# REDUCTION IN WAITING TIME OF SINGLE SERVER MARKOVIAN QUEUING ENCOURAGED ARRIVAL MODEL

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**Abstract**

*There are several other methods for improving efficiency in a control chart. The use of a control chart alone is not advised. Other process improvement methods should always be used in addition to control charts. To trace the evolution of a process variable across time, use a control chart. The variable is applicable to all industries, including service, manufacturing, non-profit, and healthcare. It illustrates how a process variable changes over time and provides information on the kinds of variations that deal with ongoing improvement. Having a solid understanding of variation is necessary for effective control chart usage. Queuing models with constant or variable sizes are extensively used in the modeling of road and transport systems, sophisticated information and computer systems, and inventory replenishment systems. The control chart technique helps in tracking the performance of these queues, because of the single-server Markovian queue with encouraged arrival (SSMQEA model) the company which is running with fewer customers can increase the number of customers and hence the company finance level increase also this SSMQEA method will improve the points in share market. The major measurable performance characteristics of any queuing system are average queue length and average waiting time. Control limits are defined in this study for the* $M^{[X]}/M/1$ *encouraged arrival queuing model where the batch size follows a geometric distribution. To highlight its uses, numerical observations are also included. Little' s law is also satisfied.*

**Keywords:** Encouraged arrival, batch size, central limit, upper control limit, Lower control limit, Little's law

## 1. Introduction

A control chart has plenty of other techniques for increasing efficiency. A control chart should not be used in isolation. Along with control charts, other process improvement techniques should always be employed. A control chart is used to track the progression of a process variable across time. The variable may occur in any sort of business or organisation, encompassing service, manufacturing, non-profit and healthcare. It depicts the process variable over time and informs the sort of variation that are dealing with continuous improvement. Understanding variance is essential for efficiently using control charts.

In general, queueing models assume that customers arrive at the service facility individually. Unfortunately, this assumption is broken in many real-world queueing scenarios. People arriving at a post office, ships coming in a convoy at a port, people attending a wedding reception are all instances of queuing scenarios in which consumers come in groups [1].

The bulk arrival queuing model from a Bayesian point of view is considered in [2]. Statistical process control is a quality control technique that was first established to monitor manufacturing operations. Investigation on server abandonment dynamics in [3]. For quality control in industrial enterprises, [4] provides a number of Shewhart control chart solutions. For the M/M/S queuing model's random queue length, [5] built a control chart.

While [6] created Shewhart control charts for the G/G/S queuing system utilizing. The M/M/1 queuing model's random queue length was controlled using a control chart made using the stacked variance method [7].

The number of customers in a M/Ek/1 wait was examined by [8] using a control chart approach. M/M/1/N queuing systems with encouraged arrival were examined by [9 and 10]. The control diagram for the $M^x$/M/1 queuing system was explored in [11].

## 2. Model Recitation

Now we describe the single-server Markovian queue with encouraged arrival (SSMQEA model) as follows:

- The arrivals occur one at a time in line with the Poisson process with parameter $\lambda$ (1+$\eta$), where indicates the percentage change in the number of customers calculated from preceding or clear vision. For example, if a business previously gave discounts and the percentage change in number of consumers was 10% or 30%, then $\eta$ = 0.1 to $\eta$ = 0.3, respectively.
- Let $p_n$ be the probability that the system now contains n customers.
- Let $d_n$ represent the batch size distribution.

The steady-state equations that control this model are as follows:

$$0=-(\lambda(1+\eta)+\mu)p_n+\mu p_{n+1}+\lambda(1+\eta)\sum_{\varepsilon=1}^{n}p_{n-\varepsilon}d_\varepsilon \quad (n\geq 1),$$

$$0=-(\lambda(1+\eta)p_0+\mu p_1. \quad (1)$$

The generating function technique may be used to solve the system of equations (1).

Define the generating functions for the steady state probability and the batch size distribution as follows:

P(a)=$\sum_{n=0}^{\infty}p_n a^n, \quad |a|\leq 1,$

D(a)=$\sum_{n=0}^{\infty}d_n a^n, \quad |a|\leq 1,$

Equation (1) is obtained by summing and multiplying by the necessary powers of z.

$$0=-\lambda(1+\eta)\sum_{n=0}^{\infty}p_n a^n-\mu\sum_{n=1}^{\infty}p_n a^n+\frac{\mu}{a}\sum_{n=1}^{\infty}p_n a^n+\lambda(1+\eta)\sum_{n=1}^{\infty}\sum_{\varepsilon=0}^{\infty}p_{n-\varepsilon}d_\varepsilon a^n, \quad (2)$$

Contemplate$\sum_{n=1}^{\infty}\sum_{\varepsilon=0}^{\infty}p_{n-\varepsilon}d_\varepsilon a^n=\sum_{\varepsilon=1}^{\infty}d_\varepsilon a^\varepsilon\sum_{n=\varepsilon}^{\infty}p_{n-\varepsilon}a^{n-\varepsilon}=d(a)p(a).$ (3)

Equation (2) becomes Equation (3).

$$0=-\lambda(1+\eta)p(a)-\mu(p(a)-p_0)+\lambda(1+\eta)d(a)p(a).$$

Solving for p(a), we get

$$\text{P(a)}=\frac{\mu p_0(1-a)}{\mu(1-a)-\lambda(1+\eta)a(1-d(a))},|a|\leq 1. \quad (4)$$

The complimentary batch size possibilities' production function P(X>X)=1-$d_x=d_x^{\sim}$ is given by $d_x^{\sim}=\sum_{n-1}^{\infty}d_x^{\sim}a^n=\frac{1-d(a)}{1-a}$.

We took R=$\frac{\lambda(1+\eta)}{\mu}$, equation (4) yields

P(a)=$\frac{p_0}{1-Rad_x^{\sim}(a)}$.

Clearly, $d_x^{\sim}(1)=E(x) and\ d_x^{\sim\prime}(1)=\frac{E(x(x-1))}{2}$.

Making use of the normalising conditions, we got $p_0=1-\rho$, where $\rho=\frac{\lambda(1+\eta)}{\mu}E(x)$.

If $K_s$ and $K_q$ are the number of consumers in the system and the queue, respectively, then

$K_s = \frac{\frac{\lambda(1+\eta)}{\mu}E(x)+RE(x^2)}{2(1-\frac{\lambda(1+\eta)}{\mu}E(x))}$,

and $K_q = K_s - \frac{\lambda(1+\eta)}{\mu}E(x)$.

Assume that the number of consumers in any arriving batch is geometrically distributed with parameter $\beta$. The probability mass function of batch size is then calculated

$d_x = (1-\beta)\beta^{x-1}, 0 < \beta < 1$.

Then d(a)=$\frac{a(1-\beta)}{1-\beta a}$,

and

E(x)=$\frac{1}{1-\beta}$ $with\ \rho = \frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}$. (5)

From equation (1) to (5), we get

$$P(a)=\left( (1-\frac{\lambda(1+\eta)}{\mu}E(x))\sum_{n=0}^{\infty}\left(\beta+(1-\beta)\frac{\lambda(1+\eta)}{\mu}E(x)\right)^n - \sum_{n=0}^{\infty}\left(\beta+(1-\beta)\frac{\lambda(1+\eta)}{\mu}E(x)\right)^n z^{n+1}\right)$$

"a" on both sides when performance is compared to results in

$$p_n = \frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta})(1-\beta)(\beta+(1-\beta)\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta})^{n-1}, n > 0.$$

Thus, the value $A_0^{-1}$ is evaluated.

To obtained the equations mean and variance

Let $K_s$represent the total number of consumers in the system (both in queue and in service). The anticipated number of clients in the system is then

$$E(K_s) = \left(\frac{\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}}{(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta})(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta})}\right). \quad (6)$$

And the variance of the number of consumers in the system is,

$$Var(K_s) = \left(\frac{\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}(1+\beta\left(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}\right))}{(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta})^2*(1-\beta)^2}\right). \quad (7)$$

According to the concept that the number of consumers in the system maintains a normal distribution, the parameters of the control chart are given by

$$\text{Upper Control Limit=E}\left(\frac{\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}}{(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta})(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta})}\right) + 3\sqrt{\left(\frac{\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}(1+\beta\left(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}\right))}{(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta})^2*(1-\beta)^2}\right)}, \quad (8)$$

$$\text{Central Limit=E}\left(\frac{\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}}{(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta})(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta})}\right), \quad (9)$$

$$\text{Lower Contral Limit= E}\left(\frac{\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}}{(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta})(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta})}\right) - 3\sqrt{\left(\frac{\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}(1+\beta\left(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}\right))}{(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta})^2*(1-\beta)^2}\right)}. \quad (10)$$

Using (6) and (7) in (8), the control chart parameters for the $M^{[X]}$/M/1 queuing model are determined.

$$\text{Central Limit} = \left(\frac{\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}}{\left(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}\right)(1-\beta)}\right),$$

$$\text{Upper Central Limit} = \left(\frac{\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}+3\sqrt{\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}\left(1+\beta\left(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}\right)\right)}}{(1-\beta)\left(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}\right)}\right),$$

$$\text{Lower Central Limit} = \left(\frac{\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}-3\sqrt{\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}\left(1+\beta\left(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}\right)\right)}}{(1-\beta)\left(1-\frac{\frac{\lambda(1+\eta)}{\mu}}{1-\beta}\right)}\right).$$

The anticipated number of waiting units

i. $L_q = \frac{\rho\lambda(1+\eta)}{\mu-\lambda(1+\eta)}$.

The average number of occupied (serviced) units

ii. $L_s = \frac{\lambda(1+\eta)}{\mu-\lambda(1+\eta)}$.

The Service time estimate.

iii. $W_s = \frac{1}{\mu-\lambda(1+\eta)}$.

The expected Waiting time in line is

iv. $W_q = \frac{\rho}{\mu-\lambda(1+\eta)}$.

## 3. Numerical Illustration

The situation that a system encountered when an organization announced incentives and discounts gave origin to the expression "encouraged arrivals." The modern application of queuing theory to consumer behavior encouraged arrivals. The "essential component" that distinguishes control charts from a conventional line graph or run chart is control limits. Your data is used to calculate control limits. They are sometimes mistaken with specification restrictions offered by your customer. A UCL can be described as an acceptable range of values for particular parameters.

The performance of the queuing system is analyzed numerically with reference to the parameters, and LCL values are negative for the specified parameter values $\lambda(1+\eta)$, $\mu$ $and$ $\beta$ they are treated as zero and are not displayed as a distinct column in the table.$\eta$ represent discounts values 10% to 30% of the table and figure.

The table displays the parameters for the control chart and traffic intensity for the number of customers in the queuing system for various values of $\lambda(1+\eta)$, $\mu$ and $\beta$

**Table 1:** *We provide Encouraged arrival 10% discount $M^{[X]}/M/1$ Control chart in the queuing system*

| S.NO | $\lambda(1+\eta)$ | $\mu$ | $\beta$ | $\rho$ | CL | UCL |
|---|---|---|---|---|---|---|
| 1 | 4.4 | 10 | 0.15 | 0.5177 | 1.2630 | 6.6994 |
| 2 | 4.4 | 10 | 0.16 | 0.5238 | 1.3095 | 6.9399 |
| 3 | 4.4 | 10 | 0.17 | 0.5301 | 1.3592 | 7.1789 |
| 4 | 4.4 | 10 | 0.18 | 0.5366 | 1.4125 | 7.4329 |
| 5 | 4.4 | 15 | 0.15 | 0.3447 | 0.6188 | 3.9329 |
| 6 | 4.4 | 15 | 0.16 | 0.3488 | 0.6377 | 4.0411 |
| 7 | 4.4 | 15 | 0.17 | 0.353 | 0.6575 | 4.1549 |
| 8 | 4.4 | 15 | 0.18 | 0.3573 | 0.6779 | 4.2721 |
| 9 | 4.4 | 20 | 0.15 | 0.2588 | 0.4108 | 2.9647 |
| 10 | 4.4 | 20 | 0.16 | 0.2619 | 0.4224 | 3.0409 |
| 11 | 4.4 | 20 | 0.17 | 0.2651 | 0.4347 | 3.1206 |
| 12 | 4.4 | 20 | 0.18 | 0.2683 | 0.4472 | 3.2024 |
| 13 | 6.6 | 10 | 0.15 | 0.7765 | 4.0874 | 18.2359 |
| 14 | 6.6 | 10 | 0.16 | 0.7857 | 4.3647 | 19.3911 |
| 15 | 6.6 | 10 | 0.17 | 0.7951 | 4.6772 | 20.6823 |
| 16 | 6.6 | 10 | 0.18 | 0.8049 | 5.0313 | 22.1489 |
| 17 | 6.6 | 15 | 0.15 | 0.5177 | 1.2628 | 6.7521 |
| 18 | 6.6 | 15 | 0.16 | 0.5238 | 1.3095 | 6.9399 |
| 19 | 6.6 | 15 | 0.17 | 0.5301 | 1.3592 | 7.1793 |
| 20 | 6.6 | 15 | 0.18 | 0.5366 | 1.4125 | 7.4329 |
| 21 | 6.6 | 20 | 0.15 | 0.3882 | 0.7466 | 4.5029 |
| 22 | 6.6 | 20 | 0.16 | 0.3929 | 0.7705 | 4.6329 |
| 23 | 6.6 | 20 | 0.17 | 0.3976 | 0.7952 | 4.7676 |
| 24 | 6.6 | 20 | 0.18 | 0.4024 | 0.8213 | 4.9086 |

**Table 2:** *We provide encouraged arrival 10% discounts Little's Law verification table*

| S.NO | $\lambda(1+\eta)$ | $\mu$ | $\rho$ | $L_s$ | $W_s$ | $L_s = \lambda(1+\eta)W_s$ |
|---|---|---|---|---|---|---|
| 1 | 4.4 | 10 | 0.44 | 0.7857 | 0.1785 | 0.7857 |
| 2 | 4.4 | 15 | 0.29 | 0.415 | 0.0943 | 0.415 |
| 3 | 4.4 | 20 | 0.22 | 0.282 | 0.0641 | 0.282 |
| 4 | 6.6 | 10 | 0.66 | 1.9411 | 0.2941 | 1.9411 |
| 5 | 6.6 | 15 | 0.44 | 0.7857 | 0.119 | 0.7857 |
| 6 | 6.6 | 20 | 0.33 | 0.4925 | 0.0746 | 0.4925 |

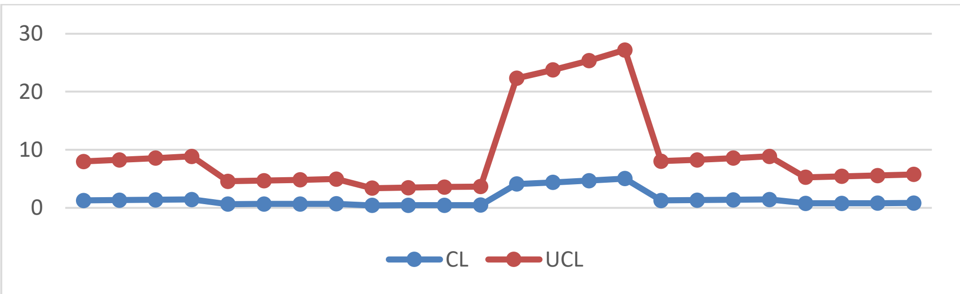


**Figure 1:** *We provide Pictorial representation encouraged arrival 10% discounts Control Chart.*

From the figure 1, with 10% discounts of encouraged arrival, the number of arrival is more the Poisson arrival Process [11].

**Table 3:** *We provide Encouraged arrival 20% discount $M^{[X]}/M/1$ Control chart in the queuing system.*

| $\lambda$ | $\mu$ | $\beta$ | $\eta$ |
|---|---|---|---|
| $4, 6$ | 10,15,20 | 0.15,0.16,0.17,0.18 | $0.2$ *or* $20\%$ |

| S.NO | $\lambda(1+\eta)$ | $\mu$ | $\beta$ | $\rho$ | CL | UCL |
|---|---|---|---|---|---|---|
| 1 | 4.8 | 10 | 0.15 | 0.5647 | 1.5262 | 7.8148 |
| 2 | 4.8 | 10 | 0.16 | 0.5714 | 1.5874 | 8.0995 |
| 3 | 4.8 | 10 | 0.17 | 0.5783 | 1.6522 | 8.3998 |
| 4 | 4.8 | 10 | 0.18 | 0.5854 | 1.7219 | 8.7209 |
| 5 | 4.8 | 15 | 0.15 | 0.3765 | 0.7104 | 4.3422 |
| 6 | 4.8 | 15 | 0.16 | 0.3809 | 0.7326 | 4.4656 |
| 7 | 4.8 | 15 | 0.17 | 0.3855 | 0.7559 | 4.594 |
| 8 | 4.8 | 15 | 0.18 | 0.3902 | 0.7804 | 4.7286 |
| 9 | 4.8 | 20 | 0.15 | 0.2824 | 0.4629 | 3.2137 |
| 10 | 4.8 | 20 | 0.16 | 0.2857 | 0.4762 | 3.2975 |
| 11 | 4.8 | 20 | 0.17 | 0.2892 | 0.4902 | 3.3855 |
| 12 | 4.8 | 20 | 0.18 | 0.2927 | 0.5047 | 3.4759 |
| 13 | 7.2 | 10 | 0.15 | 0.8471 | 6.5177 | 28.0118 |
| 14 | 7.2 | 10 | 0.16 | 0.8572 | 7.1463 | 30.5638 |
| 15 | 7.2 | 10 | 0.17 | 0.8675 | 7.8877 | 33.5764 |
| 16 | 7.2 | 10 | 0.18 | 0.878 | 8.7765 | 37.1868 |
| 17 | 7.2 | 15 | 0.15 | 0.5647 | 1.5263 | 7.8149 |
| 18 | 7.2 | 15 | 0.16 | 0.5714 | 1.5871 | 8.0982 |
| 19 | 7.2 | 15 | 0.17 | 0.5783 | 1.6522 | 8.4005 |
| 20 | 7.2 | 15 | 0.18 | 0.5854 | 1.7219 | 8.7209 |
| 21 | 7.2 | 20 | 0.15 | 0.4235 | 0.8642 | 5.0166 |
| 22 | 7.2 | 20 | 0.16 | 0.4286 | 0.8929 | 5.1677 |
| 23 | 7.2 | 20 | 0.17 | 0.4337 | 0.9227 | 5.3242 |
| 24 | 7.2 | 20 | 0.18 | 0.439 | 0.9544 | 5.4888 |

**Table 4:** *We provide encouraged arrival 20% discounts Little's law verification table*

| S.NO | $\lambda(1+\eta)$ | M | P | $L_s$ | $W_s$ | $L_s = \lambda(1+\eta)W_s$ |
|---|---|---|---|---|---|---|
| 1 | 4.8 | 10 | 0.48 | 0.923 | 0.1923 | 0.923 |
| 2 | 4.8 | 15 | 0.32 | 0.4705 | 0.098 | 0.4705 |
| 3 | 4.8 | 20 | 0.24 | 0.3157 | 0.0657 | 0.3157 |
| 4 | 7.2 | 10 | 0.72 | 2.5714 | 0.3571 | 2.5714 |
| 5 | 7.2 | 15 | 0.48 | 0.923 | 0.1282 | 0.923 |
| 6 | 7.2 | 20 | 0.36 | 0.5625 | 0.0781 | 0.5625 |

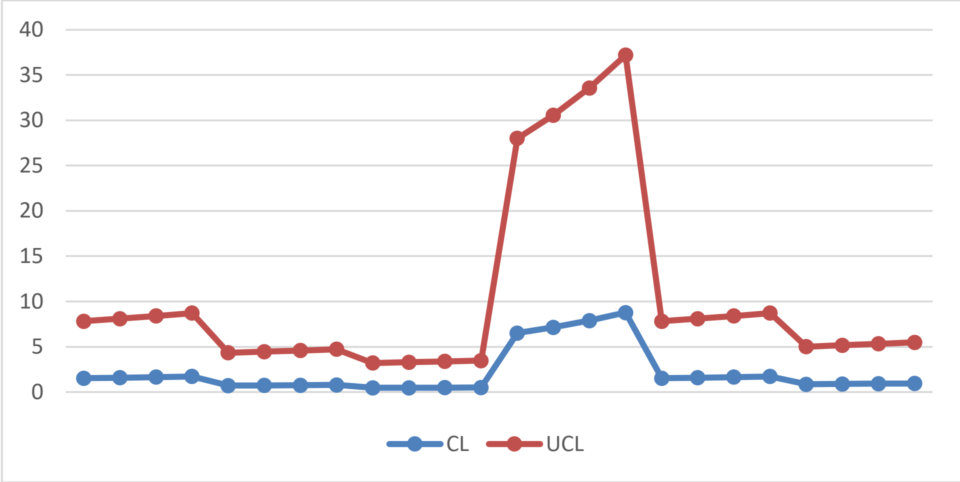


**Figure 2:** *We provide Pictorial representation encouraged arrival 20% discounts Control Chart.*

From the figure 2, with 20% discounts of encouraged arrival, the number of arrival is more the Poisson arrival Process [11].

**Table 5:** *We provide Encouraged arrival 30% discount* $M^{[X]}/M/1$ *Control chart in the queuing system*

| $\lambda$ | $\mu$ | $\beta$ | $\eta$ |
|---|---|---|---|
| $4,6$ | 10,15,20 | 0.15,0.16,0.17,0.18 | $0.3\ or\ 30\%$ |

| S.NO | $\lambda(1+\eta)$ | $\mu$ | $\beta$ | $\rho$ | CL | UCL |
|---|---|---|---|---|---|---|
| 1 | 5.2 | 10 | 0.15 | 0.6118 | 1.8541 | 9.1694 |
| 2 | 5.2 | 10 | 0.16 | 0.6191 | 1.9353 | 9.5331 |
| 3 | 5.2 | 10 | 0.17 | 0.6265 | 2.0209 | 9.9201 |
| 4 | 5.2 | 10 | 0.18 | 0.6289 | 2.0667 | 10.1416 |
| 5 | 5.2 | 15 | 0.15 | 0.4082 | 0.8115 | 4.7875 |
| 6 | 5.2 | 15 | 0.16 | 0.4131 | 0.8379 | 4.9283 |
| 7 | 5.2 | 15 | 0.17 | 0.4181 | 0.8656 | 5.0759 |
| 8 | 5.2 | 15 | 0.18 | 0.4232 | 0.8946 | 5.2298 |
| 9 | 5.2 | 20 | 0.15 | 0.3059 | 0.5185 | 3.4736 |
| 10 | 5.2 | 20 | 0.16 | 0.3095 | 0.5336 | 3.5752 |
| 11 | 5.2 | 20 | 0.17 | 0.3132 | 0.5495 | 3.6622 |
| 12 | 5.2 | 20 | 0.18 | 0.317 | 0.5662 | 3.7628 |
| 13 | 7.8 | 10 | 0.15 | 0.9177 | 13.1175 | 54.4512 |
| 14 | 7.8 | 10 | 0.16 | 0.9286 | 15.4818 | 63.9578 |
| 15 | 7.8 | 10 | 0.17 | 0.9397 | 18.7756 | 77.1462 |
| 16 | 7.8 | 10 | 0.18 | 0.9512 | 23.7756 | 97.2152 |
| 17 | 7.8 | 15 | 0.15 | 0.6118 | 1.8541 | 9.1694 |
| 18 | 7.8 | 15 | 0.16 | 0.6191 | 1.9353 | 9.5331 |
| 19 | 7.8 | 15 | 0.17 | 0.6265 | 2.0209 | 9.9201 |
| 20 | 7.8 | 15 | 0.18 | 0.6289 | 2.0667 | 10.1416 |
| 21 | 7.8 | 20 | 0.15 | 0.4588 | 0.9974 | 5.5907 |
| 22 | 7.8 | 20 | 0.16 | 0.4643 | 1.0318 | 5.7658 |
| 23 | 7.8 | 20 | 0.17 | 0.4699 | 1.0679 | 5.9479 |
| 24 | 7.8 | 20 | 0.18 | 0.4756 | 1.106 | 6.1394 |

**Table 6:** *We provide encouraged arrival 30% discounts Little's law verification table*

| S.NO | $\lambda(1+\eta)$ | $\mu$ | P | $L_s$ | $W_s$ | $L_s = \lambda(1+\eta)W_s$ |
|---|---|---|---|---|---|---|
| 1 | 5.2 | 10 | 0.52 | 10.833 | 0.2083 | 10.833 |
| 2 | 5.2 | 15 | 0.34 | 0.5306 | 0.102 | 0.5306 |
| 3 | 5.2 | 20 | 0.26 | 0.3513 | 0.0675 | 0.3513 |
| 4 | 7.8 | 10 | 0.78 | 3.545 | 0.4545 | 3.545 |
| 5 | 7.8 | 15 | 0.52 | 1.0833 | 0.1388 | 1.0833 |
| 6 | 7.8 | 20 | 0.39 | 0.6393 | 0.0819 | 0.6393 |

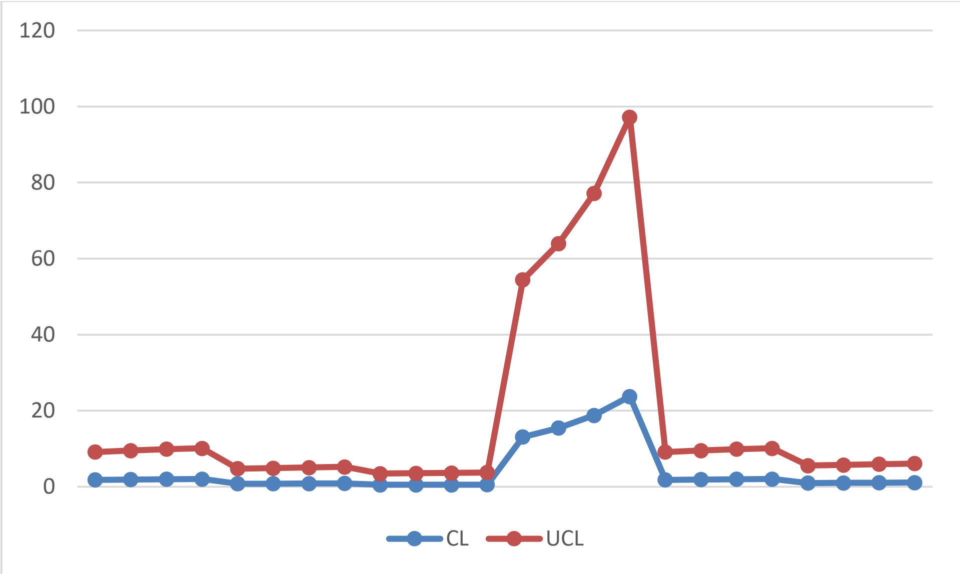


**Figure 3:** *We provide Pictorial representation encouraged arrival 30% discounts Control Chart.*

From the figure 3, with 30% discounts of encouraged arrival, the number of arrivals is more the Poisson arrival Process [11].We have identified that as the discounts rate increases the arrival rate also increases with SSMQEA model.

## 4. Result and Discussion

In this model, encouraged arrival, with a maximum 30% discount applicable in this model. Because the system size maximum has increased in this research model, we can expect a maximum profit when we apply this research concept to share markets. The average waiting time and the anticipated maximum waiting time both decrease with an increase in service rate and a constant encouraged arrival rate. The average wait time and anticipated maximum wait time are reduced with an increase in servers.

## 5. Conclusion

The model provided here has practical applications in systems such as manufacturing, telephony, share markets, and computer networks. From the figure 3, with 30% discounts of encouraged arrival, the number of arrivals is more the Poisson arrival Process [11]. Because the maximum system size in this research model has risen. In this SSMQEA model the company which are running with less customers can increase the number of customers and hence the company finance level increase also this SSMQEA method will improve the points in share market.

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