EXPLORING NOVEL EXTENSION OF SUJA DISTRIBUTION: UNVEILING PROPERTIES AND DIVERSE APPLICATIONS

C. Subramanian[1], M. Subhashree[2], Aafaq A. Rather[3,*]

•

[1,2] Department of Statistics, Annamalai University, Annamalai Nagar, Tamil Nadu, India
[3,*]Symbiosis Statistical Institute, Symbiosis International (Deemed University), Pune-411004, India
[1]manistat@yahoo.co.in, [2]Subhashreem17@gmail.com, [3,*]aafaq7741@gmail.com

Abstract

This research article introduces and explores the area biased techniques of the Suja distribution, presenting novel derivations and insights. The estimation of the one-parameter area biased Suja distribution is accomplished using maximum likelihood, providing a robust framework for modeling real-world data. A comprehensive study of several statistical properties is conducted to unveil the characteristics and behaviors of this new model. To demonstrate its practical applicability, the proposed distribution is applied to real data of weather temperature. The analysis showcases the distribution's effectiveness in capturing the intricacies of temperature patterns, revealing its potential utility in weather modeling and related applications. The research contributes to the advancement of statistical modeling techniques and enriches our understanding of the Suja distribution's versatility in handling diverse datasets.

Keywords: Area-biased distribution, Suja distribution, Maximum likelihood Estimator

1. Introduction

The concept of area- biased distribution was explained earlier by Fisher [5]. He first introduced weighted distribution which is a combination of model specification and data interpretation. Also, stated that "the estimation of frequencies based on the effective methods of ascertainment". The size-biased are the special cases of the weighted distribution. Later, Rao [16] explained that the weighted distributions as many applications such as medicine, reliability, ecology, behavioral science, finance, insurance, etc. A discrete Poisson area-biased Lindley distribution was purposed by Bashir and Rasul [3] and explained its properties. Also, applied in biological data and compared with Poisson distribution. Zahida et.al. [18] Purposed a new extension of Weibull distribution called area- biased weighted Weibull distribution. The characterization of this model was derived and shown how the model fitted to the problem of ball bearing data. Rama Shanker [10] introduced a new one- parameter Suja distribution and estimated the parameter using method of moments and maximum likelihood. The important properties were explained and finally, compared this model with other one-parameter distributions by applying a real lifetime data.

Ayesha Fazal [2] introduced an area-biased Poisson exponential distribution with its moments and other properties, Also, the goodness of fit for this model has been discussed to show how it fit in real data sets. Many studies on length biased distribution case has been published, for example; Rather and Subramanian [11], Rather and Subramanian [12], Rather and Ozel [13], Rather and Subramanian [19], Rather et al. [18]. A new generalized area- biased Aradhana distribution was introduced by Elangovan and Mohanasundari [4] and estimated the parameters by maximum likelihood estimator. Finally, applied a real lifetime data set in the model to show how it works. The new extension of Suja distribution called length-biased Suja distribution was given by Ibrahim Al-omari and Islam Khaled [6]. The various properties of this model were explained and shown the usefulness of the model in the real data set. Later, Ibrahim Al-omari et.al. [7] extended the length-biased Suja distribution to power length-biased Suja distribution with its characteristics and estimated the two parameters of this proposed distribution by maximum likelihood method. Finally, they illustrated a real data to show the performance of this model. Arun Kumar Rao and Himanshu Pandey [1] studied the parameter estimation of area-biased Rayleigh distribution by using maximum likelihood estimation and Bayesian estimation with quasi and inverted gamma priors. The weighted Suja distribution as an extension of most important Suja distribution was discussed by Islam Khaled and Ibrbhim Al-Omari [8]. Also, explained its statistical properties and application in the ball bearings data. The new generalization method as area-biased was introduced by Nuri Celik [9] for beta, Rayleigh and log-normal distributions. Also, the main statistical properties and parameters estimation were studied. At last, some of the real data examples were used. Shanker, Upadhyay and Shukla [17] were given a new two parameter quasi Suja distribution with its characteristics and parameters estimation. Also, they illustrated with real data to show its performance.

2. Area-biased Suja Distribution (ABSD)

The probability density function (p.d.f.) and cumulative distribution function (c.d.f.) of the Suja distribution is given by

$$f(y,\alpha)=\frac{\alpha^5}{\alpha^4+24}(1+y^4)e^{-\alpha z}\,; y>0,\alpha>0 \tag{1}$$

$$F(y,\alpha)=1-\left[1+\frac{(\alpha^4y^4+4\alpha^3y^3+120\alpha^2y^2+24\alpha y)}{\alpha^4+24}\right]\,; y>0,\alpha>0 \tag{2}$$

We know that, the weighted function is

$$f_w(y)=\frac{w(y)f(y)}{E(w(y))}\,; x>0 \tag{3}$$

Where, $w(x)$ be a non-negative weight function. $E\big(w(y)\big)=\int_0^\infty w(y)f(y)dy<\infty$, $w(y)=y^c$

$$f_w(y)=\frac{y^c f(y)}{E(y^c)}\,; y>0 \tag{4}$$

where, $E(y^c)=\int_0^\infty y^c\, f(y)\, dy$

For area-biased function, put c=2,

$$f_{AB}(y)=\frac{y^2 f(y)}{E(y^2)}\,; y>0 \tag{5}$$

where,

$$E(y)=\int_0^\infty y^2\, f(y)dy \tag{6}$$

substitute (1) in (6), we get

$$E(y)=\frac{2(\alpha^4+360)}{\alpha^2(\alpha^4+24)} \tag{7}$$

Thus, the p.d.f. and c.d.f. of the area-biased Suja distribution (ABSD) can be obtained as

$$f_{ABSD}(y;\alpha)=\frac{\alpha^7}{2(\alpha^4+24)}y^2(1+y^4)e^{-\alpha y};\ y>0;\alpha>0 \tag{8}$$

$$F_{ABSD}(y;\alpha) = \frac{1}{2\alpha^4+720}[\alpha^4\gamma(3,\alpha y)+\gamma(7,\alpha y)] \quad (9)$$

The graphical representation of the p.d.f. and c.d.f. of area-biased Suja distribution (ABSD) are also shown below:

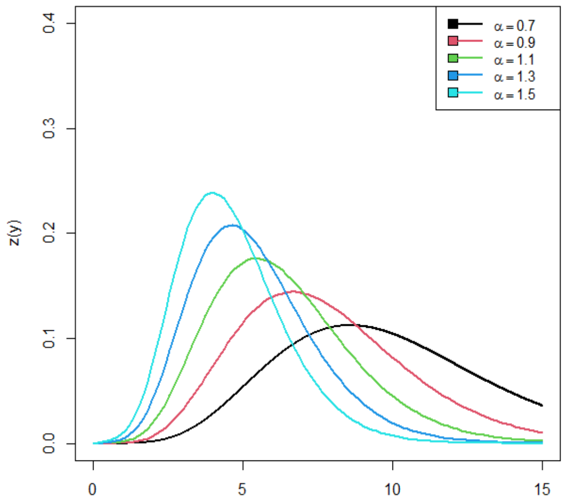

Fig. 1: *Pdf plot of ABSD*

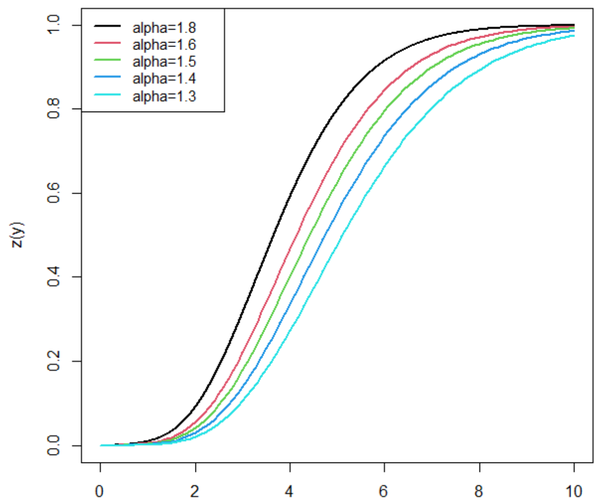

Fig. 2: *Cdf plot of ABSD*

3. Reliability Analysis

3.1 Reliability Function

The survival function of the area-biased Suja distribution is given by

$$S(y) = 1 - F(y)$$

$$S(y) = 1 - \frac{1}{2\alpha^4+720}[\alpha^4\gamma(3,\alpha y)+\gamma(7,\alpha y)]$$

3.2 Hazard Function

The hazard function is also known as the hazard rate, instantaneous failure rate or force of mortality and is given by

$$h(y) = \frac{f_{ABSD}(y;\alpha)}{1 - F_{ABSD}(y;\alpha)}$$

$$h(y) = \frac{\alpha^7 y^2(1+y^4)e^{-\alpha y}}{(2\alpha^4+720) - [\alpha^4\gamma(3,\alpha y)+\gamma(7,\alpha y)]}$$

3.3 Reverse Hazard Function

The reverse hazard function or reverse hazard rate is given by

$$h_r(y) = \frac{f_{ABSD}(y;\alpha)}{F_{ABSD}(y;\alpha)}$$

$$h_r(y) = \frac{\alpha^7 y^2(1+y^4)e^{-\alpha y}}{[\alpha^4\gamma(3,\alpha y)+\gamma(7,\alpha y)]}$$

3.4 Mill's Ratio

The Mills ratio of the area-biased Suja distribution is

$$\text{Mills Ratio} = \frac{1}{h_r(y)}$$

$$= \frac{[\alpha^4 \gamma(3, \alpha y) + \gamma(7, \alpha y)]}{\alpha^7 y^2 (1 + y^4) e^{-\alpha y}}$$

4. Moments

The moments of ABSD have been derived to describe the characteristic of the proposed model. Then, the r^{th} order moment E(y^r) of ABSD is derived as

$$\mu_r' = E(y^r) = \int_0^\infty y^r F_{ABSD}(y; \alpha) dy \tag{10}$$

$$\mu_r' = \frac{\alpha^7}{2\alpha^4 + 720} \int_0^\infty y^{r+2} (1 + y^4) e^{-\alpha y} dy \tag{11}$$

$$\mu_r' = \frac{\alpha^4 \Gamma(\text{r} + 3) + \Gamma(\text{r} + 7)}{\alpha^r (2\alpha^4 + 720)} \tag{12}$$

In equation (12), when *r*=1, the mean of ABSD which is given by

$$\mu_1' = \frac{3(\alpha^4 + 840)}{\alpha(\alpha^4 + 360)}$$

Similarly, when *r*=2, 3, 4 in equation (4.1), we will get

$$\mu_2' = \frac{12(\alpha^4 + 1680)}{\alpha^2(\alpha^4 + 360)}$$

$$\mu_3' = \frac{60(\alpha^4 + 3024)}{\alpha^3(\alpha^4 + 360)}$$

$$\mu_4' = \frac{360(\alpha^4 + 5040)}{\alpha^4(\alpha^4 + 360)}$$

The central moments about the mean of this distribution are given as

$$\mu_0 = \mu_0' = 1$$

$$\mu_0 = 0$$

$$\mu_2 = \mu_2' - (\mu_1')^2 = \frac{12(\alpha^4 + 1680)}{\alpha^2(\alpha^4 + 360)} - \left(\frac{3(\alpha^4 + 840)}{\alpha(\alpha^4 + 360)}\right)^2$$

Therefore, the variance of ABSD is

$$\mu_2 = \frac{9(\alpha^8 + 2160\alpha^4 + 571200)}{\alpha^2(\alpha^4 + 360)^2}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3 = \frac{60(\alpha^{12} + 8280\alpha^8 + 388800\alpha^4 + 108864000)}{\alpha^3(\alpha^4 + 360)^3}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^3 - 3(\mu_1')^4$$

$$\mu_4 = \frac{45(\alpha^4 + 360)(\alpha^{12} + 21048\alpha^8 + 15871680\alpha^4 + 741208320)}{\alpha^4(\alpha^4 + 360)^4}$$

The standard deviation (σ), co-efficient of variation (C.V.), co-efficient of skewness ($\sqrt{\beta_1}$), co-efficient of kurtosis (β_2) and index of dispersion (γ) of ABSD are obtained as

$$\sigma = \frac{3\sqrt{\alpha^8 + 2160\alpha^4 + 571200}}{\alpha(\alpha^4 + 360)}$$

$$C.V. = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\alpha^8 + 2160\alpha^4 + 571200}}{(\alpha^4 + 840)}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{\frac{3}{2}}} = \frac{2(\alpha^{12} + 8280\alpha^8 + 388800\alpha^4 + 108864000)}{9(\alpha^8 + 2160\alpha^4 + 571200)^{\frac{3}{2}}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2} = \frac{5(\alpha^{12} + 21048\alpha^8 + 15871680\alpha^4 + 7412083200)}{\alpha^2(\alpha^4 + 360)(\alpha^8 + 2160\alpha^4 + 571200)}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{3(\alpha^8+2160\alpha^4+571200)}{\alpha(\alpha^4+360)(\alpha^4+840)} \tag{13}$$

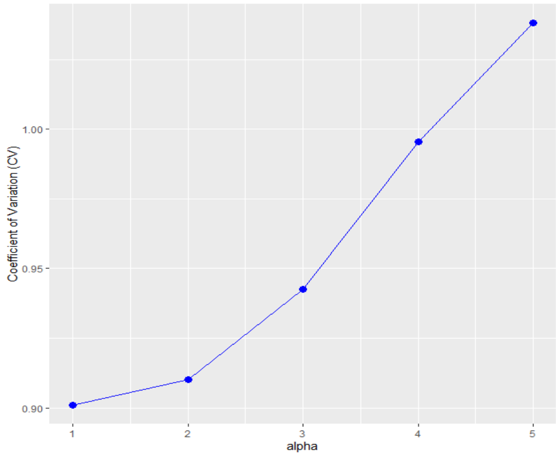

Fig. 3: *Coefficient of variation for ABSD*

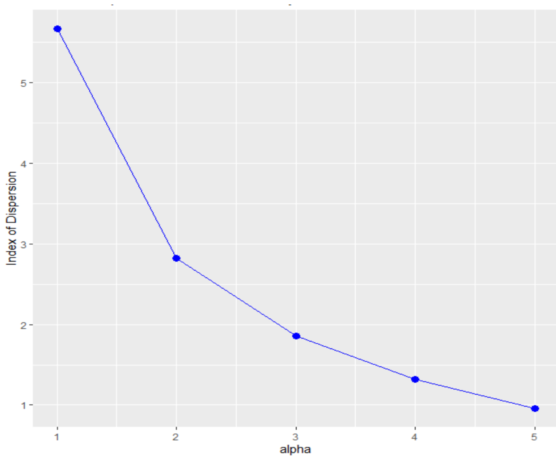

Fig. 4: *Index of dispersion for ABSD*

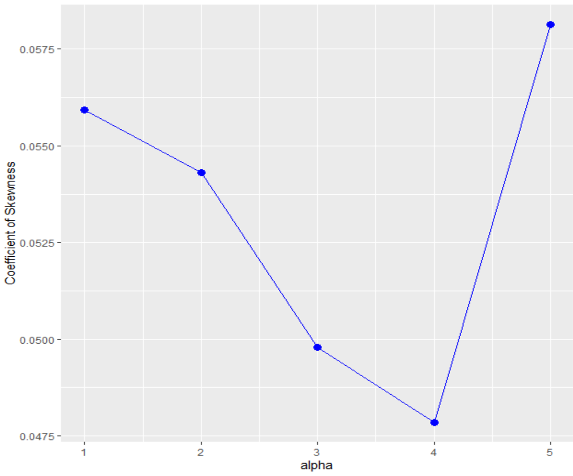

Fig. 5: *Coefficient of skewness for ABSD*

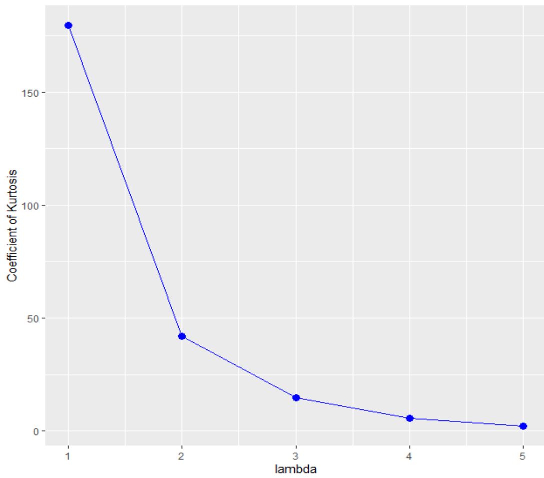

Fig. 6: *Coefficient of kurtosis for ABSD*

4.1 Harmonic Mean

The Harmonic mean of the aspired model can be derived as

$$H.M. = E\left[\frac{1}{y}\right] = \int_0^\infty \frac{1}{y} f_{ABSD}(y)dy \tag{14}$$

$$H.M. = \int_0^\infty \frac{1}{y}\frac{\alpha^7}{2(\alpha^4+360)} y^2(1+y^4)e^{-\alpha y}\, dy \tag{15}$$

Therefore,

$$H.M. = \frac{\alpha(\alpha^4+120)}{2(\alpha^4+360)}$$

4.2 Moment Generating Function and Characteristic Function

Assume Y have area-biased Suja distribution, we will get the moment generating function of Y as

$$M_y(t) = E(e^{ty}) = \int_0^\infty e^{ty} f_{ABSD}(y)dy \tag{16}$$

Using Taylor's expansion,

$$M_y(t) = \int_0^\infty \sum_{j=0}^\infty \frac{(ty)^j}{j!} f_{ABSD}(y)dy = \sum_{j=0}^\infty \frac{t^j}{j!}\mu_j' \tag{17}$$

$$M_y(t) = \frac{1}{2(\alpha^4+360)}\sum_{j=0}^\infty \frac{t^j}{j!\alpha^j}\left(\alpha^4\Gamma(\mathrm{r}+3)+\Gamma(\mathrm{r}+7)\right) \tag{18}$$

In the same way, we will get the characteristics function of SBJD can be obtained as

$$\varphi_y(t) = E[e^{ity}] = \frac{1}{2(\alpha^4+360)}\sum_{j=0}^\infty \frac{(it)^j}{j!\alpha^j}\left(\alpha^4\Gamma(\mathrm{r}+3)+\Gamma(\mathrm{r}+7)\right) \tag{19}$$

5. Order Statistics

Let $Y_1, Y_2, \ldots\ldots, Y_n$ be the random variable drawn from the continuous population. Their p.d.f. be $f_y(y)$ and cumulative density function be $F_y(y)$. Then, assume $Y_{(1)}, Y_{(2)}, \ldots\ldots, Y_{(n)}$ be the order statistics of a random sample.

Thus, the probability density function of r[th] order statistics $Y_{(r)}$ is given by

$$f_{y_{(r)}}(y) = \frac{n!}{(n-r)!\,(r-1)!} f(y)\left[F_y(y)\right]^{r-1}\left[1-F_y(y)\right]^{n-r} \tag{20}$$

Putting the equation (8) and (9) in equation (20), the probability density function of r[th] order statistics $Y_{(r)}$ of ABSD is given by

$$f_{y_{(r)}}(y) = \frac{n!}{(n-r)!(r-1)!}\left\{\frac{\alpha^7y^2(1+y^4)e^{-\alpha y}}{2(\alpha^4+360)}\right\} \mathrm{x} \left\{\frac{[\alpha^4\gamma(3,\alpha y)+\gamma(7,\alpha y)]}{2(\alpha^4+360)}\right\}^{r-1} \tag{21}$$

$$\mathrm{x}\left\{1-\frac{[\alpha^4\gamma(3,\alpha y)+\gamma(7,\alpha y)]}{2(\alpha^4+360)}\right\}^{n-r}$$

Then, the probability density function of higher order statistics $Y_{(n)}$ can be derived as

$$f_{y_{(n)}}(y) = \left\{\frac{n\alpha^7y^2(1+y^4)e^{-\alpha y}}{2(\alpha^4+360)}\right\} \mathrm{x} \left\{\frac{[\alpha^4\gamma(3,\alpha y)+\gamma(7,\alpha y)]}{2(\alpha^4+360)}\right\}^{n-1} \tag{22}$$

Hence, the probability density function of 1[st] order statistics $Y_{(1)}$ can be obtained as

$$f_{y_{(1)}}(y) = \left\{\frac{n\alpha^7y^2(1+y^4)e^{-\alpha y}}{2(\alpha^4+360)}\right\} \mathrm{x} \left\{1-\frac{[\alpha^4\gamma(3,\alpha y)+\gamma(7,\alpha y)]}{2(\alpha^4+360)}\right\}^{n-1} \tag{23}$$

6. Maximum Likelihood Estimator and Fisher Information Matrix

The maximum likelihood estimator is the best numerical stability estimator to estimate the parameters of the distribution when compared with other estimating methods. Thus, we used this method to estimate the parameters of ABSD which is derived below:

Let $Y_{(1)}, Y_{(2)}, \ldots\ldots, Y_{(n)}$ be the random sample of size n drawn from the ABSD, then, the likelihood function of ABSD is

$$L(y;\alpha) = \prod_{i=1}^n f_{ABSD}(y;\alpha) = \frac{\alpha^{7n}}{2^n(\alpha^4+360)^n}\prod_{i=1}^n y_i^2(1+y_i^4)e^{-\alpha y_i}$$

The natural log likelihood function is

$$\log L(y;\alpha) = 7n\log\alpha - n\log(2\alpha^4+720) + 2\sum_{i=1}^n \log y_i + \sum_{i=1}^n \log(1+y_i^4) - \alpha\sum_{i=1}^n y_i \tag{24}$$

By differentiating equation (24) with respect to α, the maximum likelihood estimates of α can be attained as

$$\frac{\partial \log L}{\partial\alpha} = \frac{7n}{\alpha} - \frac{8n\alpha^3}{(2\alpha^4+720)} - \sum_{i=1}^n y_i = 0 \tag{25}$$

Because of the complicated form of likelihood equation (25), algebraically it is very difficult to solve the system of non-linear equation. Therefore, we use R and wolfram Mathematica for estimating the required parameters.

7. Likelihood ratio Test

This test is used to compare the goodness of fit of the two models based on the ratio of their likelihoods. Suppose $Y_{(1)}, Y_{(2)}, \ldots\ldots, Y_{(n)}$ be a random sample from the ABSD. To test, the random sample of size n for ABSD, the hypothesis is

$$H_0: f(y) = f_{SD}(y;\alpha) \quad agaist \quad H_1: f(y) = f_{ABSD}(y;\alpha)$$

To check whether the random sample of size n comes from the Suja distribution or size-biased Suja distribution, the likelihood ratio is

$$\Delta = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{f_{ABSD}(y;\alpha)}{f_{SD}(y;\alpha)} = \left[\frac{\alpha^2(\alpha^2+24)}{2(\alpha^4+360)}\right]^n \prod_{i=1}^{n} y_i^2$$

Therefore, the null hypothesis is rejected if

$$\Delta = \left[\frac{\alpha^2(\alpha^2+24)}{2(\alpha^4+360)}\right]^n \prod_{i=1}^{n} y_i^2 > k$$

$$\Delta^* = \prod_{i=1}^{n} y_i^2 > k^* \; where \; k^* = k\left[\frac{\alpha^2(\alpha^2+24)}{2(\alpha^4+360)}\right]^n > 0$$

We can conclude, for large sample size n, 2log is distributed as chi-square distribution with one degree of freedom. Also, p-value is attained from the chi-square distribution. If p (Δ^*>k^*), where $k^* = \prod_{i=1}^{n} y_i$ is less than the specified level of significance and $\prod_{i=1}^{n} y_i$ is the observed value of the statistic Δ^*, then, reject null hypothesis.

8. Applications

It is a measure to check how a statistical model fits a data set. Here, we will discuss how the proposed model is fit to a data set which is illustrated below. Also, compare with Suja distribution to show better fit of area-biased Suja distribution. Let us represent the data set of weather temperature in Bangladesh from 2016 to 2019 by Shawkat Sujon in the website: https://www.kaggle.com/datasets/shawkatsujon/bangladesh-weather-dataset-from-1901-to2019. The data set is given as follows:

Table 1: *Data of weather temperature in Bangladesh from 2016 to 2019*

Year	2016	2016	2016	2016	2016	2016	2016
Month	1	2	3	4	5	6	7
Temperature	17.34	22.12	25.93	28.25	27.94	28.96	28.17
Year	2016	2016	2016	2016	2016	2017	2017
Month	8	9	10	11	12	1	2
Temperature	28.85	28.58	27.74	23.45	21.43	19.36	21.31
Year	2017	2017	2017	2017	2017	2017	2017
Month	3	4	5	6	7	8	9
Temperature	23.69	26.91	28.76	28.58	28.27	28.57	28.54
Year	2017	2017	2017	2018	2018	2018	2018
Month	10	11	12	1	2	3	4
Temperature	27.16	24.14	20.59	17.59	21.18	25.41	26.79
Year	2018	2018	2018	2018	2018	2018	2018
Month	5	6	7	8	9	10	11
Temperature	27.32	28.68	28.59	28.88	28.66	26.17	22.76

Year	2018	2019	2019	2019	2019	2019	2019
Month	12	1	2	3	4	5	6
Temperature	19.13	19.38	20.71	24.4	27.41	28.92	29.18
Year	2019	2019	2019	2019	2019	2019	
Month	7	8	9	10	11	12	
Temperature	28.72	29	28.28	26.94	23.88	18.51	

In order to compare the distributions, we study the criteria like Akaike Information Criterion (AIC), Akaike Information Criterion Corrected (AICC), Bayesian Information Criterion (BIC) and -2logL. The better distribution is which compatible to lesser values of AIC, BIC, AICC and -2logL.

Table 2: *Comparison of distributions*

Distribution	MLE	S.E	-2logL	AIC	BIC	AICC
Suja	$\hat{\alpha} = 0.1965312$	0.0126848	327.644	329.644	331.5152	329.730
Length-biased Suja	$\hat{\alpha} = 0.2358445$	0.0138967	319.7202	321.7202	323.5914	321.8071
Area-biased Suja	$\hat{\alpha} = 0.2751537$	0.0150105	313.2269	315.2269	317.0981	315.3138

9. Conclusion

In conclusion, this article presents a comprehensive investigation of the area-biased Suja distribution (ABSD), a noteworthy extension of the weighted distribution paradigm. The obtained probability density function (p.d.f.) and cumulative distribution function (c.d.f.) enrich the theoretical foundations of the ABSD, laying a solid groundwork for its application in various domains.

One significant advantage of the ABSD is its robust parameter resiliency, which leads to enhanced performance and more accurate results compared to other distributions. The maximum likelihood estimator proves to be an effective tool for estimating the distribution's parameter, and its validity is confirmed through the likelihood ratio test, reinforcing the reliability of our findings. The in-depth analysis of various statistical properties provides valuable insights into the ABSD's behavior and characteristics, fostering a deeper understanding of this novel model.

In particular, when applied to weather temperature data, the ABSD demonstrates superior compatibility compared to both the standard Suja distribution and the length-biased Suja distribution. The results suggest that the ABSD better captures the intricate patterns and variations inherent in weather temperature datasets.

The findings presented in this article not only enhance the understanding of the ABSD but also contribute to the broader field of statistical modeling. The improved performance in weather temperature analysis highlights the practical applicability of the ABSD in real-world scenarios. This distribution's versatility and ability to handle diverse datasets make it a promising candidate for future research and application across various domains. As the field of statistical modeling continues to evolve, the ABSD offers a valuable addition to the toolkit of data analysts and researchers alike.

References

[1] Arun Kumar Rao and Himanshu Pandey, (2020), *Parameter Estimation of Area-biased Rayleigh Distribution*, International Journal of Statistics and Applied Mathematics, Vol. 5, No. 4, pp.27-33.

[2] Ayesha Fazal, (2018), *Area-biased Poisson Exponential Distribution with Applications*, Biometrics and Biostatistics International Journal, Vol. 7, No. 3, pp. 256-261.

[3] Bahir, S. and Rasul, M., (2016), *Poisson Area-biased Lindley Distribution and its Applications on Biological Data*, Biometrics and Biostatistics International Journal, Vol. 5, No.1, pp 1-10.

[4] Elangovan, R. and Mohanasundari, R., (2019), *A New Area-biased Distribution with Applications in Cancer Data*, Journal of Science, Technology and Development, Vol.8, No. 12, pp. 1-14.

[5] Fisher, R.A., (1934), *The Effects of Methods of Ascertainment Upon the Estimation of Frequencies*, Annual Eugenics, Vol. 6, No. 1, pp. 13-25.

[6] Ibrahim Al-Omari, A. and Islam Khaled, A., (2019*), Length-biased Suja Distribution and its Applications*, Journal of Applied Probability and Statistics, Vol. 14, No.3, pp. 95-116.

[7] Ibrahim Al-Omari, A., Alhyasat, K., Ibrahim, K. and Abu Bakar, M. A., (2019), *Power Length-biased Suja Distribution: Properties and Applications*, Electronical Journal of Applied Statistical Analysis, Vol.12, no. 2, pp. 429-452.

[8] Islam Khaled, A. and Ibrahim Al-Omari, A., (2020), *Weighted Suja Distribution with Application to Bell Bearings Data*, Journal of Life Cycle Reliability and Safety Engineering, Vol. 9, pp. 195-211.

[9] Nuri Celik, (2022), *Applications in some Area-biased distribution*s, Asian Journal of Probability and Statistics, Vol. 19, No. 3, pp. 42-50.

[10] Rama Shanker, (2017), *Suja Distribution and its Application*, International Journal of Probability and Statistics, Vol. 6, No. 2, pp. 11-19.

[11] Rather, A. A. & Subramanian C. (2018). Characterization and estimation of length biased weighted generalized uniform distribution, *International Journal of Scientific Research in Mathematical and Statistical Science*, vol.5, issue. 5, pp. 72-76.

[12] Rather, A. A. & Subramanian C. (2018). Length Biased Sushila distribution, *Universal Review*, vol. 7, issue. XII, pp. 1010-1023.

[13] Rather, A. A. & Ozel G. (2021). A new length-biased power Lindley distribution with properties and its applications, *Journal of Statistics and Management Systems*, DOI: 10.1080/09720510.2021.1920665

[14] Rather, A. A. & Subramanian, C. (2019), The Length-Biased Erlang–Truncated Exponential Distribution with Life Time Data, *Journal of Information and Computational Science*, vol-9, Issue 8, pp 340-355.

[15] Rather, A. A., Subramanian, C., Shafi, S., Malik, K. A., Ahmad, P. J., Para, B. A., and Jan, T. R. (2018). A new Size Biased Distribution with applications in Engineering and Medical Science, *International Journal of Scientific Research in Mathematical and Statistical Sciences*, Vol.5, Issue.4, pp.75-85.

[16] Rao, C. R., (1955), *On Discrete Distributions Arising Out of Methods of Ascertainment*, In Classical and Contagious Discrete Distributions, Patil, G.P., Eds., Pergamon Press.

[17] Shanker, R., Upadhyay, R. and Shukla, K.K., (2022), *A Quasi Suja Distribution*, Reliability: Theory and Applications, Vol. 17, No. 3, pp. 162-178.

[18] Zahida Praveen, Zulfiqar Ahmed and Munir Ahmad, (2016), *On Area-biased Weighted Weibull Distribution*, Science International (Lahore), Vol. 28, No. 4, pp. 3669-3679.