# A COLD STANDBY SYSTEM WITH IMPERFECT SWITCH AND PREVENTIVE MAINTENANCE: A STOCHASTIC STUDY

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**Abstract**

*The aim of this paper is to develop a probabilistic model for a cold standby system that consists of an imperfect switching device and a servicing facility. The model aims to address the issue of unexpected random failures of the switch by implementing preventive maintenance measures. The system has two identical units. It starts with one unit in active operation and another unit in cold standby mode. In standby mode the unit remains in perfect state. No failure is allowed in standby mode. As the operating unit fails, the standby unit needs to be switched into operation, to keep the system working. A servicing facility is present in the system to perform necessary servicing related tasks. The servicing facility referred to as the server, also takes care of all necessary remedial activities like preventive maintenance and repairs. The switch used as switching mechanism to place the standby unit into operation may found imperfect when needed. Similarly, the server too can fail while doing job. A preventive maintenance scheme is used for the switch whereas treatment is given to server. The method of semi-Markov process and regenerative point technique is used for model developing and solving, respectively. The expressions are derived to determine different system performance measures such as mean time to system failure, availability, busy period, expected number of preventive maintenances and the profit. The distributions of random time elapsed in repairs, replacements, preventive maintenances and treatments are general. This study highlights the usefulness of switch's preventive maintenance in long run. To study the asymptotic behavior of the system model, all the expressions for system performance measures are obtained in steady state. A simulation study is conducted using a presumed data set and assuming a Weibull probability distribution. The numerical results are shown in tabular form. The simulation results serve to highlight the significance of preventive maintenance for the switch. The findings of the paper can provide guidelines to the people engaged in designing, framing and implementing standby switching systems in real applications.*

**Keywords:** Cold standby, Transition probabilities, Imperfect switch, Server, Preventive maintenance, Semi-Markov process, Steady state, Regenerative point techniques, Weibull distribution.

## 1. Introduction

The standby redundancy is always at the core of a backup system. A cold standby system is primarily characterized by the standby unit and the switch mechanism, needed to switch the standby into operation at the failure of operating unit. Implementing an appropriate standby scheme results in improved system performance. Though the operating time of a system can be enhanced by using standby redundancy but impossible to make it full failure proof. The working component of a standby system may fail due to aging, deterioration and some other factors such as shocks [1]. Such factors that are responsible for failures need appropriate repairs to improve system's functioning [2]. Different types of failures demand for different repair strategies like in-house repairs or specialized repairs, carried out by internal or external repairmen [3]. The availability of repairmen and their skills can have a significant impact on system performance and reliability.

Therefore another crucial element of a functioning system is the server; as an efficient and robust server can keep a system functioning for long. The server may get exhausted after working for a long and then needs some short of refreshment [4]. Employing additional repairmen can address the issue of system downtime caused by refreshing the server [5]. In addition to cold standby systems, the server is capable of handling failures in various other types of standby systems as well [6]. The primary task of repairmen in cold standby systems is to activate the cold standby unit which is accomplished through the switching mechanism.

The switch plays a vital role in standby system as it is responsible for switching a failed unit with a standby one. The decision to switch can be made either at a predetermined time or when the operating unit fails [7]. The functioning of the system can be impacted if the switch mechanism itself fails [8]. When a switch failure occurs the process of rebooting and repairing a standby system can have a notable impact on its overall reliability [9]. The switch in the standby system can exhibit either perfect or imperfect behavior depending on its level of functionality [10]. Due to the prevalence of switch failures the switching mechanism in standby systems is some time regarded as imperfect. [11]. The imperfections in the switching mechanism of both cold standby and warm systems have a negative impact on their reliability [12]. The combination of switch and server in a system plays a vital role in maintaining system reliability. The switch activates cold standby unit when the main unit fails while the server handles all repair activities. Such configuration effectively reduces system downtime. However, if either the server or the switch or both fail during task performance the system performance declines [13]. In case of failure the switch may be found imperfect and the server may be unreliable both adversely affect the profit of the system [14]. In such cases the repair process may be disrupted that leads to reduced system performance. So to examine impact of such failures on the profit and availability of the system a probabilistic model may be helpful [15].

A standby system requires regular maintenance and repair to ensure higher availability and reliability as it deteriorates with time [16]. Neglecting the maintenance and repair can lead to equipment failure and financial losses. So a robust maintenance or repair plan is the necessity for optimal system performance [17]. Condition-based maintenance is an effective approach among several others to optimize maintenance costs and ensure efficient operation of standby systems [18]. Though, maintenance presents challenges stemming from factors such as complexity, cost and competition but implementing suitable strategies can significantly reduce maintenance costs [19]. In most of cases the cost of breakdown maintenance is typically higher than preventive maintenance. Therefore implementing a preventive maintenance scheme can help minimize the risk of equipment failure and reduce overall maintenance costs while increasing availability and reliability [20]. Periodic system's inspections are usually conducted to decide about maintenance strategy [21]. In some cases preference is given to preventive maintenance over others [22]. Optimizing the inspection intervals is necessary to achieve the goal of effective preventive maintenance [23]. An effective maintenance plan may involves interval based inspections or multiple inspections in stages [24].

In this paper, we have developed a stochastic model for a cold standby system consisting of two identical units, a switch and a server. The switch is responsible for activating the standby unit and can experience random failures. It undergoes preventive maintenance after a specific time threshold. Similarly the server handles all remedial activities but can only perform one operation at a time and is also prone to failure during tasks. The model gives priority to switch repairs over other remedial activities. It assumes perfect repairs and server treatments. The study focuses on analyzing the impact of switch preventive maintenance on system performance. Steady state expressions for system performance measures are derived considering all random variables as statistically independent. The system model is developed using the theory of semi-Markov processes. The regenerative point technique and Laplace transforms are used to solve the model. A simulation study is conducted using a dataset and Weibull probability distribution with the results presented graphically.

## 2. Notations

| | |
|---|---|
| O: | The unit is in operative mode. |
| Cs: | The unit is kept as cold standby. |
| Sh: | The switch is ok |
| Sv: | The server is ok. |
| $Cs_w$: | The cold standby unit is in waiting. |
| p/q: | Probability that switch is operational/failed. |
| $F_{ur}$/ $F_{UR}$: | The unit is under repair/ continuously under repair from the previous state. |
| $F_{wr}$/$F_{WR}$: | The failed unit is waiting for repair/ waiting for repair continuously from the previous state. |
| $ShF_{ur}$ /$ShF_{UR}$: | The switch is under repair / under repair continuously from the previous state. |
| $Sh_{pm}$/$ShF_{PM}$: | The switch is under preventive maintenance / under continuously preventive maintenance from the previous state. |
| $ShF_{wr}$/$ShF_{WR}$: | The switch is waiting for repair / continuously from the previous state. |
| $SvF_{ut}$/$SvF_{UT}$: | The server is under treatment/ continuously from the previous state. |
| z(t)/ Z(t): | pdf/ cdf of the failure time of the unit. |
| r (t)/ R(t): | pdf / cdf of the failure time of the server. |
| f(t)/ F(t): | pdf / cdf of repair time of the failed unit. |
| h(t)/ H(t): | pdf / cdf of repair time of the failed switch. |
| s(t)/ S(t): | pdf / cdf of the treatment time of the server. |
| $q_{ij}(t)/Q_{ij}(t)$: | pdf / cdf of direct transition time from a regenerative state $S_i$ to a regenerative state $S_j$ without visiting any other regenerative state. |
| $q_{ij.k}(t)/Q_{ij.k}(t)$: | pdf / cdf of first passage time from a regenerative state $S_i$ to a regenerative state $S_j$ or to a failed state $S_j$ visiting state $S_k$ once in (0,t]. |
| (s)/(c): | Symbol for Stieltjes convolution / Laplace convolution. |
| ˜ / * : | Symbol for Laplace Stieltjes Transform(LST) / Laplace Transform(LT). |

## 3. Model Development

### 3.1. States of the System

The following are the possible states of the system model Figure 1.

The regenerative up states:

$S_0 = (O, Cs, Sh, Sv)$, $S_1 = (F_{ur}, O)$, $S_3 = (F_{wr}, O, Sv_{F_{ut}})$, $S_4 = (O, Cs_w, Sh_{pm})$

The failed regenerative down state:

$$S_2 = (F_{wr}, Cs_w, Sh_{Fur})$$

Non-regenerative states:

$S_5 = (F_{wr}, Cs_w, Sh_{PM})$, $S_6 = (F_{UR}, F_{wr})$, $S_7 = (F_{wr}, F_{WR}, Sv_{F_{ut}})$, $S_8 = (F_{WR}, Cs_w, Sh_{wr}, Sv_{F_{ut}})$, $S_9 = (F_{wr}, F_{WR}, Sv_{F_{UT}})$, $S_{10} = (F_{ur}, F_{WR})$, $S_{11} = (F_{WR}, Cs_w, Sh_{Fur})$,

### 3.2. State Transition Diagram

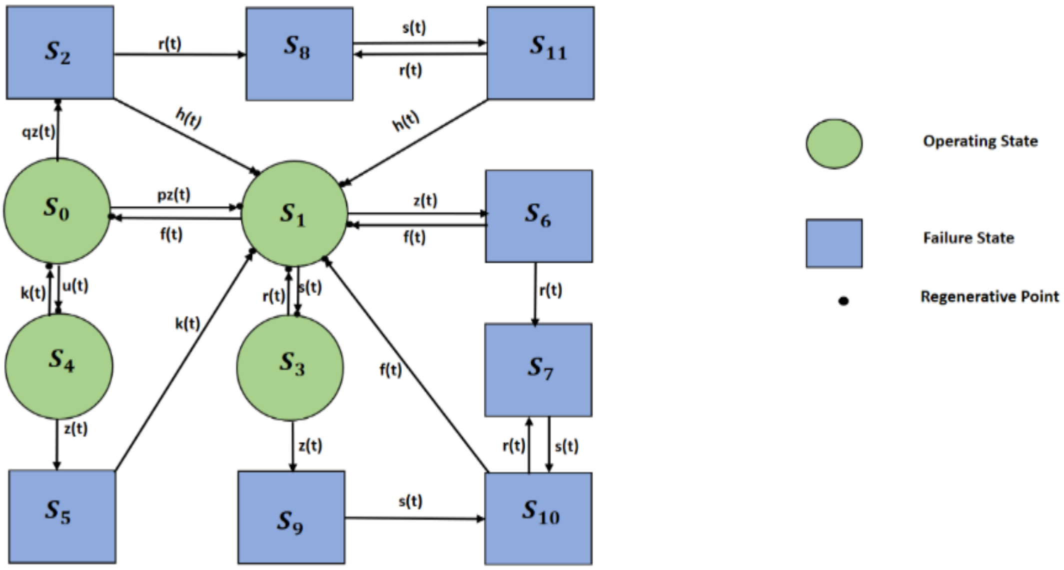


**Figure 1:** *State transition diagram*

### 3.3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \tag{1}$$

Also, the Mean Sojourn time $\mu_i$ in state $S_i$ are given by:

$$\mu_i = E(t) = \int_0^\infty P(T > t)dt \tag{2}$$

We get

$$p_{01} = \int_0^\infty pz(t)\bar{U}(t)dt,\ p_{02} = \int_0^\infty qz(t)\bar{U}(t)dt,\ \ p_{10} = \int_0^\infty f(t)\bar{R}(t)\bar{Z}(t)dt,$$

$$p_{13} = \int_0^\infty r(t)\bar{F}(t)\bar{Z}(t)dt,\ \ p_{16} = \int_0^\infty z(t)\bar{R}(t)\bar{F}(t)dt,\ p_{21} = \int_0^\infty h(t)\bar{R}(t)dt,$$

$$p_{27} = \int_0^\infty r(t)\bar{H}(t)dt,\ \ p_{31} = \int_0^\infty s(t)\bar{Z}(t)dt,\ p_{39} = \int_0^\infty z(t)\bar{S}(t)dt,$$

$$p_{40} = \int_0^\infty k(t)\bar{Z}(t)dt,\ p_{45} = \int_0^\infty z(t)\bar{F}(t)dt,\ p_{51} = \int_0^\infty k(t)dt,$$

$$p_{61} = \int_0^\infty f(t)\bar{Z}(t)dt,\ p_{67} = \int_0^\infty z(t)\bar{F}(t)dt,\ \ p_{7,10} = \int_0^\infty s(t)dt,$$

$$p_{8,11} = \int_0^\infty s(t)dt,\ p_{9,11} = \int_0^\infty s(t)dt,\ p_{10,1} = \int_0^\infty f(t)\bar{R}(t)dt,$$

$$p_{10,7} = \int_0^\infty r(t)\bar{F}(t)dt,\ p_{11,1} = \int_0^\infty h(t)\bar{R}(t)dt,\ p_{11,8} = \int_0^\infty r(t)\bar{H}(t)dt,$$

Here, it can be checked that sum of simple probabilities originating from a single state is unity. The expressions for mean sojourn times are as follows:

$$\mu_0 = \int_0^\infty \bar{Z}(t)dt,\ \ \mu_1 = \int_0^\infty \bar{F}(t)\bar{R}(t)\bar{Z}(t)dt,\qquad \mu_2 = \int_0^\infty \bar{R}(t)\bar{H}(t)dt,$$

$$\mu_3 = \int_0^\infty \bar{S}(t)\bar{Z}(t)dt,\ \ \mu_4 = \int_0^\infty \bar{K}(t)\bar{Z}(t)dt$$

# 4. System's Performance Measures

## 4.1. MTSF

Let $\emptyset_i(t)$ be the c.d.f of the first passage time from the regenerative state $S_i$ to a failed state. Regarding the failed state as an absorbing state, we have the following recursive relations for $\emptyset_i(t)$:

$$\emptyset_0(t) = Q_{01}(t)\,(s)\,\emptyset_1(t) + Q_{04}(t)\,(s)\,\emptyset_4(t) + Q_{02}(t)$$
$$\emptyset_1(t) = Q_{10}(t)\,(s)\,\emptyset_0(t) + Q_{13}(t)\,(s)\,\emptyset_3(t) + Q_{16}(t)$$
$$\emptyset_3(t) = Q_{31}(t)\,(s)\,\emptyset_1(t) + Q_{39}(t)$$
$$\emptyset_4(t) = Q_{40}(t)\,(s)\,\emptyset_1(t) + Q_{45}(t) \quad (4)$$

Taking LST of equation (4) and solving for $\widetilde{\emptyset_0}(s)$, we have

$$R^*(s) = \frac{1 - \widetilde{\emptyset_0}(s)}{s} \quad (5)$$

The reliability R(t) can be obtained by taking the inverse Laplace transition of (5) and MTSF is given by

$$MTSF = \lim_{s\to 0} R^*(s) = \lim_{s\to 0} \frac{1 - \widetilde{\emptyset_0}(s)}{s} \quad (6)$$

$$MTSF = \frac{(\mu_0 + p_{04}\mu_4)(1 - p_{13}p_{31}) + p_{01}(\mu_1 + p_{13}\mu_3)}{(1 - p_{13}p_{31})(1 - p_{04}p_{40}) - p_{01}p_{10}} \quad (7)$$

## 4.2. Steady State Availability

$M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$M_0 = \int_0^\infty \bar{Z}(t)dt,\quad M_1 = \int_0^\infty \bar{F}(t)\bar{R}(t)\bar{Z}(t)dt,\quad M_3 = \int_0^\infty \bar{S}(t)\bar{Z}(t)dt$$

Let $A_i(t)$ be the probability that the system is in up-state at an instant 't' given that the system entered regenerative state $S_i$ at t=0. The recursive relations for $A_i(t)$ are as follows:

$$A_0(t) = M_0(t) + q_{01}(t)\,(c)\,A_1(t) + q_{02}(t)(c)A_2(t)$$
$$A_1(t) = M_1(t) + q_{10}(t)\,(c)\,A_0(t) + \left(q_{1,1.6}(t)(t) + q_{1,1.6,(7,10)^n}\right)(c)A_1(t) + q_{13}(t)\,(c)\,A_3(t)$$
$$A_2(t) = \left(q_{21}(t) + q_{2,1.8,11}(t)\right)(c)A_1(t)$$
$$A_3(t) = M_3(t) + \left(q_{31}(t) + q_{3,1.9,10}(t) + q_{3,1.9,(10,7)^n}(t)\right)(c)A_1(t)$$
$$A_4(t) = M_4(t) + q_{4,0}(t)\,(c)\,A_0(t) + q_{4,1.5}(t)(c)A_1(t) \quad (8)$$

Where $S_j$ is any successive regenerative state to which the regenerative state $S_i$ can transit through n transitions. Taking LT of equation (8) and solving for $A_0^*(s)$, the steady state availability is given

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{(\mu_0 + p_{04}\mu_4)p_{1,0} + (1 - p_{13}p_{31})(\mu_1 + p_{13}\mu_3)}{(\mu_0 + p_{04}\mu_4')p_{1,0} + (1 - p_{04}p_{40})(\mu_1' + p_{13}\mu_3') + p_{10}p_{02}\mu_2'} \quad (9)$$

## 4.3. Busy period Analysis for the Server

Let $B_i(t)$ be the probability that the server is busy in repair of the unit or switch at an instant t given that the system entered regenerative state $S_i$ at t = 0. The recursive relations for $B_i(t)$ are as follows:

$$B_0(t) = q_{01}(t)\,(c)\,B_1(t) + q_{02}(t)(c)B_2(t)$$
$$B_1(t) = W_1(t) + q_{10}(t)\,(c)\,B_0(t) + q_{13}(t)\,(c)\,B_3(t) + q_{11.6}(t)(c)B_1(t) + q_{11.6,(7,10)^n}(c)B_1(t)$$
$$B_2(t) = W_2(t) + q_{21}(t)\,(c)\,B_1(t) + q_{21.8,11}(t)(c)B_1(t)$$
$$B_3(t) = q_{31}(t)\,(c)\,B_1(t) + q_{31.9,10}(t)(c)B_1(t) + q_{31.9,(10,7)^n}(t)(c)B_1(t)$$
$$B_4(t) = q_{4,0}(t)\,(c)\,B_0(t) + q_{4,1.5}(t)(c)B_1(t) \quad (10)$$

$W_i(t)$ be the probability that the server is busy in state $S_i$ due to repair of the unit or switch up to time 't' without making any transition to any other regenerative state or returning to the same via one or more non-regenerative state

$W_1(t) = \bar{Z}(t)\bar{F}(t)\bar{R}(t) + (z((t)\bar{F}(t)\bar{R}(t)©1)\bar{F}(t) + (z(t)\bar{F}(t)\bar{R}(t)©r(t)\bar{F}(t)©1)\bar{S}(t)$
$+ (z(t)\bar{F}(t)\bar{R}(t)©r(t)\bar{F}(t)©s(t)©1)\bar{F}(t)$

$W_2(t) = \bar{R}(t)\bar{H}(t) + (r(t)\bar{R}(t)©1)\bar{S}(t) + (r(t)\bar{R}(t)©s(t)©1)\bar{H}(t)$

Using LT, of equation (10) and solving for $B_0^*(s)$, the time for which server is busy due to repair of unit or switch is given by

$$B_0 = \lim_{s\to 0} sB_0^*(s) = \frac{W_1^*(0)(1-p_{04}p_{40}) + p_{02}p_{10}W_2^*(0)}{(\mu_0 + p_{04}\mu_4')p_{1,0} + (1-p_{04}p_{40})(\mu_1' + p_{13}\mu_3') + p_{10}p_{02}\mu_2'} \tag{11}$$

## 4.4. Busy Period Analysis for the Server due to Switch Preventive Maintenance

Let $B^P{}_i(t)$ be the probability that the server is busy in preventive maintenance of switch after a fixed time period at an instant t given that the system entered regenerative state $S_i$ at t = 0. The recursive relations for $B_i(t)$ are as follows:

$B^P{}_0(t) = q_{01}(t)\,(c)\,B^P{}_1(t) + q_{02}(t)(c)B^P{}_2(t) + q_{04}(t)(c)B^P{}_4(t)$
$B^P{}_1(t) = q_{10}(t)\,(c)\,B^P{}_0(t) + q_{13}(t)\,(c)\,B^P{}_3(t) + q_{11.6}(t) + q_{11.6,(7,10)^n}(c)B^P{}_1(t)$
$B^P{}_2(t) = q_{21}(t)\,(c)\,B^P{}_1(t) + q_{21.8,11}(t)(c)B^P{}_1(t)$
$B^P{}_3(t) = q_{31}(t)\,(c)\,B^P{}_1(t) + q_{31.9,10}(t)(c)B^P{}_1(t) + q_{31.9,(10,7)^n}(t)(c)B^P{}_1(t)$
$B^P{}_4(t) = W_4(t) + q_{4,0}(t)\,(c)\,B^P{}_0(t) + q_{4,1.5}(t)(c)B^P{}_1(t)$ (12)

$W_i(t)$ be the probability that the server is busy in state $S_i$ due to switch preventive maintenance up to time 't' without making any transition to any other regenerative state or returning to the same via one or more non-regenerative state

$W_4(t) = \bar{Z}(t)\bar{K}(t) + (z(t)©1)\bar{K}(t)$

Using LT, of equation (12) and solving for $B^{P^*}{}_0(s)$ , the time for which server is busy due to switch preventive maintenance is given by

$$B^P{}_0 = \lim_{s\to 0} sB^{P^*}{}_0(s) = \frac{p_{04}p_{10}W_4^*(0)}{(\mu_0 + p_{04}\mu_4')p_{1,0} + (1-p_{04}p_{40})(\mu_1' + p_{13}\mu_3') + p_{10}p_{02}\mu_2'} \tag{13}$$

## 4.5. Expected Number of Server Treatments

Let $T_i(t)$ be the expected number of treatments given to the server in (0,t ] given that the system entered regenerative state $S_i$ at t=0. The recursive relations for $T_i(t)$ are as follow:

$T_0(t) = Q_{01}(t)\,(s)\,T_1(t) + Q_{02}(t)(s)T_2(t) + Q_{04}(t)(s)T_4(t)$
$T_1(t) = Q_{10}(t)\,(s)\,T_0(t) + Q_{13}(t)\,(s)\,T_3(t) + Q_{11.6}(t)(s)T_1(t) + Q_{11.6,(7,10)^n}(s)T_1(t)$
$T_2(t) = Q_{21}(t)\,(s)\,T_1(t) + Q_{21.8,11}(t)(s)T_1(t)$
$T_3(t) = Q_{31}(t)\,(s)\,(1 + T_1(t)) + Q_{31.9,10}(t)(s)T_1(t) + Q_{31.9,(10,7)^n}(t)(s)T_1(t)$
$T_4(t) = Q_{4,0}(t)\,(s)\,T_0(t) + Q_{4,1.5}(t)(s)T_1(t)$ (14)

Using LT, of equation (14) and solving for $\widetilde{T_0}(s)$, the expected number of the treatments given to the server are given by

$$T_0 = \lim_{s\to 0} s\widetilde{T_0}(s) = \frac{p_{31}p_{13}(1-p_{04}p_{40})}{(\mu_0 + p_{04}\mu_4')p_{1,0} + (1-p_{04}p_{40})(\mu_1' + p_{13}\mu_3') + p_{10}p_{02}\mu_2'} \tag{15}$$

## 4.6. Expected Number of Switch Repairs

Let $U_i(t)$ be the expected number of repairs given to the switch in (0,t ] given that the system entered regenerative state $S_i$ at t=0. The recursive relations for $U_i(t)$ are as follow:

$U_0(t) = Q_{01}(t)\,(s)\,U_1(t) + Q_{02}(t)(s)U_2(t) + Q_{04}(t)(s)U_4(t)$
$U_1(t) = Q_{10}(t)\,(s)\,U_0(t) + Q_{13}(t)\,(s)\,U_3(t) + Q_{11.6}(t)(s)U_1(t) + Q_{11.6,(7,10)^n}(s)U_1(t)$
$U_2(t) = Q_{21}(t)\,(s)\,(1 + U_1(t)) + Q_{21.8,11}(t)(s)(1 + U_1(t))$
$U_3(t) = Q_{31}(t)\,(s)\,U_1(t) + Q_{31.9,10}(t)(s)U_1(t) + Q_{31.9,(10,6)^n}(t)(s)U_1(t)$
$U_4(t) = Q_{4,0}(t)\,(s)\,U_0(t) + Q_{4,1.5}(t)(s)U_1(t)$ (16)

Using LT, of equation (16) and solving for $\widetilde{U_0}(s)$, the expected number of the repairs given to the switch are given by

$$U_0 = \lim_{s\to 0} s\widetilde{U}_0(s) = \frac{p_{02}p_{10}}{(\mu_0 + p_{04}\mu_4')p_{1,0} + (1 - p_{04}p_{40})(\mu_1' + p_{13}\mu_3') + p_{10}p_{02}\mu_2'} \tag{17}$$

## 4.7. Expected Number of Repairs of the Unit

Let $O_i(t)$ be the expected number of repairs given to the server in (0,t] given that the system entered regenerative state $S_i$ at t=0. The recursive relations for $O_i(t)$ are as follow:

$$O_0(t) = Q_{01}(t)(s)\, O_1(t) + Q_{02}(t)(s)O_2(t) + Q_{04}(t)(s)O_4(t)$$
$$O_1(t) = Q_{10}(t)(s)\,(1 + O_0(t)) + Q_{13}(t)(s)\, O_3(t) + Q_{11.6}(t)(s)(1 + O_1(t)) + Q_{11.6,(7,10)^n}(s)(1 + O_1(t))$$
$$O_2(t) = Q_{21}(t)(s)\, O_1(t) + Q_{21.8,11}(t)(s)O_1(t)$$
$$O_3(t) = Q_{31}(t)(s)\, O_1(t) + Q_{31.9,10}(t)(s)(1 + O_1(t)) + Q_{31.9,(10,6)^n}(t)(s)(1 + O_1(t))$$
$$O_4(t) = Q_{4,0}(t)(s)\, O_0(t) + Q_{4,1.5}(t)(s)O_1(t) \tag{18}$$

Where $S_j$ is any regenerative state to which the given regenerative state $S_i$ transits. Using LT, of equation (18) and solving for $\widetilde{O_0}(s)$, the expected number of the repairs given to the unit are given by

$$O_0 = \lim_{s\to 0} s\widetilde{O}_0(s) = \frac{(p_{10} + p_{11,6} + p_{11,6,(7,10)^n})(1 - p_{04}p_{40})}{(\mu_0 + p_{04}\mu_4')p_{1,0} + (1 - p_{04}p_{40})(\mu_1' + p_{13}\mu_3') + p_{10}p_{02}\mu_2'} \tag{19}$$

## 4.8. Expected Number of Switch Preventive Maintenances

Let $N_i(t)$ be the expected number of preventive maintenance given to switch after a fixed time in (0,t] given that the system entered regenerative state $S_i$ at t=0. The recursive relations for are as follows:

$$N_0(t) = Q_{01}(t)(s)\, N_1(t) + Q_{02}(t)(s)N_2(t) + Q_{04}(t)(s)N_4(t)$$
$$N_1(t) = Q_{10}(t)(s)\, N_0(t) + Q_{13}(t)(s)\, N_3(t) + Q_{11.6}(t)(s)N_1(t) + Q_{11.6,(7,10)^n}(s)N_1(t)$$
$$N_2(t) = Q_{21}(t)(s)\, N_1(t) + Q_{21.8,11}(t)(s)N_1(t)$$
$$N_3(t) = Q_{31}(t)(s)(1 + N_1(t)) + Q_{31.9,10}(t)(s)N_1(t) + Q_{31.9,(10,7)^n}(t)(s)N_1(t)$$
$$N_4(t) = Q_{4,0}(t)(s)\,(1 + N_0(t)) + Q_{4,1.5}(t)(s)(1 + N_1(t)) \tag{20}$$

Where $S_j$ is any regenerative state to which the given regenerative state $S_i$ transits. Using LT, of equation (20) and solving for $\widetilde{N_0}(s)$, the expected number of PM of switch are given by

$$N_0 = \lim_{s\to 0} s\widetilde{N}_0(s) = \frac{p_{04}p_{10}}{(\mu_0 + p_{04}\mu_4')p_{1,0} + (1 - p_{04}p_{40})(\mu_1' + p_{13}\mu_3') + p_{10}p_{02}\mu_2'} \tag{21}$$

## 4.9. Cost Analysis

The Profit incurred to the system is given by

$$P = C_0A_0 - C_1B_0 - C_2B^P - C_3T_0 - C_4U_0 - C_5O_0 - C_6N_0 \tag{22}$$

$C_0$ = Revenue per unit up time of the system.
$C_1$ = Cost per unit time for which server is busy in repairing
$C_2$ = Cost per unit time for which server is busy in preventive maintenance.
$C_3$ = Cost per treatment of the server.
$C_4$ = Cost per repair of the switch.
$C_5$ = Cost per repair of the unit.
$C_6$ = Cost per preventive maintenance of the switch.

# 5. Simulation Study (Weibull Distribution)

Let us suppose that all the random variables follow Weibull distribution as given below:

$$z(t) = \lambda\eta t^{\eta-1}e^{-\lambda t^{\eta}}, \qquad f(t) = \alpha\eta t^{\eta-1}e^{-\alpha t^{\eta}}$$

$$r(t) = \mu\eta t^{\eta-1}e^{-\mu\eta t^{\eta}}, \qquad s(t) = \beta\eta t^{\eta-1}e^{-\beta t^{\eta}}$$

$$h(t) = \Upsilon\eta t^{\eta-1}e^{-\Upsilon t^{\eta}}, \qquad k(t) = \Theta\eta t^{\eta-1}e^{-\Theta t^{\eta}}, \quad u(t) = \xi\eta t^{\eta-1}e^{-\xi t^{\eta}}$$

For simulation, we assumed values of different parameters and costs. The impact of different parameters on system performance is shown in the tables.

**Table 1**: *Effect of various parameters on MTSF*

| MTSF η=0.5 | | | | | | | |
|---|---|---|---|---|---|---|---|
| Failure rate (λ) | $\alpha = 0.2$, $\beta = 0.7$, $\Upsilon = 0.3, \mu = 0.03$, $\Theta = 0.77, \xi = 0.4$ $p = 0.4$ | γ=0.4 | α=0.5 | β=0.8 | p=0.7 | θ=0.9 | ξ=0.8 |
| 0.01 | 896.56 | 896.56 | 918.05 | 917.72 | 917.51 | 997.48 | 1195.09 |
| 0.02 | 409.57 | 409.57 | 420.27 | 419.97 | 419.77 | 492.65 | 509.27 |
| 0.03 | 251.26 | 251.26 | 258.36 | 258.07 | 257.89 | 324.51 | 294.96 |
| 0.04 | 174.47 | 174.47 | 179.76 | 179.48 | 179.32 | 240.53 | 194.91 |
| 0.05 | 129.91 | 129.91 | 134.11 | 133.85 | 133.70 | 190.21 | 138.91 |
| η=1 | | | | | | | |
| 0.01 | 108.16 | 108.16 | 110.72 | 110.74 | 110.74 | 117.63 | 283.87 |
| 0.02 | 54.43 | 54.43 | 55.82 | 55.84 | 55.83 | 58.56 | 143.77 |
| 0.03 | 36.51 | 36.51 | 37.52 | 37.53 | 37.53 | 38.89 | 96.62 |
| 0.04 | 27.55 | 27.55 | 28.36 | 28.38 | 28.37 | 29.07 | 72.79 |
| 0.05 | 22.18 | 22.18 | 22.86 | 22.88 | 22.88 | 23.18 | 58.32 |
| η=2 | | | | | | | |
| 0.01 | 43.09 | 43.09 | 44.15 | 44.14 | 44.14 | 47.54 | 74.30 |
| 0.02 | 22.77 | 22.77 | 23.40 | 23.38 | 23.39 | 22.95 | 41.47 |
| 0.03 | 15.98 | 15.98 | 16.46 | 16.45 | 16.45 | 14.83 | 30.27 |
| 0.04 | 12.57 | 12.57 | 12.98 | 12.97 | 12.97 | 10.81 | 24.50 |
| 0.05 | 10.51 | 10.51 | 10.88 | 10.87 | 10.87 | 8.44 | 20.92 |

Table1 summarizes significant findings on Mean Time to System Failure (MTSF). It reveals the impact of different parameter values on MTSF. Higher values of failure rates (λ) lead to decreased MTSF implying shorter system lifespans. Increasing the value of shape parameter (η) also lowers MTSF and system reliability. Notably, increasing values of switch repair rate (γ) from 0.3 to 0.4, rate of switch goes under preventive maintenance (ξ) from 0.4 to 0.8, server treatment rate (β) from 0.8 to 0.9, unit repair rate (α) from 0.2 to 0.5 and switch preventive maintenance rate (Θ) from 0.77 to 0.9 all enhance the observed trends of MTSF. It implies that the switch preventive maintenance impacts the system reliability significantly.

Likewise, Tables 2 shows similar trends of system availability. The measure declines with higher values of shape parameter (η). It demonstrate that higher failure rate of the unit reduces system availability. For a fixed value of failure rates (λ) the system's availability improves with increasing values of rate with which switch goes under preventive maintenance ($\xi$), server treatment rate ($\beta$), repair rate of unit ($\alpha$) and switch preventive maintenance rate ($\Theta$). The values of system availability also rises significantly along with improved switch preventive maintenance.

**Table 2:** *Effect of various parameters on Availability*

| Availability $\eta$=0.5 | | | | | | | |
|---|---|---|---|---|---|---|---|
| Failure rate ($\lambda$) | $\alpha = 0.2$, $\beta = 0.7$, $\Upsilon = 0.3$, $\mu = 0.03$, $\Theta = 0.77$, $\xi = 0.4$ $p = 0.4$ | $\gamma$=0.4 | $\alpha$=0.5 | $\beta$=0.8 | p=0.7 | $\theta$=0.9 | $\xi$=0.8 |
| 0.01 | 0.9787 | 0.9912 | 0.9900 | 0.9902 | 0.9908 | 0.9888 | 0.9996 |
| 0.02 | 0.9574 | 0.9813 | 0.9797 | 0.9799 | 0.9806 | 0.9768 | 0.9986 |
| 0.03 | 0.9362 | 0.9704 | 0.9689 | 0.9689 | 0.9695 | 0.9640 | 0.9976 |
| 0.04 | 0.9151 | 0.9586 | 0.9578 | 0.9575 | 0.9575 | 0.9505 | 0.9961 |
| 0.05 | 0.8943 | 0.9461 | 0.9464 | 0.9456 | 0.9449 | 0.9364 | 0.9941 |
| $\eta$=1 | | | | | | | |
| 0.01 | 0.9102 | 0.9912 | 0.9921 | 0.9921 | 0.9855 | 0.9926 | 0.9940 |
| 0.02 | 0.8388 | 0.9813 | 0.9839 | 0.9837 | 0.9707 | 0.9844 | 0.9887 |
| 0.03 | 0.7807 | 0.9704 | 0.9754 | 0.9748 | 0.9557 | 0.9755 | 0.9816 |
| 0.04 | 0.7325 | 0.9586 | 0.9667 | 0.9656 | 0.9405 | 0.9659 | 0.9737 |
| 0.05 | 0.6918 | 0.9461 | 0.9578 | 0.9560 | 0.9253 | 0.9557 | 0.9651 |
| $\eta$=2 | | | | | | | |
| 0.01 | 0.8422 | 0.9901 | 0.9899 | 0.9929 | 0.9916 | 0.9884 | 0.9880 |
| 0.02 | 0.8331 | 0.9822 | 0.9816 | 0.9876 | 0.9850 | 0.9789 | 0.9785 |
| 0.03 | 0.8237 | 0.9737 | 0.9726 | 0.9819 | 0.9780 | 0.9691 | 0.9687 |
| 0.04 | 0.8141 | 0.9649 | 0.9632 | 0.9757 | 0.9705 | 0.9589 | 0.9586 |
| 0.05 | 0.8045 | 0.9557 | 0.9533 | 0.9692 | 0.9627 | 0.9485 | 0.9484 |

The table 3 highlights the effect of various parameters on system profit. It reveals that system profit declines with an increasing failure rate of unit ($\lambda$) as well as higher values of shape parameter ($\eta$). The profit expands with repairs, treatments and preventive maintenance.

The numerical simulation results have indicated that effect of preventive maintenance on overall system's performance is encouraging. All the performance measures MTSF, availability and the profit improves with switch preventive maintenance rate ($\Theta$). The higher frequency of preventive maintenance ($\xi$) too ensures improved system performance. The MTSF almost doubles with change in value of shape parameter ($\eta$) from 0.5 to 1. These results underline the importance of managing failure rates and implementing effective preventive maintenance strategies to keep a system reliable and profitable.

**Table 3:** *Effect of various parameters on system profit*

| Profit $\eta$=0.5 | | | | | | | |
|---|---|---|---|---|---|---|---|
| Failure rate ($\lambda$) | $\alpha = 0.2,\ \beta = 0.7,\ \Upsilon = 0.3, \mu = 0.03,\ \theta = 0.77, \xi = 0.4,\ p = 0.4$ | $\gamma$=0.4 | $\alpha$=0.5 | $\beta$=0.8 | p=0.7 | $\theta$=0.9 | $\xi$=0.8 |
| 0.01 | 7324.6 | 7931.7 | 8351.3 | 8796.9 | 9412.2 | 8178.6 | 9195.4 |
| 0.02 | 7048.7 | 7610.7 | 8095.4 | 8519.8 | 9097.0 | 7917.9 | 8536.6 |
| 0.03 | 6809.4 | 7338.3 | 7876.9 | 8238.1 | 8760.1 | 7663.9 | 7967.8 |
| 0.04 | 6590.2 | 7093.0 | 7678.5 | 7984.0 | 8460.9 | 7416.7 | 7470.4 |
| 0.05 | 6384.4 | 6865.4 | 7492.8 | 7747.6 | 8185.9 | 7176.4 | 7030.8 |
| $\eta$=1 | | | | | | | |
| 0.01 | 8028.0 | 8784.7 | 9666.5 | 9662.9 | 9590.1 | 9253.3 | 9759.9 |
| 0.02 | 7907.3 | 8723.4 | 9361.3 | 9379.0 | 9252.0 | 8905.2 | 9434.4 |
| 0.03 | 7780.8 | 8654.9 | 9089.5 | 9130.8 | 8958.1 | 8604.7 | 9134.0 |
| 0.04 | 7649.1 | 8580.5 | 8840.1 | 8904.5 | 8691.3 | 8332.6 | 8850.1 |
| 0.05 | 7513.2 | 8501.3 | 8607.8 | 8693.8 | 8444.0 | 8080.8 | 8601.8 |
| $\eta$=2 | | | | | | | |
| 0.01 | 8085.3 | 8534.8 | 8369.4 | 9045.3 | 9723.2 | 9664.2 | 8571.4 |
| 0.02 | 7797.9 | 8263.8 | 8108.4 | 8753.1 | 9393.2 | 8694.1 | 8547.5 |
| 0.03 | 7530.9 | 8019.6 | 7872.4 | 8488.7 | 9096.0 | 7899.1 | 7799.3 |
| 0.04 | 7280.6 | 7795.7 | 7655.6 | 8245.9 | 8823.7 | 7234.4 | 7178.6 |
| 0.05 | 7047.3 | 7588.6 | 7454.7 | 8021.0 | 8572.2 | 6670.1 | 6655.2 |

## 6. Applications

In practice the switch is an integral component that holds significant importance in various applications. It plays a critical role in diverse sectors few including Rail Tracking, Wind Power Plants, and DSLAM Networks. In Rail Tracking systems switches are responsible for ensuring the safe and efficient movement of trains by facilitating the switching of tracks. In Wind Power Plants switches are utilized to control the flow of electricity from the turbines to the grid. Similarly in DSLAM Networks the switches manage the traffic and direct data packets to their intended destinations. As the switch preventive maintenance refers to the proactive measures taken to anticipate and mitigate potential failures before they actually occur. Hence by conducting regular inspections and performing necessary preventive maintenance helps minimize the risk of sudden failures within systems using switching phenomena.

## 7. Conclusion

The occurrence of unexpected system breakdowns significantly reduces system outputs. Therefore implementing appropriate maintenance schemas are always required. These schemas aimed to enhance system performance by minimizing sudden breakdowns. In this paper the application of preventive maintenance specifically to the switch is focused on. The idea of implementing preventive maintenance effectively reduces the failure likelihood of switch. The simulation results have illustrated the impact of switch preventive maintenance on mean time to system failure, system availability and profitability. These findings indicated that switch preventive maintenance influences overall system performance and profitability.

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