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ASSESSMENT OF GENERALIZED LIFETIME PERFORMANCE INDEX FOR LINDLEY DISTRIBUTION USING PROGRESSIVE TYPE-II SAMPLES

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Abstract

A meaningful subject of discourse in manufacturing industries is the assessment of the lifetime performance index. In manufacturing industries, the lifetime performance index is used to measure the performance of the product. A generalized lifetime performance index (GLPI) is defined by taking into consideration the median of the process measurement when the lifetime of products follow a parametric distribution may serve better the need of quality engineers and scientists in industry. The present study constructs various point estimators of the GLPI based on progressive type II right censored data for the Lindley distributed lifetime in both classical and Bayesian setup. We perform Monte Carlo simulations to compare the performances of the maximum likelihood and Bayes estimates with a gamma prior of $C_Y(L)$ under progressive type-II right censoring scheme. Finally, the validity of the model is adjudged through analysis of a data set.

Keywords: Bayesian estimation, Metropolis-Hastings method, Process Capability Index, Maximum likelihood estimator.

1. Introduction

Process Capability Indices (PCIs) have wide use in industries for evaluating a manufacturing process and whether or not it can produce articles within the specified limits. PCIs aim to quantify the capability of a process (X) to meet some specifications related to a measurable characteristic of its produced items. These specifications are determined through the lower specification limit (L), the upper specification limit(U) and the target value (T). PCI is an effective means to measure a process's performance and potential capabilities. In the manufacturing industry, PCIs are utilized to assess whether product quality meets customer expectations. Since capability is typically defined in dictionaries as the ability to carry out a task or achieve a goal, a better process capability implies better product quality. If the process capability is evaluated with product survival lifetime, it is clear that a larger lifetime means better product quality, higher reliability, and the process is capable. Hence, the lifetime of products exhibit the larger- the better quality characteristic of time orientation. It should be noted that the lifetime of products does not follow a normal distribution. For instance, [14], [34], [6], [24] and [17] pointed out that the product lifetime possesses an exponential distribution. The description of the lifetime by the Weibull distribution was noted by [34], [40], [41] and [16]. In addition, [34] also mentioned that lifetime follows a gamma distribution. Since the lifetime of products exhibits the larger-the-better quality characteristic of time orientation, [35] and [23] recommended the use of the capability index (lifetime performance index) for evaluating the lifetime performance of electronic components, where L is the lower specification limit. Also, there are many different PCIs available in the literature. The hypothesis testing procedures are developed by [20] and [45] using the maximum likelihood estimator of PCI for Pareto distribution under type-II censored sampling and progressive type-I interval censoring, respectively. A hypothesis testing procedure was developed by [42] for the PCI of the Gompertz distribution based on the progressive type-I interval censored sample. The MLE was used by [46] to estimate the PCI of Rayleigh distribution based on the progressive type-I interval censored sample and developed a new hypothesis testing procedure utilizing an asymptotic distribution of this estimator. Some classical estimations and bootstrap confidence interval methods for the PCIs are derived by [36, 37] when the process follows exponentiated exponential and normal distributions, respectively. The classical and Bayes estimates of PCIs are obtained by [12, 13] for generalized exponential and normal distributions, respectively. For an expository review, the reader may follow the following articles of bibliography of the literature on PCIs, viz., [25], [38], [48] and [1].

The lifetime performance index (LPI) is defined by [23], denoted by C_L, which mainly originated from the concept of symmetry of lifetime distributions. The uniformly minimum variance unbiased estimator (UMVUE) for C_L was obtained by [39] and considered the problem of the hypothesis testing procedure for the exponential distribution. The UMVUE of C_L to develop the confidence interval under exponential distribution was obtained by [9]. The maximum likelihood estimator of the lifetime performance index based on first-failure progressive right type-II censored sample for Lindley distribution was obtained by [18]. The maximum likelihood estimates of the lifetime performance index based on progressive first failure censoring scheme Weibull, exponential and two-parameter exponential distributions, were obtained by [2, 3, 4] respectively. The lifetime performance index of products based on progressively Type-II censored for the Pareto samples was evaluated by [5]. The MLE, some Bayesian estimators, and credible intervals were given by [49] for the lifetime performance index of the Pareto distribution based on the general progressive type-II censored data. Approximate and exact parametric bootstrap confidence intervals are proposed by [50] for the process performance index of power-normal distribution. Most of the time, lifetime distributions are not necessarily symmetric. In this case, the median of the process distribution plays an important role than the process mean (μ). Therefore, it should be better if the index deals with the distribution's median (μ_e). If μ is replaced by μ_e, the inferential aspects and their property studies will be somewhat complicated. Using the median, [32] proposed a generalized process capability index (GPCI) that is the ratio of the proportion of specification conformance (or process yield) to the proportion of desired (or natural) conformance. In the same tune, [33] defined a GLPI given as

$$\begin{aligned} C_Y(L) &= \frac{0.5 - F(L)}{0.5 - \alpha} \\ &= \frac{1 - 2F(L)}{1 - 2\alpha}. \end{aligned} \tag{1}$$

Here $F(\cdot)$ denotes the cumulative distribution function (CDF) of the process distribution and $\alpha = P(X < LDL)$ with LDL being the lower desirable limit (practitioners sometimes take it as a lower tolerance limit). Here $(1 - \alpha)$ is the confidence level close to unity. Statistical inference for $C_Y(L)$, viz., properties of GLPI order, testing procedure for GLPI and parametric bootstrap confidence intervals based on a complete sample for the Lindley and in particular for exponential distribution have been obtained by [33].

In the case of a complete sample, it is necessary to continue the experiment until the last item (or product) fails. Sometimes, many articles have very long lifetimes, and the experiment continues for a very long period, so that the results may be of little interest or use. Then, it may be desirable to terminate the test before all the items under test fail, and the resulting observations will be called the censored sample. Various types of censored samples exist, including type II, progressive type II, and progressive first-failure censored samples. The testing of the hypothesis problem was proposed by [43] based on the maximum likelihood estimator (MLE) of C_L for two-parameter exponential distribution under type-II right censored sample. Based on a type-II right censored sample, the confidence interval using Pareto distribution was obtained by [20, 21]. A hypothesis testing procedure was proposed by [27, 28] based on MLE and UMVUE with the exponential distribution under progressive type-II right censored samples, respectively. The MLE of C_L was obtained by [19] under progressive first-failure censored samples from two-parameter exponential distributions. The C_L for the exponential lifetime products are evaluated by [29, 30] based on type-II censored data. They obtained Bayes's estimate of C_L for the Rayleigh lifetime products based on upper record values, respectively. Recently, [11] assessed the lifetime performance index for Weibull distributed products based on progressive type-II right censored samples.

In this article, we consider a progressive type-II right censoring scheme, which is helpful in a specific fraction of individuals at risk that may be removed from the experiment at each of several ordered failure times. Therefore, a progressive censoring scheme allows us to incorporate the removals before the experiment's termination into analysis, which is a very common situation in life-testing experiments. To the best of our knowledge thus far, an attempt has yet to be made to study the GLPI $C_Y(L)$ based on a progressive type-II right censoring scheme. Filling up this gap is the aim of the present study. In this article, we consider the GLPI $C_Y(L)$, introduced by [33] that could be used for either normal or non-normal and either continuous or discrete characteristics and is very simple and could be used comfortably by the practitioners.
The paper is arranged as follows. In section 2, The MLE and the Bayes estimate of $C_Y(L)$ are suggested based on progressive type II right censored sample for the Lindley distributed lifetime. In section 3, the testing procedure due to the GLPI is done. In section 4, an extensive Monte Carlo study is carried out to compare the performances of $C_Y(L)$ based on considered methods of estimation (MLE and Bayes) in terms of their corresponding mean squared errors (MSEs). A real-world application has been discussed to illustrate the proposed index under progressive type-II right censored samples in section 5. A brief concluding remark is made in section 6.

2. Estimation of $C_Y(L)$ under progressively type-II censored sample for Lindley products

Suppose that the lifetime of products may be modeled by Lindley distribution and let X denote the lifetime of such product. Hence, the probability density function (PDF) and cumulative distribution function (CDF), specified by Lindley distribution (see, [31]) are given as

$$f(x) = \frac{\theta^2}{\theta+1}(1+x)e^{-x\theta}, \quad x>0,\ \theta>0 \tag{2}$$

and

$$F(x) = 1-\frac{1+\theta+x\theta}{\theta+1}e^{-x\theta}, \quad x>0,\ \theta>0 \tag{3}$$

respectively, where, θ is the parameter. Now, for a process whose distribution can be regarded as Lindley, the GLPI is given as

$$\begin{aligned} C_Y(L) &= \left\{ \frac{1-2F(L)}{1-2\alpha} \right\} \\ &= \left\{ \frac{1-2\left(1-\frac{1+\theta+\theta L}{1+\theta}e^{-\theta L}\right)}{1-2\alpha} \right\}. \end{aligned} \tag{4}$$

Here, in the following subsections, we derived the maximum likelihood estimate (MLE) and the Bayes estimate of $C_Y(L)$ under progressively type-II right censoring scheme for Lindley distributed products, respectively.

2.1. Maximum likelihood estimate of $C_Y(L)$

The experimenter may not always observe the lifetimes of all the products (or items) put on tests for conducting life testing experiments. The reason may be time limitation and/or other restrictions (such as money, mechanical or experimental difficulties, material resources, etc.) on data collection. Therefore, censored samples may arise in practice. In an industrial experiment, products (or items) may break accidentally. These lead us into the area of progressive type-II censoring. Under this scheme, n units are placed on test at time zero, and m failures are observed. When the first failure is observed, r_1 of the surviving units are randomly selected and removed. At the second observed failure, r_2 of the surviving units are randomly selected and removed. Termination of the experiment occurs when the m-th failure is observed, and the remaining $r_m = n - \sum_{j=1}^{m-1} r_j - m$ surviving units are all removed. Inferences for the data obtained by progressive censoring have been investigated, among others, by [10], [7], [15], and [44]. So, in this paper, we consider the case of the progressive type-II right censoring.

Let $x_{1:m:n}$, $x_{2:m:n}$, ..., $x_{m:m:n}$ be a progressive type-II right censored sample where $x_{1:m:n}$, $x_{2:m:n}$, ..., $x_{m:m:n}$ denote the observed failure times and r_1, r_2, ..., r_m denote the corresponding numbers of items removed (withdrawn) from the test. If m be the number of failures observed before termination, then $x_{1:m:n} \leq x_{2:m:n} \leq$ ..., $\leq x_{m:m:n}$ be the observed ordered lifetimes. For convenience, we will write $x_{i:m:n}$ as $x_{(i)}$. Let r_i denote the number of items removed at the time of the ith failure, $0 \leq r_i \leq n - \sum_{j=1}^{i-1} r_j - i,\ i = 2, 3, ..., m-1$ with $0 \leq r_1 \leq n-1$ and $r_m = n - \sum_{j=1}^{m-1} r_j - m$, where r_i's and m are pres-specified integers [see, Viveros and Balakrishnan (1994)]. The complete sample ($r_1 = r_2 = ... = r_m = 0$) and type-II right censored samples ($r_1 = r_2 = ... = r_{m-1} = 0,\ r_m = n-m$) are special cases of this scheme. For further details and for relevant references the reader may follow the article of [7]. The likelihood function of θ under progressive type-II right censoring scheme is given by

$$\begin{aligned} l(\theta) &= A\prod_{i=1}^{m} f(x_{(i)};\theta)[1-F(x_{(i)};\theta)]^{r_i}, \text{ where } A = n(n-r_1-1)...(n-\sum_{j=1}^{m-1} r_j - m + 1) \\ &= A.\frac{\theta^{2m}}{(1+\theta)^{m+\sum_{i=1}^{m} r_i}} e^{-\theta\sum_{i=1}^{m}(1+r_i)x_{(i)}} \prod_{i=1}^{m}(1+x_{(i)}) \prod_{i=1}^{m}(1+\theta+\theta x_{(i)})^{r_i} \end{aligned} \tag{5}$$

Therefore, the log-likelihood function is given by

$$\begin{aligned} L(\theta) = \ln l(\theta) &= k + 2m\ln\theta - (m+\sum_{i=1}^{m} r_i)\ln(1+\theta) - \theta\sum_{i=1}^{m}(1+r_i)x_{(i)} \\ &\quad + \sum_{i=1}^{m} r_i \ln(1+\theta+\theta x_{(i)}), \end{aligned} \tag{6}$$

where k is constant, independent of θ. Now, for MLE of the parameter θ, $\frac{\partial L(\theta)}{\partial \theta} = 0$

$$\implies \frac{2m}{\theta} - \frac{m + \sum_{i=1}^{m} r_i}{1+\theta} - \sum_{i=1}^{m}(1+r_i)x_{(i)} + \sum_{i=1}^{m} \frac{r_i(1+x_{(i)})}{1+\theta+\theta x_{(i)}} = 0. \quad (7)$$

The explicit solution for the parameter θ through the above non-linear equation is not possible. Hence, to solve this equation for θ, we have to proceed by some numerical method, from the previous equation,

$$\frac{\partial^2 L(\theta)}{\partial \theta^2} = -\frac{2m}{\theta^2} + \frac{m + \sum_{i=1}^{m} r_i}{(1+\theta)^2} - \sum_{i=1}^{m} \frac{r_i(1+x_{(i)})^2}{(1+\theta+\theta x_{(i)})^2}.$$

Hence, the Fisher's information $I(\theta)$ is obtained as;

$$I(\theta) = -E(\frac{\partial^2 L(\theta)}{\partial \theta^2}) = \frac{2m}{\theta^2} - \frac{m + \sum_{i=1}^{m} r_i}{(1+\theta)^2} + \sum_{i=1}^{m} r_i E\left[\frac{(1+X_{(i)})^2}{(1+\theta+\theta X_{(i)})^2}\right] \quad (8)$$

Further, after obtaining the solution above mentioned non-linear equation, the MLE of the index $C_Y(L)$ can be directly computed using invariance property of MLE. Let $\hat{\theta}$ be the solution of the non-linear equation, then the MLE $\hat{C}_Y(L)$ of $C_Y(L)$ is given by

$$\hat{C}_Y(L) = \left\{ \frac{1 - 2\left(1 - \frac{1+\hat{\theta}+\hat{\theta}L}{1+\hat{\theta}} e^{-\hat{\theta}L}\right)}{1-2\alpha} \right\}. \quad (9)$$

The asymptotic distribution of MLE $\hat{\theta}$ is normal distribution $N(\theta, I^{-1}(\theta))$. Hence, the two-sided tail asymptotic $100(1-\alpha)\%$ confidence interval for the parameter θ is given by

$$\left\{ \hat{\theta} \mp \tau_{\alpha/2} \sqrt{I^{-1}(\hat{\theta})} \right\},$$

where $I(\hat{\theta}) = \frac{2m}{\hat{\theta}^2} - \frac{m+\sum_{i=1}^{m} r_i}{(1+\hat{\theta})^2} + \sum_{i=1}^{m} \frac{r_i(1+x_{(i)})^2}{(1+\hat{\theta}+\hat{\theta}x_{(i)})^2}$ and $\tau_{\alpha/2}$ is the upper $\alpha/2$-point of standard normal deviate. The asymptotic distribution of MLE $\hat{C}_Y(L)$ is also normal distribution $N(C_Y(L)$, $Var(\hat{C}_Y(L))$ with

$$Var(\hat{C}_Y(L)) = \left[\frac{\partial C_Y(L)}{\partial \theta}\right]^2 \times Var(\hat{\theta}) = \left[\frac{-2\theta L\{1+(1+\theta)(1+L)\}e^{-\theta L}}{(1-2\alpha)(1+\theta)^2}\right]^2 \times I^{-1}(\theta).$$

The asymptotic variance is approximated as

$$\hat{V}ar(\hat{C}_Y(L)) \approx \left[\frac{-2\hat{\theta} L\{1+(1+\hat{\theta})(1+L)\}e^{-\hat{\theta} L}}{(1-2\alpha)(1+\hat{\theta})^2}\right]^2 \times I^{-1}(\hat{\theta}).$$

2.2. Bayes estimate of $C_Y(L)$

In this section, we obtain the Bayes estimator of $C_Y(L)$ under the assumption that the parameter θ is random variable and follows some prior distribution. Let the prior distribution of θ is assumed to be Gamma with parameter (k, a). Then the distribution of θ is given as

$$g(\theta) = \frac{a^k}{\Gamma(k)} e^{-a\theta}\theta^{k-1}, \quad \theta > 0 \quad (10)$$

Now, the posterior distribution of θ by using Equations (2.5) and (2.9) is given as

$$g(\theta \mid \underline{x}) \propto \frac{\theta^{2m+k-1}}{(1+\theta)^{m+\sum_{i=1}^{m} r_i}} e^{-\theta(a+\sum_{i=1}^{m}(1+r_i)x_{(i)})} \prod_{i=1}^{m}(1+\theta+\theta x_{(i)})^{r_i} \quad ; \quad \theta > 0. \quad (11)$$

Hence, the Bayes estimate of the parameter θ under squared error loss function is obtained by the following Equation:

$$E_\theta(\theta) = \zeta \int_\theta \frac{\theta^{2m+k}}{(1+\theta)^{m+\sum_{i=1}^m r_i}} e^{-\theta(a+\sum_{i=1}^m (1+r_i)x_{(i)})} \prod_{i=1}^m (1+\theta+\theta x_{(i)})^{r_i} \, d\theta \tag{12}$$

where ζ is a proportionality constant. The computation of the Bayes estimate of the index $C_Y(L)$ under the same assumption of prior and loss function is not possible directly from the above posterior distribution. Since, the explicit form of the posterior PDF is not available but the associated plot exhibit a more or less assume the shape of normal probability distribution. Thus, the Metropolis-Hastings method with normal proposal distribution is to be used to generate random numbers from respective posterior distribution using the Gibbs algorithm. The following steps are taken to generate the posterior random deviates from the above posterior is as follows:

- Start with an initial guess $\theta^{(0)}$.
- Set $t = 1$.
- Using the Metropolis-Hastings, generate $\theta^{(t)}$ from $g(\theta \mid \underline{x})$ with the $N\left(\theta^{(t-1)},\ 1\right)$ proposal distribution.
- Compute $C_Y(L)^{(t)}$ from Equation (1)
- Set $t = t + 1$.
- Repeat steps 3-5, T times.

Note that in step 3, we use the Metropolis-Hastings algorithm with $q(\theta^{(t-1)},\ \sigma^2)$ proposal distribution as follows:

1. Let $x = \theta^{(t-1)}$.
2. Generate y from the proposal distribution q.
3. Let $p(x, y) = \min\left(1, \dfrac{g_\theta(y)q(x)}{g_\theta(x)q(y)}\right)$.
4. Accept y with the probability $p(x, y)$ or accept x with the probability $1 - p(x, y)$.

The posterior deviates for $C_Y(L)$ using the above mentioned steps is simulated using the random deviates of θ by plug-in principal. Let $C_Y(L)^1, C_Y(L)^2, \cdots, C_Y(L)^T$ be the T simulated posterior deviates, then the approximate posterior mean, and posterior variance of $C_Y(L)$ are given by

$$\hat{E}(C_Y(L)|\underline{x}) = \frac{1}{T}\sum_{t=1}^{T} C_Y(L)^t$$

and

$$MSE(C_Y(L)|\underline{x}) = \frac{1}{T}\sum_{t=1}^{T} \left(C_Y(L)^t - C_Y(L)\right)^2$$

respectively.

3. Testing procedure for the generalized lifetime performance index using progressive type-II samples

In this section, following statistical hypothesis testing will be performed to access whether the $C_Y(L)$ adheres the required level. The proposed hypothesis testing procedure using progressive type-II samples can be performed for $C_Y(L)$, summarized as follows:

1. Determine the lower specification limit L and the GLPI, C_Y^0.
2. Construct the null hypothesis $H_0 : C_Y(L) \leq C_Y^0$ against the alternative $H_1 : C_Y(L) > C_Y^0$.
3. Specify the level of significant α.
4. Compute $\hat{C}_Y(L)$ and $\hat{V}ar(\hat{C}_Y(L)) \approx [\frac{-2\hat{\theta}L\{1+(1+\hat{\theta})(1+L)\}e^{-\hat{\theta}L}}{(1-2\alpha)(1+\hat{\theta})^2}]^2.I^{-1}(\hat{\theta})$.
5. Set critical region $\omega : \hat{C}_Y(L) > C_Y^0 + \tau_\alpha \sqrt{\hat{V}ar(\hat{C}_Y(L))}$.

4. Simulation and discussion based on progressively type-II censored sample.

In this section, a comparison study has been carried out through simulation study under progressively type II censoring scheme between maximum likelihood and the Bayes estimate of $C_Y(L)$ in terms of their corresponding mean squared errors (MSEs) for the considered set up. All the calculations have been made by using R software (see, [22]). The progressive type-II right censored sample from the considered lifetime distribution for the different variation of the parameter (θ), censoring parameters (n, m), censoring schemes $(r_i' s)$ and lower desired limit (L) is generated by following the algorithm suggested by [8] algorithm, as stated below:

1. Generate m independent Uniform$(0, 1)$ observations $W_1,\ W_2,\ ...,\ W_m$.
2. Set $V_i = W_i^{1/(i+r_m+r_{m-1}+...+r_{m-i+1})}$ for $i = 1, 2, ..., m$.
3. Set $U_i = 1 - V_m.V_{m-1}...V_{m-i+1}$ for $i = 1, 2, ..., m$. Then $U_1, U_2, ..., U_m$ is the required progressive type-II censored sample from the Uniform $(0, 1)$ distribution.
4. Finally, we set $U_i = 1 - \frac{1+\theta+X_i\theta}{\theta+1}e^{-X_i\theta}$, and solve this equation by Newton-Raphson method to get X_i for $i = 1, 2, ..., m$. Then $X_1, X_2, ..., X_m$ is the required progressive type-II censored sample from the distribution (2).

The simulated MSE of the MLE and the Bayes estimate of $C_Y(L)$ have been presented in Tables 1-6 for different censoring schemes with some particular choices of θ and L. From the Tables 1-6, it is found that the MSE of each estimator is decreasing with n, the sample size. This verifies the consistency property of all the estimators. It is also observed that the performance of the Bayes estimation is relatively better than the MLE under all the considered choices.

5. Applications

A real data set is cited to illustrate the MLE, Bayes estimate of LPI C_L (see, [23]) and the proposed GLPI $C_Y(L)$ for progressive type-II right censoring scheme for the Lindley distributed lifetime. The considered data set is described in detail by [33] (also available in [26]), and the goodness of fit test to Lindley distribution is discussed therein. The data set is primarily fitted to the exponential model in Lawless. [33] have checked the data set with the Lindley distribution and found it to be a better fit. Thus, from the same data set, the progressive type-II censored data are generated for the different values of $m = 10, 15$, L=100 and the censoring schemes, and the corresponding MLE and Bayes estimates of $C_Y(L)$ is reported in the Table 7. The Bayes estimates for the real data set is computed under non-informative prior.
Now, the proposed testing procedure of the GLPI $C_Y(L)$ is performed for the above chosen schemes as follows. For the considered data set, the progressive type-II censored data are generated for the same censoring schemes, mentioned in Table 8, respectively. The MLE of the parameter θ for the Lindley distribution is obtained from the Eqn. (2.7) for all the considered schemes and the same are reported in Table 8. The statistical test for testing the null hypothesis $H_0 : C_Y(L) \leq 1$ against the alternative hypothesis $H_1 : C_Y(L) > 1$ has been performed for the

Table 1: *MLEs of $C_Y(L)$ and their MSEs with $L = 0.1$ and $L = 0.3$, samples generated from the Lindley distribution for $\theta = 0.50$ under progressively type-II censoring scheme [True $C_Y(L) = 1.073193$ when $L = 0.1$ and $C_Y(L) = 0.992842$ when $L = 0.3$].*

2[0]*n,m	2[0]*Schemes	L=0.1		L=0.3	
		$\hat{C}_Y(L)_{MLE}$	$MSE[\hat{C}_Y(L)_{MLE}]$	$\hat{C}_Y(L)_{MLE}$	$MSE[\hat{C}_Y(L)_{MLE}]$
7[0]*10, 8	0*10	1.07058	0.00149	0.98230	0.01431
	2, 0*7	1.06040	0.00316	0.99986	0.01086
	1,1, 0*6	1.06686	0.00326	0.99877	0.01036
	0*7, 2	1.06560	0.00272	0.98734	0.01451
	0*6, 1*2	1.07203	0.00191	0.99791	0.00939
	1, 0*6, 1	1.06470	0.00424	0.97192	0.01438
	0*3, 1*2, 0*3	1.06315	0.00349	0.99770	0.00973
7[0]*20, 16	0*20	1.07081	0.00072	0.99550	0.00439
	4, 0*15	1.06834	0.00083	1.00409	0.00546
	1*4, 0*12	1.06991	0.00108	0.99523	0.00572
	0*15, 4	1.07087	0.00106	0.98304	0.00653
	0*12, 1*4	1.07045	0.00138	1.00387	0.00463
	2,0*14,2	1.06791	0.00096	1.06689	0.00177
	0*7, 2*2, 0*7	1.06902	0.00092	0.99288	0.00449
7[0]*30, 24	0*30	1.07523	0.00048	0.99840	0.00301
	6,0*23	1.06878	0.00068	0.97993	0.00392
	1*6, 0*18	1.06788	0.00077	0.99505	0.00416
	0*23, 6	1.06747	0.00088	0.99042	0.00439
	0*18, 1*6	1.06858	0.00057	1.00475	0.00349
	3, 0*22, 3	1.06891	0.00063	0.99489	0.00358
	0*9, 1*6, 0*9	1.07182	0.00050	0.98809	0.00379
7[0]*50, 40	0*50	1.07370	0.00028	0.99244	0.00218
	10, 0*39	1.06526	0.00050	0.98831	0.00264
	2*5, 0*35	1.07372	0.00028	0.98873	0.00256
	0*39, 10	1.07123	0.00045	0.99057	0.00281
	0*30, 1*10	1.06765	0.00052	0.99992	0.00220
	5, 0*38, 5	1.07045	0.00032	0.98705	0.00375
	0*18, 2*5, 0*17	1.07152	0.00038	0.98859	0.00329

Note: In the table, Scheme $(0 * 3, r)$ indicates that at 1st, 2nd and 3rd failure, no active unit is withdrawn or removed but at 4th failure, r active units are drawn or removed.

Table 2: *Bayes estimates of $C_Y(L)$ and their MSEs with $L = 0.1$ and $L = 0.3$, samples generated from the Lindley distribution for $\theta = 0.50$ under progressively type-II censoring scheme [True $C_Y(L) = 1.073193$ when $L = 0.1$ and $C_Y(L) = 0.992842$ when $L = 0.3$].*

2[0]*n,m	2[0]*Schemes	L=0.1		L=0.3	
		$\hat{C}_Y(L)_{Bayes}$	$MSE[\hat{C}_Y(L)_{Bayes}]$	$\hat{C}_Y(L)_{Bayes}$	$MSE[\hat{C}_Y(L)_{Bayes}]$
7[0]*10, 8	0*10	1.07597	0.00135	0.97546	0.01372
	2, 0*7	1.06649	0.00283	0.99215	0.01049
	1,1, 0*6	1.07202	0.00310	0.99148	0.00999
	0*7, 2	1.07260	0.00245	0.97842	0.01403
	0*6, 1*2	1.07847	0.00168	0.98854	0.00911
	1, 0*6, 1	1.07092	0.00416	0.96363	0.01415
	0*3, 1*2, 0*3	1.06895	0.00315	0.99046	0.00958
7[0]*20, 16	0*20	1.07378	0.00066	0.99127	0.00437
	4, 0*15	1.07176	0.00077	0.99953	0.00533
	1*4, 0*12	1.07317	0.00101	0.99098	0.00568
	0*15, 4	1.07475	0.00098	0.97784	0.00658
	0*12, 1*4	1.07413	0.00128	0.99871	0.00449
	2,0*14,2	1.07175	0.00088	1.06302	0.00169
	0*7, 2*2, 0*7	1.07239	0.00085	0.98837	0.00439
7[0]*30, 24	0*30	1.07714	0.00047	0.99551	0.00297
	6,0*23	1.07127	0.00063	0.97659	0.00390
	1*6, 0*18	1.07023	0.00072	0.99216	0.00409
	0*23, 6	1.07036	0.00081	0.98673	0.00420
	0*18, 1*6	1.07131	0.00052	1.00128	0.00341
	3, 0*22, 3	1.07160	0.00059	0.99127	0.00345
	0*9, 1*6, 0*9	1.07408	0.00048	0.98499	0.00360
7[0]*50, 40	0*50	1.07486	0.00028	0.99075	0.00209
	10, 0*39	1.06679	0.00047	0.98640	0.00240
	2*5, 0*35	1.07516	0.00027	0.98682	0.00247
	0*39, 10	1.07291	0.00044	0.98830	0.00273
	0*30, 1*10	1.06939	0.00049	0.99764	0.00193
	5, 0*38, 5	1.07212	0.00030	0.98480	0.00357
	0*18, 2*5, 0*17	1.07286	0.00037	0.98670	0.00292

Table 3: *MLEs of $C_Y(L)$ and their MSEs with $L = 0.1$ and $L = 0.3$, samples generated from the Lindley distribution for $\theta = 0.75$ under progressively type-II censoring scheme [True $C_Y(L) = 1.038898$ when $L = 0.1$ and $C_Y(L) = 0.891517$ when $L = 0.3$].*

2[0]*n,m	2[0]*Schemes	L=0.1		L=0.3	
		$\hat{C}_Y(L)_{MLE}$	$MSE[\hat{C}_Y(L)_{MLE}]$	$\hat{C}_Y(L)_{MLE}$	$MSE[\hat{C}_Y(L)_{MLE}]$
10, 8	0*10	1.02408	0.00490	0.90298	0.01973
	2, 0*7	1.03005	0.00589	0.88015	0.02835
	1,1, 0*6	1.02637	0.00580	0.90158	0.02385
	0*7, 2	1.02057	0.00629	0.91253	0.02451
	0*6, 1*2	1.02306	0.00557	0.92649	0.02055
	1, 0*6, 1	1.03016	0.00484	0.88536	0.03157
	0*3, 1*2, 0*3	1.01554	0.00776	0.90067	0.01950
20, 16	0*20	1.03593	0.00230	0.89003	0.01112
	4, 0*15	1.03237	0.00296	0.88569	0.01545
	1*4, 0*12	1.03649	0.00246	0.90247	0.01130
	0*15, 4	1.02421	0.00318	0.89690	0.01192
	0*12, 1*4	1.02827	0.00291	0.88864	0.01237
	2,0*14,2	1.03430	0.00229	1.03440	0.00274
	0*7, 2*2, 0*7	1.03789	0.00200	0.89309	0.01031
30, 24	0*30	1.03208	0.00159	0.88711	0.00637
	6,0*23	1.03609	0.00180	0.89093	0.00982
	1*6, 0*18	1.03655	0.00173	0.88934	0.00759
	0*23, 6	1.03475	0.00164	0.89663	0.00958
	0*18, 1*6	1.03212	0.00182	0.89877	0.00774
	3, 0*22, 3	1.03816	0.00202	0.89079	0.01046
	0*9, 1*6, 0*9	1.03469	0.00183	0.89383	0.00753
50, 40	0*50	1.03908	0.00077	0.89749	0.00487
	10, 0*39	1.03609	0.00114	0.89273	0.00453
	2*5, 0*35	1.03278	0.00128	0.89426	0.00483
	0*39, 10	1.03953	0.00110	0.90031	0.00463
	0*30, 1*10	1.03555	0.00109	0.89815	0.00530
	5, 0*38, 5	1.03578	0.00128	0.89571	0.00574
	0*18, 2*5, 0*17	1.03867	0.00089	0.89162	0.00504

Table 4: *Bayes estimates of $C_Y(L)$ and their MSEs with $L = 0.1$ and $L = 0.3$, samples generated from the Lindley distribution for $\theta = 0.75$ under progressively Type-II censoring scheme [True $C_Y(L) = 1.038898$ when $L = 0.1$ and $C_Y(L) = 0.891517$ when $L = 0.3$].*

2[0]*n,m	2[0]*Schemes	L=0.1		L=0.3	
		$\hat{C}_Y(L)_{Bayes}$	$MSE[\hat{C}_Y(L)_{Bayes}]$	$\hat{C}_Y(L)_{Bayes}$	$MSE[\hat{C}_Y(L)_{Bayes}]$
10, 8	0*10	1.03297	0.00455	0.89338	0.01833
	2, 0*7	1.03916	0.00579	0.87174	0.02609
	1,1, 0*6	1.03618	0.00560	0.89210	0.02173
	0*7, 2	1.03254	0.00582	0.90036	0.02258
	0*6, 1*2	1.03506	0.00509	0.91270	0.01865
	1, 0*6, 1	1.04124	0.00444	0.87482	0.02897
	0*3, 1*2, 0*3	1.02560	0.00733	0.88980	0.01799
20, 16	0*20	1.04061	0.00223	0.88503	0.01078
	4, 0*15	1.03777	0.00285	0.88032	0.01488
	1*4, 0*12	1.04167	0.00237	0.89705	0.01079
	0*15, 4	1.03095	0.00294	0.88995	0.01139
	0*12, 1*4	1.03464	0.00272	0.88194	0.01196
	2,0*14,2	1.04039	0.00220	1.02836	0.00293
	0*7, 2*2, 0*7	1.04343	0.00193	0.88711	0.00992
30, 24	0*30	1.03536	0.00152	0.88355	0.00628
	6,0*23	1.03974	0.00175	0.88705	0.00954
	1*6, 0*18	1.04011	0.00168	0.88558	0.00743
	0*23, 6	1.03937	0.00157	0.89173	0.00930
	0*18, 1*6	1.03660	0.00174	0.89408	0.00747
	3, 0*22, 3	1.04224	0.00197	0.88630	0.01015
	0*9, 1*6, 0*9	1.03840	0.00177	0.88977	0.00734
50, 40	0*50	1.04110	0.00076	0.89530	0.00479
	10, 0*39	1.03837	0.00111	0.89024	0.00445
	2*5, 0*35	1.03504	0.00124	0.89184	0.00473
	0*39, 10	1.04223	0.00109	0.89728	0.00452
	0*30, 1*10	1.03823	0.00106	0.89516	0.00518
	5, 0*38, 5	1.03836	0.00125	0.89290	0.00565
	0*18, 2*5, 0*17	1.04098	0.00088	0.88919	0.00497

Table 5: *MLEs of $C_Y(L)$ and their MSEs with $L = 0.1$ and $L = 0.3$, samples generated from the Lindley distribution for $\theta = 1.50$ under progressively type-II censoring scheme [True $C_Y(L) = 0.916334$ when $L = 0.1$ and $C_Y(L) = 0.560892$ when $L = 0.3$].*

2[0]*n,m	2[0]*Schemes	L=0.1		L=0.3	
		$\hat{C}_Y(L)_{MLE}$	$MSE[\hat{C}_Y(L)_{MLE}]$	$\hat{C}_Y(L)_{MLE}$	$MSE[\hat{C}_Y(L)_{MLE}]$
7[0]*10, 8	0*10	0.92111	0.01876	0.59670	0.06439
	2, 0*7	0.92292	0.02211	0.58234	0.07653
	1,1, 0*6	0.92693	0.01922	0.55731	0.06999
	0*7, 2	0.92967	0.01968	0.57833	0.07527
	0*6, 1*2	0.93835	0.01868	0.60658	0.06007
	1, 0*6, 1	0.92819	0.02075	0.59201	0.06345
	0*3, 1*2, 0*3	0.91772	0.02080	0.57585	0.06962
7[0]*20, 16	0*20	0.92597	0.00852	0.57983	0.02738
	4, 0*15	0.91773	0.01063	0.56602	0.03939
	1*4, 0*12	0.92573	0.01097	0.57893	0.03167
	0*15, 4	0.91871	0.01025	0.58104	0.03375
	0*12, 1*4	0.91498	0.01218	0.59792	0.03509
	2,0*14,2	0.91435	0.01054	0.58679	0.03709
	0*7, 2*2, 0*7	0.91880	0.00977	0.57989	0.03197
7[0]*30, 24	0*30	0.91462	0.00617	0.56864	0.01894
	6,0*23	0.91960	0.00780	0.57341	0.02488
	1*6, 0*18	0.92418	0.00739	0.57963	0.02501
	0*23, 6	0.92321	0.00829	0.57843	0.02103
	0*18, 1*6	0.92471	0.00667	0.56706	0.02135
	3, 0*22, 3	0.91766	0.00781	0.57184	0.01984
	0*9, 1*6, 0*9	0.91798	0.00590	0.55905	0.02281
7[0]*50, 40	0*50	0.91497	0.00387	0.57058	0.01225
	10, 0*39	0.91995	0.00436	0.56369	0.01427
	2*5, 0*35	0.92503	0.00390	0.56944	0.01577
	0*39, 10	0.92130	0.00401	0.56377	0.01520
	0*30, 1*10	0.91975	0.00443	0.57915	0.01139
	5, 0*38, 5	0.91687	0.00477	0.56287	0.01436
	0*18, 2*5, 0*17	0.91364	0.00370	0.55129	0.01093

Table 6: *Bayes estimates of $C_Y(L)$ and their MSEs with $L = 0.1$ and $L = 0.3$, samples generated from the Lindley distribution for $\theta = 1.5$ under progressively Type-II censoring scheme [True $C_Y(L) = 0.916334$ when $L = 0.1$ and $C_Y(L) = 0.560892$ when $L = 0.3$].*

2[0]*n,m	2[0]*Schemes	L=0.1		L=0.3	
		$\hat{C}_Y(L)_{Bayes}$	$MSE[\hat{C}_Y(L)_{Bayes}]$	$\hat{C}_Y(L)_{Bayes}$	$MSE[\hat{C}_Y(L)_{Bayes}]$
7[0]*10, 8	0*10	0.90365	0.01723	0.58724	0.05271
	2, 0*7	0.90402	0.01989	0.57706	0.05956
	1,1, 0*6	0.90828	0.01743	0.55465	0.05433
	0*7, 2	0.90508	0.01810	0.57052	0.05563
	0*6, 1*2	0.91331	0.01689	0.59403	0.04676
	1, 0*6, 1	0.90561	0.01861	0.58232	0.05013
	0*3, 1*2, 0*3	0.89825	0.01857	0.57098	0.05346
7[0]*20, 16	0*20	0.91626	0.00821	0.57459	0.02488
	4, 0*15	0.90710	0.01022	0.56201	0.03504
	1*4, 0*12	0.91540	0.01042	0.57481	0.02815
	0*15, 4	0.90527	0.01001	0.57409	0.02974
	0*12, 1*4	0.90220	0.01193	0.59034	0.03104
	2,0*14,2	0.90213	0.01034	0.57034	0.03124
	0*7, 2*2, 0*7	0.90799	0.00946	0.57521	0.02814
7[0]*30, 24	0*30	0.90805	0.00612	0.56529	0.01775
	6,0*23	0.91221	0.00761	0.57023	0.02298
	1*6, 0*18	0.91709	0.00714	0.57655	0.02317
	0*23, 6	0.91407	0.00807	0.57355	0.01931
	0*18, 1*6	0.91574	0.00648	0.56287	0.01980
	3, 0*22, 3	0.90917	0.00764	0.56766	0.01829
	0*9, 1*6, 0*9	0.91040	0.00581	0.55630	0.02116
7[0]*50, 40	0*50	0.91096	0.00385	0.56855	0.01179
	10, 0*39	0.91526	0.00428	0.56163	0.01363
	2*5, 0*35	0.92053	0.00379	0.56757	0.01504
	0*39, 10	0.91552	0.00393	0.56076	0.01451
	0*30, 1*10	0.91423	0.00437	0.57613	0.01084
	5, 0*38, 5	0.91159	0.00473	0.56055	0.01368
	0*18, 2*5, 0*17	0.90905	0.00370	0.54963	0.01048

pre-chosen level of significance $\alpha = 5\%$ and $L = 100$. Also, the estimate of GLPI $C_Y(L)$ and the one-sided 95% confidence interval for $C_Y(L)$, i.e., $[L_B, \infty)$ are computed and are reported in Table 8. From the obtained result it can be verified that the value of $C_Y^0 = 1$ does not belong to the one-sided confidence interval, thus the null hypothesis H_0 is rejected. Hence, the rejection of the null hypothesis indicates that the GLPI for the considered censored observations meets the required level.

Table 7: *Real data estimates of C_L and $C_Y(L)$ for different censoring schemes where $L = 100$.*

n	m	Schemes	$\hat{C}_{LMLE}$	$\hat{C}_{LBayes}$	$\hat{C}_Y(L)_{MLE}$	$\hat{C}_Y(L)_{Bayes}$
6[0]*19	3[0]*10	0,1*9	1.29976	1.31230	1.06033	1.06053
		1*9,0	1.32879	1.30256	1.07422	1.07421
		0*4,3*3,0*3	1.33256	1.32036	1.08983	1.08686
	3[0]*15	0*11,1*4	1.26546	1.27359	1.05995	1.05736
		1*4,0*11	1.24328	1.25370	1.06700	1.05697
		0*6,1*4,0*5	1.25042	1.23734	1.08076	1.07896

Table 8: *Testing of hypothesis of C_L, $C_Y(L)$ for different censoring schemes based on considered data set.*

n	m	Schemes	$\hat{\theta}$	$\hat{C}(L)_{MLE}$	L'_B	$\hat{V}ar(\hat{C}(L))$	$\hat{C}_Y(L)_{MLE}$	L_B	$\hat{V}ar(\hat{C}_Y(L))$
6[0]*19	3[0]*10	0,1*9	0.001602602	1.29976	1.145873	0.00032479	1.06033	1.06053	0.00012319
		1*9,0	0.001193257	1.32879	1.267304	0.00025823	1.07422	1.07421	0.00004212
		0*4,3*3,0*3	0.001438218	1.33256	1.192645	0.00025178	1.08983	1.08686	0.00008373
	3[0]*15	0*11,1*4	0.002093421	1.26546	1.238082	0.0000432	1.05995	1.05736	0.00030325
		1*4,0*11	0.002393421	1.24328	1.253402	0.00089531	1.06700	1.05697	0.00024422
		0*6,1*4,0*5	0.002293421	1.25042	1.210789	0.000045286	1.08076	1.07896	0.00030897

6. Conclusions

The present article considers the problem of estimating the GLPI, introduced by [32], under progressive type-II right censored sample where the lower specification limit is given for the Lindley distributed products. The model parameter and the GLPI are obtained by the MLE and Bayes estimation methods, respectively. A comparison study has been carried out through the Monte Carlo simulation study under a progressive type-II censoring scheme between MLE and the Bayes estimate of GLPI in their corresponding MSEs. A real data set is analyzed to study the performance of the proposed index. Though the approach of classical estimation and Bayes estimation are different in direction, assuming the gamma prior and using MCMC method, we discussed the Bayes estimation of $C_Y(L)$. The Bayes estimate of the performance index is relatively better than MLE in terms of corresponding MSEs. The proposed procedure can be extended to obtain the confidence interval of $C_Y(L)$ based on MLE and Bayes estimate to evaluate whether the product quality meets the required level. In our upcoming course of work, the problem will be attempted. We may use the MLE and the Bayes estimate of $C_Y(L)$ based on the progressive type-II right censored sample to draw conclusions about additional lifetime distributions in future.

Declarations

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