MOVING AVERAGE AND DOUBLE MOVING AVERAGE CONTROL CHARTS FOR PROCESS VARIABILITY USING AUXILIARY INFORMATION

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Abstract

The memory type control charts based on auxiliary information have been introduced in the literature for improved monitoring of the process parameters for normally distributed process. In this paper, we design moving average and double moving average control charts based on auxiliary information for efficient monitoring the shifts in the process variability. Regression estimator of process variance in the form of auxiliary and study variables is considered to construct charting statistics for the proposed charts. The average run length (ARL) and standard deviation of run length (SDRL) performance of the proposed charts is investigated using simulation study and is compared with the originally proposed Shewhart control charts based on auxiliary information and without auxiliary information. The proposed auxiliary information based moving average and double moving average charts are found to be efficient for monitoring the process variance of normally distributed process. An illustrative example based on simulated data set is provided to show the implementation of the proposed charts in detecting shifts in the process standard deviation.

Keywords: Control chart, average run length, auxiliary information, moving average, double moving average.

1. Introduction

Statistical process control (SPC) is a powerful statistical technique used to determine the performance of the process accurately. It has been widely used in manufacturing and service industries. A control chart is one of the most widely known tools in SPC which is extensively used to monitor process quality. It is designed to identify and detect timely assignable causes in the process. In general, control chart is used to detect changes in the process parameters. Two types of control charts are generally used to monitor production processes namely the location chart and the variability chart. The location chart is used to monitor process mean and the variability chart is used to monitor process variability. It is a standard practice to use Shewhart $\bar{X}$ chart for monitoring the process mean and R or S charts for monitoring the process variability. Some practioners recommend a control chart based directly on the sample variance S^2 control chart for monitoring process variability. A major disadvantage of Shewhart type control charts is that they use only information of last sample observation and ignores the past information of the process which makes it insensitive to small shifts in process parameters.

Memory type control charts are most commonly used in process monitoring to detect small to moderate shifts in the process parameters. They are constructed using past information regarding the production process and are more sensitive to monitor the small and moderate shifts in the process parameters. These control charts includes cumulative sum (CUSUM), exponentially weighted moving average (EWMA) and moving average (MA). Relative to CUSUM chart, the EWMA and MA charts are quite basic. The EWMA chart uses a weighted average as the chart statistic while the time weighted MA chart is based on simple moving average. The moving average statistics of width w is simply the average of the w most recent observations and are more sensitive to monitor the small and moderate shifts in the process parameters. Wong *et al*. [1] developed simple procedures for the design of an individual MA chart and a combined MA-Shewhart scheme. Khoo and Yap [2] proposed the use of single MA chart for joint monitoring of the process mean and variance by combining $\bar{X}$ and S charts into a single chart. Adeoti and Olaomi [3] proposed a moving average control chart based on sample standard deviation for detecting small shifts in process variability. Ghute and Rajmanya [4] developed moving average control chart based on Downton's D statistic and Gini's mean difference G statistic for detecting small shifts in the process standard deviation. The proposed moving average control charts are found to be more efficient for monitoring process variability. In multivariate setup, Ghute and Shirke [5] developed a multivariate moving average T^2 control chart for monitoring mean vector of multivariate process. It was shown that the proposed chart performs better than the Hotelling's T^2 chart in the detection of small to moderate shifts in the process mean vector. Ghute and Shirke [6] also developed a multivariate moving average $|S|$control chart for monitoring covariance matrix of multivariate normally distributed process. It was shown that proposed chart performs better than Shewhart-type $|S|$ chart in the detection of small to moderate changes in the process covariance matrix.

The double moving average (DMA) control charts have been proposed in the literature to further improve the performance of moving average (MA) control charts. Khoo and Wong [7] introduced the DMA chart by computing moving averages twice for early detection of small to moderate shifts in the process mean. It was shown that the DMA control chart performs better than the MA chart for the detection of small to moderate shifts in the mean. Adeoti et al. [8] proposed a DMA control chart based on sample standard deviation for detecting shifts in the process variability. Sukparungsee et al. [9] developed a mixed Tukey-Double moving average control chart for monitoring process mean of symmetric and asymmetric processes. They compared ARL performance of the proposed chart with the existing charts. It was shown that the proposed chart is an effective competitor to the existing counterparts. Taboran and Sukparungsee [10] designed a new EWMA-DMA control chart for detecting change of mean of the process with normal, Laplace, exponential and gamma distributions. They compared ARL performance of the proposed chart with other existing charts. It was shown that the proposed chart has best detection ability for certain level of shifts in process mean.

In order to increase the sensitivity of the traditional control charts many new modifications and improvements in the control charting procedure have been suggested in the SPC literature. One of such modifications is the development of auxiliary-information-based (AIB) control charts which have an excellent speed in detecting shift in the process parameters than those based without it. Such control charts are based on a statistic that utilizes information from both the study and auxiliary variables. The information on auxiliary variable is generally known prior to the sampling procedure and it assists in estimating the study variable with increased accuracy. There are many AIB control charts available in literature with Shewhart type and memory type charting structures for monitoring process mean and process variability. Riaz [11] first suggested AIB-

Shewhart that is based on regression-type mean estimator for monitoring shifts in process mean. It was shown that the AIB-Shewhart chart using the regression estimator performs better than the Shewhart chart for detecting shifts in the process mean. Riaz and Does [12] developed a Shewhart-type variability chart using ratio-type variance estimator for the Phase I quality control. It was shown that AIB Shewhart variability chart is more powerful than the existing Shewhart variance chart. Riaz [13] proposed a Shewhart-type control chart for an improved monitoring of mean level of quality characteristic of interest Y using the information on a single auxiliary characteristic X on product difference pattern. Riaz et al. [14] suggested new AIB-Shewhart chart based on regression estimator for monitoring the process variability. They have shown that the Shewhart chart using regression estimator outperforms the other Shewhart charts when detecting increase in the process variability. Abbas et al. [15] introduced the EWMA chart with the auxiliary information, using regression estimator for monitoring location of the process. It was shown that proposed chart performs better than its existing control charts. Abbasi and Riaz [16] made dual use of auxiliary information to propose new chart for process location. Riaz et al. [17] made dual use of auxiliary information to propose a chart for process variability. Sanusi et al. [18] studied CUSUM charts using different estimators based on auxiliary information. Haq [19] proposed new EWMA charts using auxiliary information for efficiently monitoring the process dispersion. Recently, Amir et al. [20] developed auxiliary information based moving average control chart (denoted as AB-MA chart) for effective monitoring of shifts in the process location parameter. They compared the performance of proposed chart with existing control chart and shown that chart outperforms in detecting small and medium shifts in the process location parameter. Amir et al. [21] designed auxiliary information based double moving average control chart (denoted as ADMA chart) for effective monitoring of the process location parameter. They compared the performance of the proposed ADMA chart with its memory-type counterparts and shown that the proposed ADMA chart performs uniformly better than the EWMA and CUSUM charts when correlation between auxiliary variable and study variable is high.

Most of the studies on AIB memory-type charts have been concentrated on monitoring process mean. Often, monitoring shifts in the variance of related study variable is also important. The purpose of this paper is to develop auxiliary information based MA and DMA control charts for efficient monitoring of process variability of normally distributed process in Phase II. By getting motivation of improved performance of auxiliary information based MA and DMA charts for monitoring process mean recently proposed by Amir et al. [20] and Amir et al. [21] respectively, in the present paper, we develop new MA and DMA charts using auxiliary information for efficiently monitoring the process variability in phase II. We expect that the proposed MA and DMA control charts will be more sensitive for efficiently monitoring process variability. The regression estimator of the process variance in the form of auxiliary and study variables is considered to construct charting statistics for the proposed MA and DMA charts. The performance of the proposed charts is evaluated in terms of the average run length (ARL) and standard deviation of run length (SDRL) criteria. Monte Carlo simulations are used to study the run length profiles of the proposed AIB-MA-V and AIB-DMA-V charts. The performance of the proposed charts is compared with its Shewhart-type counterparts.

Rest of the paper is organized as follows: In Section 2, traditional S^2 chart for monitoring process variability is discussed. The auxiliary information based Shewhart-type control chart for monitoring process variability is presented in Section 3. The design procedure of proposed auxiliary information based moving average and double moving average control charts are presented in Section 4 and Section 5 respectively. In Section 6, a simulation study is conducted to evaluate the performance of proposed control charts in comparison to that of Shewhart-type charts. An illustrative example is presented in Section 7 to demonstrate the implementation of the proposed charts. Finally, Section 8 concludes the findings of the paper.

2. Shewhart control chart for process variability.

In this Section, we discuss the traditional S^2control chart for monitoring process variability that has been constructed without using the auxiliary information. Let $Y_i = (Y_{i1}, Y_{i2}, \dots Y_{in})\ i - 1,2, \dots.$ be independent random samples of size n $(n \geq 2)$ from a normally distributed process with mean μ_y and variance σ_y^2. Here our objective is to monitor the changes in the process variance. We assume that $\mu_y = \mu_{y,0}$ and $\sigma_y = \sigma_{y,0}$ are known, even if, in practice, these parameters have to be estimated from an in-control population. It is assumed that the process is in-control with variance $\sigma_{y,0}^2$. When shift in process variance $\sigma_{y,0}^2$ occurs, we have change from in-control value $\sigma_{y,0}^2$ to the out-of-control value$\sigma_{y,1}^2$. Let $\lambda = \sigma_{y,1}/\sigma_{y,0} (0 < \lambda \leq 1)$ denotes the amount of shift in the in-control process standard deviation$\sigma_{y,1}$. When$\lambda = 1$, the process is considered to be in-control, otherwise the process is considered to be out-of-control. Let $\bar{Y}_i = \frac{1}{n}\sum_{j=1}^{n} Y_{ij}$ the sample mean at stage i. The S^2 chart can be constructed by plotting sample variances

$$S_i^2 = \frac{1}{n-1}\sum_{j=1}^{n}\left(Y_{ij} - \bar{Y}\right)^2, i = 1,2, \dots. \tag{1}$$

With lower and upper control limits set as

$$LCL = \frac{\sigma^2}{n-1}\chi_{n-1,1-\alpha/2}^2, UCL = \frac{\sigma^2}{n-1}\chi_{n-1,\alpha/2}^2 \tag{2}$$

Where $\chi_{\alpha/2,}^2\ \chi_{1-\alpha/2,}^2$ denotes the upper and lower $\alpha/2$percentage points of the Chi-square distribution with $n - 1$degrees of freedom. The S^2chart for monitoring process variability gives a signal if S_i^2exceeds the control limits.

3. AIB Shewhart-type V Chart for Process variability

In this Section, we discuss AIB Shewhart-type control chart using regression estimator of the variance for monitoring shifts in process variability. Assume that a process has a quality characteristic of interest Y which is correlated with an auxiliary variable X. The pairs $(Y_i,\ X_i)$are assumed to follow bivariate normal distribution with $N_2(\mu_y, \mu_x, \sigma_y^2, \sigma_x^2, \rho)$. Here ρ represents the correlation coefficient between study variable Y and auxiliary variable X. The observations of Y and X are obtained in the paired form for each sample and the population parameters are assumed to be known.

Let $(Y_1, X_1), (Y_2, X_2), \dots. (Y_n, X_n)$ represents a sample of size n from the bivariate normal distribution. The auxiliary information based unbiased regression estimator of population variance σ_y^2 of study variable Y using a single auxiliary variable X for a bivariate random sample of size n is given by (Haq,(2017))

$$V = S_y^2 + \rho^2\frac{\sigma_y^2}{\sigma_x^2}(\sigma_x^2 - S_x^2) \tag{3}$$

Where $S_y^2 = \frac{1}{n-1}\sum_{j=1}^{n}(Y_i - \bar{Y})^2$, $S_x^2 = \frac{1}{n-1}\sum_{j=1}^{n}(X_i - \bar{X})^2$ represent sample variances of Y and X respectively and $\bar{Y} = \frac{1}{n}\sum_{j=1}^{n} Y_i$, $\bar{X} = \frac{1}{n}\sum_{j=1}^{n} X_i$ represent sample means of Y and X respectively.
The mean and variance of V are as follows:

$$E(V) = \sigma_y^2 \text{ and } Var(V) = \sigma_V^2 = \frac{2\sigma_y^4}{n-1}(1 - \rho^4)$$

Riaz et al. (2014) suggested AIB-Shewhart chart based on regression estimator V of process variance σ_y^2 for monitoring process variability. We denote the chart as AIB-Shewhart-V chart. In the construction of AIB-Shewhart-V control chart for monitoring the process variability, the statistic V is plotted on the chart against the sample number. The control limits of the AIB-Shewhart-V chart are

$$LCL = \sigma_y^2 - L\sigma_y^2\sqrt{\frac{2(1-\rho^4)}{n-1}} \text{ and } UCL = \sigma_y^2 + L\sigma_y^2\sqrt{\frac{2(1-\rho^4)}{n-1}} \quad (4)$$

Where the value of L determines the in-control ARL of the AIB-Shewhart-V chart.

4. AIB Moving Average Control Chart for Process Variability

In this Section, we develop moving average control chart for detecting changes produced in the process variability. The proposed moving average chart is based on auxiliary information based V statistic and chart is denoted as AIB-MA-V chart. The construction of the chart is based on computing the moving averages of V statistics given in Eq. (3). The moving average statistic of span w at time i for a sequence of V statistics are computed as

$$MA_i = \frac{V_i + V_{i-1} + \cdots + V_{i-w+1}}{w}, for\ i \geq w \quad (5)$$

For periods $i < w$ we compute the average of available charting statistic. In other words, average of all V observations up to period i defines moving average. For$i > w$, mean and variance of MA_istatistic for in-control process are given as

$$E(MA_i) = \sigma_y^2 \text{ and } Var(MA_i) = \frac{2\sigma_y^4}{w\,(n-1)}(1-\rho^4)$$

The control limits of MA-V chart are as follows:

$$UCL/LCL = \begin{cases} \sigma_y^2 \pm L\sigma_y^2\sqrt{\frac{2(1-\rho^4)}{w(n-1)}}, & for\ i \geq w \\ \sigma_y^2 \pm L\sigma_y^2\sqrt{\frac{2(1-\rho^4)}{i(n-1)}}, & for\ i < w \end{cases} \quad (6)$$

Where L is a constant chosen to specify in-control ARL for the AIB-MA-V chart. The AIB-MA-V chart is constructed by plotting the MA_i statistics on the chart against the sample number i. An out-of-control signal is issued when $MA_i$is smaller than LCL or larger than the UCL.

5. AIB Double Moving Average Control Chart for Process Variability

In this Section, we present the design of double moving average control chart for detecting changes produced in the process variability. The proposed chart is denoted as AIB-DMA-V chart. The double moving average (DMA) statistic is based on the twice the subgroup average of the MA statistic. The moving average statistic for sequence of subgroup variances with time i and width w is given in Eq. (5). For interval $i < w$ the DMA statistic can be calculated as the mean of all subgroup variances up to interval i while for interval $i \geq w$, the plotting statistic of AIB-DMA-V chart is given by

$$DMA_i = \frac{MA_i + MA_{i-1} + \cdots + MA_{i-w+1}}{w}, for\ i \geq w \quad (7)$$

where w represents the span at time i for computing DMA_istatistic. The in-control mean of DMA_istatistic calculated for $i \geq w$ is given by,

$$E(DMA_i) = \frac{1}{w}E\left(\sum_{j=i-w+1}^{i} MA_j\right) = \frac{1}{w}\left(w\sigma_y^2\right) = \sigma_y^2$$

The in-control variance of DMA_i is given by

$$Var(DMA_i) = \begin{cases} \frac{\sigma_y^4 2(1-\rho^4)}{i^2(n-1)} \sum_{j=1}^{i} \frac{1}{j} & , i \leq w \\ \frac{\sigma_y^4 2(1-\rho^4)}{w^2(n-1)} \sum_{j=i-w+1}^{w-1} \frac{1}{j} + (i-w+1)\frac{1}{w} & , w < i < 2w-1 \\ \frac{\sigma_y^4 2(1-\rho^4)}{w^2(n-1)}, & i \geq 2w-1 \end{cases} \tag{8}$$

For the $w = 2$, the variance of the DMA is calculated using the 1st and 3rd lines of the above Eq. (8). The control limits of the proposed AIB-DMA-V chart are given as

$$UCL/LCL = \begin{cases} \sigma_y^2 \pm \frac{L\,\sigma_y^2}{i} \sqrt{\frac{2(1-\rho^4)}{n-1} \sum_{j=1}^{i} \frac{1}{j}} & , i \leq w \\ \sigma_y^2 \pm \frac{L\,\sigma_y^2}{w} \sqrt{\frac{2(1-\rho^4)}{n-1} \sum_{j=i-w+1}^{w-1} \frac{1}{j} + (i-w+1)\frac{1}{w}} & , w < i < 2w-1 \\ \sigma_y^2 \pm \frac{L\,\sigma_y^2}{w} \sqrt{\frac{2(1-\rho^4)}{n-1}} & , i \geq 2w-1 \end{cases} \tag{9}$$

The control limits for $w = 2$ are calculated based on the using the 1st and 3rd lines of the above Eq. (9). L is the constant that is set according to the desired in-control ARL for the AIB-DMA-V control chart.

6. Performance Evaluation and Comparison

Performance of a control chart is typically measured in terms of average run length (ARL) and standard deviation of run length ($SDRL$). The ARL is the average number of sample points that is plotted on a chart before the first out-of-control signal is detected, whereas, the $SDRL$ measures the spread of the run length distribution. When the process is out-of-control, it is desirable to have small values of ARL and $SDRL$. The performance of the control chart is measured in terms of in-control ARL (denoted asARL_0) and out-of-control ARL (denoted asARL_1). To compare the efficiency of two control charts for detecting the shift in process parameter, the general practice is to adjust their control limits so that their ARL_0values become same and then compare ARL_1values at various shifts in process parameter. A chart with smaller ARL_1 is considered to be more efficient to detect pre-assigned shift in the process parameter.

In this Section, we evaluate the performance of the proposed AIB-MA-V and AIB-DMA-V charts for detecting shifts in Phase II monitoring. Monte Carlo simulation approach is used to evaluate the run length performance of the proposed charts in comparison with that of the AIB-Shewhart-V and S^2charts. Simulation study based on sample of size $n = 10,15,20$ with $\rho = 0.3,0.6,0.9$ is carried out to assess the performance of the proposed AIB-MA-V, AIB-DMA-V and AIB-Shewhart-V charts. It is assumed that the in-control process is a bivariate normal distribution. Without loss of generality a standard bivariate normal distribution (i.e.$N_2(0,0,1,1,\rho)$ is considered as in-control process distribution. The out-of-control process is a bivariate normal with the same means of both auxiliary and study variables with changed variance of study variable$Y$. That is, we consider out-of-control procedure as$N_2(0,0,\lambda,1,\rho)$. Using the simulation, the control limit constant L of the AIB-MA-V and AIB-DMA-V charts are obtained for $w = 2,3$ and 4 so thatARL_0 of the chart is approximately 200. The ARL and $SDRL$ values of the chart are found with 50000 simulations for each shift of magnitude λ in the process variance of the study variable Y. The magnitude of shift in the standard deviation of the study variable is considered as$\lambda = 1.0(0.1)2.0,2.5,3.0$. To compare the performance of the proposed AIB-MA-V and AIB-DMA-V charts with AIB-Shewhart-V chart and Shewhart S^2chart, each chart is designed so that ARL_0 is approximately 200. The control limits, ARL and $SDRL$ values of the AIB-Shewhart-V chart are obtained using simulation. The exact ARL values of the traditional S^2chart are computed using Excel.

Tables 1 to 3 represent the ARL and $SDRL$ (shown in parenthesis) values of the proposed control chart with $n = 10,15,20$ according to the variance shift when the correlation coefficient is

0.3, 0.6 and 0.9 respectively. Here control limit constant L is chosen so that the value of ARL_0 is close to 200.

Table 1: *Run length profiles of the charts with* $n = 10$ *and* $ARL_0 = 200$

Shift λ	S^2chart	AIB-V chart	AIB-MA-V chart $w = 2$	$w = 3$	$w = 4$	AIB-DMA-V chart $w = 2$	$w = 3$	$w = 4$
				$\rho = 0.3$				
1.0	200.65	199.95	199.03	200.54	200.93	199.90	200.73	200.38
	(197.87)	(197.86)	(200.71)	(200.13)	(200.79)	(199.54)	(200.53)	(198.33)
1.1	72.98	46.54	36.11	31.68	29.23	34.65	30.36	29.01
	(72.48)	(45.94)	(35.64)	(30.92)	(28.56)	(33.80)	(29.01)	(26.72)
1.2	25.23	16.68	12.11	10.38	9.46	11.69	10.40	10.25
	(24.18)	(16.25)	(11.42)	(9.56)	(8.65)	(10.68)	(8.76)	(8.01)
1.3	11.00	8.03	5.89	5.18	4.83	5.89	5.55	5.80
	(10.40)	(7.48)	(5.22)	(4.46)	(4.04)	(4.89)	(4.04)	(3.76)
1.4	6.09	4.66	3.62	3.29	3.12	3.75	3.76	4.03
	(5.52)	(4.11)	(2.99)	(2.57)	(2.42)	(2.76)	(2.43)	(2.37)
1.5	3.92	3.17	2.58	2.40	2.28	2.76	2.87	3.12
	(3.34)	(2.61)	(1.93)	(1.72)	(1.61)	(1.84)	(1.72)	(1.82)
1.6	2.83	2.38	2.02	1.91	1.85	2.22	2.34	2.52
	(2.26)	(1.82)	(1.37)	(1.23)	(1.19)	(1.36)	(1.35)	(1.48)
1.7	2.20	1.92	1.70	1.62	1.58	1.87	1.99	2.12
	(1.66)	(1.33)	(1.05)	(0.95)	(0.91)	(1.07)	(1.12)	(1.23)
1.8	1.81	1.63	1.49	1.44	1.41	1.64	1.73	1.83
	(1.22)	(1.02)	(0.82)	(0.75)	(0.72)	(0.88)	(0.95)	(1.04)
1.9	1.57	1.46	1.36	1.32	1.29	1.48	1.55	1.62
	(0.96)	(0.82)	(0.67)	(0.62)	(0.59)	(0.74)	(0.81)	(0.88)
2.0	1.42	1.33	1.26	1.23	1.21	1.35	1.40	1.46
	(0.77)	(0.67)	(0.55)	(0.51)	(0.49)	(0.63)	(0.69)	(0.74)
2.5	1.10	1.06	1.06	1.05	1.05	1.09	1.11	1.13
	(0.33)	(0.24)	(0.26)	(0.24)	(0.23)	(0.31)	(0.34)	(0.37)
3.0	1.03	1.33	1.02	1.02	1.01	1.03	1.03	1.04
	(0.17)	(0.66)	(0.13)	(0.13)	(0.12)	(0.17)	(0.18)	(0.20)
L	---	3.431	3.090	2.909	2.789	3.742	4.045	4.342
				$\rho = 0.6$				
1.0	200.65	200.97	200.80	199.06	200.21	200.76	199.56	200.79
	(197.87)	(201.34)	(201.53)	(200.18)	(201.40)	(200.75)	(198.82)	(198.20)
1.1	72.98	42.69	33.12	28.76	26.55	31.50	27.92	26.79
	(72.48)	(42.50)	(32.96)	(28.04)	(25.97)	(30.78)	(26.32)	(24.47)
1.2	25.23	14.79	10.66	9.29	8.55	10.37	9.35	9.21
	(24.18)	(14.34)	(9.95)	(8.54)	(7.63)	(9.36)	(7.75)	(6.93)
1.3	11.00	6.94	5.22	4.59	4.36	5.26	5.08	5.33
	(10.40)	(6.43)	(4.52)	(3.84)	(3.62)	(4.23)	(3.60)	(3.34)
1.4	6.09	4.14	3.26	2.98	2.83	3.40	3.45	3.77
	(5.52)	(3.59)	(2.61)	(2.30)	(2.12)	(2.42)	(2.15)	(2.18)

Table 1 continued

Shift λ	S^2chart	AIB-V chart	AIB-MA-V chart $w = 2$	$w = 3$	$w = 4$	AIB-DMA-V chart $w = 2$	$w = 3$	$w = 4$
				$\rho = 0.6$				
1.5	3.92	2.83	2.34	2.19	2.11	2.52	2.68	2.92
	(3.34)	(2.29)	(1.68)	(1.52)	(1.42)	(1.62)	(1.55)	(1.69)
1.6	2.83	2.14	1.87	1.78	1.72	2.05	2.19	2.36
	(2.26)	(1.56)	(1.22)	(1.11)	(1.06)	(1.22)	(1.25)	(1.38)
1.7	2.20	1.76	1.58	1.52	1.49	1.74	1.86	1.97
	(1.66)	(1.16)	(0.92)	(0.84)	(0.81)	(0.96)	(1.03)	(1.13)
1.8	1.81	1.52	1.40	1.37	1.34	1.54	1.63	1.72
	(1.22)	(0.89)	(0.72)	(0.67)	(0.64)	(0.80)	(0.88)	(0.96)
1.9	1.57	1.37	1.29	1.26	1.24	1.39	1.47	1.53
	(0.96)	(0.72)	(0.59)	(0.56)	(0.53)	(0.66)	(0.74)	(0.80)
2.0	1.42	1.27	1.21	1.19	1.18	1.29	1.35	1.40
	(0.77)	(0.58)	(0.49)	(0.46)	(0.45)	(0.57)	(0.63)	(0.68)
2.5	1.10	1.05	1.05	1.04	1.04	1.07	1.09	1.10
	(0.33)	(0.22)	(0.23)	(0.21)	(0.20)	(0.28)	(0.30)	(0.33)
3.0	1.03	1.26	1.01	1.01	1.01	1.02	1.02	1.03
	(0.17)	(0.57)	(0.12)	(0.11)	(0.10)	(0.14)	(0.16)	(0.17)
L	---	3.360	3.045	2.877	2.774	3.680	4.015	4.332
				$\rho = 0.9$				
1.0	200.65	200.46	200.25	200.41	199.09	199.40	199.88	199.83
	(197.87)	(199.21)	(198.55)	(199.40)	(200.01)	(198.33)	(199.47)	(198.62)
1.1	72.98	24.31	17.98	15.48	14.28	17.01	14.96	14.38
	(72.48)	(23.72)	(17.24)	(14.62)	(13.42)	(16.04)	(13.29)	(11.85)
1.2	25.23	6.72	5.02	4.51	4.20	5.00	4.86	5.16
	(24.18)	(6.19)	(4.30)	(3.68)	(3.33)	(3.91)	(3.26)	(3.08)
1.3	11.00	3.17	2.60	2.42	2.33	2.76	2.93	3.21
	(10.40)	(2.62)	(1.92)	(1.69)	(1.59)	(1.78)	(1.66)	(1.75)
1.4	6.09	2.04	1.78	1.72	1.68	1.97	2.14	2.32
	(5.52)	(1.47)	(1.11)	(1.02)	(0.97)	(1.11)	(1.15)	(1.28)
1.5	3.92	1.56	1.44	1.41	1.39	1.59	1.71	1.83
	(3.34)	(0.94)	(0.75)	(0.71)	(0.68)	(0.82)	(0.89)	(0.98)
1.6	2.83	1.33	1.26	1.24	1.23	1.37	1.46	1.53
	(2.26)	(0.66)	(0.55)	(0.52)	(0.51)	(0.63)	(0.70)	(0.76)
1.7	2.20	1.20	1.16	1.15	1.14	1.24	1.29	1.35
	(1.66)	(0.48)	(0.42)	(0.40)	(0.39)	(0.50)	(0.56)	(0.61)
1.8	1.81	1.13	1.11	1.09	1.09	1.16	1.20	1.23
	(1.22)	(0.38)	(0.33)	(0.31)	(0.31)	(0.40)	(0.46)	(0.49)
1.9	1.57	1.08	1.07	1.06	1.06	1.10	1.13	1.16
	(0.96)	(0.30)	(0.26)	(0.25)	(0.25)	(0.33)	(0.37)	(0.41)
2.0	1.42	1.05	1.04	1.04	1.04	1.07	1.09	1.11
	(0.77)	(0.24)	(0.21)	(0.21)	(0.20)	(0.27)	(0.30)	(0.33)
2.5	1.10	1.01	1.01	1.01	1.01	1.01	1.02	1.02
	(0.33)	(0.10)	(0.08)	(0.08)	(0.07)	(0.11)	(0.12)	(0.14)
3.0	1.03	1.05	1.00	1.00	1.00	1.00	1.00	1.00
	(0.17)	(0.24)	(0.04)	(0.04)	(0.04)	(0.05)	(0.06)	(0.06)
L	---	3.176	2.952	2.842	2.763	3.579	3.984	4.333

Table 2: *Run length profiles of the charts with $n = 15$ and $ARL_0 = 200$*

Shift λ	S^2chart	AIB-V chart	AIB-MA-V chart $w = 2$	$w = 3$	$w = 4$	AIB-DMA-V chart $w = 2$	$w = 3$	$w = 4$
				$\rho = 0.3$				
1.0	200.49	199.97	199.21	199.29	199.17	200.77	199.40	200.93
	(201.15)	(200.70)	(200.38)	(198.87)	(199.22)	(199.73)	(198.08)	(199.18)
1.1	59.44	37.14	27.73	23.65	21.79	25.84	22.80	21.72
	(58.56)	(36.65)	(27.04)	(22.94)	(20.94)	(24.94)	(21.19)	(19.33)
1.2	17.34	11.87	8.41	7.20	6.64	8.03	7.33	7.48
	(16.74)	(11.39)	(7.75)	(6.38)	(5.71)	(6.92)	(5.71)	(5.20)
1.3	7.29	5.39	4.02	3.60	3.41	4.10	4.06	4.38
	(6.76)	(4.82)	(3.32)	(2.84)	(2.63)	(3.08)	(2.56)	(2.51)
1.4	3.95	3.16	2.52	2.34	2.24	2.69	2.83	3.10
	(3.38)	(2.63)	(1.85)	(1.64)	(1.52)	(1.71)	(1.60)	(1.72)
1.5	2.59	2.19	1.87	1.77	1.73	2.04	2.20	2.40
	(2.02)	(1.62)	(1.19)	(1.07)	(1.02)	(1.17)	(1.21)	(1.33)
1.6	1.93	1.69	1.52	1.46	1.44	1.68	1.80	1.94
	(1.34)	(1.08)	(0.84)	(0.77)	(0.74)	(0.88)	(0.96)	(1.05)
1.7	1.56	1.43	1.33	1.29	1.28	1.45	1.54	1.63
	(0.93)	(0.78)	(0.63)	(0.58)	(0.56)	(0.70)	(0.78)	(0.84)
1.8	1.35	1.27	1.21	1.19	1.18	1.30	1.37	1.43
	(0.70)	(0.58)	(0.48)	(0.45)	(0.44)	(0.57)	(0.63)	(0.68)
1.9	1.23	1.17	1.14	1.12	1.11	1.20	1.25	1.29
	(0.52)	(0.45)	(0.38)	(0.36)	(0.34)	(0.47)	(0.51)	(0.55)
2.0	1.15	1.11	1.09	1.08	1.07	1.14	1.17	1.20
	(0.41)	(0.35)	(0.30)	(0.28)	(0.27)	(0.38)	(0.42)	(0.46)
2.5	1.02	1.01	1.01	1.01	1.01	1.02	1.03	1.03
	(0.14)	(0.12)	(0.10)	(0.10)	(0.09)	(0.13)	(0.16)	(0.18)
3.0	1.00	1.11	1.00	1.00	1.00	1.00	1.00	1.00
	(0.05)	(0.35)	(0.04)	(0.04)	(0.03)	(0.06)	(0.06)	(0.07)
L	---	3.268	2.991	2.837	2.750	3.608	3.964	4.299
				$\rho = 0.6$				
1.0	200.49	200.57	200.93	200.57	200.71	199.40	199.46	199.77
	(201.15)	(198.84)	(199.00)	(198.47)	(199.68)	(198.47)	(198.53)	(198.65)
1.1	59.44	33.38	24.94	21.55	19.84	23.27	20.92	19.95
	(58.56)	(33.15)	(24.29)	(20.67)	(18.89)	(22.06)	(19.16)	(17.47)
1.2	17.34	10.23	7.41	6.44	5.97	7.13	6.66	6.84
	(16.74)	(9.67)	(6.74)	(5.60)	(5.08)	(6.03)	(5.03)	(4.57)
1.3	7.29	4.69	3.61	3.25	3.10	3.68	3.74	4.05
	(6.76)	(4.16)	(2.95)	(2.49)	(2.32)	(2.64)	(2.28)	(2.25)
1.4	3.95	2.77	2.28	2.13	2.08	2.46	2.65	2.90
	(3.38)	(2.22)	(1.61)	(1.43)	(1.36)	(1.52)	(1.46)	(1.59)
1.5	2.59	1.96	1.71	1.64	1.61	1.89	2.06	2.23
	(2.02)	(1.37)	(1.03)	(0.95)	(0.92)	(1.05)	(1.10)	(1.23)
1.6	1.93	1.55	1.42	1.39	1.36	1.57	1.70	1.81
	(1.34)	(0.92)	(0.73)	(0.68)	(0.65)	(0.80)	(0.88)	(0.97)
1.7	1.56	1.34	1.26	1.24	1.23	1.37	1.46	1.53
	(0.93)	(0.67)	(0.54)	(0.52)	(0.50)	(0.63)	(0.70)	(0.76)

Table 2 continued

Shift λ	S^2chart	AIB-V chart	AIB-MA-V chart			AIB-DMA-V chart		
			$w = 2$	$w = 3$	$w = 4$	$w = 2$	$w = 3$	$w = 4$
				$\rho = 0.6$				
1.8	1.35	1.21	1.16	1.15	1.14	1.25	1.30	1.36
	(0.70)	(0.50)	(0.42)	(0.40)	(0.39)	(0.51)	(0.57)	(0.62)
1.9	1.23	1.13	1.11	1.09	1.09	1.16	1.20	1.24
	(0.52)	(0.38)	(0.33)	(0.31)	(0.31)	(0.41)	(0.46)	(0.50)
2.0	1.15	1.08	1.06	1.06	1.06	1.10	1.13	1.16
	(0.41)	(0.30)	(0.25)	(0.25)	(0.24)	(0.33)	(0.37)	(0.41)
2.5	1.02	1.01	1.01	1.01	1.01	1.01	1.02	1.02
	(0.14)	(0.10)	(0.09)	(0.08)	(0.08)	(0.11)	(0.13)	(0.15)
3.0	1.00	1.08	1.00	1.00	1.00	1.00	1.00	1.00
	(0.05)	(0.30)	(0.03)	(0.03)	(0.03)	(0.04)	(0.05)	(0.06)
L	---	3.206	2.958	2.826	2.746	3.563	3.950	4.289
				$\rho = 0.9$				
1.0	200.49	199.36	199.00	200.65	199.10	199.60	200.81	200.19
	(201.15)	(199.21)	(198.65)	(200.28)	(198.94)	(200.21)	(199.91)	(198.89)
1.1	59.44	17.49	12.61	10.81	9.83	11.87	10.41	10.13
	(58.56)	(16.93)	(11.79)	(9.81)	(8.69)	(10.73)	(8.60)	(7.65)
1.2	17.34	4.44	3.42	3.10	2.94	3.49	3.58	3.92
	(16.74)	(3.93)	(2.70)	(2.27)	(2.11)	(2.40)	(2.04)	(2.05)
1.3	7.29	2.15	1.85	1.78	1.74	2.05	2.25	2.47
	(6.76)	(1.58)	(1.15)	(1.05)	(1.00)	(1.13)	(1.15)	(1.27)
1.4	3.95	1.48	1.37	1.34	1.33	1.53	1.66	1.78
	(3.38)	(0.84)	(0.67)	(0.62)	(0.61)	(0.74)	(0.82)	(0.900
1.5	2.59	1.23	1.18	1.17	1.16	1.28	1.35	1.43
	(2.02)	(0.53)	(0.43)	(0.41)	(0.40)	(0.53)	(0.59)	(0.65)
1.6	1.93	1.11	1.09	1.08	1.08	1.15	1.19	1.24
	(1.34)	(0.34)	(0.30)	(0.29)	(0.28)	(0.38)	(0.44)	(0.49)
1.7	1.56	1.05	1.05	1.04	1.04	1.08	1.10	1.13
	(0.93)	(0.23)	(0.22)	(0.20)	(0.19)	(0.28)	(0.32)	(0.36)
1.8	1.35	1.03	1.02	1.02	1.02	1.04	1.06	1.07
	(0.70)	(0.16)	(0.15)	(0.14)	(0.14)	(0.20)	(0.24)	(0.27)
1.9	1.23	1.01	1.01	1.01	1.01	1.02	1.03	1.04
	(0.52)	(0.12)	(0.11)	(0.10)	(0.10)	(0.15)	(0.18)	(0.20)
2.0	1.15	1.01	1.01	1.01	1.00	1.01	1.02	1.02
	(0.41)	(0.09)	(0.08)	(0.07)	(0.07)	(0.11)	(0.13)	(0.15)
2.5	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	(0.14)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.04)
3.0	1.00	1.01	1.00	1.00	1.00	1.00	1.00	1.00
	(0.05)	(0.09)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
L	---	3.061	2.895	2.809	2.741	3.505	3.934	4.299

Table 3: *Run length profiles of the charts with $n = 20$ and $ARL_0 = 200$*

Shift λ	S^2chart	AIB-V chart	AIB-MA-V chart $w = 2$	$w = 3$	$w = 4$	AIB-DMA-V chart $w = 2$	$w = 3$	$w = 4$
				$\rho = 0.3$				
1.0	199.83	200.49	199.47	199.24	200.78	200.14	200.10	200.53
	(200.31)	(200.63)	(200.17)	(200.61)	(200.42)	(199.60)	(198.72)	(198.39)
1.1	49.93	30.87	22.43	19.05	17.72	20.97	18.30	17.63
	(49.41)	(30.30)	(21.83)	(18.29)	(16.74)	(19.91)	(16.66)	(15.13)
1.2	13.04	9.05	6.38	5.56	5.17	6.21	5.80	6.07
	(12.55)	(8.54)	(5.67)	(4.68)	(4.24)	(5.08)	(4.19)	(3.79)
1.3	5.26	4.03	3.09	2.80	2.70	3.21	3.32	3.64
	(4.72)	(3.50)	(2.38)	(2.06)	(1.92)	(2.20)	(1.90)	(1.96)
1.4	2.91	2.40	2.00	1.89	1.85	2.18	2.36	2.60
	(2.41)	(1.83)	(1.33)	(1.18)	(1.12)	(1.25)	(1.25)	(1.39)
1.5	1.95	1.72	1.53	1.48	1.46	1.70	1.84	1.99
	(1.38)	(1.12)	(0.84)	(0.76)	(0.75)	(0.88)	(0.95)	(1.04)
1.6	1.53	1.39	1.30	1.27	1.25	1.42	1.52	1.62
	(0.89)	(0.73)	(0.59)	(0.55)	(0.53)	(0.66)	(0.74)	(0.80)
1.7	1.29	1.21	1.17	1.15	1.14	1.25	1.32	1.39
	(0.62)	(0.51)	(0.42)	(0.40)	(0.39)	(0.51)	(0.57)	(0.63)
1.8	1.17	1.12	1.09	1.08	1.08	1.15	1.20	1.24
	(0.44)	(0.36)	(0.31)	(0.29)	(0.29)	(0.39)	(0.45)	(0.49)
1.9	1.10	1.07	1.05	1.05	1.05	1.09	1.12	1.15
	(0.33)	(0.27)	(0.23)	(0.22)	(0.21)	(0.30)	(0.35)	(0.39)
2.0	1.05	1.04	1.03	1.03	1.03	1.05	1.07	1.09
	(0.24)	(0.20)	(0.18)	(0.17)	(0.16)	(0.23)	(0.27)	(0.30)
2.5	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.01
	(0.06)	(0.05)	(0.04)	(0.04)	(0.01)	(0.06)	(0.07)	(0.08)
3.0	1.00	1.04	1.00	1.00	1.00	1.00	1.00	1.00
	(0.02)	(0.20)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.03)
L	---	3.173	2.934	2.808	2.735	3.545	3.926	4.279
				$\rho = 0.6$				
1.0	199.83	199.36	199.39	199.37	199.88	199.44	199.19	200.34
	(200.31)	(199.22)	(197.10)	(199.03)	(198.13)	(198.89)	(196.75)	(198.49)
1.1	49.93	27.68	19.98	17.16	15.97	18.90	16.74	16.07
	(49.41)	(27.23)	(19.26)	(16.30)	(15.05)	(17.97)	(14.94)	(13.58)
1.2	13.04	7.80	5.61	4.94	4.63	5.49	5.28	5.57
	(12.55)	(7.26)	(4.86)	(4.10)	(3.73)	(4.38)	(3.64)	(3.36)
1.3	5.26	3.53	2.76	2.57	2.45	2.90	3.07	3.38
	(4.72)	(2.99)	(2.07)	(1.81)	(1.67)	(1.89)	(1.70)	(1.78)
1.4	2.91	2.13	1.84	1.75	1.71	2.03	2.21	2.41
	(2.41)	(1.56)	(1.14)	(1.04)	(0.99)	(1.13)	(1.16)	(1.27)
1.5	1.95	1.57	1.43	1.39	1.37	1.59	1.73	1.86
	(1.38)	(0.94)	(0.72)	(0.68)	(0.66)	(0.80)	(0.87)	(0.96)
1.6	1.53	1.30	1.23	1.21	1.20	1.35	1.43	1.52
	(0.89)	(0.62)	(0.50)	(0.48)	(0.46)	(0.60)	(0.67)	(0.73)
1.7	1.29	1.16	1.13	1.12	1.11	1.20	1.26	1.32
	(0.62)	(0.43)	(0.36)	(0.35)	(0.33)	(0.46)	(0.51)	(0.57)
1.8	1.17	1.09	1.07	1.06	1.06	1.12	1.15	1.19
	(0.44)	(0.31)	(0.26)	(0.25)	(0.25)	(0.34)	(0.39)	(0.44)

Table 3 continued

Shift λ	S^2chart	AIB-V chart	AIB-MA-V chart $w=2$	$w=3$	$w=4$	AIB-DMA-V chart $w=2$	$w=3$	$w=4$
				$\rho=0.6$				
1.9	1.10	1.05	1.04	1.03	1.03	1.07	1.09	1.12
	(0.33)	(0.22)	(0.20)	(0.18)	(0.18)	(0.26)	(0.30)	(0.34)
2.0	1.05	1.03	1.02	1.02	1.02	1.04	1.05	1.07
	(0.24)	(0.16)	(0.15)	(0.14)	(0.14)	(0.20)	(0.23)	(0.26)
2.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	(0.06)	(0.04)	(0.04)	(0.03)	(0.03)	(0.05)	(0.06)	(0.07)
3.0	1.00	1.03	1.00	1.00	1.00	1.00	1.00	1.00
	(0.02)	(0.16)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)
L	---	3.118	2.906	2.799	2.731	3.510	3.919	4.277
				$\rho=0.9$				
1.0	199.83	200.67	199.75	199.31	200.17	199.85	200.14	199.57
	(200.31)	(200.68)	(197.77)	(198.84)	(199.80)	(200.61)	(197.25)	(196.52)
1.1	49.93	13.74	9.66	8.19	7.45	9.04	8.07	7.99
	(49.41)	(13.20)	(8.88)	(7.18)	(6.32)	(7.85)	(6.29)	(5.47)
1.2	13.04	3.33	2.62	2.43	2.36	2.78	2.97	3.27
	(12.55)	(2.77)	(1.90)	(1.62)	(1.53)	(1.70)	(1.54)	(1.61)
1.3	5.26	1.69	1.52	1.48	1.46	1.71	1.89	2.05
	(4.72)	(1.07)	(0.82)	(0.75)	(0.72)	(0.85)	(0.92)	(1.01)
1.4	2.91	1.25	1.20	1.19	1.17	1.31	1.41	1.51
	(2.41)	(0.56)	(0.46)	(0.44)	(0.42)	(0.55)	(0.63)	(0.69)
1.5	1.95	1.09	1.08	1.07	1.07	1.14	1.19	1.24
	(1.38)	(0.32)	(0.28)	(0.27)	(0.26)	(0.37)	(0.43)	(0.48)
1.6	1.53	1.04	1.03	1.03	1.03	1.06	1.08	1.11
	(0.89)	(0.20)	(0.17)	(0.16)	(0.16)	(0.24)	(0.28)	(0.32)
1.7	1.29	1.01	1.01	1.01	1.01	1.02	1.04	1.05
	(0.62)	(0.12)	(0.11)	(0.10)	(0.10)	(0.15)	(0.19)	(0.22)
1.8	1.17	1.01	1.00	1.00	1.00	1.01	1.01	1.02
	(0.44)	(0.08)	(0.07)	(0.06)	(0.06)	(0.10)	(0.12)	(0.15)
1.9	1.10	1.00	1.00	1.00	1.00	1.00	1.01	1.01
	(0.33)	(0.05)	(0.05)	(0.04)	(0.04)	(0.06)	(0.08)	(0.10)
2.0	1.05	1.00	1.00	1.00	1.00	1.00	1.00	1.01
	(0.24)	(0.04)	(0.02)	(0.03)	(0.02)	(0.04)	(0.06)	(0.07)
2.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	(0.06)	(0.01)	(0.01)	(0.00)	(0.01)	(0.00)	(0.01)	(0.01)
3.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	(0.02)	(0.03)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
L	---	3.002	2.868	2.790	2.733	3.471	3.917	4.280

Observations from Tables 1-3:

- For any range of shifts in process standard deviation, the proposed AIB-DMA-V and AIB-MA-V charts consistently produces smaller out-of-control ARL values than the AIB-Shewhart-V chart and Shewhart S^2 chart. For example, in Table 1, with $\lambda = 1.3$, the traditional S^2chart requires on an average 11 samples to signal, the Shewhart AIB-V chart requires 8.03 samples to signal, the ARL reduces to 5.89 with $w = 2$, 5.18 with $w = 3$ and 4.83 with $w = 4$ for proposed AIB-MA-V chart and ARL reduces to 5.89 with $w = 2$, 5.55 with $w = 3$ and 5.80 with $w = 4$ for proposed AIB-DMA-V chart. That means, proposed

AIB-DMA-V and AIB-MA-V charts early detect shifts in process standard deviation earlier than the other two charts.

- The sensitivity of the proposed control charts increases as the span of moving average increases. For example, in Table 3, with $\lambda = 1.2$ and $w = 2$, the proposed AIB-DMA-V chart requires on an average 6.21 samples to signal, the ARL reduces to 5.80 and 6.07 with $w = 3$ and $w = 4$ respectively. The similar performance is observed for AIB-MA-V chart.
- The out-of-control ARL values of proposed AIB-DMA-V chart decreases when the correlation between study and auxiliary variables increases. For example, from Table 1, with fixed $\lambda = 1.3$and $w = 2$, the proposed AIB-DMA-V chart requires on an average 5.89 samples to signal when $\rho = 0.3$, the ARL reduces to 5.26 and 2.76 when $\rho = 0.6$ and $\rho = 0.9$ respectively. The similar performance is observed for AIB-MA-V and Shewhart-AIB-V charts.
- The performance of the proposed AIB-DMA-V and AIB-MA-V charts keeps improving with an increase in sample size n. For example, from Tables 1-3, with fixed $\lambda = 1.2$, $w = 3$ and $\rho = 0.6$, the proposed AIB-DMA-V chart requires on an average 9.35 samples to signal when $n = 10$, the ARL reduces to 6.66 and 5.28 when $n = 15$ and $n = 20$ respectively.

7. An Example

In this Section, we provide an illustrative example in order to demonstrate the practical application of AIB-MA-V, AIB-DMA-V charts for monitoring process variability using auxiliary information. Here we consider a simulated dataset to present implementation of AIB-Shewhart-V, AIB-MA-V and AIB-DMA-V control charts. To identify the performance of these control charts, the in-control ARL value is set as$ARL_0 = 200$. We have considered the paired information on (Y, X) where X is used as auxiliary variable and Y as the study variable of interest. The bivariate data set in the form of 15 sub-groups each of size $n = 10$ are simulated from$N_2(\mu_y, \mu_x, \sigma_y^2, \sigma_x^2, \rho)$ distribution. For in-control state, the first 7 samples are generated from$N_2(0,0,1,1,0.6)$. Thus the process is stable with respect to process variability for first 7 samples. We add 8 samples to simulate an out-of-control process. Starting from sample 8, new samples are generated from a process by introducing a shift $\lambda = 1.5$ in σ_y. Based on 15 subgroups, the values of the control chart statistic for AIB-MA-V and AIB-DMA-V control charts for span $w = 3$ and AIB-Shewhart-V chart are displayed in Table 4. The control limits of AIB-Shewhart-V chart are computed using Eq. (4) while those of AIB-MA-V and AIB-DMA-V charts are computed using Eq. (6) and Eq. (9) respectively. Implementation of the said charts is presented in Figure 1.

From Figure 1, it can be seen that the process remains in-control at the first seven samples. For detecting shift of size $\lambda = 1.5$ in process standard deviation, the AIB-Shewhart-V chart does not produce any out-of-control signal for detecting the shift. So AIB-Shewhart-V chart fail to detect a shift in process standard deviation when the shift occur. The AIB-MA-V chart shows first out-of-control signal at point 13. AIB-DMA-V chart shows first out-of-control signal at point 11 which is earlier than that of AIB-MA-V chart. Hence the proposed AIB-DMA-V chart is effective in detecting shifts in process standard deviation than AIB-MA-V and AIB-Shewhart-V charts.

Table 4: *Chart statistics based on simulated data*

Sample Number	AIB-V	AIB-MA-V	AIB-DMA-V
1	0.91	0.91	0.91
2	0.63	0.77	0.84
3	1.56	1.03	0.91
4	1.65	1.28	1.03
5	0.74	1.31	1.21

Table 4 continued

6	0.95	1.11	1.23
7	0.56	0.75	1.06
8	1.88	1.13	1.00
9	2.30	1.58	1.15
10	0.80	1.66	1.46
11	2.07	1.72	1.65
12	1.71	1.53	1.64
13	2.02	1.93	1.73
14	1.54	1.76	1.74
15	1.03	1.53	1.74

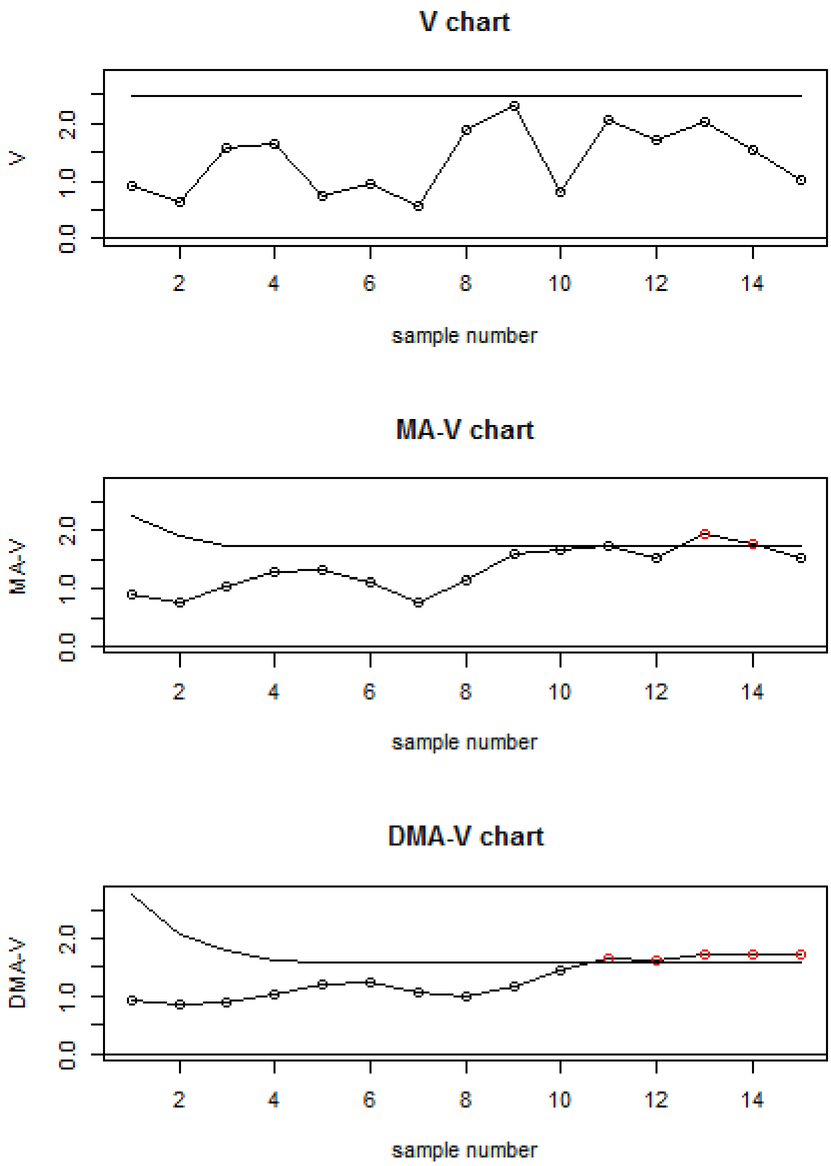

Figure 1: *AIB-Shewhart, AIB-MA and AIB-DMA Charts for process variability*

8. Conclusions

In this paper, we have proposed the AIB-MA and AIB-DMA control charts to efficiently monitoring the variability of normally distributed process. These charts are based on regression estimator of the process variance that utilizes information on a study variable as well as any related auxiliary variable. The construction, performance assessment and an illustrative example of proposed charts are presented in this paper. Using extensive Monte Carlo simulations, ARL and SDRL of the proposed AIB-MA-V and AIB-DMA-V chart has been computed with various choices of correlation coefficient ρ and span w. From the simulation results it is observed that with an increase in the value of w, the performance of the AIB-MA-V and AIB-DMA-V charts is improved. The performance of the proposed charts keep improving with an increase in the values of sample size n, level of correlation between study variable and auxiliary variable ρ and size of shift λ in process standard deviation at a fixedARL_0. The SDRL values of the charts are approximately the same as ARL values. The performance of the proposed charts is also compared to its existing counterparts incorporated in this study. It has been found that AIB-DMA-V and AIB-MA-V charts perform uniformly better than the AIB-Shewhart-V andS^2 charts for different kind of shifts in the process variability. Among AIB-DMA-V perform better than the AIB-MA-V chart for a span of $w = 2, 3$; while for span of $w = 4$, performance of both proposed charts is similar.

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