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OPTIMIZING MULTI-OBJECTIVE MULTI-INDEX TRANSPORTATION PROBLEMS: A SMART ALGORITHMIC SOLUTION WITH LINDO SOFTWARE

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Abstract

In the present paper, we create an algorithm to address the transportation problem with numerous objectives and indexes. The transportation problem exists when there are more supply points, more demand points, and various means of transportation are used to meet demand or when moving certain types of goods. The transportation problem may frequently be more complex than the typical form of transportation problem. We create a model that blends fuzzy multi-objective programming and the multi-index transportation problem' by using LINDO software to resolve all related problems. Additionally, the decision-maker may present a variety of data and it may be further improved. The new algorithm for addressing transport problems in fuzzy environments is demonstrated numerically.

Key Words: Fuzzy transportation problem (FTP), Linear Programming Problem(LP P), Multi-index transportation problem (MITP)

1. Introduction

The transportations prolem has a extensive range of real-world applications and can be seen as a specific example of the LPP. It is one of the best optimization techniques and has a wide range of real-world applications. a combination of various goods from any of the m origins to any of the n destination places. In order to reduce the overall cost of a transportation issue, we control the amount to be transported from all origins to all destinations. We may not have focused on a single objective function in this situation, which is multi-objective. All of the objectives of MOTP are in competition with one another, and all of the restrictions are of the equality kind. The technique for multi-objective fuzzy linear programming with uncertain goals. The multi-objective transportation problem has the best compromise solution when applied to fuzzy linear programming.

Numerous academics have evaluated the use of the MITP to optimization, mathematical modelling and industry. Wang et al. [10] established a decomposition technique for handling the standard three-index transportation problem that is entirely dependent on the successive adjustment of the optimality criterion. They looked at the transportation problem's solution using a linear and quadratic objective function. In addition to recommending the adoption of such transportation efficiency, Rautman et al. [8] find a solution to the shipment scheduling conflict by utilising a multi-index transportation problem method to optimize the integral system Bit et al. [4] used a fuzzy programming method with a hyperbolic membership function to solve the Multi-Objective Capacitated Transportation Problem, in which the targets are non-commensurable and incompatible and the deliver and demand constraints are all of the same kind. In order to identify the best and most effective compromise solution to a multi-objective capacitated transportation problem, fuzzy programming with hyperbolic membership function was applied. For the first time, all parameters are taken into consideration when using fuzzy multi index bi-criteria constant fee bottleneck transportation (FMIBCFCBTP) by Sungeeta et al. [9]. An algorithm was created to detect FMIBCFCBTP fuzzy time-value change-off pairings. A numerical example was provided to explain the said algorithm. An exponential membership function was employed in the fuzzy programming method by Kaur et al. [5] to resolve a multi-objective and multi-index transportation problem. The main emphasis is on reducing the prices, decreasing rates, and underutilized capacity of transporting raw materials via various modes of transportation from various points of origin to various destination sites. Each target function is given a unique form of non-linear membership function by employing the fuzzy programming technique to solve actual transport problems using an exponential function and creating a non-dominated compromise solution. To tackle the multi-index fixed charge bi-criterion transportation problem , Archana and Veena [2] provided a method for determining the ideal trade-off pair amongst efficient cost-time trade-off pairs.

The linear-multi-objective-solid transportation problem was approached from a fuzzy-linear programming perspective by Bit et al. [3]. The outcome is a compromise approach that is both cost-effective and ideal. The fuzzy linear programming approach was used to develop the FORTRAN programme. With profit maximisation and time minimization as the objectives, Anjana et al. [1] developed a multi-objective multi-object strong transportation problem (MMSTP) with fuzzy inequality constraints. Fuzzy chance programming and the additional specificity opportunity-necessity principle are used to create a method for changing the ambiguous MMSTP to the equivalent-deterministic shape. The fuzzy interactive satisfied method was used to develop optimal compromise solutions for the MMSTP through a generalised reduced gradient strategy. The finest non-dominating solution was created using the technique for order preference by similar to perfect solution. An algorithm was created by Porchelvi and Anitha [7] to address the multi-objective transportation problem. The source and destination parameters , along with the cost coefficients of the goal function, are expressed as interval costs. To solve MOTP, they used fuzzy programming techniques with linear membership functions for various costs.

2. Mathematical Model

The MOMITP,we assume p_{ijl} be a multidimensional array $1 \le i \le m, 1 \le j \le n$ *and* $1 \le l \le k$ and let $P = p_{ij}$, $Q = q_{jl}$ and $R = r_{il}$ be multi-matrices then the MITP is as fallows

$$\text{Min } Z = \sum_i \sum_j \sum_l p_{ijl} X_{ijl} \tag{1}$$

Such that

$$\sum_i X_{ijl} = p_{ij} \quad \textit{for all } i, j$$

$$\sum_{j} X_{ijl} = r_{il} \quad for\,all\; i,l$$

$$\sum_{j} X_{ijl} = q_{jl} \quad for\,all\; j,l\,,\ \&\ X_{ijl} \geq 0,\ for\,all\; i,j,l \tag{2}$$

it further, implies that

$$\sum_{i} p_{ij} = \sum_{l} q_{jl}\ ;\ \sum_{j} p_{ij} = \sum_{l} r_{il}\ ;\ \sum_{j} q_{jl} = \sum_{l} r_{il} \tag{3}$$

All the 3-conditions are necessary but not sufficient. MOMITP is defined as fallows

$$\text{Minimize } Z_k = \sum_{i}^{m}\sum_{j}^{n} v_{ij}^{(1)}\, x_{ij}^{(1)} + \sum_{i}^{m}\sum_{j}^{n} v_{ij}^{(2)}\, x_{ij}^{(2)} \tag{4}$$

Such that

$$\sum_{j}^{n} x_{ij}^{(1)} = p_{1i}\,,\ \forall i \tag{5}$$

$$\sum_{j}^{n} x_{ij}^{(2)} = p_{2i}\,,\ \forall i \tag{6}$$

$$\sum_{i}^{m} x_{ij}^{(1)} = q_{1j}\,,\ \forall j \tag{7}$$

$$\sum_{i}^{m} x_{ij}^{(2)} = q_{2j}\,,\ \forall j \tag{8}$$

$$x_{ij}^{(1)} + x_{ij}^{(2)} = r_{ij}\,,\ \forall i,j \tag{9}$$

$$x_{ij}^{(1)} \geq 0,\ x_{ij}^{(2)} \geq 0 \tag{10}$$

The following set of conditions are necessary for the existance solution.

$$\sum_{j}^{n} r_{il} = p_{1i} + p_{2i},\ \forall i, \tag{11}$$

$$\sum_{j}^{n} r_{il} = q_{1j} + q_{2j},\ \forall j \tag{12}$$

$$\sum_{i}^{m} p_{1i} = \sum_{j}^{n} q_{1j}\,,\ \forall i,j \tag{13}$$

$$\sum_{i}^{m} p_{2i} = \sum_{j}^{n} q_{2j}\,,\ \forall i,j \tag{14}$$

$$\sum_{j}^{n} r_{il} \leq \min\,(p_{1i} + q_{1j}) + \min\,(p_{2i} + q_{2j}),\ \forall i,j \tag{15}$$

3. Proposed Algorithm

Step 1: Formulate a FTP.
Step 2: Solving the MOTP, k times, taking, one at a time, we first develop a matrix form inorder to get corresponding values for each objectives for each solution.

$$\begin{array}{c} \\ X_1 \\ X_2 \\ \vdots \\ X_k \end{array}\begin{array}{c} \begin{array}{cccc} Z_1 & Z_2 & \cdots & Z_k \end{array} \\ \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1k} \\ Z_{21} & Z_{22} & \cdots & Z_{2k} \\ \vdots & \vdots & & \vdots \\ Z_{k1} & Z_{k2} & \cdots & Z_{kk} \end{bmatrix} \end{array}$$

Where, each $X^i, i=1,2,..k$ represent the isolated optimal solutions to the K distinct transportation problems for k distict objective functions $Z_{ij} = Z_j(X^i)\ \forall i,j,\ \ where\ i,j=1,2,3........k$, respectavelly the ith row and jth column members of the matrix.

Step 3: Using step 2, we set upper and lower bounds for each objective and defining the range of values for the membership function that represents the degree of acceptance and rejection for a perticular solution. The values of such functions can be calculated as.

$U_k^\mu = Max(Z_r(X_r))$

$L_k^\mu = Min(Z_r(X_r)),\ 0 \le r \le k,$

Where U_k^μ and L_k^μ are rtespectivelly the upper and lower bound for the (k^{th} objective function Z_k) $k=1,2,3........K$, $d_k = U_k^\mu - L_k^\mu$, the degradation allowance for objective k.

Step 4: We define the membership function as:

$$\mu_k\{Z(X)_k\} = \begin{cases} 1, & L_k^\mu \ge Z_k(X) \\ 1-\dfrac{(Z_1(X)-L_k^\mu)}{d_k}, & L_k^\mu \le Z_k(X) \le U_k^\mu\ ,\ where\ d_k = U_k^\mu - L_k^\mu \\ 0, & Z_k(X) \ge U_k^\mu \end{cases} \tag{16}$$

Step 5: We use a LMF for the initial fuzzy model, the crisp model can be simplified as:

Minimize α

Subject to

$$Z_k(X) + \alpha d_k \le U_k^\mu, \tag{17}$$

$$\sum_j^n x_{ij}^{(1)} = p_{1i},\ \forall i,j$$

$$\sum_j^n x_{ij}^{(2)} = p_{2i},\ \ \forall j$$

$$\sum_i^m x_{ij}^{(1)} = q_{1j},\ \ \forall i$$

$$\sum_i^m x_{ij}^{(2)} = q_{2j},\ \ \forall j$$

$$x_{ij}^{(1)} + x_{ij}^{(2)} = r_{ij},\ \forall i,j$$

$$x_{ij}^{(1)} \ge 0,\ \ x_{ij}^{(2)} \ge 0$$

Above system of LP can be solved by using LONDO statistical software.

Step 6: Using the precise mathematical Programming approach, we are able to solve the crisp model.

Min α

$$C_{ij}^k x_{ij} + \alpha d_k \le U_k^\mu,\ k=1,2,..........K, \tag{18}$$

$$\sum_j^n x_{ij}^{(1)} = p_{1i},\ \ \forall i,j$$

$$\sum_j^n x_{ij}^{(2)} = p_{2i},\ \ \forall i,$$

$$\sum_i^m x_{ij}^{(1)} = q_{1j},\ \ \forall j$$

$$\sum_i^m x_{ij}^{(2)} = q_{2j},\ \ \forall j$$

$$x_{ij}^{(1)} + x_{ij}^{(2)} = r_{ij},\ \forall i,j$$

$$x_{ij}^{(1)} \geq 0,\ x_{ij}^{(2)} \geq 0$$

Another membership function, like the Hyperbolic Tangent function, is one that we utlise.
Min α

$$\alpha \geq \frac{1}{2} + \frac{1}{2}\tanh\{\frac{U_k^{\mu} + L_k^{\mu}}{2} + Z_k\}\tau_k, \tag{19}$$

Where, $\tau_k = \dfrac{S}{U_k^{\mu} - L_k^{\mu}}$, where S is number of constraints.

$$x_{ij} \geq 0, \forall i, j \text{ and } \alpha \geq 0.$$

Step 7: An intuitionistic fuzzy optimization for MOLP is defined as

$$\mu_k^e\{Z_k(X)\} = \begin{cases} 1, & L_k \geq Z_k(X) \\ e^{-\frac{1}{2}\left(\frac{Z_k - L_k^{\mu}}{d_k}\right)}, & L_k^{\mu} \leq Z_k(X) \leq U_k^{\mu} \quad where \quad d_k = U_k^{\mu} - L_k^{\mu} \\ 0, & Z_k(X) \geq U_k^{\mu} \end{cases} \tag{20}$$

where $k = 1, 2, \ldots\ldots\ldots.K$.

4. Illustrations

Example 4.1: Consider the fuzzy MOMITP discussed by Lohgaonkar et al. [6]

$$\begin{aligned} Minimize\ Z_1 = {} & 4Y_{11}^{(1)} + 3Y_{12}^{(1)} + 5Y_{13}^{(1)} + 8Y_{21}^{(1)} + 6Y_{22}^{(1)} + 2Y_{23}^{(1)} + 7Y_{31}^{(1)} + 4Y_{32}^{(1)} \\ & + Y_{33}^{(1)} + 9Y_{41}^{(1)} + 10Y_{42}^{(1)} + 12Y_{43}^{(1)} + 8Y_{11}^{(2)} + 6Y_{12}^{(2)} + 3Y_{13}^{(2)} + 5Y_{21}^{(2)} \\ & + 4Y_{22}^{(2)} + Y_{23}^{(2)} + 9Y_{31}^{(2)} + 2Y_{32}^{(2)} + 6Y_{33}^{(2)} + 4Y_{41}^{(2)} + 9Y_{42}^{(2)} + 3Y_{43}^{(2)} \end{aligned}$$

Subject to

$$\begin{aligned} & Y_{11}^{(1)} + Y_{12}^{(1)} + Y_{13}^{(1)} = 9 \\ & Y_{21}^{(1)} + Y_{22}^{(1)} + Y_{23}^{(1)} = 14 \\ & Y_{31}^{(1)} + Y_{32}^{(1)} + Y_{33}^{(1)} = 6 \\ & Y_{41}^{(1)} + Y_{42}^{(1)} + Y_{43}^{(1)} = 7 \\ & Y_{11}^{(2)} + Y_{12}^{(2)} + Y_{13}^{(2)} = 6 \\ & Y_{21}^{(2)} + Y_{22}^{(2)} + Y_{23}^{(2)} = 7 \\ & Y_{31}^{(2)} + Y_{32}^{(2)} + Y_{33}^{(2)} = 5 \\ & Y_{41}^{(2)} + Y_{42}^{(2)} + Y_{43}^{(2)} = 6 \\ & Y_{11}^{(1)} + Y_{21}^{(1)} + Y_{31}^{(1)} + Y_{41}^{(1)} = 14 \\ & Y_{12}^{(1)} + Y_{22}^{(1)} + Y_{32}^{(1)} + Y_{42}^{(1)} = 12 \\ & Y_{13}^{(1)} + Y_{23}^{(1)} + Y_{33}^{(1)} + Y_{43}^{(1)} = 10 \\ & Y_{11}^{(2)} + Y_{21}^{(2)} + Y_{31}^{(2)} + Y_{41}^{(2)} = 5 \\ & Y_{12}^{(2)} + Y_{22}^{(2)} + Y_{32}^{(2)} + Y_{42}^{(2)} = 8 \\ & Y_{13}^{(2)} + Y_{23}^{(2)} + Y_{33}^{(2)} + Y_{43}^{(2)} = 11 \\ & Y_{11}^{(1)} + Y_{11}^{(2)} = 5 \\ & Y_{12}^{(1)} + Y_{12}^{(2)} = 7 \end{aligned}$$

$$\begin{aligned}
&Y_{13}^{(1)}+Y_{13}^{(2)}=3\\
&Y_{21}^{(1)}+Y_{21}^{(2)}=8\\
&Y_{22}^{(1)}+Y_{22}^{(2)}=4\\
&Y_{23}^{(1)}+Y_{23}^{(2)}=9\\
&Y_{31}^{(1)}+Y_{31}^{(2)}=4\\
&Y_{32}^{(1)}+Y_{32}^{(2)}=1\\
&Y_{33}^{(1)}+Y_{33}^{(2)}=6\\
&Y_{41}^{(1)}+Y_{41}^{(2)}=2\\
&Y_{42}^{(1)}+Y_{42}^{(2)}=8\\
&Y_{43}^{(1)}+Y_{43}^{(2)}=3\\
&Y_{ij}^{(1)},Y_{ij}^{2}\geq 0,\ i=1,2,3,4\quad and\quad j=1,2,3
\end{aligned} \tag{21}$$

Example 4.2:

$$\begin{aligned}
Minimize\,Z_2 = {} & 5Y_{11}^{(1)}+6Y_{12}^{(1)}+7Y_{13}^{(1)}+4Y_{21}^{(1)}+5Y_{22}^{(1)}+2Y_{23}^{(1)}+1Y_{31}^{(1)}+3Y_{32}^{(1)}\\
&+4Y_{33}^{(1)}+4Y_{41}^{(1)}+2Y_{42}^{(1)}+3Y_{43}^{(1)}+10Y_{11}^{(2)}+9Y_{12}^{(2)}+9Y_{13}^{(2)}+7Y_{21}^{(2)}\\
&+9Y_{22}^{(2)}+2Y_{23}^{(2)}+8Y_{31}^{(2)}+7Y_{32}^{(2)}+9Y_{33}^{(2)}+8Y_{41}^{(2)}+4Y_{41}^{(2)}+5Y_{43}^{(2)}
\end{aligned}$$

Subject to

$$\begin{aligned}
&Y_{11}^{(1)}+Y_{12}^{(1)}+Y_{13}^{(1)}=9\\
&Y_{21}^{(1)}+Y_{22}^{(1)}+Y_{23}^{(1)}=14\\
&Y_{31}^{(1)}+Y_{32}^{(1)}+Y_{33}^{(1)}=6\\
&Y_{41}^{(1)}+Y_{42}^{(1)}+Y_{43}^{(1)}=7\\
&Y_{11}^{(2)}+Y_{12}^{(2)}+Y_{13}^{(2)}=6\\
&Y_{21}^{(2)}+Y_{22}^{(2)}+Y_{23}^{(2)}=7\\
&Y_{31}^{(2)}+Y_{32}^{(2)}+Y_{33}^{(2)}=5\\
&Y_{41}^{(2)}+Y_{42}^{(2)}+Y_{43}^{(2)}=6\\
&Y_{11}^{(1)}+Y_{21}^{(1)}+Y_{31}^{(1)}+Y_{41}^{(1)}=14\\
&Y_{12}^{(1)}+Y_{22}^{(1)}+Y_{32}^{(1)}+Y_{42}^{(1)}=12\\
&Y_{13}^{(1)}+Y_{23}^{(1)}+Y_{33}^{(1)}+Y_{43}^{(1)}=10\\
&Y_{11}^{(2)}+Y_{21}^{(2)}+Y_{31}^{(2)}+Y_{41}^{(2)}=5\\
&Y_{12}^{(2)}+Y_{22}^{(2)}+Y_{32}^{(2)}+Y_{42}^{(2)}=8\\
&Y_{13}^{(2)}+Y_{23}^{(2)}+Y_{33}^{(2)}+Y_{43}^{(2)}=11\\
&Y_{11}^{(1)}+Y_{11}^{(2)}=5\\
&Y_{12}^{(1)}+Y_{12}^{(2)}=7\\
&Y_{13}^{(1)}+Y_{13}^{(2)}=3\\
&Y_{21}^{(1)}+Y_{21}^{(2)}=8\\
&Y_{22}^{(1)}+Y_{22}^{(2)}=4\\
&Y_{23}^{(1)}+Y_{23}^{(2)}=9
\end{aligned}$$

$$Y_{31}^{(1)} + Y_{31}^{(2)} = 4$$
$$Y_{32}^{(1)} + Y_{32}^{(2)} = 1$$
$$Y_{33}^{(1)} + Y_{33}^{(2)} = 6$$
$$Y_{41}^{(1)} + Y_{41}^{(2)} = 2$$
$$Y_{42}^{(1)} + Y_{42}^{(2)} = 8$$
$$Y_{43}^{(1)} + Y_{43}^{(2)} = 3$$
$$Y_{ij}^{(1)}, Y_{ij}^{2} \geq 0, \; i = 1, 2, 3, 4 \quad and \quad j = 1, 2, 3$$

If no errors are found, then the LINGO Solver status window appears of the illustration 4.1 is given below (by changing variable Y to X and also, taking $X_{11}^{(1)}$ and $X_{11}^{(2)}$ respectively X11M and X11N so on. In LINGO solver)

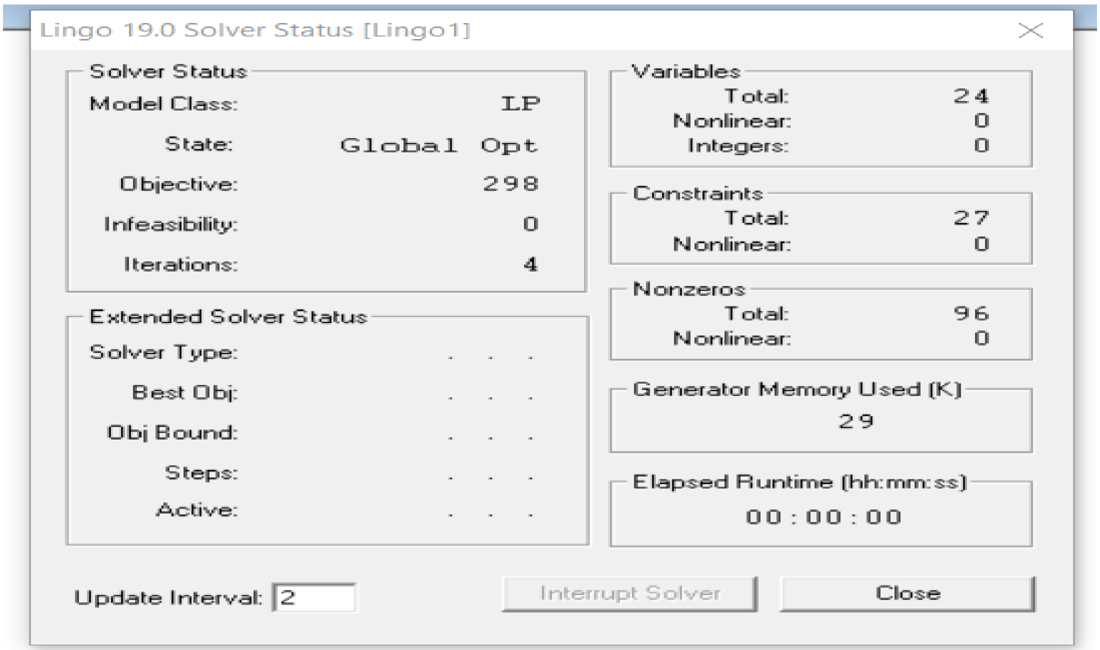

Figure 1: *Illustration of model 4.1 on LINGO window*

By closing above window, we can view the solution of the Model 4.1 is given (Global optimal solution found).

Objective value:	298.0000
Infeasibilities:	0.000000
Total solver iterations:	4
Model Class:	LP
Total variables:	24
Nonlinear variables:	0
Integer variables:	0
Total constraints:	27
Nonlinear constraints:	0
Total nonzeros	96
Nonlinear nonzeros:	0

Table 1: *Optimal Solution of model 4.1*

Variable	Value	Reduced Cost
X11M	5.000000	0.000000
X12M	4.000000	0.000000
X13M	0.000000	6.000000
X21M	8.000000	0.000000
X22M	1.000000	0.000000
X23M	5.000000	0.000000
X31M	1.000000	0.000000
X32M	0.000000	6.000000
X33M	5.000000	0.000000
X41M	0.000000	2.000000
X42M	7.000000	0.000000
X43M	0.000000	9.000000
X11N	0.000000	3.000000
X12N	3.000000	0.000000
X13N	3.000000	0.000000
X21N	0.000000	1.000000
X22N	3.000000	0.000000
X23N	4.000000	0.000000
X31N	3.000000	0.000000
X32N	1.000000	0.000000
X33N	1.000000	0.000000
X41N	2.000000	0.000000
X42N	1.000000	0.000000
X43N	3.000000	0.000000

If no errors are found, then the LINGO Solver status window appears for the illustration 4.2 is given below

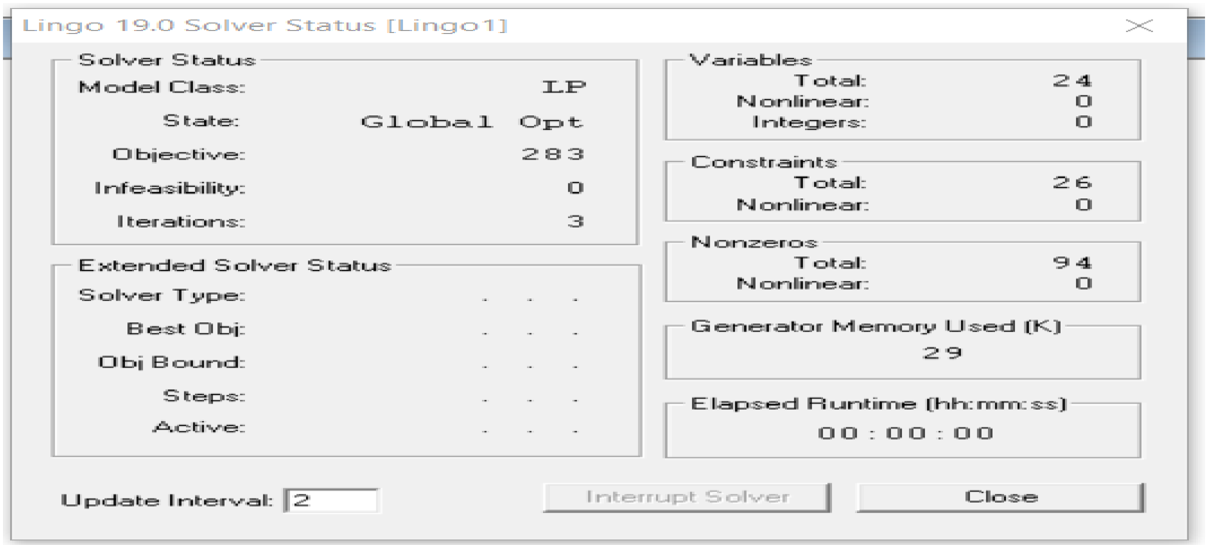

Figure 2: *Illustration of model 4.2 on LINGO window*

By closing above window, we can view the solution of the Model 4.2 is given
(Global optimal solution found).

Objective value:	283.0000
Infeasibilities:	0.000000
Total solver iterations:	3
Model Class:	LP
Total variables	24
Nonlinear variables:	0
Integer variables:	0
Total constraints:	27
Nonlinear constraints:	0
Total nonzeros:	96
Nonlinear nonzeros:	0

Table 2: *Optimal Solution of model 4.2*

Variable	Value	Reduced Cost
X11M	3.000000	0.000000
X12M	6.000000	0.000000
X13M	0.000000	1.000000
X21M	8.000000	0.000000
X22M	4.000000	0.000000
X23M	2.000000	0.000000
X31M	1.000000	0.000000
X32M	0.000000	1.000000
X33M	5.000000	0.000000
X41M	2.000000	0.000000
X42M	2.000000	0.000000
X43M	3.000000	0.000000
X11N	2.000000	0.000000
X12N	1.000000	0.000000
X13N	3.000000	0.000000
X21N	0.000000	1.000000
X22N	0.000000	4.000000
X23N	7.000000	0.000000
X31N	3.000000	0.000000
X32N	1.000000	0.000000
X33N	1.000000	0.000000
X41N	0.000000	0.000000
X42N	6.000000	0.000000
X43N	0.000000	0.000000

Now, we can determine Z_1 and Z_2, for $(X^{(2)}, X^{(1)})$ respectively as given below
$Z_2(X^{(1)}) = 295$ and $Z_1(X^{(2)}) = 335$, and written in the form of matrix.

$$\begin{array}{c} \\ X^{(1)} \\ \\ X^{(2)} \end{array}\begin{array}{c} \begin{array}{cc} Z_1 & Z_2 \end{array} \\ \begin{bmatrix} 298 & 295 \\ & \\ 335 & 283 \end{bmatrix} \end{array}$$

From the above, we have $U_1^{\mu} = 335$ $U_2^{\mu} = 295$ $L_1^{\mu} = 298$ $L_2^{\mu} = 283$

We define Z_1 and Z_2 respectively

$$\mu_1(X) = \begin{cases} 1, & if \ Z_1(X) \le 298 \\ 1 - \dfrac{Z_1(X) - 298}{335 - 298}, & if \ 298 \le Z_1(X) \le 335 \\ 0, & if \ Z_1(X) \ge 335 \end{cases}$$

$$d_{k_1} = 37$$

$$\mu_2(X) = \begin{cases} 1, & if \ Z_1(X) \le 283 \\ 1 - \dfrac{Z_2(X) - 283}{295 - 283}, & if \ 283 \le Z_1(X) \le 295 \\ 0, & if \ Z_1(X) \ge 295 \end{cases}$$

$$d_{k_2} = 12$$

We find

Minimize α

$$Z_1(X) + 37a \le 335 \tag{22}$$

$$Z_2(X) + 12a \le 295 \tag{23}$$

We Solve the above crisp model and the Solver window appears given below

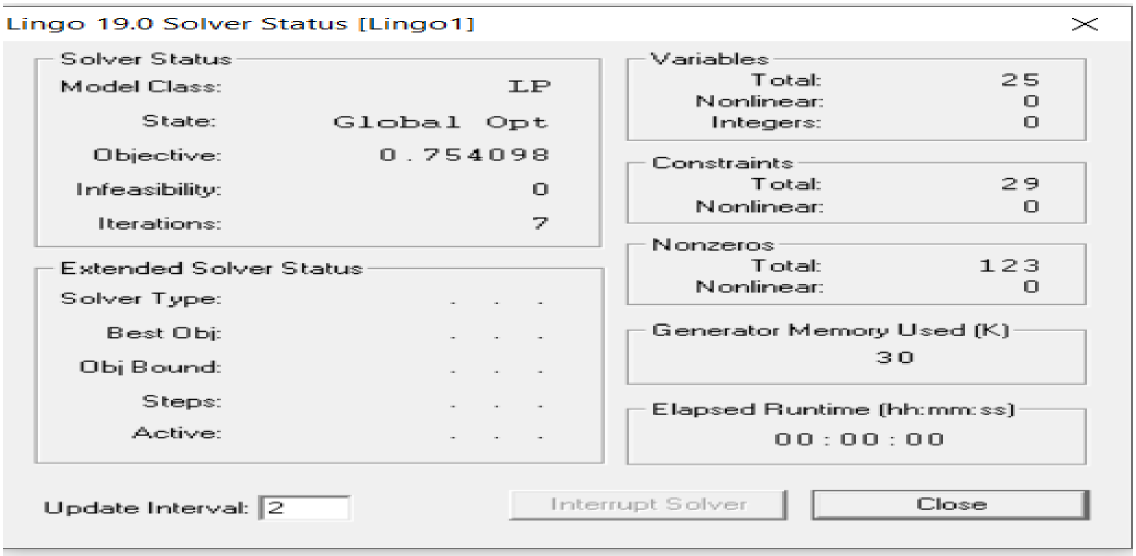

Figure 3: *Illustration of model 22 and 23 on LINGO window*

By closing above window, we can view the solution of the Model 22 and 23 is given (Global optimal solution found).

Objective value: 0.7540984

Infeasibilities: 0.000000

Total solver iterations:	7
Elapsed runtime seconds:	0.11
Model Class:	LP
Total variables:	25
Nonlinear variables:	0
Integer variables:	0
Total constraints:	29
Nonlinear constraints:	0
Total nonzeros:	123
Nonlinear nonzeros:	0

Table 3: *Optimal Solution of crisp model 22 and 23*

Variable	Value	Reduced Cost
A	0.7540984	0.000000
X11M	5.000000	0.000000
X12M	3.316940	0.000000
X13M	0.6830601	1.000000
X21M	7.000000	0.000000
X22M	3.683060	0.000000
X23M	3.316940	0.000000
X31M	0.000000	0.4918033E-01
X32M	0.000000	0.2622951
X33M	6.000000	0.000000
X41M	2.000000	0.000000
X42M	5.000000	0.000000
X43M	0.000000	0.1639344E-01
X11N	0.000000	0.1311475
X12N	3.683060	0.000000
X13N	2.316940	0.000000
X21N	1.000000	0.000000
X22N	0.3169399	0.000000
X23N	5.683060	0.000000
X31N	4.000000	0.000000
X32N	1.000000	0.000000
X33N	0.000000	0.000000
X41N	0.000000	0.4918033E-01
X42N	3.000000	0.000000
X43N	3.000000	0.000000

$$Z_1^* = 307.04 \quad Z_2^* = 285.92 \qquad \alpha = 0.754$$

Maximize α

$$\alpha \geq \frac{1}{2} + \frac{1}{2}\tanh\{\frac{U_k^{\mu} + L_k^{\mu}}{2} - Z_k\}\tau_k$$

Further implies that

$$Z_k \tau_k + \tanh^{-1}(2\alpha - 1) \leq \frac{U_k^{\mu} + L_k^{\mu}}{2}\tau_k \tag{24}$$

Maximize w

$$6(4\cdot X_{11}^{(1)}+3\cdot X_{12}^{(1)}+5\cdot X_{13}^{(1)}+8\cdot X_{21}^{(1)}+6\cdot X_{22}^{(1)}+2\cdot X_{23}^{(1)}+7\cdot X_{31}^{(1)}+4\cdot X_{32}^{(1)}+1\cdot X_{33}^{(1)}+9\cdot X_{41}^{(1)}$$
$$+10\cdot X_{42}^{(1)}+12\cdot X_{43}^{(1)}+8\cdot X_{11}^{(2)}+6\cdot X_{12}^{(2)}+3\cdot X_{13}^{(2)}+5\cdot X_{21}^{(2)}+4\cdot X_{22}^{(2)}+1\cdot X_{23}^{(2)}+9\cdot X_{31}^{(2)}+2\cdot X_{32}^{(2)}$$
$$+6\cdot X_{33}^{(2)}+4\cdot X_{41}^{(2)}+9\cdot X_{42}^{(2)}+3\cdot X_{43}^{(2)})+37\cdot\alpha\leq 1899$$

$$6(5\cdot X_{11}^{(1)}+6\cdot X_{12}^{(1)}+7\cdot X_{13}^{(1)}+4\cdot X_{21}^{(1)}+5\cdot X_{22}^{(1)}+2\cdot X_{23}^{(1)}+1\cdot X_{31}^{(1)}+3\cdot X_{32}^{(1)}+4\cdot X_{33}^{(1)}+4\cdot X_{41}^{(1)}$$
$$+2\cdot X_{42}^{(1)}+3\cdot X_{43}^{(1)}+10\cdot X_{11}^{(2)}+9\cdot X_{12}^{(2)}+9\cdot X_{13}^{(2)}+7\cdot X_{21}^{(2)}+9\cdot X_{22}^{(2)}+2\cdot X_{23}^{(2)}+8\cdot X_{31}^{(2)}+7\cdot X_{32}^{(2)}$$
$$+9\cdot X_{33}^{(2)}+8\cdot X_{41}^{(2)}+4\cdot X_{41}^{(2)}+5\cdot X_{43}^{(2)})+12\cdot\alpha\leq 1734$$

Subject to the condition (21)
Solver window of model (24) appears below

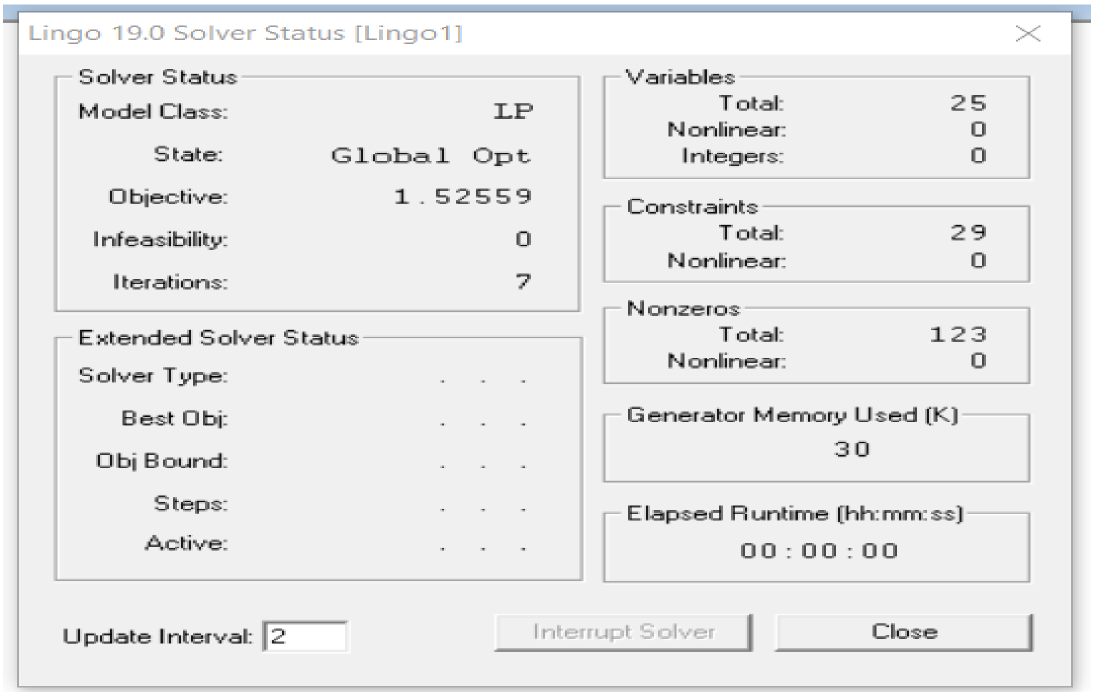

Figure 4: *Illustration of model 24 on LINGO window*

Solution of above model (24) is given by
(Global optimal solution found).

Objective value:	1.525591
Infeasibilities:	0.000000
Total solver iterations:	7
Elapsed runtime seconds:	0.17
Model Class:	LP
Total variables:	25
Nonlinear variables:	0
Integer variables:	0
Total constraints:	29
Nonlinear constraints:	0
Total nonzeros:	123
Nonlinear nonzeros:	0

Table 4 : *Optimal solution of model 24*

Variable	Value	Reduced Cost
A	1.525591	0.000000
X11M	5.000000	0.000000
X12M	3.316273	0.000000
X13M	0.6837270	0.000000
X21M	7.000000	0.000000
X22M	3.683727	0.000000
X23M	3.316273	0.000000
X31M	0.000000	0.4918033E-01
X32M	0.000000	1.574803
X33M	6.000000	0.000000
X41M	2.000000	0.000000
X42M	5.000000	0.000000
X43M	0.000000	0.9842520E-0
X11N	0.000000	0.7874016
X12N	3.683727	0.000000
X13N	2.316273	0.000000
X21N	1.000000	0.000000
X22N	0.3162730	0.000000
X23N	5.683727	0.000000
X31N	4.000000	0.000000
X32N	1.000000	0.000000
X33N	0.000000	0.000000
X41N	0.000000	0.2952756
X42N	3.000000	0.000000
X43N	3.000000	0.000000

$$Z_1^* = 307.04 \quad Z_2^* = 285.92$$

Where $w = \tanh^{-1}(2\alpha - 1)$ *and* w=1.52, $\alpha = 0.92$

$$\mu_1^e\{Z_1(X)\} = \begin{cases} 1, & 298 \le Z_1(X) \\ e^{-\frac{1}{2}\left(\frac{Z_1 - 298}{d_k}\right)} & 298 \le Z_1(X) \le 335, \qquad d_{k_1} = 37 \\ 0, & Z_1(X) \ge 335 \end{cases}$$

$$\mu_2^e\{Z_2(X)\} = \begin{cases} 1, & 283 \le Z_1(X) \\ e^{-\frac{1}{2}\left(\frac{Z_2 - 283}{d_k}\right)} & 283 \le Z_2(X) \le 295, \qquad d_{k_2} = 12 \\ 0, & Z_2(X) \ge 295 \end{cases}$$

Maximize α

Such that

$$\alpha \le e^{-\frac{1}{2}\left(\frac{307.04 - 298}{37}\right)} \text{ and } \alpha \le e^{-\frac{1}{2}\left(\frac{285.92 - 283}{12}\right)}$$

The solution of the problem is given by $\alpha = 0.89$

5. Conclusion

A fuzzy MOMIT algorithm is constructed in this paper, and with the help of numerical examples, a solution is demonstrated using three various kinds of membership functions, including linear, hyperbolic, and exponential membership functions. The numerous modes of transporting goods between points of origin and destination are represented by the multi-index transportation problem. The crisp model becomes a linear one when the hyperbolic membership functionis used. When compared to the linear membership function and hyperbolic membership function, the optimum compromise solution is drastically different. However, there is no significant different between the linear membership function's solution and the exponential membership function's ideal compromise solution.

There are numerous methods that future research in the field of fuzzy programming might be carried out. For problems involving many scales and several objectives in linear programming, employing fuzzy programming to design decision support systems will be particularly beneficial in real-world scenarios. Future research may take into account the use of fuzzy programming to solve MOTP's when the supply and demand factors are simply made up of fuzzy integers. There is still room for more research into duality theory and post optimality analysis in multi-objective two- and three-dimensional transportation problems. The demand parameters, agency capacity, and mode of transportation capacity can be anticipated as random variables that follow specific probability distributions in multi-index transportation problems, in addition to the two index transportation problem.

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