^{L}_{G} = \tilde{y}_{..} = \sum_{i=1}^{t}\tilde{y}_{i.}\,;\ cf = \frac{\tilde{y}^2_{..}}{N}\,;\tilde{y}_{i.} = \sum_{j=1}^{n_i}\tilde{y}_{ij} \tag{7}$$

$$\tilde{Q}^{L}_{sst} = \sum_{i=1}^{t}\sum_{j=1}^{n_i}\tilde{y}^2_{ij} - \frac{\tilde{y}^2_{..}}{N} \tag{8}$$

$$\tilde{Q}^{L}_{sstr} = \sum_{i=1}^{t}\frac{\tilde{y}^2_{i.}}{n_i} - \frac{\tilde{y}^2_{..}}{N} \tag{9}$$

and

$$\tilde{Q}^{L}_{sse} = \tilde{Q}^{L}_{sst} - \tilde{Q}^{L}_{sstr} \tag{10}$$

Table 2: *ANOVA table for lower level Fuzzy CRD*

sv	*df*	*ss*	*mss*	$\tilde{F}-ratio$
Between Treatmen	$(t-1)$	$\tilde{Q}^{L}_{sstr}$	$msstr^{L}$	$\tilde{F}^{L} = \frac{msstr}{msse}$
Within Treatmen	$(N-t)$	$\tilde{Q}^{L}_{sse}$	$msse^{L}$	
Total	$(N-1)$	$\tilde{Q}^{L}_{sst}$		

$$\tilde{Q}^{U}_{ssro} = \sum_{i=1}^{t}\sum_{j=1}^{n_i}\tilde{y}^2_{ij}\,;\tilde{Q}^{U}_{G} = \tilde{y}_{..} = \sum_{i=1}^{t}\tilde{y}_{i.}\,;\ cf = \frac{\tilde{y}^2_{..}}{N}\,;\tilde{y}_{i.} = \sum_{j=1}^{n_i}\tilde{y}_{ij} \tag{11}$$

$$\tilde{Q}^{U}_{sst} = \sum_{i=1}^{t}\sum_{j=1}^{n_i}\tilde{y}^2_{ij} - \frac{\tilde{y}^2_{..}}{N} \tag{12}$$

$$\tilde{Q}^{U}_{sstr} = \sum_{i=1}^{t}\frac{\tilde{y}^2_{i.}}{n_i} - \frac{\tilde{y}^2_{..}}{N} \tag{13}$$

and

$$\tilde{Q}^{U}_{sse} = \tilde{Q}^{U}_{tss} - \tilde{Q}^{U}_{sstr} \tag{14}$$

Table 3: *ANOVA table for upper level Fuzzy CRD*

sv	*df*	*ss*	*mss*	$\tilde{F}-ratio$
Between Treatmen	$(t-1)$	$\tilde{Q}^{U}_{sstr}$	$msstr^{U}$	$\tilde{F}^{U} = \frac{msstr}{msse}$
Within Treatmen	$(N-t)$	$\tilde{Q}^{U}_{sse}$	$msse^{U}$	
Total	$(N-1)$	$\tilde{Q}^{U}_{sst}$		

3.2 Fuzzy Decision Rules of $\tilde{F}$ -Test

Suppose that if at α level of significance, the null hypothesis of the lower level model is accepted for $0 \le h \le \tilde{F}_t$ where $0 \le \tilde{F}_t \le 1$ and the null hypothesis of the upper level model is accepted for $0 \le r \le \tilde{F}_t$ where $0 \le \tilde{F}_t \le 1$ then, the fuzzy null hypothesis of the fuzzy ANOVA model is accepted for and at α level of significance. Otherwise, the fuzzy alternative hypothesis of the fuzzy ANOVA model is accepted at α level of significance.

4. Applications

Following application to each of the three types of paddy in a CRD, the yield in kilograms (kgs.) per four plots. Due to some unforeseen circumstances, it is impossible to record the precise amount of yields in kgs. in a sample; nonetheless, there is some fuzzy information available. Below are the triangular fuzzy data:

Table 4: *Fuzzy CRD table forTFNs*

Yields in kgs. (*i*)	Varieties of paddy (*j*)		
	V1	V2	V3
Y1	4,6,8	6,8,10	-
Y2	5,7,10	4,6,8	6,8,10
Y3	7,9,11	8,10,12	9,12,14
Y4	5,9,12	7,9,11	-

Test that there is a significant difference in the varieties of paddy performance of the yields in kgs. per plots.

Let $\tilde{\mu}_i$ be the mean number of varieties of paddy for the i^{th} yields in kgs. per plots.

Now, the null hypothesis, $\tilde{H}_0 : \tilde{\mu}_1 = \tilde{\mu}_2 = \tilde{\mu}_3 = \tilde{\mu}_4$ and the alternative hypothesis, $\tilde{H}_A :$ not all $\tilde{\mu}_i$'s are equal.

Now, the ANOVA model for " r is the lower level of pessimistic value" and " h is the upper level of optimistic value" the interval model for the triangular fuzzy number is given below:

Table 5: *Fuzzy CRD table for lower and upper level models*

Yields in kgs. (*i*)	Varieties of paddy (*j*)		
	V1	V2	V3
Y1	$2r+4, 8-2h$	$2r+6, 10-2h$	-
Y2	$2r+5, 10-3h$	$2r+4, 8-2h$	$2r+6, 10-2h$
Y3	$2r+7, 11-2h$	$2r+8, 12-2h$	$9r+3, 14-2h$
Y4	$4r+5, 12-3h$	$2r+7, 11-2h$	-

Table 6: *Fuzzy CRD table for lower level model*

Yields in kgs. (*i*)	Varieties of paddy (*j*)		
	V1	V2	V3
Y1	$2r+4$	$2r+6$	-
Y2	$2r+5$	$2r+4$	$2r+6$
Y3	$2r+7$	$2r+8$	$3r+9$
Y4	$4r+5$	$2r+7$	-

The null hypothesis $H_0^{LL} : \mu_1^{LL} = \mu_2^{LL} = \mu_3^{LL} = \mu_4^{LL}$ against the alternative hypothesis H_A^{LL} : not all μ_i^{LL}' s are equal.

Here, $N = 10$ and $n_i = 2, 3, 3, 2$ the yields in kgs. per plot and the varieties of paddy for the 1,2,3,4 respectively.

Total sum of squares for lower level model is

$$sst_r^L = 4.1r^2 + 1.4r + 24.9$$

Treatment sum of squares of lower level model is

$$sstr_r^L = 1.4r^2 + 3.4r + 16.9$$

Error sum of squares of lower level model is

$$sse_r^L = 2.7r^2 - 2r + 8$$

MSTR and *MSE* lower level model is

$$msstr_r^L = 0.47r^2 + 1.13r + 5.63 \text{ and } msse_r^L = 0.45r^2 - 0.33r + 1.33$$

$\tilde{F}$ – Ratio of lower level model is

$$\tilde{F}_r^L = \frac{0.47r^2 + 1.13r + 5.63}{0.45r^2 - 0.33r + 1.33}$$

All these values are referring in *ANOVA* table and inference is drawn.

Table 7: *ANOVA table for Lower Level of Fuzzy CRD*

sv	*df*	*ss*	*mss*	$\tilde{F} - ratio$
Between Treatments	3	$1.4r^2 + 3.4r + 16.9$	$0.47r^2 + 1.13r + 5.63$	$\frac{0.47r^2 + 1.13r + 5.63}{0.45r^2 - 0.33r + 1.33}$
Within Treatments	6	$2.7r^2 - 2r + 8$	$0.45r^2 - 0.33r + 1.33$	
Total	9	$4.1r^2 + 1.4r + 24.9$	-	-

Now, $F_r^L > F_t^L$, for all $r; 0 \le r \le 0.33$ where $F_t^L = 4.76$ is the F table value of α at 5% level of significance with $(3,6)$ degrees of freedom. Therefore, the null hypothesis H_0^L of the lower level model is accepted for the $r; 0 \le r \le 0.33$.

Table 8: *Fuzzy CRD table for upper level model*

Yields in kgs. (*i*)	Varieties of paddy (*j*)		
	V1	V2	V3
Y1	$8-2h$	$10-2h$	-
Y2	$10-3h$	$8-2h$	$10-2h$
Y3	$11-2h$	$12-2h$	$14-2h$
Y4	$12-3h$	$11-2h$	-

The null hypothesis $H_0^{UL} : \mu_1^{UL} = \mu_2^{UL} = \mu_3^{UL} = \mu_4^{UL}$ against the alternative hypothesis H_A^{UL} : not all μ_i^{UL} ' s are equal.

Here, $N = 10$ and $n_i = 2,3,3,2$ the varieties of yields in kg. 1,2,3,4 respectively.

Total sum of squares for upper level model is

$$sst_h^U = 1.6h^2 - 1.6h + 30.4$$

Treatment sum of squares of upper level model is

$$sstr_h^U = 0.43h^2 + 0.73h + 20.57$$

Error sum of squares of upper level model is

$$sse_h^U = 1.17h^2 - 2.33h + 9.83$$

MSTR and *MSE* upper level model is

$$msstr_h^U = 0.14h^2 + 0.24h + 6.85 \text{ and } msse_h^U = 0.19h^2 - 0.38h + 1.64$$

$\tilde{F}$ – Ratio of upper level model is

$$\tilde{F}_h^U = \frac{0.14h^2 + 0.24h + 6.85}{0.19h^2 - 0.38h + 1.64}$$

All these values are referring in *ANOVA* table and inference is drawn.

Table 9: *ANOVA table for Upper Level of Fuzzy CRD*

sv	*df*	*ss*	*mss*	$\tilde{F} - ratio$
Between Treatments	3	$0.43h^2 + 0.73h + 20.57$	$0.14h^2 + 0.24h + 6.85$	$\frac{0.14h^2 + 0.24h + 6.85}{0.19h^2 - 0.38h + 1.64}$
Within Treatments	6	$1.17h^2 - 2.33h + 9.83$	$0.19h^2 - 0.38h + 1.64$	
Total	9	$1.6h^2 - 1.6h + 30.4$	-	-

Now, $F_h^U < F_t^U$, for all $h; 0 \le h \le 0.61$ where $F_t^U = 4.76$ is the F table value of 5% at α level of significance with $(3,6)$ degrees of freedom. Therefore, the null hypothesis H_0^U of the upper level model is accepted for the $h; 0 \le h \le 0.61$.

Thus, since the null hypothesis H_0^L and H_0^U of the lower level data and upper level data are accepted for all $r; 0 \le r \le 0.33$ and $h; 0 \le h \le 0.61$ (note that null hypotheses are not rejected at $r = 1$ and $h = 1$, that is the centre level), the fuzzy null hypothesis $\tilde{H}_0$ of the fuzzy ANOVA model is accepted for all $r; 0 \le r \le 0.33$ and $h; 0 \le h \le 0.61$. Thus, we conclude that four yields of kgs. per plots are equal only if $r; 0 \le r \le 0.33$ and $h; 0 \le h \le 0.61$. That is, the maximum level of pessimistic value is 0.33 and the maximum level of optimistic value is 0.61. From the applications, thus observe that the acceptance of the fuzzy null hypothesis for not all *r* and *h* always, but for some specific levels of *r* and *h*, that is $r; 0 \le r \le 0.42$ and $h; 0 \le h \le 0.61$.

5. Conclusion

In this paper, the propose a new statistical fuzzy hypothesis testing of completely randomized design model with the fuzzy data. In the proposed technique, do transfer the fuzzy completely randomized design model into two crisp CRD models. Based on the decisions of hypotheses of two crisp CRD models, to take a decision on the fuzzy hypothesis of the fuzzy CRD model. Since our fuzzy test is more flexible than the traditional test of significance, it seems to be a competitive tool in situations with imprecise data, especially of the linguistic type. Since the proposed technique in this paper is mainly based only on the crisp models, the proposed technique can be extended to the experimental design analysis having fuzzy data and RBD, LSD etc.

References

[1] Buckley J.J. Fuzzy statistics, Springer-Verlag, New York, 2005.
[2] Chanas S. (1999). On the interval approximation of a fuzzy numbers, *Fuzzy sets and*

Systems, 102, 221-226.

[3] Chandrasekaran S. and Tamilmani E. (2015). Arithemetic Operation of Fuzzy Numbers using - cut Method, *International Journal of Innovative Science, Engineering and Technology*, vol. 2, Issue 10.

[4] Cochran W.G. and Cox G.M., Experimental Designs (2nd ed.), New York: Wiley, 1957.

[5] Dubois D. and Prade H., Fuzzy Sets and Systems: Theory and Application, Academic Press, New York, 1980.

[6] Dubois D. and Prade H. (1978). Operations on fuzzy numbers, *Int. J. Syst. Sci.*, 9, 613-626.

[7] Filzmoser P. and Viertl R. (2009). Testing Hypotheses with Fuzzy Data: The Fuzzy P value, Metrika, 59, 21-29.

[8] George.J. Klir and Bo Yuan, Fuzzy sets and fuzzy logic, Theory and Applications, Prentice-Hall, New Jersey, 2008.

[9] Gil et al. (2006). Bootstrap Approach to the Multi-sample Test of Means with Imprecise Data *Computer Statistics and Data Analysis* 51, 148-162.

[10] Gupta S.C. and Kapoor V.K., Fundamentals of applied statistics, Sultan Chand & Sons, New Delhi, India, 2007.

[11] Hocking R.R., Methods and applications of linear models: regression and the analysis of variance, New York: John Wiley & Sons, 1996.

[12] Koul, S., Awasthi, A. K., & Garov, A. K. (2019). Experimental model approach for decision making in Stock Index. *Think India Journal*, *22*(37), 1272-1276.

[13] Kumari, S. (2022). A Study on Neutrosophic Completely Randomised Design. *Mathematical Statistician and Engineering Applications*, *71*(4), 3738-3747.

[14] Ma M. et al. (2000). A new approach for defuzzification, *Fuzzy Sets and Systems*, 111, 351-356.

[15] Manton K.G. et al., Statistical applications using fuzzy sets, New York: John Wiley & Sons, 1994.

[16] Montgomery D.C., Design and Analysis of Experiments (8th ed.), New York: John Wiley & Sons, 1991.

[17] Nguyen H.T. and Walker E.A., A First Course in Fuzzy Logic (3rd ed.), Paris: Chapman Hall/CRC, 2005.

[18] Viertl R. (2006). Univariate statistical analysis with fuzzy data, *Computational Statistics and Data Analysis,* 51, 33-147.

[19] Viertl R., Statistical methods for fuzzy data, John Wiley and Sons, 2011.

[20] Wu H.C., (2005). Statistical hypotheses testing for fuzzy data, *Information Sciences,* 175, 30 -56.

[21] Wu H.C., (2007). Analysis of variance for fuzzy data, *International Journal of Systems Science*, 38, 235-246.

[22] Zadeh L.A., (1965). Fuzzy sets, Information and Control, 8, 338-353.

[23] Zadeh L.A., (1975). The Concept of a Linguistic Variable and its application to approximate reasoning, *Information Sciences*, 8, 199-249.

[24] Zimmermann H.J., Fuzzy set theory and its applications, Academic Publishers, Kluwer, 1991.