RELIABILITY MODELLING OF UTENSILS MANUFACTURING SYSTEM WITH TEMPERATURE DEPENDENT MAINTENANCE

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Abstract

In this paper, a stochastic model for utensils manufacturing system with preventive maintenance (PM) is analysed in detail. The operation is affected by variation in the temperature dependent maintenance. The entire manufacturing process of utensils goes through four subsystems viz., Circle cutting subsystem 1, Pressing subsystem 2, Spinning subsystem 3 and Polishing & Packing 4. The system has series structure of all the subsystems. The system is put under PM on the winter time and after PM it operates as new. The PM time distributions are considered as arbitrary and the time to failure as well as repair of each subsystem follows a negative exponential distribution. All random variables are statistically independent. Several measures for evaluating the effectiveness of a system, including mean time to system failure (MTSF), system availability (in summer and winter), busy period of repairman and expected number of repairs (in summer and winter) are derived using a regenerative point technique and Markov process. The system is also analysed for particular values of the parameters.

Keywords: Utensils Manufacturing System; MTSF; Availability; Regenerative Point Technique, and Preventive Maintenance (PM).

1. Introduction

Over the years researchers have made significant contribution to the reliability field. With the advent of advanced technological system, the expectations of the people have increased extremely for the use of flawless system at least for considerably period. To cater the demand

and expectation of the people, researcher have developed various stochastic models considering the aspects of different repair and obtaining performance affecting measures which include Teng et al. [1], Yusuf and Yusuf [2], Manocha and Taneja [3], Gupta et al. [4], Fagge et al. [5], Rajesh et al. [6], Kumar and Malik [7], Rajesh and Taneja [8], Jain and Malik [9], Rahbi et al. [10], Sheetal and Taneja [11, 12], Sachdeva et al. [13], Rizwan et al. [14, 15]. Singh and Mahajan [16] studied reliability of utensils manufacturing plant-a case study. Kumar and Kumar [17] studied mathematical modelling of stainless-steel utensils manufacturing plant using fuzzy reliability. This research investigated several failure modes in order to increase system reliability. There may be instances where the system's operation is impacted by temperature-dependent maintenance. Thus, the objective of this paper is to make a contribution in this regard because none of the research described above examined the impact of temperature-dependent maintenance on the operation of the system.

Despite playing a significant part in our daily lives, the manufacturing facility for utensils with preventative maintenance has not yet been covered. With this in mind, the present study took into account four subsystems of the manufacturing facility for utensils, with constant rates of subsystem failure and repair, and it covered stochastic modelling of the facility for utensils with preventive maintenance using the regenerative point technique and Markov process. As a result, PM of the unit is required after a certain amount of time to increase reliability and availability. Also, an effort was made to discuss the plant's availability in relation to various failure and repair rates.

The system needs to be maintained since low temperature harm the quality of utensils. By employing lubrications, replacement of a nut, screw, or other component of the system, cleaning, or other techniques to create high-quality utensils, the system is made operational as quickly as feasible. Preventive maintenance can therefore be used to increase system reliability and availability. It's also interesting to note that there hasn't been much work documented in the reliability literature so far on reliability modelling of the utensil manufacturing facility subject to preventive maintenance. Utensils plant can have a variety of parts but mainly the plant consists for four subsystems like cutting system, pressing system, spinning system and polishing and packing system. In winter there is low temperature spinning system goes in reduced capacity because of breakdown of rubber plate which is used in dye.

The mean time to system failure (MTSF), availability (in summer and winter), busy period analysis of repairman for repair (in summer and winter), expected number of visits of the repairman for repair (in summer and winter), and expected number of visits of the repairman for preventive maintenance are a few of the several ways that system efficacy can be measured that are obtained. Further the profit incurred to the system is obtained. Graphical representations of various intriguing system efficacy behaviours have been produced.

2. System Description, Notations and Assumptions

Utensils manufacturing plants are widely used to produce various kinds of utensils. Utensils plant can have a variety of parts but mainly the plant consists of four subsystems like cutting system, pressing system, spinning system and polishing and packing system. Manufacturing of utensils entails the press or spin forming of metal, which frequently involves complex geometries with straight sides and as well as curves of various radii. Below is a list of every system and notation needed for the mathematical formulation.

2.1. Description of the System

Sub-system M_C (Circle Cutting Machine)
As needed, sheets are cut into circular shapes.
Sub-system M_P (Pressing Machine)
The circle that was cut using a circular saw is now being sent to a pressing machine. Here, it is

pressed using various dies in accordance with the size and shape of various types of kitchenware. Due to their shallow depth, some products, including as plates and bojanthal are ready for polishing right away.

Sub-system M_S (Spinning Machine)

According to their dies, the product created by pressing is sent for spinning. Some goods don't require further annealing before polishing, but others require it because of their deeper shapes. To eliminate contaminants, these items must be subjected to acid cleaning (Acid is a combination of Sulphuric and nitric acid).

Sub-system M_D (Polishing & Packing)

The final process has produced a product that is polished-ready. This stage involves packing and polishing the final product.

2.2. Notations

$m_1(t), M_1(t)$	probability and cumulative density functions by which the system go for preventive maintenance
$m_2(t), M_2(t)$	probability and cumulative density functions for completion of preventive maintenance time
$w_1(t), W_1(t)$	probability and cumulative density functions for changing the summer to winter season
$w_2(t), W_2(t)$	probability and cumulative density functions for changing the winter to summer season
a_1, a_2, a_3, a_4	rate of failure for subsystem $M_C, M_P, M_S, M_{\overline{S}}$
$b_1, b_2, b_3, , b_4$	rate of repair for subsystem $M_C, M_P, M_S, M_{\overline{S}}$
M_C, M_P, M_S	subsystem M_C, M_P, M_S operative
$M_{CUR}, M_{PUR}, M_{SUR}, M_{\overline{S}UR}$	subsystem $M_C, M_P, M_S, M_{\overline{S}}$ under repair
$M_{\overline{S}}$	subsystem M_S in reduced state
M_{DPM}	subsystem M_D under preventive maintenance
$\odot$	Laplace Stieltjes Convolution
©	Laplace Convolution
q_{ij}	probability density function of the first passage time from regenerative state i to regenerative state j
p_{ij}	steady state transition probability from state i to state j
m_{ij}	the unconditional mean time taken to transit to any regenerative state i from the epoch of entry into regenerative state j
μ_i	mean sojourn time in the regenerative state i before transiting to any other state
$\phi_i(t)$	cumulative distribution function (c.d.f.) of the first passage time from a regenerative state i to a failed state
AS_0, AW_0	availability in summer, winter
BS_0, BW_0	busy period of the repairman due to repair in summer, winter
VS_0, VW_0	expected number of visits of repairman for repair during summer, winter
PM_0	expected number of visits of the repairman for preventive maintenance

2.3. Assumptions

- The failure and repair rates are independent and exponential in general.
- None of the sub-systems are experiencing simultaneous failures.

- Subsystem M_D has never failed.
- The repaired system works just like the new system.
- Subsystems are only repaired when they are in reduced or failed state.

3. Analysis of Model

In Fig. 1, the system model's possible transition diagram is shown.

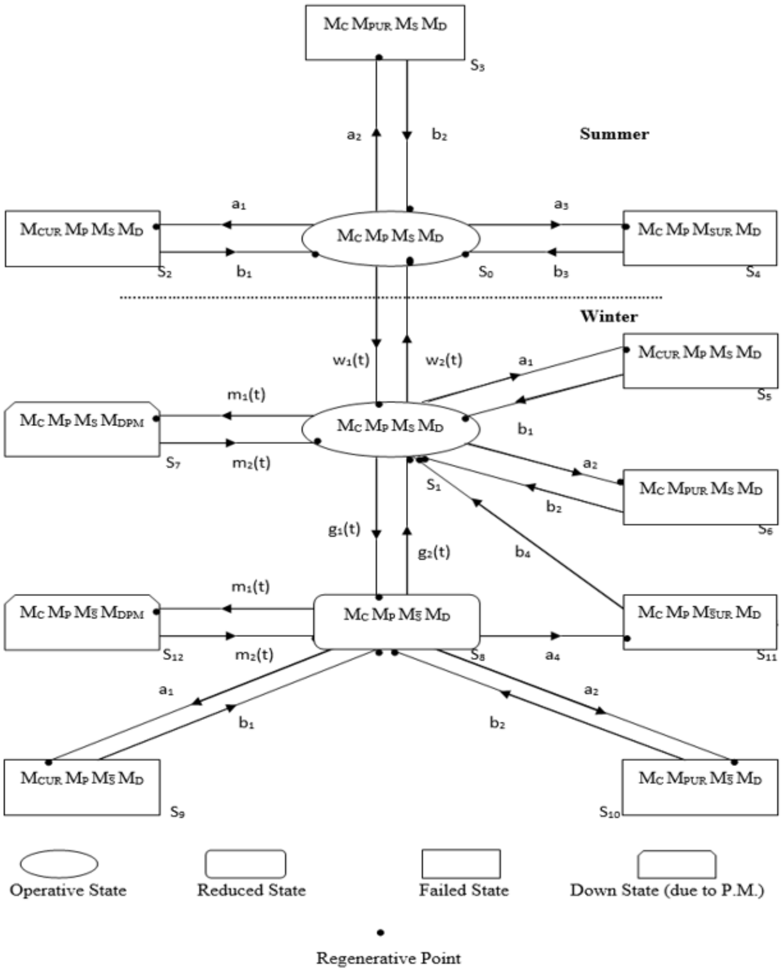

Figure 1: *State Transition Diagram*

3.1. Description of the Model and Transition Probabilities

3.1.1 Description of the model

Various states of the model for the system consisting four subsystems with season wise (summer and winter) and the state transition diagram is displayed in above Fig. 1. States 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 of the state transition diagram are regeneration points and hence these states are regenerative states for the model. States 0 and 1 are the states four subsystem work and so represents operative state during summer and winter respectively. States 2 and 5 are the states where the sub-system M_C go in failed states during summer and winter respectively so represents failed state. States 3 and 6 are the states where sub-system M_P go in failed states during summer and winter respectively so represents failed state. States 4 and 11 are the states where sub-system M_S go in failed states during summer and winter respectively so represents failed state. State 8 is the state where sub-system M_S go in reduced capacity during winter so represents reduced state. States 9 and 10 are the states where sub-system M_C and sub-system M_P go in failed states respectively so represents failed state also in those states sub-system M_S is in reduced capacity. States 7 and 12 are the states where sub-system M_D under preventive maintenance. Also, in the state 7 the sub-system M_S work in full capacity, in the state 12 the sub-system M_S work in reduced capacity.

3.1.2 State Transition Probabilities

In Fig. 1, the system's transition diagram is depicted, indicating the various states of the system. Expressions for $q_{ij}(t)$ (for all required combinations of i and j) are found based on the state transition diagram, and same are provided below:

$q_{01}(t) = e^{-(a_1+a_2+a_3)t} w_1(t)$ $\quad q_{02}(t) = a_1 e^{-(a_1+a_2+a_3)t}\overline{W_1}(t)$
$q_{03}(t) = a_2 e^{-(a_1+a_2+a_3)t}\overline{W_1}(t)$ $\quad q_{04}(t) = a_3 e^{-(a_1+a_2+a_3)t}\overline{W_1}(t)$
$q_{10}(t) = e^{-(a_1+a_2)t} F_{10}(t)$ $\quad q_{15}(t) = a_1 e^{-(a_1+a_2)t} F_1(t)$
$q_{16}(t) = a_2 e^{-(a_1+a_2+a_3)t} F_1(t)$ $\quad q_{17}(t) = a_3 e^{-(a_1+a_2+a_3)t} F_{17}(t)$
$q_{18}(t) = e^{-(a_1+a_2+a_3)t} F_{18}(t)$ $\quad q_{20}(t) = b_1 e^{-b_1 t}$
$q_{30}(t) = b_2 e^{-b_2 t}$ $\quad q_{40}(t) = b_3 e^{-b_3 t}$
$q_{51}(t) = b_1 e^{-b_1 t}$ $\quad q_{61}(t) = b_2 e^{-b_2 t}$
$q_{71}(t) = m_2(t)$ $\quad q_{81}(t) = e^{-(a_1+a_2+a_4)t} G_{8,1}(t)$
$q_{89}(t) = a_1 e^{-(a_1+a_2+a_3)t} G_8(t)$ $\quad q_{8,10}(t) = a_2 e^{-(a_1+a_2+a_3)t} G_8(t)$
$q_{8,11}(t) = a_4 e^{-(a_1+a_2+a_4)t} G_8(t)$ $\quad q_{8,12}(t) = e^{-(a_1+a_2+a_4)t} G_{8,12}(t)$
$q_{98}(t) = b_1 e^{-b_1 t}$ $\quad q_{10,8}(t) = b_2 e^{-b_2 t}$
$q_{11,1}(t) = b_4 e^{-b_4 t}$ $\quad q_{12,8}(t) = m_2(t)$

where

$G_8(t) = \overline{M_1}(t)\overline{G_2}(t)$ $\quad G_{8,1}(t) = \overline{M_1}(t) g_2(t)$
$G_{8,12}(t) = m_1(t)\overline{G_2}(t)$ $\quad F_{10}(t) = \overline{M_1}(t)\overline{G_1}(t) w_2(t)$
$F_{17}(t) = m_1(t)\overline{G_1}(t)\overline{W_2}(t)$ $\quad F_{18}(t) = \overline{M_1}(t) g_1(t)\overline{W_2}(t)$
$F_1(t) = \overline{M_1}(t)\overline{G_1}(t)\overline{W_2}(t)$

Transition probabilities $p_{ij}(t)$ from state i to state j can be calculated by taking Laplace transform of above obtained values of $q_{ij}(t)$ and then using the following mathematical relationship between p_{ij} and $q^*_{ij}(s)$
$p_{ij} = \lim_{s\to 0} q^*_{ij}(s)$
values of for all required combinations of i and j are obtained and the same are given as follows:

$p_{01} = w_1^*(a_1+a_2+a_3)$ $\quad p_{02} = \frac{a_1}{(a_1+a_2+a_3)}[1 - w_1^*(a_1+a_2+a_3)]$
$p_{03} = \frac{a_2}{(a_1+a_2+a_3)}[1 - w_1^*(a_1+a_2+a_3)]$ $\quad p_{04} = \frac{a_3}{(a_1+a_2+a_3)}[1 - w_1^*(a_1+a_2+a_3)]$
$p_{10} = F_{10}^*(a_1+a_2)$ $\quad p_{15} = a_1 F_1^*(a_1+a_2)$
$p_{16} = a_2 F_1^*(a_1+a_2)$ $\quad p_{17} = F_{17}^*(a_1+a_2)$
$p_{18} = F_{18}^*(a_1+a_2)$ $\quad p_{71} = m_2^*(0)$
$p_{81} = G_{81}^*(a_1+a_2+a_4)$ $\quad p_{89} = a_1 G_8^*(a_1+a_2+a_4)$
$p_{8,10} = a_2 G_8^*(a_1+a_2+a_4)$ $\quad p_{8,11} = a_4 G_8^*(a_1+a_2+a_4)$
$p_{8,12} = G_{8,12}^*(a_1+a_2+a_4)$ $\quad p_{12,8} = m_2^*(0)$
$p_{20} = p_{30} = p_{40} = p_{51} = p_{61} = p_{98} = p_{10,8} = p_{11,1} = 1$
We may verify that
$p_{01} + p_{02} + p_{03} + p_{04} = 1$
$p_{10} + p_{15} + p_{16} + p_{17} + p_{18} = 1$
$p_{81} + p_{89} + p_{8,10} + p_{8,11} + p_{8,12} = 1$
$p_{20} = p_{30} = p_{40} = p_{51} = p_{61} = p_{71} = p_{98} = p_{10,8} = p_{11,1} = p_{12,8} = 1$

3.1.3 Mean Sojourn time (μ_i)

If T_i denotes the stay time of the system in state i, then using the following mathematical relationship between μ_i and T_i
$\mu_i = \int_0^\infty P[T_i > t]dt$
values of μ_i for all required values of i are found, and the same are provided as:
$\mu_0 = \int_0^\infty e^{-(a_1+a_2+a_3)t}\overline{W_1}(t)dt$
$\mu_1 = \int_0^\infty e^{-(a_1+a_2)t}\overline{F_1}(t)$
$\mu_2 = \mu_5 = \mu_9 = \frac{1}{b_1}$
$\mu_3 = \mu_6 = \mu_{10} = \frac{1}{b_2}$
$\mu_4 = \frac{1}{b_3}$
$\mu_7 = \mu_{12} = \int_0^\infty \overline{M_2}(t)dt$
$\mu_{11} = \frac{1}{b_4}$
The unconditional mean time (m_{ij}) which the system under consideration takes to move to state j where counting of the time starts as soon as it enters into state i can be obtained using the following mathematical relationship between m_{ij} and $q_{ij}(t)$
$m_{ij} = \int_0^\infty t q_{ij}(t)dt,$
values of for all required combinations of i and j thus obtained and given as follows:

$\mu_0 = m_{01} + m_{02} + m_{03} + m_{04}$
$\mu_1 = m_{10} + m_{15} + m_{16} + m_{17} + m_{18}$
$\mu_2 = m_{20} = \frac{1}{b_1}$
$\mu_3 = m_{30} = \frac{1}{b_2}$
$\mu_4 = m_{40} = \frac{1}{b_3}$
$\mu_5 = m_{51} = \frac{1}{b_1}$
$\mu_6 = m_{61} = \frac{1}{b_2}$
$\mu_7 = m_{71} = \int_0^\infty t m_2(t)dt$
$\mu_8 = m_{81} + m_{89} + m_{8,10} + m_{8,11} + m_{8,12}$
$\mu_9 = m_{98} = \frac{1}{b_1}$
$\mu_{10} = m_{10,8} = \frac{1}{b_2}$
$\mu_{11} = m_{11,1} = \frac{1}{b_4}$
$\mu_{12} = m_{12,8} = \int_0^\infty t m_2(t)dt$

4. System Performance Measures

4.1. Mean Time to System Failure

We retain failed states as absorbing states in order to calculate the system's MTSF. Using recursive relations for $\phi_i(t)$ can be obtained and the same are given as:
$\phi_0(t) = Q_{01}(t) \odot \phi_1(t) + Q_{02}(t) + Q_{03}(t) + Q_{04}(t)$
$\phi_1(t) = Q_{10}(t) \odot \phi_0(t) + Q_{15}(t) + Q_{16}(t) + Q_{17}(t) + Q_{18}(t) \odot \phi_8(t)$
$\phi_7(t) = Q_{71}(t) \odot \phi_1(t)$
$\phi_8(t) = Q_{81}(t) \odot \phi_1(t) + Q_{89}(t) + Q_{8,10}(t) + Q_{8,11}(t) + Q_{8,12}(t) \odot \phi_{12}(t)$
$\phi_{12}(t) = Q_{12,8}(t) \odot \phi_8(t)$
By solving these relations for $\phi_0^{**}(s)$ using the Laplace Stieltjes transformation of these relations, we get
$\phi_0^{**}(s) = \frac{N(s)}{D(s)}$,
where

$$\begin{aligned}N(s) = {} & q_{8,12}^*(s)q_{12,8}^*(s)[(q_{02}^*(s) + q_{03}^*(s) + q_{04}^*(s))(q_{17}^*(s)q_{71}^*(s) - 1) - q_{01}^*(s)(q_{15}^*(s) + q_{16}^*(s))] \\ & + (q_{02}^*(s) + q_{03}^*(s) + q_{04}^*(s))(1 - q_{18}^*(s)q_{81}^*(s)) + q_{17}^*(s)q_{71}^*(s)) + q_{01}^*(s)(q_{15}^*(s) + q_{16}^*(s)) \\ & + q_{01}^*(s)q_{18}^*(s)(q_{89}^*(s) + q_{8,10}^*(s) + q_{8,11}^*(s))\end{aligned}$$

$$\begin{aligned}D(s) = {} & 1 + q_{01}^*(s)q_{10}^*(s)q_{8,12}^*(s)q_{12,8}^*(s) - q_{01}^*(s)q_{10}^*(s)q_{17}^*(s)q_{71}^*(s)q_{8,12}^*(s)q_{12,8}^*(s) - q_{17}^*(s)q_{71}^*(s) \\ & - q_{18}^*(s)q_{81}^*(s) - q_{8,12}^*(s)q_{12,8}^*(s)\end{aligned}$$

Using above calculated value of $\phi_0^{**}(s)$, MTSF can be obtained when the system under consideration starts from the state 0 and the same is given as follows:

$T_0 = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{N}{D}$,
where
$N = \mu_0[(1-p_{17})(1-p_{8,12}) - p_{18}p_{81}] - \mu_1 p_{01}p_{8,12} + \mu_8 p_{01}p_{18} + \mu_7[p_{18}p_{8,12} + (1-p_{8,12})p_{17}]$
$D = (1-p_{8,12})(1-p_{01}p_{10}-p_{17}) - p_{18}p_{81}$

4.2. Availabilities in Summer and Winter

During Summer
To determine the availability in summer $AS_0(t)$ of the system, recursive relations thus obtained using probabilistic arguments, are given as:
$AS_0(t) = M_0(t) + q_{01}(t)©AS_1(t) + q_{02}(t)©AS_2(t) + q_{03}(t)©AS_3(t) + q_{04}(t)©AS_4(t)$
$AS_1(t) = q_{10}(t)©AS_0(t) + q_{15}(t)©AS_5(t) + q_{16}(t)©AS_6(t) + q_{17}(t)©AS_7(t) + q_{18}(t)©AS_8(t)$
$AS_2(t) = q_{20}(t)©AS_0(t)$
$AS_3(t) = q_{30}(t)©AS_0(t)$
$AS_4(t) = q_{40}(t)©AS_0(t)$
$AS_5(t) = q_{51}(t)©AS_1(t)$
$AS_6(t) = q_{61}(t)©AS_1(t)$
$AS_7(t) = q_{71}(t)©AS_1(t)$
$AS_8(t) = q_{81}(t)©AS_1(t) + q_{89}(t)©AS_9(t) + q_{8,10}(t)©AS_10(t) + q_{8,11}(t)©AS_{11}(t) + q_{8,12}(t)©AS_{12}(t)$
$AS_9(t) = q_{98}(t)©AS_8(t)$
$AS_{10}(t) = q_{10,8}(t)©AS_8(t)$
$AS_{11}(t) = q_{11,1}(t)©AS_1(t)$
$AS_{12}(t) = q_{12,8}(t)©AS_8(t)$
where,
$M_0(t) = e^{-(a_1+a_2+a_3)t}\overline{W_1}(t)$
By solving these relations for $AS_0^*(s)$ using the Laplace transform of these relations, we get
$AS_0^*(s) = \frac{N_1(s)}{D_1(s)}$
where,

$$\begin{aligned} N_1(s) = & M_0^*(s)[1 - q_{18}^*(s)q_{11,1}^*(s)q_{8,11}^*(s) - q_{15}^*(s)q_{51}^*(s) - q_{16}^*(s)q_{61}^*(s) - q_{17}^*(s)q_{71}^*(s) - q_{18}^*(s)q_{81}^*(s) \\ & - q_{89}^*(s)q_{98}^*(s) - q_{8,10}^*(s)q_{10,8}^*(s) - q_{8,12}^*(s)q_{12,8}^*(s) + (q_{15}^*(s)q_{51}^*(s) + q_{16}^*(s)q_{61}^*(s) \\ & + q_{17}^*(s)q_{71}^*(s))(q_{89}^*(s)q_{98}^*(s) + q_{8,10}^*(s)q_{10,8}^*(s) + q_{8,12}^*(s)q_{12,8}^*(s))] \end{aligned}$$

$$\begin{aligned} D_1(s) = & [q_{02}^*(s)q_{20}^*(s) + q_{03}^*(s)q_{30}^*(s) + q_{04}^*(s)q_{40}^*(s)][q_{15}^*(s)q_{51}^*(s) + q_{16}^*(s)q_{61}^*(s) + q_{17}^*(s)q_{71}^*(s) \\ & + q_{18}^*(s)q_{81}^*(s) + q_{89}^*(s)q_{98}^*(s) + q_{8,10}^*(s)q_{10,8}^*(s) + q_{8,12}^*(s)q_{12,8}^*(s) - 1] + (q_{01}^*(s)q_{10}^*(s) + q_{15}^*(s)q_{51}^*(s) \\ & + q_{16}^*(s)q_{61}^*(s) + q_{17}^*(s)q_{71}^*(s))(q_{89}^*(s)q_{98}^*(s) + q_{8,10}^*(s)q_{10,8}^*(s) + q_{8,12}^*(s)q_{12,8}^*(s) - 1) + q_{18}^*(s) \\ & q_{11,1}^*(s)q_{8,11}^*(s)(q_{02}^*(s)q_{20}^*(s) + q_{03}^*(s)q_{30}^*(s) + q_{04}^*(s)q_{40}^*(s) - 1) + (q_{02}^*(s)q_{20}^*(s) + q_{03}^*(s)q_{30}^*(s) \\ & + q_{04}^*(s)q_{40}^*(s))(q_{15}^*(s)q_{51}^*(s) + q_{16}^*(s)q_{61}^*(s) + q_{17}^*(s)q_{71}^*(s))(q_{89}^*(s)q_{98}^*(s) + q_{8,10}^*(s)q_{10,8}^*(s) \\ & + q_{8,12}^*(s)q_{12,8}^*(s)) \end{aligned}$$

Using above calculated value of $AS_0^*(s)$ availability in summer can be obtained in steady-state and the same is given as follows:
$AS_0 = \lim_{s\to 0} sAS_0^*(s) = \frac{N_1}{D_1}$
where,
$N_1 = \mu_0 p_{10}(p_{81} + p_{8,11})$

$$\begin{aligned} D_1 = & (\mu_0 + \mu_2 p_{02} + \mu_3 p_{03} + \mu_4 p_{04})p_{10}(p_{81} + p_{8,11}) + (\mu_1 + \mu_5 p_{15} + \mu_6 p_{16} + \mu_7 p_{17})p_{01}(p_{81} + p_{8,11}) \\ & + (\mu_8 + \mu_9 p_{89} + \mu_{10} p_{8,10} + \mu_{12} p_{8,12})p_{01}p_{18} \end{aligned}$$

During Winter
Similarly, steady-state availability during winter are given as follows:
$AW_0 = \lim_{s\to 0} sAW_0^*(s) = \frac{N_2}{D_1}$
where,
D_1 already defined and

$N_2 = \mu_1 p_{01}(p_{81} + p_{8,11})$

4.3. Busy Period Analysis

Busy period of the repairman due to repair in summer
Similarly, steady-state Busy period of the repairman due to repair in summer are given as follows:
$BS_0 = \lim_{s\to 0} sBS_0^*(s) = \frac{N_3}{D_1}$
where,
D_1 already defined and
$N_3 = (\mu_2 p_{02} + \mu_3 p_{03} + \mu_4 p_{04})p_{10}(p_{81} + p_{8,11})$
During Winter
Similarly, steady-state Busy period of the repairman due to repair in winter are given as follows:
$BW_0 = \lim_{s\to 0} sBW_0^*(s) = \frac{N_4}{D_1}$
where,
D_1 already defined and
$N_4 = p_{01}[p_{18}(\mu_9 p_{89} + \mu_{10} p_{8,10} + \mu_{11} p_{8,11}) + (\mu_5 p_{15} + \mu_6 p_{16})(p_{81} + p_{8,11})]$

4.4. Expected Number of Visits of the Repairman for Repair

During summer
Similarly, steady-state number of visits of the repairman during summer are given as follows:
$VS_0 = \lim_{s\to 0} sVS_0^*(s) = \frac{N_5}{D_1}$
where,
D_1 already defined and
$N_5 = p_{10}(1 - p_{01})(p_{81} + p_{8,11})$
During Winter
Similarly, steady-state number of visits of the repairman during winter are given as follows:
$VW_0 = \lim_{s\to 0} sVW_0^*(s) = \frac{N_6}{D_1}$
where,
D_1 already defined and
$N_6 = p_{01}p_{18}(1 - p_{81} - p_{8,12}) + p_{01}(1 - p_{10} - p_{17} - p_{18})(p_{81} + p_{8,11})$

4.5. Expected Number of Visits of the Repairman for Preventive Maintenance

Similarly, steady-state number of visits of the repairman for preventive maintenance are given as follows:
$PM_0 = \lim_{s\to 0} sPM_0^*(s) = \frac{N_7}{D_1}$
where,
D_1 already defined and
$N_7 = p_{01}p_{17}(p_{81} + p_{8,11}) + p_{01}p_{18}p_{8,12}$

5. Cost-Benefit Analysis

Profit of the system under consideration can be obtained by subtracting the costs due to repair, per visit charges of the repairman for repair in summer and winter and per visit charges of the repairman for preventive maintenance. The same can expressed in terms of the various performance measures obtained through the model developed in this given as follows:
$Profit = CS_0 AS_0 + CW_0 AS_0 - CS_1 BS_0 - CW_1 BW_0 - CS_2 VS_0 - CW_2 VW_0 - C_3 PM_0$
where,

CS_0 : revenue during summer, per unit uptime
CW_0 : revenue during winter, per unit uptime
CS_1 : revenue during summer per unit time for repair
CW_1 : revenue during winter per unit time for repair
CS_2 : Cost per visit during summer for repair
CW_2 : Cost per visit during winter for repair
C_3 : Cost per visit for preventive maintenance

6. Numerical Interpretation

To obtain various numerical outcomes, the following specific case is used:

$w_1(t) = \alpha e^{-\alpha t}$ $\quad w_2(t) = \beta e^{-\beta t}$
$m_1(t) = \gamma e^{-\gamma t}$ $\quad m_2(t) = \delta e^{-\delta t}$
$g_1(t) = \lambda e^{-\lambda t}$ $\quad g_2(t) = \mu e^{-\mu t}$
$\mu_0 = \frac{1}{a_1+a_2+a_3+\alpha}$ $\quad \mu_1 = \frac{1}{a_1+a_2+a_3+\beta+\gamma}$
$\mu_2 = \frac{1}{b_1}$ $\quad \mu_3 = \frac{1}{b_2}$
$\mu_4 = \frac{1}{b_3}$ $\quad \mu_5 = \frac{1}{b_1}$
$\mu_6 = \frac{1}{b_2}$ $\quad \mu_7 = \frac{1}{\delta}$
$\mu_8 = \frac{1}{a_1+a_2+a_4+\mu+\gamma}$ $\quad \mu_9 = \frac{1}{b_1}$
$\mu_{10} = \frac{1}{b_2}$ $\quad \mu_{11} = \frac{1}{b_4}$
$\mu_{12} = \frac{1}{\delta}$ $\quad p_{01} = \frac{\alpha}{a_1+a_2+a_3+\alpha}$
$p_{02} = \frac{a_1}{a_1+a_2+a_3+\alpha}$ $\quad p_{03} = \frac{a_2}{a_1+a_2+a_3+\alpha}$
$p_{04} = \frac{a_3}{a_1+a_2+a_3+\alpha}$ $\quad p_{10} = \frac{\beta}{a_1+a_2+a_3+\gamma+\beta}$
$p_{15} = \frac{a_1}{a_1+a_2+a_3+\gamma+\beta}$ $\quad p_{16} = \frac{a_2}{a_1+a_2+a_3+\gamma+\beta}$
$p_{17} = \frac{a_3}{a_1+a_2+a_3+\gamma+\beta}$ $\quad p_{18} = \frac{\gamma}{a_1+a_2+a_3+\gamma+\beta}$
$p_{81} = \frac{\mu}{a_1+a_2+a_4+\gamma+\mu}$ $\quad p_{89} = \frac{a_1}{a_1+a_2+a_4+\gamma+\mu}$
$p_{8,10} = \frac{a_2}{a_1+a_2+a_4+\gamma+\mu}$ $\quad p_{8,11} = \frac{a_4}{a_1+a_2+a_4+\gamma+\mu}$
$p_{8,12} = \frac{\gamma}{a_1+a_2+a_4+\gamma+\mu}$
$p_{20} = p_{30} = p_{40} = p_{51} = p_{61} = p_{71} = p_{98} = p_{10,8} = p_{11,1} = p_{12,8} = 1$

where
$a_1 = 0.235, a_2 = 0.0381, a_3 = 0.01589, a_4 = 0.02673, b_1 = 0.887, b_2 = 0.793, b_3 = 0.821,$
$b_4 = 0.896, \alpha = 0.615, \beta = 0.83, \lambda = 0.034, \gamma = 0.00937, \delta = 0.870, \mu = 0.875, CS_0 = 15000,$
$CS_1 = 1500, CW_0 = 15000, CW_1 = 1600, CS_2 = 1450, CW_2 = 1550, C_3 = 1400.$
Various graphs have been plotted but all the graphs have not been shown here to use minimum space and to avoid repetition of similar interpretations. However, the users of such systems may plot graph of their interest as per the requirement and may take important decision regarding profitability of the system. Regarding the availability and nature of MTSF, various rates have been depicted as shown in Fig. 2 ,3, 4 and 5 which reveal that MTSF, Availability and profit decreases as failure rates increases. However, their values go in the direction δ and b_2. Some of the plotted graphs are shown as follows:
The MTSF behaviour for different values of β is shown in Fig. 2. MTSF decreases as the failure rate value (a_2) rises. Higher values of β correspond to higher values in it.

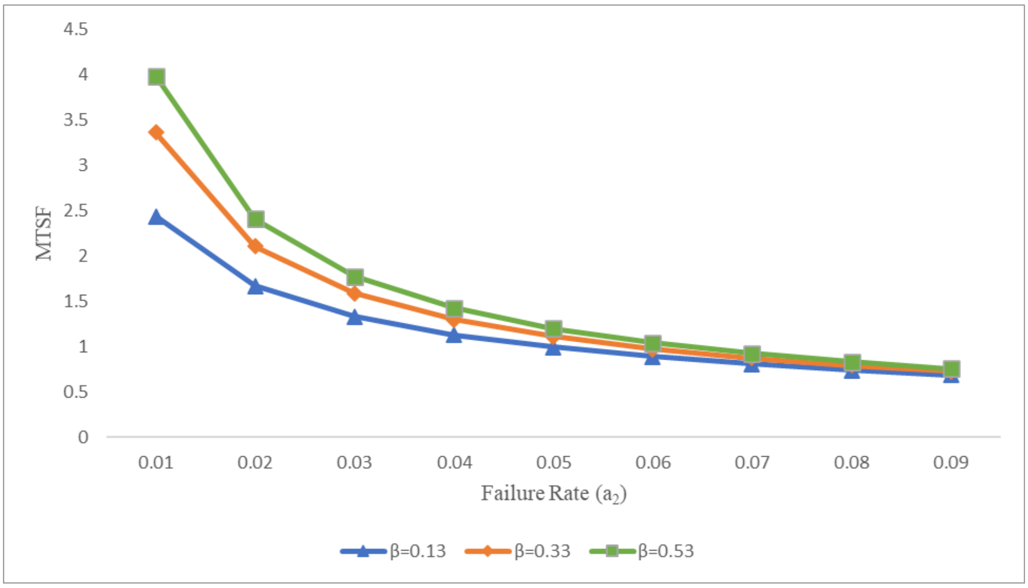

Figure 2: *MTSF versus Failure Rate* (a_2) *for different values of* (β)

The availability behaviour in the summer for various repair rate values (b_2) is displayed in Fig. 3. As the failure rate (a_2) rises, summer availability decreases. Additionally, it has been noted that when b_2 values rise, so does availability.

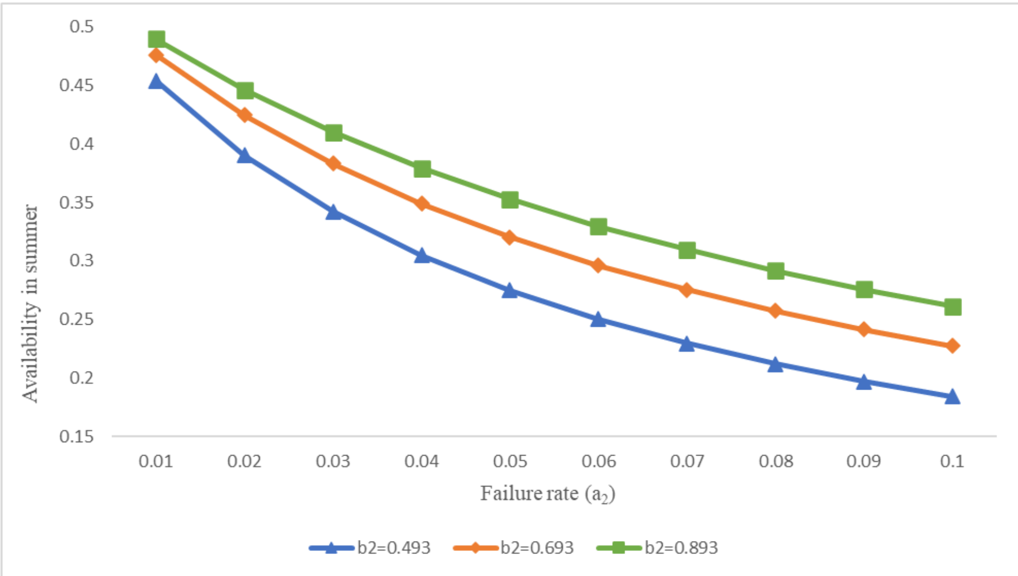

Figure 3: *Availability in Summer versus Failure rate* (a_2) *for different values of Repair rate* (b_2)

The way that profit acts in relation to revenue in the summer (CS_0) for various values of the cost paid for repair in the summer (CS_1) is shown in Fig. 4. As revenue values rise in the summer (CS_0), profit rises as well. Additionally, it has been seen that as (CS_1) values rise, the profit falls.

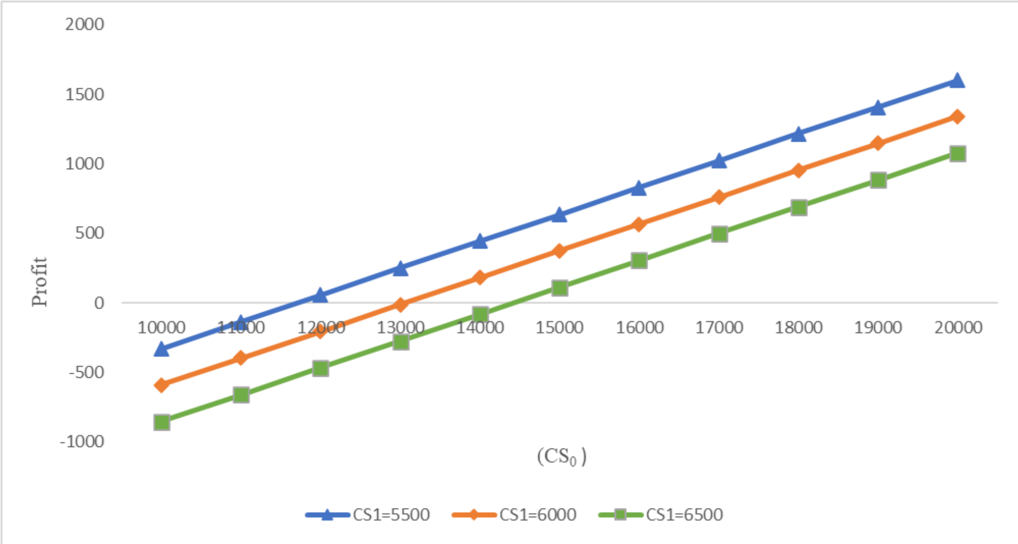

Figure 4: *Profit versus Revenue in Summer (CS_0) for different values of cost paid for paid in Summer (CS_1)*

Fig. 5 illustrates the behaviour of profit in relation to revenue during the winter (CW_0) for various costs (C_3) associated with preventive maintenance. With an increase in winter revenue values (CW_0), profit rises. Additionally, it has been seen that as C_3 values rise, the profit falls.

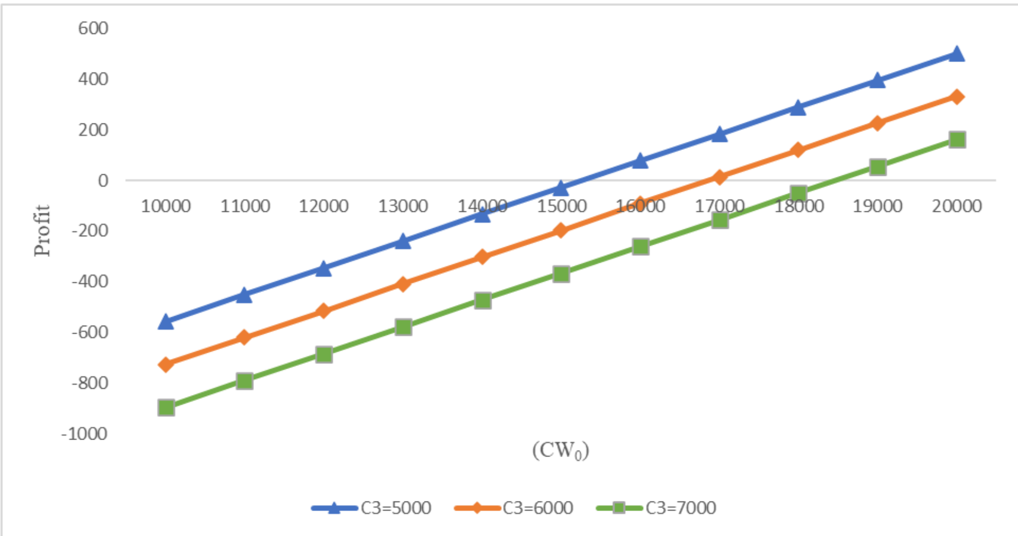

Figure 5: *Profit versus Revenue in Winter (CW_0) for different values of cost (C_3) paid for paid for Preventive Maintenance*

Values of parameters taken and cut-off points obtained from the above figures are tabulated as follows:

Fig	Varied Parameters	Condition	Interpretation
4	CS_1 = 5500	CS_0> 11700.3105	System is profitable
	CS_1 = 6000	CS_0> 13059.0062	System is profitable
	CS_1 = 6500	CS_0> 14417.1842	System is profitable
5	CW_1 =5000	CW_0>15261.2412	System is profitable
	CW_1 =6000	CW_0>16863.2744	System is profitable
	CW_1 =7000	CW_0>18465.3067	System is profitable

7. Conclusion

In the current study, a reliability model is developed using a system for producing utensils. The findings for a specific situation demonstrate the relevance of research since cut-off points may be used to set lower and upper limits for a variety of factors. For instance, setting a product's pricing so that the system is profitable depends on the cut-off point for revenue per unit uptime. The cut-off points facilitate many crucial judgments for the profits according to revenue.

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Disclosure statement

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