# STUDY OF THE FUNCTIONING OF A MULTI-COMPONENT AND MULTI-PHASE QUEUING SYSTEM ON THE EXAMPLE OF A VEHICLE REPAIR ENTERPRISE

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#### Abstract

The purpose of the work is to build, on the basis of multi-component and multi-phase models of queuing systems (QS), mathematical models of maintenance and repair of vehicles by repair enterprises to increase the efficiency of their use. Results. The article considers multi-component and multi-stage mathematical models of QSs with the distribution of the arrival flow simultaneously between the system components, which consist of a certain number of service channels and waiting places in the queue. The same service channels can have different performance depending on the type of customers which they serve. Customers go through several stages of service and waiting. Considered are service of customers without a lack of time to stay in the service channel and waiting and with a lack of such time. The service process in the QS of each component consists of several  $(k_{\rho})$  stages with the corresponding duration, the full-service period will be equal to the sum of such time intervals. Stage durations have certain probability distributions with appropriate parameters, then the total duration of the service process will have a generalized Erlang distribution with parameters of probability distributions of stages of order  $k_{\rho}$ . The number of components and their parameters correspond to the similar characteristics of the production divisions of the repair enterprise. The study of the effectiveness of the repair enterprise operation as a multi-component and multi-stage QS consists in determining the probability of service and the probability of failure of QS components and the system as a whole, the number of service channels, the number of customers in components, the number of customers in component queues, the duration of maintenance of customers in components and the system, the duration of being customers in queues of components and QSs, duration of stay of requirements in QSs and duration of customer waiting in QS queues. The model is implemented using Any Logic University Researcher. The AnyLogic University Researcher development environment allowed to combine the principles of system dynamics with the paradigms of agent and discrete-event modeling. In addition, thanks to the built-in Java SE compiler, a library of ready-made solutions is available, including generators of random variables, which significantly expands the possibilities of developing and implementing experiments. In particular, experiments on optimization (relative to a defined criterion), sensitivity of the model, stability of the model, etc. are available.

**Keywords:** vehicle repair enterprise, maintenance and repair, queuing theory application, multi-component and multi-phase queuing system, simulation, modeling, reliability

# I. Introduction

The problem of ensuring the resource and reliability of vehicles is part of the general problem of transport safety and the efficiency of the use of such vehicles. Ensuring high reliability primarily depends on the effectiveness of the maintenance and repair strategy and the quality of work.

Maintenance and repair of railway, including special rolling stock, is carried out in accordance with the requirements of the Regulations [1] at repair plants, depots, track engineering stations, workshops and maintenance points.

Requirements for the condition of rolling stock, the procedure for its maintenance and repair, sending it to repair bodies, as well as technical instructions and typical technological processes for maintenance and repair of rolling stock are determined by Ukrzaliznytsia [1].

The aircraft maintenance system is designed to maintain and restore the airworthiness and serviceability of aircraft and prepare them for flight. Technical operation is carried out by operators, aviation and technical bases, maintenance and repair enterprises, repair enterprises, aviation and technical services of airports [2].

Aircraft maintenance is carried out during major and other repairs (or during equivalent works), inspections, modifications, upgrade, elimination of defects, which are carried out by aircraft repair enterprises both individually and collectively in the relevant workshops, production divisions and areas, laboratories, stands, etc. [3].

Maintenance and repair of motor vehicles and their components is performed in order to maintain them in proper condition and ensure the technical characteristics established by the manufacturer for use, storage or maintenance during the period of operation [4]. Requirements for maintenance and repair of motor vehicles and the services provided by them (work to be performed) are established by technical regulations. The work is carried out in workshops of motor vehicle enterprises and car repair enterprises.

The work [5] is devoted to the creation and introduction into the practice of air transport of information and advisory systems for the maintenance of passenger aircraft based on modern computer technologies and mathematical methods of information processing.

In [6], the structure of the methodical apparatus for ensuring a given level of serviceability of on-board equipment products, in particular optoelectronic sighting systems of military aircraft of the Air Force of Ukraine, is proposed.

Methodical approaches to the structural and parametric determination of general requirements for ground flight maintenance facilities are considered in [7], which can also be used to develop a methodology for conducting tests and assessing the quality of modern weapons systems and military equipment at all stages of the life cycle.

Based on the analysis of the existing methods of calculating the durability indicators of the radio-electronic system of the aircraft, the factors affecting its reliability were identified in [8], and measures were also proposed to improve the existing scientific and methodological apparatus for calculating such indicators.

Works [9] and [10] are devoted to the analysis of the causes of failure situations at the airport. The aircraft maintenance system was analyzed, it was shown that ensuring uninterrupted operation of the airport, execution of the daily flight plan in extraordinary situations is possible only by introducing into the control circuit of the aircraft ground handling system an intelligent decision support system for dispatchers, which will take into account the positive experience of their actions in typical, emergency and failure situations. This will allow, in particular, to reduce the time to get out of a malfunctioning situation and to optimize the operational planning of the ground maintenance of aircraft, considering the available equipment and special technical means.

In work [11], organizational measures are given with the help of which it is possible to

minimize the lack of transport aviation during the transportation of general and oversized cargo. Data on incidents related to aircraft ground handling are given, the causes of the events are indicated. The main ways of eliminating the problems of standardization of airfield technical support in the conditions of interaction with NATO and in the processes of international integration are defined.

The work [12] is devoted to the solution of the problem of minimizing the risks of import substitution in the process of factory repair of military aviation equipment in the conditions of a special period, the issue of providing post-repair military aviation equipment by adjusting the production of necessary component parts by domestic enterprises in the process of import substitution is analyzed.

The work [13] presents the results of the quality of repair of aircraft equipment at aircraft repair enterprises. A significant proportion of the failures detected during the operation of aviation equipment after major (medium) repairs are the result of manufacturing defects of components (parts) that were installed on aircraft. Technological methods for ensuring sufficient repair quality and significantly reducing the risks of production defects are proposed.

The work [14] is devoted to the problem of mathematical modeling of the processes of technical operation of military aircraft. The results of the analysis show that the most acceptable modeling method in terms of the compliance of the models with the proposed requirements is the simulation modeling method, and the more accepted class for creating a stochastic model of aircraft maintenance and repair processes is the class of semi-Markov models.

In work [15], a three-dimensional model of an aircraft skin element with riveted seams was built using the Sold Works software, wind load simulation was carried out in the ANSYS software package, which allowed to determine the stress-strain state of the aircraft skin elements in the presence of multifocal damage to the riveted seams.

Modern methods and approaches to modeling technological systems are considered in [16]. Basic definitions and concepts are given. New approaches to solving problems that arise during the development of models of mechanisms, systems and processes of machine-building production are proposed.

In work [17], it is proposed to consider the production of car service enterprises as an open multi-channel QS, in which random processes occur due to the combined action of random variables. As a result of the experimental study, information was obtained about the indicators characterizing maintenance and repair, as well as affecting the change in the parameters of these processes. The developed model makes it possible to take into account the specifics of managing car maintenance stations.

The paper [18] considered a model for assessing the technical condition of radio-electronic elements of water transport vehicles using control and diagnostic equipment as a QS with a limited number of channels and a storage of customers to be served. On the basis of various optimization criteria, it is possible to establish a rational system for assessing the technical condition of such elements, to determine the expediency (rational, optimal) of developing a number of different types of control and diagnostic equipment and the effectiveness of new assessment methods.

The work [19] is devoted to the development of a simulation model of the influence of an accurate assessment of the readiness factor of mobile control and diagnostic complexes on the reliability of control of radio-electronic systems of marine transport.

In the paper [20], a new model of the task of managing the processes of diagnosis and monitoring of automation tools is proposed for the objects of rail-water transport connection, compiled on the basis of the experimental research results and mathematical description using Markov chains with an informative parameter in the form of damage intensity, aimed at increasing the efficiency of forecasting the technical condition of automation equipment.

The work [21] describes workshops for repairing locomotives in the form of multi-channel QS

with a limited queue. A simulation model of such a workshop as a QS was developed, which allowed rational use of equipment, labor force, as well as distribution of repair work time.

In work [22], the issue of modeling the processes of maintenance and repair of technical systems of a distributed information system is considered. The model is based on a joint presentation of the serviced system and its technical operation system in the form of a closed non-homogeneous QS consisting of two types of QS. The QS of the first type simulates the functioning processes of repair bodies to meet the received requirements.

In work [23], a study of the actions of emergency units of railway transport as processes of functioning of mass service systems was carried out. The authors established quantitative relationships between the intensity of the influence of dangerous factors of a railway emergency situation, the time of arrival, deployment and productivity of actions of emergency aftermath liquidation units and the effectiveness of liquidation works due to the implementation of the principles of network-centric management of complex dynamic hierarchical transport systems.

Thus, to improve the management processes of material, human, financial and informational resources during the maintenance and repair of aviation equipment, in particular on-board power supply systems, a wide range of operations research methods, the theory of mass service systems and simulation modeling are currently used.

## II. Methods

The on-hand practical experience of the organization of maintenance and repair of vehicle equipment indicates that certain types of their technical systems that require various types of repair work, modifications, upgrade, inspections, elimination of defects, etc. are sent to specific production divisions of the repair enterprise, which, according to their purpose, carry out the necessary types of work according to the specified technologies.

To simulate the processes of maintenance and repair of vehicle, which are carried out by the production divisions of the repair enterprise, it is advisable to use multi-component and multi-phase QSs, which can be of both Markov and non-Markov types, capable of serving the arriving flows of non-priority, in general, heterogeneous (mixed) customers. At the same time, the system can have an arbitrary number of common service channels of the same type, and each component can also have an arbitrary number of places in the queue.

The same service channels can have different performance depending on the type of requirements for which they are involved: when the j-component of the system receives uniform requirements with the rate  $\lambda$ j determined by the overall rate  $\lambda$  of the source, in the general case, of mixed customers. The magnitude of the source of mixed customers entering the system has an intensity (arrival rate) of

$$\lambda = \sum_{j=1}^{L} \lambda_{j}, \qquad j = \overline{1, L},$$

where L is the number of components in the QS.

The service process in each component of the QS consists of several stages (phases) with the corresponding duration  $T_i$ , then the full-service period  $T_s$  is equal to

$$\mathbf{T}_s = \sum_{i=1}^{\mathbf{K}_p} T_i$$

where  $K_p$  is the number of such phases.

All  $T_i$  durations have certain probability distributions with the appropriate parameters, then  $T_s$  will have a generalized Erlang distribution with the parameters of the probability distributions of order  $K_p$ .

The number of components and their parameters correspond to similar characteristics of the repair enterprise.

The study of the operation effectiveness of a vehicle repair enterprise as a multi-component QS will consist in determining the probability and time characteristics of each component and the QS as a whole.

Let's consider several examples.

**Example 1.** Two-component QS with  $M/E_4/2/3$  in the first component and  $M/E_3/1/2$  in the second component without restrictions.

The graph of states of such a QS is presented in fig. 1.



**Figure 1:** State graph of QS of M/E<sub>4</sub>/2/3 type in the first component and M/E<sub>3</sub>/1/2 type in the second component without constraints

There are no constraints in the QS because customers do not leave the service channel during service and the queue during the service waiting period due to the lack of time for them to be in service and in the queue.

Using the well-known algorithm for solving the Kolmogorov equations, we obtain the probabilities of the system states  $P'_{1i}$ ,  $P'_{2i}$ ,  $P'_{3i}$ ,  $P'_{4i}$ ,  $P'_{5i}$  ( $i = \overline{1,4}$ ) ta  $P''_{1i}$ ,  $P''_{2i}$ ,  $P''_{3i}$  ( $i = \overline{1,3}$ ).

Service in the QS of the first component consists of four phases of duration  $T'_1$ ,  $T'_2$ ,  $T'_3$  and  $T'_4$ , the full-service time  $T'_s$  has a generalized Erlang distribution of the 4th order with a mathematical expectation of  $1/\mu_1$ .

$$\mathbf{T'}_s = \sum_{i=1}^4 \mathbf{T'}_i,$$

where  $T'_i$  have an exponential distribution with parameter  $\mu' = 4\mu_1$ .

Service in the second component of the QS consists of three phases of duration  $T'_1$ ,  $T'_2$  and  $T'_3$ , the full-service time  $T''_s$  has a generalized Erlang distribution of the 3rd order with a mathematical expectation of  $1/\mu_2$ .

$$\mathbf{T}_{s}^{"}=\sum_{i=1}^{3}T_{i}^{"},$$

where  $T_i^{"}$  have an exponential distribution with parameter  $\mu^{"} = 3\mu_1$ .

The QS states of the first component are characterized by the following probabilities [24]:

$$P'_{c} = \sum_{j=1}^{4} P'_{cj}$$
,  $c = \overline{1,5}$ ;  $j = \overline{1,4}$ ,

where:

is the probability of one server busy in the QS component (1 customer in component 1);

is the probability of two servers busy in the QS component (2 customers in component 1);

is the probability of 3 customers being in the component, of which 2 are served, one is in the queue;

is the probability of 4 customers being in the component, of which 2 are served, 2 are in the queue;

is the probability of 5 customers being in the component, of which 2 are served, 3 are in the queue.

Similarly, for the QS states of the second component:

$$P_{c}^{"} = \sum_{j=1}^{3} P_{cj}^{"}, c = \overline{1,3}; j = \overline{1,3},$$

where:

 $P_{1}^{"} = \sum_{\substack{j=1\\K_{p}^{"}}}^{K_{p}^{"}} P_{1}^{"}j$  $P_{2}^{"} = \sum_{\substack{j=1\\H_{n}^{"}}}^{K_{p}^{"}} P_{2}^{"}j$ 

is the probability of one customer being served in the component;

is the probability of two customers being in the component, one served, one in the queue;

$$P_3^{"} = \sum_{i=1}^{n_p} P_3^{"} j$$
 is the probability of three customers being in the component, one served, two in the queue.

The number of busy service channels in the component:

$$\overline{k_1} = P_1' + 2\sum_{c=2}^5 P_c' / 4; \quad \overline{k_2} = \sum_{c=1}^3 P_c'' / 3.$$

 $P'_{1} = \sum_{j=1}^{K'_{p}} P'_{1}j$   $P'_{2} = \sum_{j=1}^{K'_{p}} P'_{2}j$   $P'_{3} = \sum_{j=1}^{K'_{p}} P'_{3}j$   $P'_{4} = \sum_{j=1}^{K'_{p}} P'_{4}j$   $P'_{5} = \sum_{j=1}^{K'_{p}} P'_{5}j$ 

Probability of service in the first component:

$$P'_{s} = 1 - P'_{ls} - \sum_{c=1}^{(n+m)"} P^{"}_{c} = 1 - P'_{fl} - \sum_{c=1}^{(n+m)"} P^{"}_{c},$$

where  $P'_{ls}$  is the probability of loss of a customer;

 $P'_{fl}$  is the probability of failure of serving a customer.

Here, "failure" means the impossibility of servicing a customer (client) in a system component (for example, due to insufficient service rate compared to the arrival flow rate). "Loss" means leaving the queue for service by a customer (loss of a client by the system) due to the impossibility of waiting for service. Example, cars needing gasoline and service arrive to the gas or car service station. However, if the station already is being used, fully busy with servicing other cars, these potential customers may balk to another service station.

$$P_{c}^{"} = \sum_{j=1}^{k_{E}^{"}} P_{cj}^{"}, c = \overline{1, (n+m)}; \quad P_{fl}' = P_{(n+m)'}' = \sum_{c=2}^{(n+m)'} P_{cj}' / k'_{E};$$

$$P_{bo}' = \sum_{\substack{d=2\\j=2}}^{k_{E}} (d-1)P_{(b-1)j}', \quad b = \overline{2, (n+m)'}$$

$$P_{ls}' = P_{5}' - \sum_{b=5}^{5} P_{bo}' / 4,$$

$$P_{20}' = P_{12}' + 2P_{13}' + 3P_{14}'; \quad P_{30}' = P_{22}' + 2P_{23}' + 3P_{24}';$$

$$P_{40}' = P_{32}' + 2P_{33}' + 3P_{44}'; \quad P_{50}' = P_{42}' + 2P_{43}' + 3P_{44}';$$

$$P_{1}^{"} = P_{11}^{"} + P_{12}^{"} + P_{13}^{"}; \quad P_{2}'' = P_{21}^{"} + P_{22}^{"} + P_{23}^{"}; \quad P_{3}'' = P_{31}'' + P_{32}'' + P_{33}''.$$

Then

$$P'_{s} = 1 - P'_{5} + \sum_{b=2}^{5} P'_{bo} / 4 - \sum_{i=1}^{3} P'_{c} \cdot P'_{s} = 1 - P'_{ls} - \sum_{c=1}^{5} P'_{c} \cdot P'_{c}$$

The probability of service in the second component  $P_s^{"}$  is determined considering the fact that

$$P_{ls}^{"} = P_{fl}^{"} = P_{3}^{"} - \frac{P_{20}^{"} + P_{30}^{"}}{3}; P_{20}^{"} = P_{12}^{"} + 2P_{13}^{"}; P_{30}^{"} = P_{22}^{"} + 2P_{23}^{"}.$$

Whence

$$P_s'' = 1 - P_3'' + \frac{P_{20}'' + P_{30}''}{3} - \sum_{c=1}^5 P_c'.$$

The average number of customers  $\overline{N}^{(i)}$  in the i-component:

$$\overline{N}^{(1)} = \sum_{i=1}^{(n+m)'} i P_i' / k_E' = (1 \cdot P_1' + 2 \cdot P_2' + 3 \cdot P_3' + 4 \cdot P_4' + 5 \cdot P_5') / 4,$$
$$\overline{N}^{(2)} = \sum_{i=1}^{(n+m)''} i P_i' / k_E'' = (1 \cdot P_1' + 2 \cdot P_2' + 3 \cdot P_3' / 3.$$

The average number of requests  $\overline{N}_q^{(i)}$  that are in the queue and waiting for service in the i-component:

$$\overline{N}_{q}^{(1)} = \sum_{q=1}^{m'} q P_{(n+q)}' / k_{E}' = (1 \cdot P_{3}' + 2 \cdot P_{4}' + 3 \cdot P_{5}') / 4:$$
$$\overline{N}_{q}^{(1)} = \sum_{q=1}^{m''} q P_{(n+q)}' / k_{E}'' = (1 \cdot P_{2}'' + 2 \cdot P_{3}'') / 3.$$

Duration of waiting time for the customer in the queue for the i-component equals:

$$\overline{W}_{q}^{(1)} = \frac{\overline{N}_{q}^{(1)}}{\lambda_{1}}; \ \overline{W}_{q}^{(2)} = \frac{\overline{N}_{q}^{(2)}}{\lambda_{2}}.$$

Customer service time in the QS:

$$\overline{t}_{sqs} = \frac{\lambda_1}{\lambda_1 + \lambda_2} \overline{t}_s^{(1)} + \frac{\lambda_2}{\lambda_1 + \lambda_2} \overline{t}_s^{(2)},$$
$$t_s^{(1)} = \frac{\overline{N}^{(1)}}{\lambda_1}; \quad t_s^{(2)} = \frac{\overline{N}^{(2)}}{\lambda_2}.$$

Duration of waiting for customers in QS queues:

$$\overline{w}_{wqs} = \frac{\lambda_1}{\lambda_1 + \lambda_2} \overline{w}_q^{(1)} + \frac{\lambda_2}{\lambda_1 + \lambda_2} \overline{w}_q^{(2)}.$$

Probability of QS failure:

$$P_{fl}^{qs} = \frac{\lambda_1}{\lambda_1 + \lambda_2} P_{fl_1} + \frac{\lambda_1}{\lambda_1 + \lambda_2} P_{fl_2}.$$

It should be noted that in the L-component QS in a steady mode, the value of the probability of customer service in the j-component can be determined as follows [24]:

$$P_{s_j} = 1 - P_{ls_j} - \sum_{\psi \neq j}^{L} \left( \sum_{i=1}^{n+m} P_i \right)_{\psi},$$
  

$$\psi = \overline{1, L} = 1, 2, 3 \dots, j \dots, L;$$
  

$$i = \overline{0, (n+m)};$$
  

$$P_{S_{qs}} = \sum_{j=1}^{L} \frac{\lambda_j}{\lambda} P_{S_j},$$
  

$$\lambda = \sum_{j=1}^{L} \lambda_j.$$

During the maintenance and repair of aircraft, force majeure circumstances may arise, related to the time limitations of customer service for maintenance and waiting in the queue, so let's consider the following example. **Example 2.** Two-component QS of M/E<sub>4</sub>/2/3 type in the first component and of M/E<sub>3</sub>/1/2 in the second component with restrictions on the time spent in the service period  $\beta_{1,2}$  and waiting  $\gamma_{1,2}$ .

The graph of the states of this QS coincides with the graph of the states of the QS presented in fig. 1.

At each service stage of the first channel, the service duration has an exponential distribution with the parameter  $\mu'+\beta$ , in the second service channel  $2\mu'+2\beta$ , in queues  $2\mu'+2\gamma_1$ .

In the second component, the parameter  $\mu$ "+ $\beta_2$  at the service stages, and  $\mu$ "+ $\gamma_2$  at the stages of waiting in the queue.

The expressions for the probabilities of the QS states of the first and second components are similar to the QS considered above.

Service probability for the first component [24]:

$$P'_{S} = 1 - P'_{ls} - \sum_{i=1}^{3} P'_{c},$$
$$P'_{ls} = P'_{fl} + P'_{lv} + P'_{lv},$$

where

 $P'_{lv}$  is the probability of the customer leaving the system in the service channel;

 $P'_{lv}$  is the probability of the customer leaving the system in the queue.

$$P'_{fl} = P'_{5} - \sum_{b=1}^{5} P'_{bo} / 5$$
$$P'_{lv} = \frac{\beta_{1}}{\lambda_{1}} \overline{k_{1}}.$$

The average number of customers in the queue

$$\overline{N}_{q}^{(1)} = \sum_{q=1}^{m'} q P'_{(n+q)} / k'_{E} = (1 \cdot P'_{3} + 2 \cdot P'_{4} + 3 \cdot P'_{5})/4.$$

If  $\beta_1 = \gamma_1$ , then

$$P_{lv}'_{q} = \frac{\beta_1}{\lambda_1} \overline{N}_q^{(1)} \,.$$

Hence

$$P_{ls}' = P_5' - \sum_{b=2}^{k_e'} P_{bo}' / k_E' + \frac{\beta_1}{\lambda_1} \overline{k_1} + \frac{\gamma_1}{\lambda_1} \overline{N}_q^{(1)},$$
$$P_S' = P_{ls}' - \sum_{c=1}^3 P_c^c.$$

The probability of customer service in the second component is

$$P_{S}^{"} = 1 - P_{ls}^{"} - \sum_{c=1}^{5} P_{c}^{\prime}$$
,

where

$$P_{ls}^{"} = P_{fl}^{"} + P_{lv}^{"} + P_{lv}^{"},$$

$$P_{fl}^{"} = P_{3}^{"} - \frac{P_{20}^{"} + P_{30}^{"}}{\lambda_{2}},$$

$$P_{lv}^{"} = \frac{\beta_{2}}{\lambda_{2}} \overline{k}_{2},$$

$$\overline{N}_{q}^{(2)} = \frac{1 \cdot P_{2}^{"} + 2 \cdot P_{3}^{"}}{3}.$$

Then

$$P_{lv}^{"} = \frac{\gamma_2}{\lambda_2} \overline{N}_q^{(2)}.$$

Provided that  $\beta_2 = \gamma_2$ ,

$$P_{lv}^{"} = \frac{\beta_2}{\lambda_2} \overline{N}_q^{(2)}.$$

When applying the proposed mathematical models, it is advisable to consider the following:

- in multi-component QSs, the performance of any component decreases compared to a singlecomponent system at the same rates of service stages. With the same values of the parameters of each component of the QS, the performance of multi-component and single-component systems will be the same;

- if one of the components is a QS with a queue, and the second component is a QS with failures, then the QS with a queue has a higher performance, simultaneously reducing the performance of the second component;

- with small values ( $0 \le P_s \le 0.1$ ), the impact on the system as a whole or on a separate component of the intensities of customers leaving the system during the service period and being in the queue is insignificant. When these intensities change, the Ps value will fluctuate relative to its average value.

Example 3. Consider a two-component QS of M/E<sub>2</sub>/1/2 type in each component.

Factors of the effectiveness of the functioning of such a QS include: service probabilities  $P_{S_1}$  and  $P_{S_2}$ , failure probabilities  $P_{fl_1}$ ,  $P_{fl_2}$  and  $P_{fl}^{(QS)}$ , the number of service channels  $\overline{k}_1$  and  $\overline{k}_2$ , the total number of customers in components  $\overline{N}^{(1)}$  and  $\overline{N}^{(2)}_q$ , the number of customers in component queues  $\overline{N}_q^{(1)}$  and  $\overline{N}_q^{(2)}$ , service time in components  $\overline{t}_1$  Ta  $\overline{t}_2$ , waiting time in component queues  $\overline{w}_1$  and  $\overline{w}_2$  and the customers service time duration in channels and waiting time in the queues of the QS.

### III. Results of experiments and discussion of the results

The values of these factors are determined according to the formulas given above.

Graphs of dependencies of some factors' impact on the QS functioning effectiveness are constructed using AnyLogic production process simulation software and presented in fig. 2 - 8 for one of the 15 series of computer experiments.



**Figure 2:** Dependence of probabilities  $P_{S_1}$  and  $P_{S_2}$  on service rate  $\mu''$  at  $\mu' = 0.05$ ; 0.1; 0.15



**Figure 3:** Dependence of number of service channels  $\overline{k}_1$  and  $\overline{k}_2$  on service rate  $\mu''$  at  $\mu' = 0.05$ ; 0.1; 0.15



**Figure 4:** Dependence of number of customers  $\overline{N}^{(1)}$  and  $\overline{N}^{(2)}$  in components on service rate  $\mu''$  at  $\mu' = 0.05$ ; 0.1; 0.15



**Figure 5:** Dependence of number of customers in queues by component  $\overline{N}_q^{(1)}$  and  $\overline{N}_q^{(2)}$  on service rate  $\mu''$  at  $\mu' = 0.05$ ; 0.1; 0.15



**Figure 6**: Dependence of waiting time in queues in components  $\overline{w}_1$  and  $\overline{w}_1$  on service rate  $\mu''$  at  $\mu' = 0.05$ ; 0.1; 0.15



**Figure 7:** Dependence of customer service time T and waiting time in QS queues  $\overline{W}$  on service rate  $\mu''$  at  $\mu' = 0.05$ ; 0.1; 0.15



**Figure 8:** Dependence of failure probabilities  $P_{fl_1}$  ma  $P_{fl_2}$  in components and in QS  $P_{fl_{QS}}$  on service rate  $\mu''$ at  $\mu' = 0.05$ ; 0.1; 0.15

The AnyLogic University Researcher development environment allows you to combine the principles of system dynamics with the paradigms of agent and discrete-event modeling. In addition, thanks to the built-in Java SE compiler, a library of ready-made solutions is available, including generators of random variables, which significantly expands the possibilities of developing and implementing experiments. In particular, experiments on optimization (relative to a defined criterion), model sensitivity, model stability, etc. are available [23], [24], [25], [26], [27].

From the graphs presented in fg. 2 – 8, it can be seen that with a time interval between the arriving customers of 8 hours ( $\lambda_1 = \lambda_2 = 0.125$ ), with an increase in  $\mu'$ , the values of  $P_{S_1}$  and  $P_{S_2}$  increase from 0.24 to 0.38 and from 0.19 to 0.37, respectively, but are at an insufficient level.

The average number of busy service channels of the first component  $\overline{k}_1$  decreases from 0.4 to 0.28, and the values of  $\overline{k}_2$  increase from 0.1 to 0.19, which indicates a weak system utilisation.

The total number of customers in the first component  $\overline{N}^{(1)}$  decreases from 2.1 to 1.48 due to small values of  $\mu'$ , and  $\overline{N}^{(2)}$ , on the contrary, increases from 0.48 to 1.6.

The average number of customers in the queue of the first component  $\overline{N}^{(1)}$  decreases from 1.4 to 0.75, and in the second component  $\overline{N}^{(2)}$  increases from 0.26 to 0.48.

The duration of waiting time  $\overline{w}_1$  in the queue of the first component decreases from 10.1 hours to 6 hours, and the value of  $\overline{w}_2$  increases from 2.2 hours to 4 hours.

The duration of service time  $\overline{t}_{qs}^s$  in the QS is reduced from 10 h to 8.5 h, as well as the duration of waiting time in queues  $\overline{w}_{qs}$  from 6.1 h to 3 h.

The probability of failure  $P_{fl}$  decreases from 0.34 to 0.19.

As we can see, the availability of the presented mathematical model contributes to the determination of measures aimed at increasing the operation effectiveness of the QS, which simulates the work of a vehicle maintenance and repair enterprise, that will ultimately allow to organize more efficient work of such an enterprise.

# Conclusions

The theoretical approach proposed by the authors is implemented on the example of the modeling of vehicle maintenance and repair processes by production divisions of a repair enterprise as a multi-component and multi-phase queuing system (QS) and allows to determine the effectiveness of the functioning of such a QS, identify "bottlenecks", unreliable components and obtain arguments for improving the efficiency of the enterprise in rapidly changing conditions.

The presented mathematical apparatus and used simulation modeling tools show their relevance to real processes and can be applied to improve the performance of not only vehicle repair enterprises, but also a greater variety of objects that can be described as QSs of various types, according to conditions of their functioning.

The conducted comprehensive research and its results make it possible to increase the reliability and efficiency of the functioning of a wide class of objects and systems, as they allow to evaluate the quantitative, qualitative and probabilistic characteristics of various technological processes and organizational measures as processes in mass service QSs, and therefore to minimize delays, failures in maintenance and related losses and optimize production, transport-logistics, military and other complex systems.

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