A NOVEL METHODOLOGY IN DEVELOPING STRESS-STRENGTH RELIABILITY MODEL FOR WEIBULL DISTRIBUTION: A COMPARISON OF ARTIFICIAL NEURAL NETWORK (ANN) AND RESPONSE SURFACE ANALYSIS (RSA)

Dr. Saurabh L. Raikar[1], Prof. Rajesh S. Prabhu Gaonkar[2]

[1]Mechanical Engineering Department, Goa College of Engineering (affiliated to Goa University), Goa
[2]Indian Institute of Technology Goa (IIT Goa), Farmagudi, Goa
[1]saurabhr3003@gmail.com , [2]rpg@iitgoa.ac.in

Abstract

Stress strength interference theory is widely used in evaluating the reliability of mechanical components. Various interference models have been developed when stress and strength follow a wide range of distributions. But when stress and strength follow Weibull distribution, a closed form of interference model is not available. This paper deals with developing a methodology for obtaining a closed form interference model for a given application when the stress and strength follow Weibull distribution. The method of artificial neural network (ANN) and response surface analysis (RSA) are used in modelling and analysis. The validation experiment has been conducted and the error obtained shows that the proposed methodology performs reasonably well.

Keywords: Weibull distribution, stress strength interference, reliability, ANN, RSM

I. Introduction

Reliability is gaining increasing importance in recent years as it takes into account the uncertainty present in the properties. One of the most common theories used in estimating reliability is the stress strength interference theory. The theory says that if X and Y follow a particular distribution, then their interference area gives the probability of failure [1]. Many interference models have been developed when stress and strength follow various distributions like normal, lognormal, exponential, etc. But, when stress and strength follow Weibull distribution, the closed form of interference model is not available. Weibull distribution is widely used statistical and reliability studies because of its ability to fit a wide range of data.

II. Stress Strength interference models

Reliability models have been developed when strength and stress are seen to be following normal, lognormal, exponential distribution, etc. [2]. This section presents reliability models for some of the most widely used distributions.

I. Reliability when stress and strength follow normal distribution

Consider the strength (random variable X) and stress (random variable Y) follow normal distribution with pdf

$$f(x) = \frac{1}{\delta_x\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{x-\mu_{nx}}{\delta_x}\right)^2\right) \tag{1}$$

and

$$f(y) = \frac{1}{\delta_y\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{y-\mu_{ny}}{\delta_y}\right)^2\right) \tag{2}$$

respectively, where μ_{nx} is the mean of strength, δ_x is the standard deviation of strength, μ_{ny} is the mean of stress and δ_y is the standard deviation of strength. As per the interference theory, the reliability of the system will be equal to [3]

$$R = \int_0^\infty \frac{1}{\delta_x\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{x-\mu_{nx}}{\delta_x}\right)^2\right)\left(\int_0^x \frac{1}{\delta_y\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{y-\mu_{ny}}{\delta_y}\right)^2\right)dy\right)dx \tag{3}$$

On simplifying the above equation, the reliability can be obtained as

$$R = \phi\left(\frac{\mu_{nx}-\mu_{ny}}{\sqrt{{\delta_x}^2+{\delta_y}^2}}\right) \tag{4}$$

II. Reliability when stress and strength follow lognormal distribution

Consider that the strength (random variable X) and stress (random variable Y) follow lognormal distribution with pdf

$$f(x) = \frac{1}{x.\delta_x\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{\ln(x)-\mu_{nx}}{\delta_x}\right)^2\right) \tag{5}$$

and

$$f(y) = \frac{1}{y.\delta_y\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{\ln(y)-\mu_{ny}}{\delta_y}\right)^2\right) \tag{6}$$

respectively, where μ_{nx} is the mean and δ is the standard deviation of ln(X), μ_{ny} is the mean and δ_y is the standard deviation of ln(Y). As per the interference theory, the reliability of the system will be equal to [4]

$$R = \int_0^\infty \frac{1}{x.\delta_x\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{\ln(x)-\mu_{nx}}{\delta_x}\right)^2\right)\left(\int_0^x \frac{1}{y.\delta_y\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{\ln(y)-\mu_{ny}}{\delta_y}\right)^2\right)dy\right)dx \tag{7}$$

On simplifying the above equation, the reliability can be obtained as

$$R = \phi\left(\frac{\mu_{nx}-\mu_{ny}}{\sqrt{{\delta_x}^2+{\delta_y}^2}}\right) \tag{8}$$

III. Reliability when stress and strength follow exponential distribution

Consider that the strength (random variable X) and stress (random variable Y) follow lognormal distribution with pdf

$$f(x) = \lambda_x \, e^{-\lambda_x x} \tag{9}$$

and

$$f(y) = \lambda_y \, e^{-\lambda_y y} \tag{10}$$

where λ is the rate parameter.

As per the interference theory, the reliability of the system will be equal to

$$R = \int_0^{\infty} \lambda_x \, e^{-\lambda_x x} \left(\int_0^{x} \lambda_y \, e^{-\lambda_y y} dy \right) dx \tag{11}$$

On simplifying the above equation, the reliability can be obtained as [5]

$$R = \frac{\lambda_y}{\lambda_x + \lambda_y} \tag{12}$$

IV. Reliability when stress and strength follow Weibull distribution

Consider that the strength (random variable X) and stress (random variable Y) follow Weibull distribution with pdf

$$f(x) = \frac{p_1}{{\sigma_1}^{p_1}} (x - \mu_1)^{p_1 - 1} \exp\left\{ -\left(\frac{x - \mu_1}{\sigma_1} \right)^{p_1} \right\}, x > \mu_1, \sigma_1 > 0, p_1 > 0 \tag{13}$$

and

$$f(y) = \frac{p_2}{{\sigma_2}^{p_2}} (y - \mu_2)^{p_2 - 1} \exp\left\{ -\left(\frac{y - \mu_2}{\sigma_2} \right)^{p_2} \right\}, y > \mu_2, \sigma_2 > 0, p_2 > 0 \tag{14}$$

where μ_1, σ_1 *and* p_1 are the location, scale and shape parameter respectively for strength and μ_2, σ_2 *and* p_2 are location, scale and shape parameter respectively for stress. As per the interference theory, the reliability of the system will be equal to

$$R = \int_0^{\infty} \frac{p_1}{{\sigma_1}^{p_1}} (x - \mu_1)^{p_1 - 1} \exp\left\{ -\left(\frac{x - \mu_1}{\sigma_1} \right)^{p_1} \right\} \left(\int_0^{x} \frac{p_2}{{\sigma_2}^{p_2}} (y - \mu_2)^{p_2 - 1} \exp\left\{ -\left(\frac{y - \mu_2}{\sigma_2} \right)^{p_2} \right\} dy \right) dx \tag{15}$$

$$R = \int_0^{\infty} \frac{p_1}{{\sigma_1}^{p_1}} (x - \mu_1)^{p_1 - 1} \exp\left\{ -\left(\frac{x - \mu_1}{\sigma_1} \right)^{p_1} \right\} \left(e^{-\frac{e^{p_2 \ln(-\mu_2)}}{{\sigma_2}^{p_2}}} - e^{-\frac{e^{p_2 \ln(x - \mu_2)}}{{\sigma_2}^{p_2}}} \right) dx \tag{16}$$

Equation 16 cannot be solved further and hence, the calculation of stress-strength reliability for Weibull distribution has to be solved using numerical or graphical methods [6]. This can sometimes lead to complications and is time consuming.

Similarly, the reliability models have been developed for other distributions of stress and strength. S. Nadarajah, 2003 [7] developed stress-strength interference for stress and strength following lifetime distributions i.e. exponential and gamma distribution. He has also developed a reliability model for stress and strength following bivariate gamma distribution [8]. Patowary et al., 2013 [9] studied and proposed a mathematical model for stress-strength reliability for stress and strength following mixture of distributions. An inference on reliability was also drawn, stating standby redundancy aids in achieving high reliability. An et al., 2008 [10] developed a discrete stress-strength interference model based on universal generating function. K. Shen, 1992 [11] proposed a new empirical approach based on the subinterval probabilities of stress and strength in the interference region to compute the unreliability bounds. Kotz et al., 2003 [12] reviewed the stress-strength interference models and showed practical results in application of stress-strength interference concepts in industrial systems. Many studies have been carried out in developing stress-strength reliability models for various distributions. However, it has been identified that the reliability model for stress and strength following Weibull distributions has not been developed yet.

III. Design of experiments

Design of experiments (DOE) is a systematic tool to find the relations between the input variables and the response. DOE gives a significant experimental setup sufficient to find the relation between input variables and output response which helps in saving time, cost and resources. Taguchi method and response surface analysis (RSA) are some of the widely used techniques in DOE. Taguchi method is used to obtain a set of significant experiments and to find the most influential parameter towards the response. RSA is also used to obtain a set of significant parameters and analyze for the influencing parameters. Additionally, RSA gives a prediction model of input variables and the output response. Khare et al, 2018 [13] used DOE in optimizing the surface roughness of AA 6061 material in turning operation. The cutting speed, feed rate, depth of cut and rake angle were taken as the input parameters while the surface roughness was taken as the response. Taguchi's method was used for carrying out the DOE and analysis. It was found that the rake angle was the most influential parameter towards the surface roughness followed by cutting speed. The set of optimum parameters was also found. Similarly Taguchi analysis has been used by many researchers in their studies [14]–[17]. Laghari et al., 2018 [18] developed prediction models for tool wear and surface roughness in turning of Al/SiCp workpiece using response surface analysis (RSA). Cutting speed, feed rate and depth of cut were taken as the independent variables towards the response. The response surface analysis was proved to be effective in modeling the prediction equation. Ammeri et al., 2015 [19] combined Taguchi method and RSA in determining the optimal lot size for the manufactured product in supply chain. RSA has been used to develop models and carrying out analysis of parameters and response [20]–[23]. Nair et al., 2004 [24] used design of experiments for design of accelerated test experiments for reliability improvement. Rigdon et al., 2022 [25] studied on the use of design of experiments to understand and improve product reliability. A detailed description of the statistical distributions, methods to model reliability, various DOE methods that can be used and the analysis that can be carried out has been made in this research.

IV. Weibull stress strength model

Weibull distribution is most commonly used to describe mechanical systems. The interference model of reliability when the stress and strength follow Weibull distribution is given as:

$$R = \int_0^\infty \frac{p_2}{\sigma_2}\left(\frac{s-\mu_2}{\sigma_2}\right)^{p_2-1} e^{-\frac{(s-\mu_2)}{\sigma_2}} \left[\int_0^s \left(\frac{l-\mu_1}{\sigma_1}\right)^{p_1-1} e^{-\frac{(l-\mu_1)}{\sigma_1}} dl\right] ds \tag{17}$$

The integration of the above model is complicated and does not have a closed form. This study attempts to obtain the closed form of stress-strength reliability using design of experiments (DOE) when the stress and strength follow Weibull distribution. Minitab 16 was the software used to conduct design of experiments and analysis. The design chosen was L27 for 6 factors of 3 level each. The parameters chosen for DOE are shown in Table 1. The results of design of experiments with response are displayed in Table 2. The stress-strength equation was partly solved manually and partly using Wolfram Mathematica software.

Table 1 *Factors and levels for stress and strength following Weibull distribution*

Distribution	Factors	Levels
Stress	Shape Parameter: p_1	0.5, 1.5, 2.5
	Scale Parameter: σ_1	1, 1.5, 2
	Location Parameter: μ_1	0, 0.5, 1
Strength	Shape Parameter: p_2	1.5, 2.5, 3.5
	Scale Parameter: σ_2	1, 1.5, 2
	Location Parameter: μ_2	1.5, 2, 2.5

Table 2 *Design of experiments for stress and strength following Weibull distribution*

Stress			Strength			Reliability
p1	σ1	μ1	p2	σ2	μ2	R
0.5	1	0	1.5	1	1.5	0.78155
0.5	1	0	1.5	1.5	2	0.83273
0.5	1	0	1.5	2	2.5	0.86709
0.5	1.5	0.5	2.5	1	1.5	0.67035
0.5	1.5	0.5	2.5	1.5	2	0.74267
0.5	1.5	0.5	2.5	2	2.5	0.79106
0.5	2	1	3.5	1	1.5	0.56323
0.5	2	1	3.5	1.5	2	0.65846
0.5	2	1	3.5	2	2.5	0.72007
1.5	1	0.5	3.5	1	2	0.97066
1.5	1	0.5	3.5	1.5	2.5	0.996097
1.5	1	0.5	3.5	2	1.5	0.979634
1.5	1.5	1	1.5	1	2	0.722195
1.5	1.5	1	1.5	1.5	2.5	0.8851964
1.5	1.5	1	1.5	2	1.5	0.743035
1.5	2	0	2.5	1	2	0.815387
1.5	2	0	2.5	1.5	2.5	0.91893406
1.5	2	0	2.5	2	1.5	0.85073635
2.5	1	1	2.5	1	2.5	0.99773487
2.5	1	1	2.5	1.5	1.5	0.91938117
2.5	1	1	2.5	2	2	0.993511113
2.5	1.5	0	3.5	1	2.5	0.99883877
2.5	1.5	0	3.5	1.5	1.5	0.97996489
2.5	1.5	0	3.5	2	2	0.998682329
2.5	2	0.5	1.5	1	2.5	0.87792976
2.5	2	0.5	1.5	1.5	1.5	0.6762277
2.5	2	0.5	1.5	2	2	0.87038164

I. Response surface analysis

Response surface analysis has been successfully implemented in modelling studies. In this article, the response surface analysis was carried out to model an equation for stress and strength following Weibull distribution and to study the two-parameter interaction towards the reliability. The equation for reliability model developed using response surface analysis is

$$
\begin{aligned}
R = {} & 0.879 + 0.1346\, p1 - 0.6489\, \sigma1 - 0.3621\, \mu1 + 0.2496\, p2 + 0.5987\, \sigma2 - 0.4066\, \mu2 \\
& - 0.04577\, p1{*}p1 + 0.0502\, \sigma1{*}\,\sigma1 + 0.0215\, \mu1{*}\,\mu1 - 0.01542\, p2{*}\, p2 - 0.0504\, \sigma2{*}\,\sigma2 \\
& + 0.0009\, \mu2{*}\,\mu2 - 0.00085\, p1{*}\,\sigma2 + 0.04883\, p1{*}\mu2 - 0.0652\, \sigma1{*}\,\sigma2 + 0.2211\, \sigma1{*}\,\mu2 \\
& - 0.0930\, \mu1{*}\,\sigma2 + 0.1933\, \mu1{*}\mu2 - 0.0925\, p2{*}\,\sigma2
\end{aligned} \tag{18}
$$

The R-sq value for the above equation is 99.82% which shows that the equation can predict reliability with significantly less variability. Figure 1 shows the main effects plot for reliability. It can be seen that the reliability increases with increase in location and scale parameter of strength, and decrease with increase in location and scale parameter of stress. A notable observation that can be made is that reliability increases with increase in shape parameter of both the stress and strength distribution. Figure 2 shows the interaction plot for reliability. Figure 3 shows contour plot of two parameter interaction towards reliability for stress and strength following Weibull distribution. When the parameters are held at middle values from the levels considered, a high reliability greater than 0.9 is obtained when parameter set lies in a region inscribed by the origin and the following as shown in the figure: p1 greater than 1.4 and σ1 on the minimum side preferably lesser than 1.4 in p1 x σ1 interaction, μ1 lesser than 0.25 and p1 greater than 1.7 in μ1 x p1 interaction, μ2 greater than 2.2 and p1 greater than 1.7 in μ2 x p1 interaction, μ1 lesser than 0.5 and σ1 lesser than 1.4 in μ1 x σ1 interaction, p2 greater than 2.4 and σ1 on the minium side in p2 x σ1 interaction, σ2 greater than 1.5 and σ1 lesser than 1.25 in σ2 x σ1 interaction, σ1 close to 1 in μ2 x σ1 interaction, and σ2 greater than 1.75 and μ1 lesser than 0.2 σ2 x μ1 interaction.

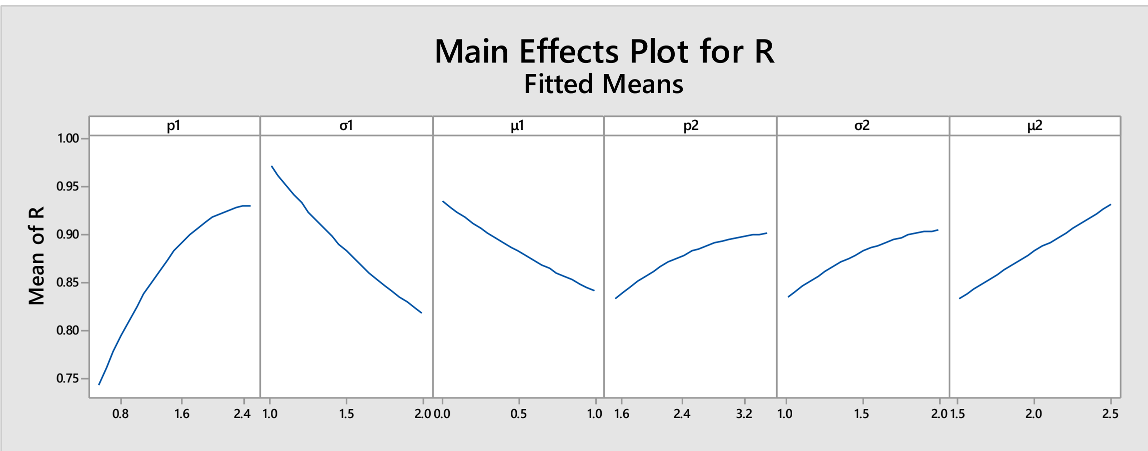

Figure 1 *Main effects plot for reliability for Weibull stress and strength*

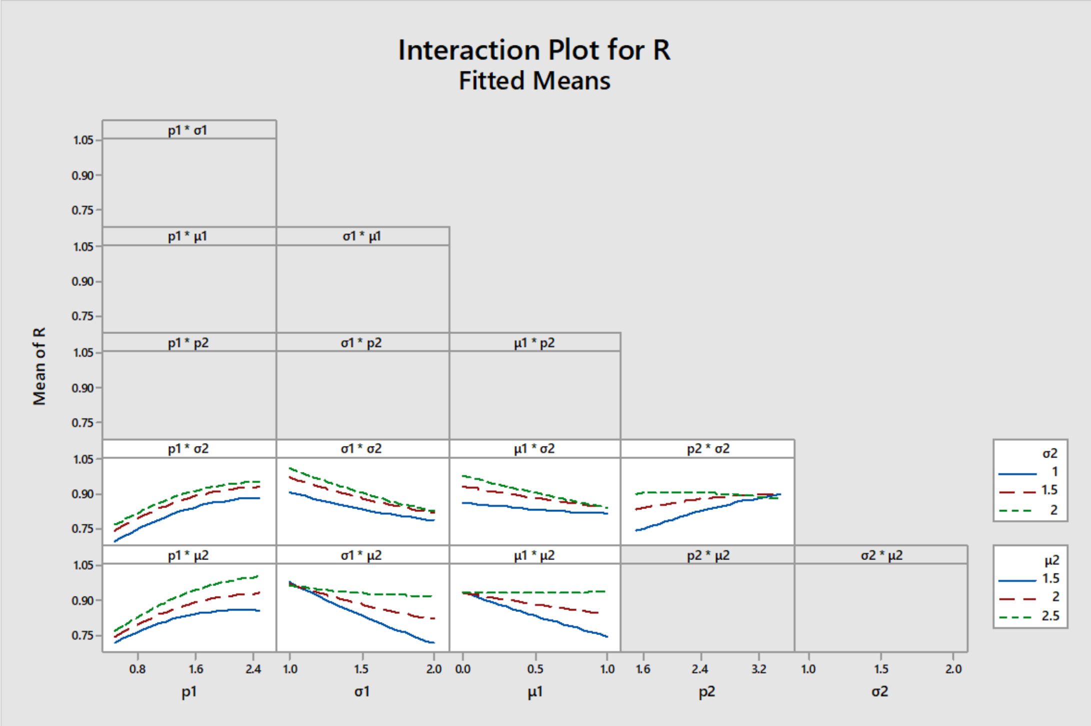

Figure 2 *Interaction plot for reliability for Weibull stress and strength*

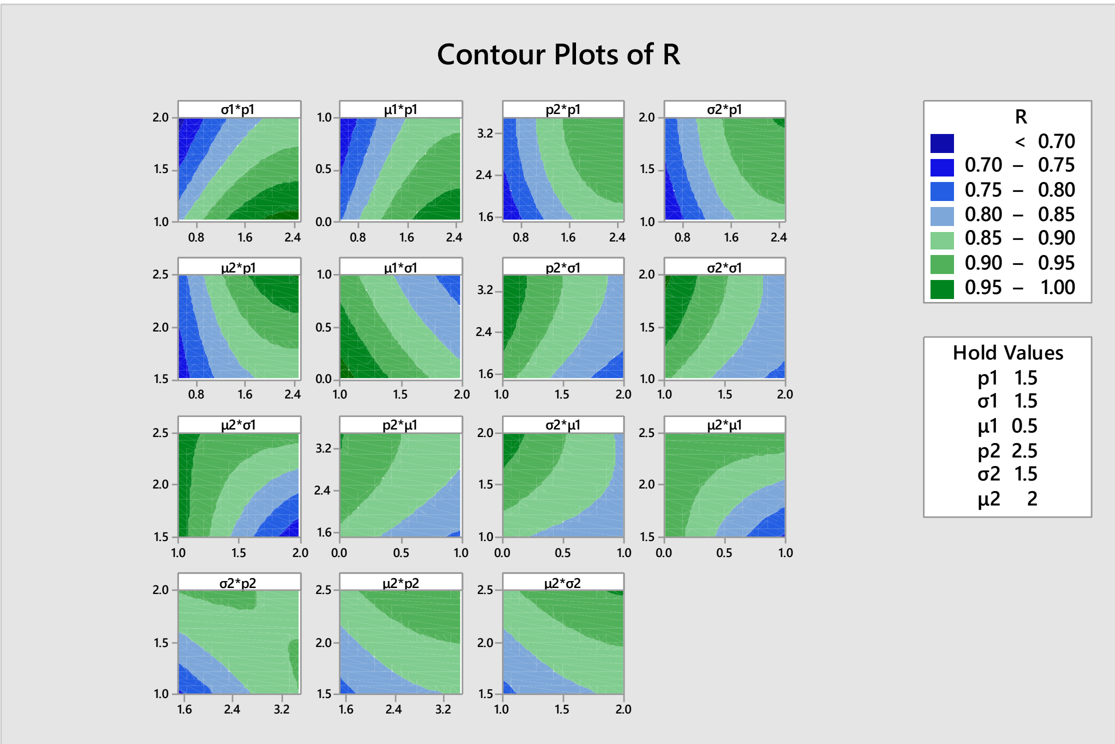

Figure 3 *Contour plot for reliability in case of stress and strength following Weibull distribution*

II. Artificial Neural Network (ANN)

Artificial Neural Network is a modelling technique based on artificial neurons which are a set of interconnected units or nodes that loosely resemble the neurons in a biological brain. Each link has

the ability to communicate with other neurons, much like the synapses in a human brain. An artificial neuron receives a signal, processes it and sends a signal to the neurons connected to it. Each neuron computes its output using a non-linear function of the sum of its inputs, where the "signal" at a connection is a real value. The method of ANN has been used in many recent applications and has shown great performance in modelling studies [26]. Abbasi et al., 2008 [27] deployed ANN method in estimation of the three parameters of Weibull distribution and obtained satisfactory results. Also, the authors compared the method with other techniques used in the application and concluded that the application of ANN is easier compared to the other methods.
In this study, the inputs to ANN model are the parameters of the distribution and the output is stress-strength reliability. ANN model consisting of 6 nodes in the input layer, a single hidden layer with 5 nodes and a single node output layer has been selected and is depicted in Figure 4.

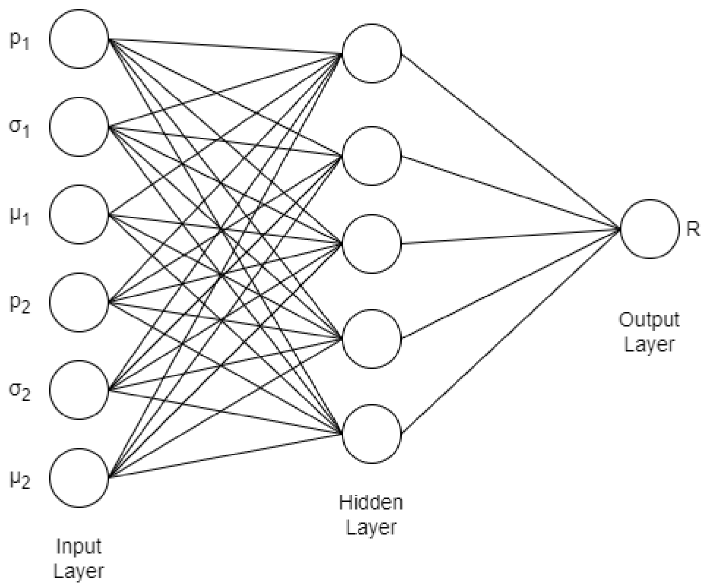

Figure 4 *ANN model with an input layer, hidden layer and output layer*

The model equation is given by

$$y(output) = b_2 + \sum_{k=1}^{h} \left[w_k f_{sig} \left(b_{1k} + \sum_{i=1}^{m} w_{ik} x_i \right) \right] \quad (19)$$

where,
y is the output, xi is the input of ith node, k is the number of nodes in hidden layer,
k is the number of nodes in the hidden layer.
wik is the weight connecting ith node in the input layer and kth node in the hidden layer.
wk is the weight connecting the kth node in the hidden layer and the output node.
b1k is the bias of the kth node in the hidden layer, b2 is the bias of the output node.
The function f_{sig} is given by

$$f_{sig}(a) = \frac{2}{1 + e^{-2a}} - 1 \quad (20)$$

Around 70% of the data points have been used in training, 15% for validation and 15% for testing. Levenberg-Marquardt algorithm has been used and the training is conducted till the regression coefficients for training, validation and testing are more than 0.98 and the MSE is less than 10-3. The regression plots for training, validation and testing are shown on Figure 5. Weights and biases of the trained neural network are as follows:

$$W_{ik} = \begin{bmatrix} 0.643333 & 0.821043 & 0.609942 & -0.82412 & -0.56263 & -1.03838 \\ -1.41695 & 0.404932 & -1.0738 & -0.43627 & 0.54569 & 1.152087 \\ 2.417915 & -0.47916 & 0.179956 & 1.587887 & 0.295953 & 0.896151 \\ 0.212642 & -0.16882 & -2.00698 & -0.05127 & 0.636396 & 0.046588 \\ 1.813323 & -0.9575 & -0.69993 & -0.90945 & 0.643434 & -0.25748 \end{bmatrix}$$

$$b_{1k} = \begin{bmatrix} -1.79789 \\ 0.793825 \\ -0.1424 \\ -1.06508 \\ 1.602905 \end{bmatrix} \qquad b_2 = [-0.31431] \qquad W_k = \begin{bmatrix} -0.50302 \\ 0.031225 \\ 0.308233 \\ -0.02231 \\ 0.502355 \end{bmatrix}$$

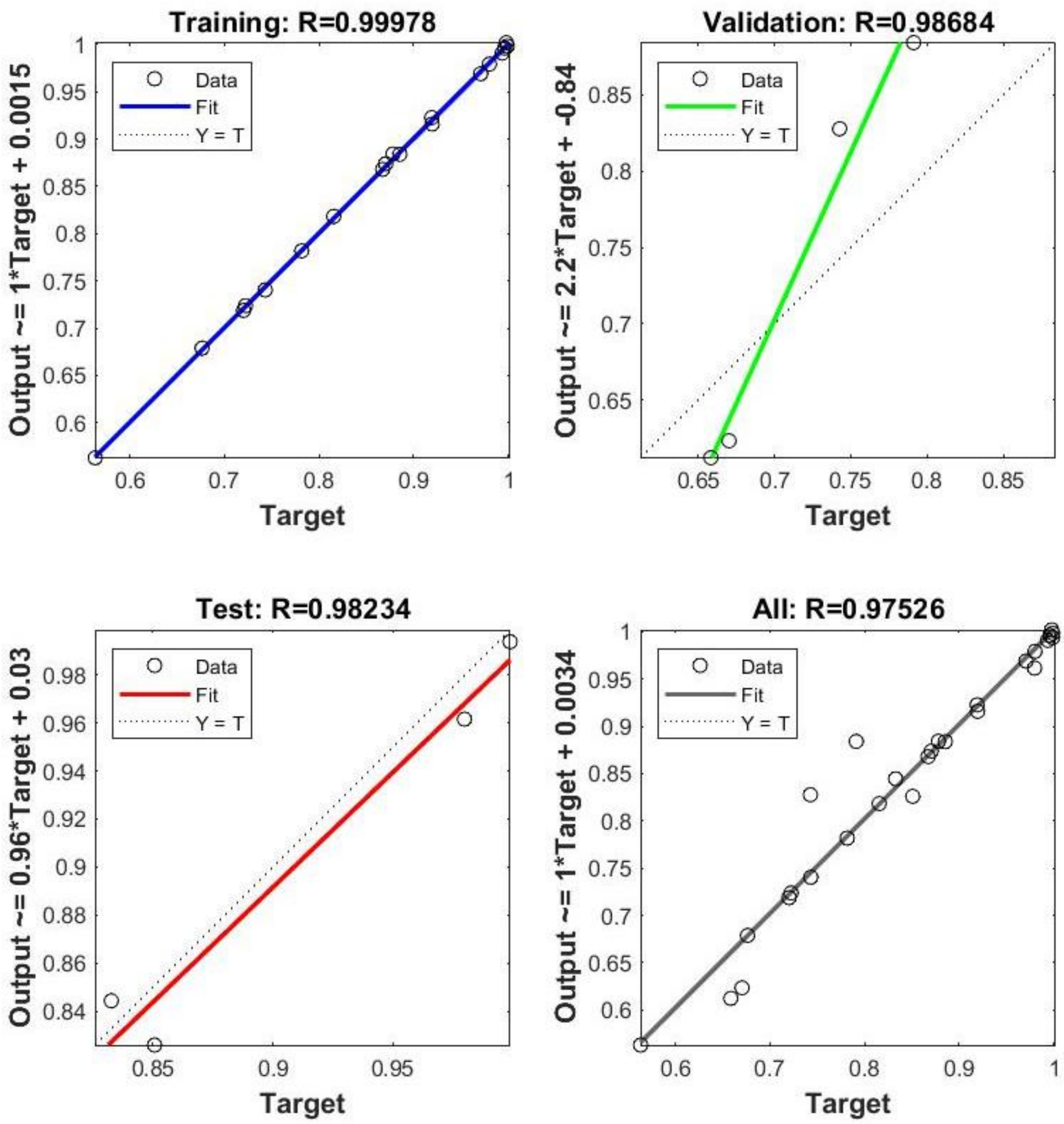

Figure 5 *Regression plot for ANN model*

III. Validation experiment

Two parameter sets are considered for the validation experiment. The parameter values are as shown below:
p1 = 2.5   σ1 = 1   μ1 = 0   p2 = 3.5   σ2 = 2   μ2 = 2
p1 = 0.5   σ1 = 1   μ1 = 0   p2 = 2.5   σ2 = 2   μ2 = 2.5
The results of the validation experiment using RSA and ANN have been depicted in Table 3 & Table 4 respectively. A set of 100, 1000 and 10000 numbers were generated for both the stress and strength distributions. Simulation was carried out using Matlab software. The distribution plot of corresponding stress-strength interference is shown in Figure 6. The error in probability obtained from both the models and that obtained from simulation is below 1% and is depicted in Figure 7 for RSA and Figure 8 for ANN. The optimization of equation 18 was carried out and the reliability obtained was 0.99999 with the parameter set p1 = 2.5, σ1 = 1, μ1 = 0, p2 = 1.5, σ2 = 1, μ2 = 2.5. The distribution plot with the optimum set of parameters is shown in Figure 6.

Table 3 Results of validation experiment for stress and strength following Weibull distribution using RSM

Sr No.	Parameters	Sample Size	Reliability using Simulation	R estimated as per proposed model	Bias	Error (%)
1	p1 = 2.5, σ1 = 1 μ1 = 0, p2 = 3.5 σ2 = 2, μ2 = 2	100	0.9876	0.99633	0.00873	0.88396
		1000	0.99143	0.99633	0.0049	0.49424
		10000	0.99612	0.99633	0.00021	0.02108
2	p1 = 0.5, σ1 = 1 μ1 = 0, p2 = 2.5 σ2 = 2, μ2 = 2.5	100	0.8732	0.8687	-0.0045	0.5153
		1000	0.87078	0.8687	-0.0021	0.24116
		10000	0.86973	0.8687	-0.0010	0.11843

Table 4 Results of validation experiment for stress and strength following Weibull distribution using ANN

Sr No.	Parameters	Sample Size	Reliability using Simulation	R estimated as per proposed model	Bias	Error (%)
1	p1 = 2.5, σ1 = 1 μ1 = 0, p2 = 3.5 σ2 = 2, μ2 = 2	100	0.9876	0.9961	0.0085	0.86067
		1000	0.99143	0.9961	0.00467	0.471037
		10000	0.99612	0.9961	-0.00002	0.002008
2	p1 = 0.5, σ1 = 1 μ1 = 0, p2 = 2.5 σ2 = 2, μ2 = 2.5	100	0.8732	0.8663	-0.0069	0.790197
		1000	0.87078	0.8663	-0.00448	0.514481
		10000	0.86973	0.8663	-0.00343	0.394375

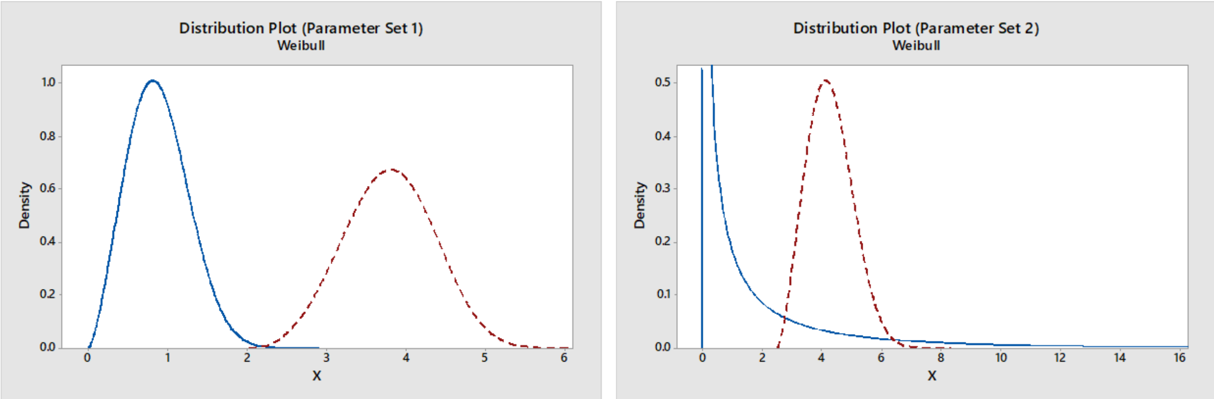

Figure 6 *Distribution plots for stress and strength following Weibull distribution*

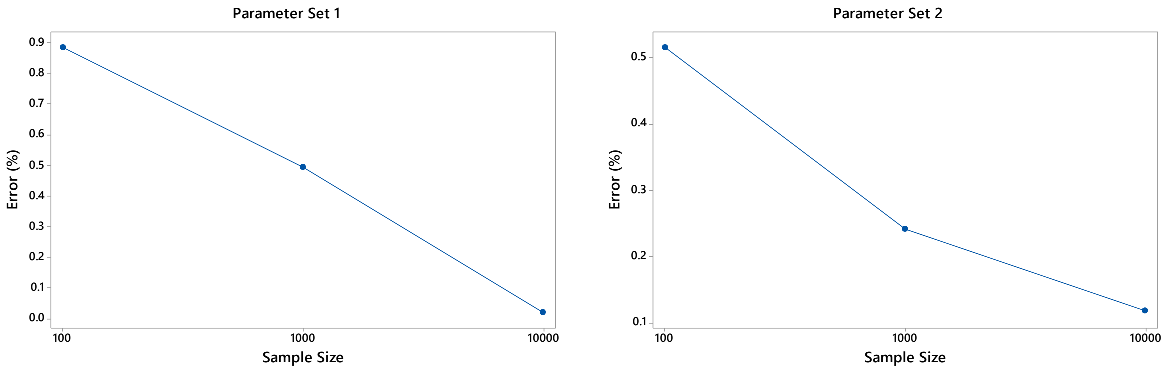

Figure 7 *Plot of error in estimation of stress-strength reliability for Weibull distribution using RSA*

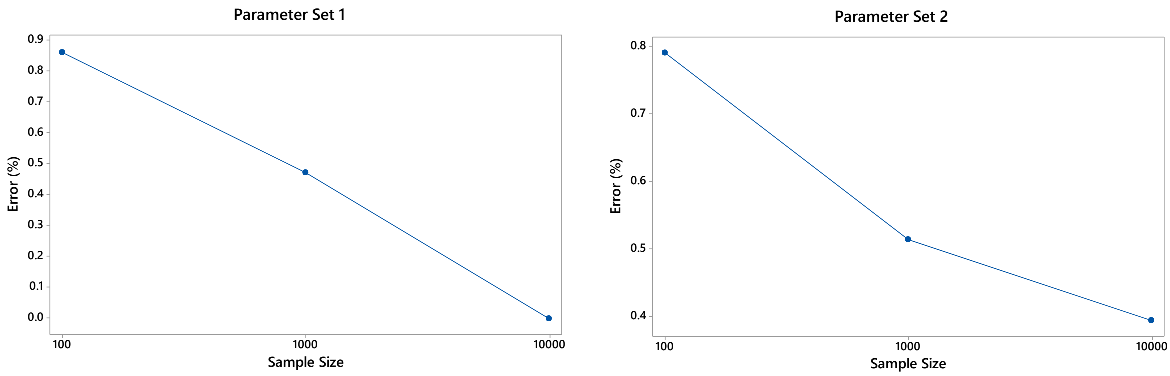

Figure 8 *Plot of error in estimation of stress-strength reliability for Weibull distribution using ANN*

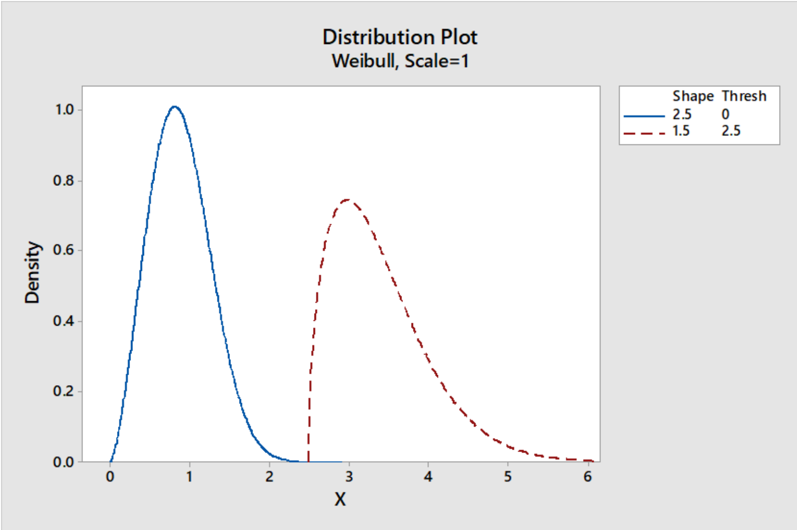

Figure 8 *Stress-strength interference with optimum set of parameters for Weibull distribution*

V. Conclusion

In this study an attempt has been made to derive a methodology in obtaining the closed form stress strength interference model for stress and strength following Weibull distribution. Design of experiments approach has been used and analysis has been carried out using Taguchi and RSA method. The reliability model has been developed for the considered set of parameters. The validation experiments with errors obtained less than 1% shows that the proposed method performs well. The optimum set of parameters was determined for the given range having reliability of 0.9999. Further studies can be carried out in this area by considering the different range of parameters and its effect on the reliability model. Also, considering dynamic nature of strength as seen in many cases, the studies can be accordingly modified.

References

[1] Raikar, S. L., & Gaonkar, R. S. P. (2022). Jaya algorithm in estimation of P[X>Y] for two parameter weibull distribution. *AIMS Mathematics*, *7*(2), 2820–2839.

[2] B. S. Dhillon. (1988). Mechanical reliability : theory, models, and applications. *American Institute of Aeronautics and Astronautics*, Wiley.

[3] Patrick D.T. O'Connor. (2013). Practical Reliability Engineering. *Wiley India*, 4th edition.

[4] Bridget, E., & Abiodun, M. (2017). Inference on Stress-Strength Reliability for Log-Normal Distribution based on Lower Record Values, *AMSE IIETA*, 22(July), 77–97.

[5] Dhillon, B. S. (1980). Stress-Strength Reliability Models. *Miroelectronics Reliability*, 20(2), 513–516.

[6] Rao, S. S. (2016). Reliability Engineering. Pearson.

[7] Nadarajah, S. (2003). Reliability for lifetime distributions. *Mathematical and Computer Modelling*, 37(7–8), 683–688.

[8] Nadarajah, Saralees. (2004). Reliability for laplace distributions. 2(October 2003), 169–183.

[9] Patowary, A. N., Hazarika, J., & Sriwastav, G. L. (2013). Interference theory of reliability: A review. *International Journal of Systems Assurance Engineering and Management*, 4(2), 146–158.

[10] An, Z. W., Huang, H. Z., & Liu, Y. (2008). A discrete stress-strength interference model based on universal generating function. *Reliability Engineering and System Safety*, 93(10), 1485–1490.

[11] Shen, K. (1992). An empirical approach to obtaining bounds on the failure probability

through stress/strength interference. *Reliability Engineering and System Safety*, 36(1), 79–84.

[12] Samuel Kotz, Lumelskii, Y., & Marianna Pensky. (2003). The Stress–Strength Model and Its Generalizations. *World Scientific*.

[13] Khare, S. K., Agarwal, S., & Srivastava, S. (2018). Analysis of Surface Roughness during Turning Operation by Taguchi Method. *Materials Today: Proceedings*, *5*(14), 28089–28097

[14] Oemar, B., & Chang, W. C. (2020). Taguchi method for optimizing process parameters in the production of activated carbon from rubber seed shell. *International Journal of Advanced Manufacturing Technology*, *107*(11–12), 4609–4620

[15] Jagatheesan, K., & Babu, K. (2020). Taguchi optimization of minimum quantity lubrication turning of AISI-4320 steel using biochar nanofluid. *Biomass Conversion and Biorefinery*.

[16] Sah, S. K., Murugesan, K., & Elangovan, R. (2021). Optimization of energy consumption for indoor climate control using Taguchi technique and utility concept. *Science and Technology for the Built Environment*, *27*(10), 1473–1491.

[17] Bement, T. R. (1989). Taguchi Techniques for Quality Engineering. *Technometrics*, *31*(2), 253–255.

[18] Laghari, R. A., Li, J., Xie, Z., & Wang, S. qi. (2018). Modeling and Optimization of Tool Wear and Surface Roughness in Turning of Al/SiCp Using Response Surface Methodology. *3D Research*, *9*(4).

[19] Ammeri, A., Chabchoub, H., Hachicha, W., & Masmoudi, F. (2015). Combining Taguchi approach and response surface methodolgy for lot-sizing problem in MTO environnement. *2015 4th IEEE International Conference on Advanced Logistics and Transport, IEEE ICALT 2015*, 252–256.

[20] Bayuo, J., Abukari, M. A., & Pelig-Ba, K. B. (2020). Optimization using central composite design (CCD) of response surface methodology (RSM) for biosorption of hexavalent chromium from aqueous media. *Applied Water Science*, *10*(6).

[21] Hazir, E., & Ozcan, T. (2019). Response Surface Methodology Integrated with Desirability Function and Genetic Algorithm Approach for the Optimization of CNC Machining Parameters. *Arabian Journal for Science and Engineering*, *44*(3), 2795–2809.

[22] Patel, K. A., & Brahmbhatt, P. K. (2018). Response Surface Methodology based Desirability Approach for Optimization of Roller Burnishing Process Parameter. *Journal of The Institution of Engineers (India): Series C*, *99*(6), 729–736.

[23] Myers, R. H., Montgomery, D. C., Geoffrey Vining, G., Borror, C. M., & Kowalski, S. M. (2004). Response Surface Methodology: A Retrospective and Literature Survey. *Journal of Quality Technology*, *36*(1), 53–78.

[24] Nair, V. N., Escobar, L. A., & Hamada, M. S. (2004). Design and Analysis of Experiments for Reliability Assessment and Improvement. In *Mathematical Reliability: An Expository Perspective*. Springer Science + Business Media New York.

[25] Rigdon, S. E., Pan, R., Montgomery, D. C., & Borror, C. M. (2022). *Design of Experiments for Reliability Achievement*, Wiley.

[26] Mohan, S., Dinesha, P., & Campana, P. E. (2022). ANN-PSO aided selection of hydrocarbons as working fluid for low-temperature organic Rankine cycle and thermodynamic evaluation of optimal working fluid. *Energy*, *259*(June), 124968.

[27] Abbasi, B., Rabelo, L., & Hosseinkouchack, M. (2008). Estimating parameters of the three-parameter Weibull distribution using a neural network. *European Journal of Industrial Engineering*, *2*(4), 428–445.