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PROPERTIES OF QUADRASOPHIC FUZZY SET AND ITS APPLICATIONS

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Abstract

Fuzzy set theory is a distinctive way of approaching ambiguous information. In this artifact, we introduce a new extension of fuzzy set known as Quadrasophic Fuzzy set and its properties. The Quadrasophic Fuzzy set has four parameters. The attributes and operations of the Quadrasophic Fuzzy sets are defined with pertinent examples. The arithmetic aggregator operators with a redefined level of 0.5 are introduced. The theorems of aggregator operators of Quadrasophic Fuzzy sets are explained using mathematical formulations. Suitable results and examples are provided to enlighten the proposed method. The arithmetic aggregator operators of the proposed method have been used in decision- making to get the optimal solution with supplementary statistics. Additionally, the selection of appropriate fertilizer in farming is demonstrated using the operators of the suggested model. A decision making approach is also used to develop the proposed method in order to identify the ideal solution. An illustration is provided to examine the unique feature of the proposed method to resolve the decision-making problems with a perfect solution.

Keywords: Quadrasophic Fuzzy set, Operations, Aggregated operator, Decision making , QF-VIKOR

1. Introduction

The most crucial factor in analyzing the vagueness state is the fuzzy set. The theory of fuzzy sets and its numerous extensions help in solving decision-making issues and are useful in real-world circumstances that involve uncertain data. To address the newly introduced non-membership parameter the Intuitionistic fuzzy set was introduced by Atanassov [1] . R. Yager [7] develops the idea of the Pythagorean fuzzy set (PFS) in order to address the IFS deficiency. To tackle a bipolar environment, Zhang [8] proposed the concept of a bipolar fuzzy set. Cuong proposed the idea of a Picture fuzzy set to manage the neutral environment. The Neurosophic fuzzy set was a concept that Florentin Smarandache [2] presented to handle three new parameters .

There is a great deal of unclear information available today. Poor choices can cause a person to suffer in life. A successful existence depends on making decisions. Every field relies heavily on decisions to move forward in the right direction. For the examination of decision-making, fuzzy set theory is useful. The authors devised a variety of strategies to find the MCDM problem's ideal answer. Some of the operators used in MCDM problems comprise the fuzzy weighted technique, arithmetic and geometric operators, power operators, Yager's operator, Dombi's operator, weighted ordered arithmetic and geometric, and hybrid operator. To obtain precise outcomes, authors today use cutting-edge techniques to tackle decision-making problems. And also, in order to obtain the results in an appropriate manner, several authors have incorporated the current models into new methodologies. Also, various extensions of the fuzzy set are used in decision-making environments to produce the best results. The use of MDCM is widespread, including applications in areas like business management, operations research, neural-network, and medical science.

C. Jana et al. [5] analyzed the role of bipolar fuzzy Dombi aggregation operators in the multi-attribute decision-making process. K. Mohana and R. Jansi [6] developed the weighted arithmetic operator and applied it to the MCDM problem in a bipolar-Pythagorean environment. The VIKOR technique [10] for decision-making was employed by several authors.

A new idea in fuzzy set design, known as a Quadrasophic fuzzy set (QFS), employs two new parameters. Restricted membership values are a QFS exclusive feature that helps it effectively manage uncertain situations. QFS has four parameters: positive membership functions, restricted positive, restricted negative, and negative. The system's advantages and disadvantages are revealed by the restricted value. As a result, QFS consists of techniques with built-in values that make it simpler to understand the situation and deliver accurate results.

In this artifact, Section 2 provides the preliminary definitions. Basic operations of QFS and results are given in Section 3, certain properties, operations and theorems of QFS are presented in Section 4 with pertinent examples. The comparison study with other current models is presented in Section 5 to corroborate our findings. Section 6 illustrates the use of QFS to analyze fertilizer in agricultural decision-making. The novel application of the QF-VIKOR technique in decision-making is shown in Section 7 and Section 8 supplies the conclusion with its scope for further research.

2. Preliminaries

Intuitionistic Fuzzy sets [4]: An intuitionistic fuzzy set I in the non-empty set X is defined as $I = \{(x, \mu(x), \nu(x); x \in X)\}$ where the function $\mu(x), \nu(x) : X \to [0,1]$ represents the membership degree and the non- membership degree value with the condition $0 \leq \mu(x) + \nu(x) \leq 1$. The value $\pi(x) = 1 - \mu(x) - \nu(x)$ is named as the degree of indeterminacy $\forall x \in X$.

Pythagorean Fuzzy set [7] : A Pythagorean fuzzy set P, is defined as $P = \{(x, \mu(x), \nu(x); x \in X\}$ where the function $\mu(x), \nu(x){:}X \to [0,1]$ represents the membership degree and the non- membership degree value with the condition $0 \leq \mu_P^2(x) + \nu_P^2(x) \leq 1$. The value $\pi_P(x) = \sqrt{1 - (\mu_P^2(x) + \nu_P^2(x))}$ is named as the degree of indeterminacy for $\forall x \in X$.

Picture fuzzy sets [3]: A picture fuzzy set (PFS) on the universe U is defined as $P = \{(x, \mu(x), \eta(x), \nu(x))\}$ where $\mu(x) \in [0,1]$ is the positive membership degree of x in P, $\eta(x) \in [0,1]$ is the neutral membership degree and $\nu(x) \in [0,1]$ is the negative membership degree.

Bipolar fuzzy sets [8] : Let X be the non-empty fuzzy set. The bipolar fuzzy set $B = \{B^-, B^+\}$ is defined in X. $B^+ \in [0,1]$ is the satisfaction degree of x in B, $B^- \in [-1,0]$ is the satisfaction degree of the implicit counter property of x in B.

Properties of Pythagorean fuzzy set [7]: Let $PF_1, PF_2 \in PFS(x)$ then the result as follows,

$$PF_1 + PF_2 = \{x, \sqrt{(\mu_{PF_1}(x))^2 + (\mu_{PF_2}(x))^2 - (\mu_{PF_1}(x))^2(\mu_{PF_2}(x))^2},$$
$$\nu_{PF_1}(x)\,\nu_{PF_2}(x)\}, \forall x \in X.$$

3. Quadrasophic Fuzzy Set

Quadrasophic fuzzy set: The Quadrasophic fuzzy set (QFS) on the universal set X is defined as

$$Q = \{(x, \eta(x), \lambda_\eta(x), \lambda_\mu(x), \mu(x)) | x \in X\}$$

where $\mu(x) : X \to [0,1]$ is the degree of high positive membership of x in Q, $\eta(x) : X \to [-1,0]$ is the degree of high negative membership of x in Q, $\lambda_\mu(x) : X \to [0,0.5]$ is the degree of restricted positive membership of x in Q,$\lambda_\eta(x) : X \to [-0.5,0]$ is the degree of restricted negative membership of x in Q. And it satisfies the following condition: for all $x \in X$, $-1 \leq \mu(x) + \eta(x) \leq 1$, $-0.5 \leq \lambda \leq 0.5$ and $0 \leq \mu^2 + \eta^2 + \lambda^2 \leq 3$ where $\lambda =$Length of $(\lambda_\mu, \lambda_\eta)$. Let $QFS(x)$ denotes the collection of all Quadrasophic fuzzy set on X.

3.1. Operations of Quadrasophic Fuzzy Set

Intersection of QFS : The intersection of two quadrasophic fuzzy set Q_1 and Q_2 in QFS is defined as:

$$Q_1 \cap Q_2 = \{max(\eta_{Q_1}(x), \eta_{Q_2}(x)), max(\lambda_{\eta_{Q_1}}(x), \lambda_{\eta_{Q_2}}(x)), \\ min(\lambda_{\mu_{Q_1}}(x), \lambda_{\mu_{Q_2}}(x)), min(\mu_{Q_1}(x), \mu_{Q_2}(x))\} \forall x \in X.$$

Union of QFS: The union of Q_1 and Q_2 in Quadrasophic fuzzy set is defined as:

$$Q_1 \cup Q_2 = \{min(\eta_{Q_1}(x), \eta_{Q_2}(x)), min(\lambda_{\eta_{Q_1}}(x), \lambda_{\eta_{Q_2}}(x)), \\ max(\lambda_{\mu_{Q_1}}(x), \lambda_{\mu_{Q_2}}(x)), max(\mu_{Q_1}(x), \mu_{Q_2}(x))\} \forall x \in X.$$

Subset: Let $Q_1, Q_2 \in Q$ defined on the non - empty set X then Q_1 is the subset of Q_2 denoted by $Q_1 \subseteq Q_2$, if for each $x \in X$; $\eta_{Q_1}(x) \geq \eta_{Q_2}(x)$, $\lambda_{\eta Q_1}(x) \geq \lambda_{\eta Q_2}(x)$, $\lambda_{\mu Q_1}(x) \leq \lambda_{\mu Q_2}(x)$, $\mu_{Q_1}(x) \leq \mu_{Q_2}(x)$.

Complement of QFS: The complement of the set $Q_1 \in Q$ in X is represented as Q_1^C and is defined as $Q_1^C = (\eta^C, \lambda_\eta^C, \lambda_\mu^C, \mu^C)$, where $\eta^C = -1 - \eta$, $\lambda_\eta^C = -0.5 - \lambda_\eta$, $\lambda_\mu^C = 0.5 - \lambda_\mu$ and $\mu^C = 1 - \mu$.

Equal Set: Let $Q_1, Q_2 \in Q$ be defined on the non empty set X then Q_1 is equal set to Q_2 denoted by $Q_1 = Q_2$ if for each $x \in X$;

$$\eta_{Q_1}(x) = \eta_{Q_2}(x), \lambda_{\eta_{Q_1}}(x) = \lambda_{\eta_{Q_2}}(x), \lambda_{\mu_{Q_1}}(x) = \lambda_{\mu_{Q_2}}(x), \mu_{Q_1}(x) = \mu_{Q_2}(x).$$

Distance Metric of QFS : The normalized Hamming distance between any QFS set $Q_1, Q_2 \in Q(x)$ is defined as,

$$d_{Qh}(Q_1, Q_2) = \frac{1}{2n}\sum_{i=1}^{n}[\left|\eta_{Q_1}(x_i))^2 - (\eta_{Q_2}(x_i))^2\right| + \left|(\lambda_{\eta_{Q_1}}(x_i))^2 - (\lambda_{\eta_{Q_2}}(x_i))^2\right| \\ + \left|(\lambda_{\mu_{Q_1}}(x_i))^2 - (\lambda_{\mu_{Q_2}}(x_i)))^2\right| + \left|(\mu_{Q_1}(x_i))^2 - \mu_{Q_2}(x_i))^2\right|].$$

The normalized Euclidean distance between any QFS set $Q_1, Q_2 \in Q(x)$ is defined as,

$$d_{Qh}(Q_1, Q_2) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}[\eta_{Q_1}(x_i))^2 - (\eta_{Q_2}(x_i))^2]^2 + [](\lambda_{\eta_{Q_1}}(x_i))^2 - (\lambda_{\eta_{Q_2}}(x_i))^2]^2 \\ + [(\lambda_{\mu_{Q_1}}(x_i))^2 - (\lambda_{\mu_{Q_2}}(x_i)))^2]^2 + [(\mu_{Q_1}(x_i))^2 - \mu_{Q_2}(x_i))^2]^2}$$

Proposition 1. The QFS is not the simplification of bipolar fuzzy set.

Proof. Let $Q_1 = (\eta_{Q_1}, \lambda_{\eta Q_1}, \lambda_{\mu Q_1}, \mu_{Q_1})$ be the set in QFS and $B_1 = \{b_1^-, b_1^+\}$ be the set in BFS. Although positive (b_1^+) and negative (b_1^-) degree of membership exists in BFS, there is no restricted level. It does not give the information about the partial belongingness or level of influential. In QFS, separate parameter is fixed to trace the level of restricted. Hence, QFS differs from BFS. ■

Remark 1. If both membership grade of positive and negative restricted value is zero. Then, QFS is equal to BFS.

4. Certain Properties of Quadrasophic Fuzzy Set

In this segment, the operations and Quadrasophic fuzzy set weighted arithmetic operator $(QFWA)$ is defined and proved with theorems and illustrated with model.
Algebraic Sum : Let $Q_1, Q_2 \in Q$
i) If $Q_1 = (\eta_{Q_1}, \lambda_{\eta Q_1}, \lambda_{\mu Q_1}, \mu_{Q_1})$ and $Q_2 = (\eta_{Q_2}, \lambda_{\eta Q_2}, \lambda_{\mu Q_2}, \mu_{Q_2})$ be any QFS, then the algebraic sum is defined as:

$$Q_1 \oplus Q_2 = \{-(-\eta_{Q_1})(-\eta_{Q_2}), -(-\lambda_{\eta_{Q_1}})(-\lambda_{\eta_{Q_2}}),$$
$$\sqrt{0.5\lambda^2_{\mu Q_1} + 0.5\lambda^2_{\mu Q_2} - (0.5\lambda^2_{\mu Q_1})(0.5\lambda^2_{\mu Q_2})}, \sqrt{\mu^2_{Q_1} + \mu^2_{Q_2} - \mu^2_{Q_1}\mu^2_{Q_2}}\}$$

Then ,

$$Q_1 \oplus Q_2 = \{-(-\eta_{Q_1})(-\eta_{Q_2}), -(-\lambda_{\eta_{Q_1}})(-\lambda_{\eta_{Q_2}}),$$
$$((1-(1-0.5\lambda^2_{\mu Q_1}) + (1-(1-0.5\lambda^2_{\mu Q_2}) - (1-(1-0.5\lambda^2_{\mu Q_1}).(1-(1-0.5\lambda^2_{\mu Q_2}))^{\frac{1}{2}},$$
$$((1-(1-\mu^2_{Q_1}) + (1-(1-\mu^2_{Q_2}) - (1-(1-\mu^2_{Q_1}).(1-(1-\mu^2_{Q_2}))^{\frac{1}{2}}\}$$

ii) For $\alpha \geq 0$,

$$\alpha Q_1 = \{-(-\eta_{Q_1})^{\alpha}, -(-\lambda_{\eta Q_1})^{\alpha}, (1-(1-0.5\lambda^2_{\mu Q_1})^{\alpha})^{\frac{1}{2}}, (1-(1-\mu^2_{Q_1})^{\alpha})^{\frac{1}{2}}\}.$$

Theorem 1. If $Q_1 = (\eta_{Q_1}, \lambda_{\eta Q_1}, \lambda_{\mu Q_1}, \mu_{Q_1})$ and $Q_2 = (\eta_{Q_2}, \lambda_{\eta Q_2}, \lambda_{\mu Q_2}, \mu_{Q_2})$ be any two set in QFS defined in the non-empty set X and for $\alpha_1, \alpha_2 \geq 0$ the results follow:
i) $Q_1 \oplus Q_2 = Q_2 \oplus Q_1$

$$Q_1 \oplus Q_2 = \{-(-\eta_{Q_1})(-\eta_{Q_2}), -(-\lambda_{\eta_{Q_1}})(-\lambda_{\eta_{Q_2}}),$$
$$\sqrt{0.5\lambda^2_{\mu Q_1} + 0.5\lambda^2_{\mu Q_2} - (0.5\lambda^2_{\mu Q_1})(0.5\lambda^2_{\mu Q_2})}, \sqrt{\mu^2_{Q_1} + \mu^2_{Q_2} - \mu^2_{Q_1}\mu^2_{Q_2}}\}$$
$$= \{-(-\eta_{Q_2})(-\eta_{Q_1}), -(-\lambda_{\eta_{Q_2}})(-\lambda_{\eta_{Q_1}}),$$
$$\sqrt{0.5\lambda^2_{\mu Q_2} + 0.5\lambda^2_{\mu Q_1} - (0.5\lambda^2_{\mu Q_2})(0.5\lambda^2_{\mu Q_1})}, \sqrt{\mu^2_{Q_2} + \mu^2_{Q_1} - \mu^2_{Q_2}\mu^2_{Q_1}}\}$$
$$= Q_2 \oplus Q_1$$

ii) $\alpha(Q_1 \oplus Q_2) = (\alpha Q_1 \oplus \alpha Q_2)$

$$\alpha(Q_1 \oplus Q_2) = \alpha\{-(-\eta_{Q_1})(-\eta_{Q_2}), -(-\lambda_{\eta_{Q_1}})(-\lambda_{\eta_{Q_2}}),$$
$$(1-(1-(0.5\lambda^2_{\mu Q_1} + 0.5\lambda^2_{\mu Q_2} - (0.5\lambda^2_{\mu Q_1} \times 0.5\lambda^2_{\mu Q_2}))))^{\frac{1}{2}},$$
$$(1-(1-(\mu^2_{Q_1} + \mu^2_{Q_2} - \mu^2_{Q_1}.\mu^2_{Q_2})))^{\frac{1}{2}}\}$$
$$= \{-(-\eta_{Q_1})^{\alpha}(-\eta_{Q_2})^{\alpha}, -(-\lambda_{\eta_{Q_1}})^{\alpha}(-\lambda_{\eta_{Q_2}})^{\alpha},$$
$$(1-(1-(0.5\lambda^2_{\mu Q_1} + 0.5\lambda^2_{\mu Q_2} - (0.5\lambda^2_{\mu Q_1} \times 0.5\lambda^2_{\mu Q_2})))^{\alpha})^{\frac{1}{2}},$$
$$(1-(1-(\mu^2_{Q_1} + \mu^2_{Q_2} - \mu^2_{Q_1}.\mu^2_{Q_2}))^{\alpha})^{\frac{1}{2}}\}$$

$$\begin{aligned}(\alpha Q_1 \oplus \alpha Q_2) &= \{-(-\eta_{Q_1})^{\alpha}, -(-\lambda_{\eta_{Q_1}})^{\alpha}, (1-(1-0.5\lambda^2_{\mu Q_1})^{\alpha})^{\frac{1}{2}}, (1-(1-\mu^2_{Q_1})^{\alpha})^{\frac{1}{2}}\}\\
&\oplus \{-(-\eta_{Q_2})^{\alpha}, -(-\lambda_{\eta_{Q_2}})^{\alpha}, (1-(1-0.5\lambda^2_{\mu Q_2})^{\alpha})^{\frac{1}{2}}, (1-(1-\mu^2_{Q_2})^{\alpha})^{\frac{1}{2}}\}\\
&= \{-(-\eta_{Q_1})^{\alpha}(-\eta_{Q_2})^{\alpha}, -(-\lambda_{\eta_{Q_1}})^{\alpha}(-\lambda_{\eta_{Q_2}})^{\alpha},\\
&(1-(1-0.5\lambda^2_{\mu Q_1})^{\alpha}(1-0.5\lambda^2_{\mu Q_2})^{\alpha})^{\frac{1}{2}}, (1-(1-\mu^2_{Q_1})^{\alpha}(1-\mu^2_{Q_2})^{\alpha})^{\frac{1}{2}}\}\\
&= \{-(-\eta_{Q_1})^{\alpha}(-\eta_{Q_2})^{\alpha}, -(-\lambda_{\eta_{Q_1}})^{\alpha}(-\lambda_{\eta_{Q_2}})^{\alpha},\\
&(1-(1-(0.5\lambda^2_{\mu Q_1}+0.5\lambda^2_{\mu Q_2}-(0.5\lambda^2_{\mu Q_1}\times 0.5\lambda^2_{\mu Q_2})))^{\alpha})^{\frac{1}{2}},\\
&(1-(1-(\mu^2_{Q_1}+\mu^2_{Q_2}-\mu^2_{Q_1}.\mu^2_{Q_2})))^{\alpha})^{\frac{1}{2}}\}\end{aligned}$$

Hence, $\alpha(Q_1 \oplus Q_2) = (\alpha Q_1 \oplus \alpha Q_2)$

iii) $\alpha_1 Q_1 \oplus \alpha_2 Q_1 = (\alpha_1 \oplus \alpha_2) Q_1$

$$\begin{aligned}\alpha_1 Q_1 \oplus \alpha_2 Q_1 &= \alpha_1\{(-\eta_{Q_1}), (-\lambda_{\eta_{Q_1}}), (1-(1-0.5\lambda^2_{\mu Q_1}))^{\frac{1}{2}}, (1-(1-(\mu^2_{Q_1}))^{\frac{1}{2}}\}\\
&\oplus \alpha_2\{(-\eta_{Q_1}), (-\lambda_{\eta_{Q_1}}), (1-(1-0.5\lambda^2_{\mu Q_1}))^{\frac{1}{2}}, (1-(1-(\mu^2_{Q_1}))^{\frac{1}{2}}\}\\
&= \{-(-\eta_{Q_1})^{\alpha_1}(-\eta_{Q_1})^{\alpha_2}, -(-\lambda_{\eta_{Q_1}})^{\alpha_1}(-\lambda_{\eta_{Q_1}})^{\alpha_2},\\
&(1-(1-0.5\lambda^2_{\mu Q_1})^{\alpha_1}(1-0.5\lambda^2_{\mu Q_1})^{\alpha_2})^{\frac{1}{2}}, (1-(1-\mu^2_{Q_1})^{\alpha_1}(1-\mu^2_{Q_1})^{\alpha_2})^{\frac{1}{2}}\}\\
&= \{-(-\eta_{Q_1})^{\alpha_1+\alpha_2}, -(-\lambda_{\eta_{Q_1}})^{\alpha_1+\alpha_2},\\
&(1-(1-0.5\lambda^2_{\mu Q_1})^{\alpha_1+\alpha_2})^{\frac{1}{2}}, (1-(1-\mu^2_{Q_1})^{\alpha_1+\alpha_2})^{\frac{1}{2}}\}\end{aligned}$$

$$\begin{aligned}(\alpha_1 \oplus \alpha_2) Q_1 &= (\alpha_1 \oplus \alpha_2)\{(-\eta_{Q_1}), (-\lambda_{\eta_{Q_1}}), (1-(1-0.5\lambda^2_{\mu Q_1}))^{\frac{1}{2}}, (1-(1-(\mu^2_{Q_1})))^{\frac{1}{2}}\}\\
&= \{-(-\eta_{Q_1})^{\alpha_1+\alpha_2}, -(-\lambda_{\eta_{Q_1}})^{\alpha_1+\alpha_2},\\
&(1-(1-0.5\lambda^2_{\mu Q_1})^{\alpha_1+\alpha_2})^{\frac{1}{2}}, (1-(1-\mu^2_{Q_1})^{\alpha_1+\alpha_2})^{\frac{1}{2}}\}\end{aligned}$$

Hence, $\alpha_1 Q_1 \oplus \alpha_2 Q_1 = (\alpha_1 \oplus \alpha_2) Q_1$

Example 1: Consider $Q_1 = (-0.5, -0.2, 0.4, 0.7)$ and $Q_2 = (-0.6, -0.4, 0.5, 0.8)$
i) Since, $-0.5 \geq -0.6$, $-0.2 \geq -0.4$, $0.4 \leq 0.5$, $0.7 \leq 0.8$
$\implies Q_1 \subseteq Q_2$.
Assume $n = 0.5$ then $-0.25 \geq -0.3$, $-0.1 \geq -0.2$, $0.2 \leq 0.25$, $0.35 \leq 0.4$
$\implies nQ_1 \subseteq nQ_2$(for any positive integer n).
ii) The union of Q_1 and Q_2 is: $Q_1 \cup Q_2 = (-0.6, -0.4, 0.5, 0.8)$
Assume $n = 0.5$ then $n(Q_1 \cup Q_2) = (-0.3, -0.2, 0.25, 0.4)$
$nQ_1 = (-0.25, -0.1, 0.2, 0.35)$, $nQ_2 = (-0.3, -0.2, 0.25, 0.4)$
$\implies nQ_1 \cup nQ_2 = (-0.3, -0.2, 0.25, 0.4)$
Thus, $n(Q_1 \cup Q_2) = nQ_1 \cup nQ_2$.
iii) The intersection of Q_1 and Q_2 is: $Q_1 \cap Q_2 = (-0.5, -0.2, 0.4, 0.7)$
If $n = 0.5$, then $n(Q_1 \cap Q_2) = (-0.25, -0.1, 0.2, 0.35)$
$\implies nQ_1 \cap nQ_2 = (-0.25, -0.1, 0.2, 0.35)$
Thus, $n(Q_1 \cap Q_2) = nQ_1 \cap nQ_2$.

Theorem 2. Let $Q_1 = (\eta_{Q_1}, \lambda_{\eta Q_1}, \lambda_{\mu Q_1}, \mu_{Q_1})$ and $Q_2 = (\eta_{Q_2}, \lambda_{\eta Q_2}, \lambda_{\mu Q_2}, \mu_{Q_2})$ be any two QFS set defined in the non-empty set X and for $n \geq 0$ the results follow:
i) If $Q_1 \subseteq Q_2$ then $nQ_1 \subseteq nQ_2$.
ii) $n(Q_1 \cup Q_2) = nQ_1 \cup nQ_2$.
iii) $n(Q_1 \cap Q_2) = nQ_1 \cap nQ_2$.

Proof. The proof is obvious by Example 1.

■

QFWA Operator: Let $Q_s = (\eta_s, \lambda_{\eta_s}, \lambda_{\mu_s}, \mu_s)$ $(s = 1, 2, \ldots, n)$ be the set of QFS. Then $QFWA$ (Quadrasophic Fuzzy Weighted Arithmetic operator) with respect to α_i=$(\alpha_1, \alpha_2, \ldots, \alpha_n)$ is the weight vector, where $\alpha_i \in [0, 1]$ such that $\sum_{i=1}^{n} \alpha_i = 1$ is a function defined from $Q^n \to Q$. Then $QFWA$ is defined as:

$$\begin{aligned} QFWA(Q_1, Q_2, \ldots, Q_n) &= \oplus_{i=1}^{n} \alpha_i Q_i \\ &= (\alpha_1 Q_1 + \alpha_2 Q_2 + \cdots + \alpha_n Q_n) \end{aligned}$$

Theorem 3. Let $Q_s = (\eta_s, \lambda_{\eta_s}, \lambda_{\mu_s}, \mu_s)$ $(s = 1, 2, 3 \ldots, n)$ be the set of QFS defined in the non-empty set X. Then $QFWA$ operator of QFS is defined as

$$\begin{aligned} QFWA(Q_1, Q_2, \ldots, Q_s) &= (-\prod_{k=1}^{s}(-\eta_{Q_k})^{\alpha_k}, -\prod_{k=1}^{s}(-\lambda_{\eta_{Q_k}})^{\alpha_k}, \\ &(1 - \prod_{k=1}^{s}(1 - 0.5\lambda^2_{\mu_{Q_k}})^{\alpha_k})^{\frac{1}{2}}, (1 - \prod_{k=1}^{s}(1 - \mu^2_{Q_k})^{\alpha_k})^{\frac{1}{2}}) \end{aligned} \tag{1}$$

Proof. The proof follows the method of mathematical induction. Assume that $n = 2$ then,

$$QFWA(Q_1, Q_2) = \alpha_1 Q_1 \oplus \alpha_2 Q_2 = \alpha_1(\eta_{Q_1}, \lambda_{\eta_{Q_1}}, \lambda_{\mu_{Q_1}}, \mu_{Q_1}) \oplus \alpha_2(\eta_{Q_2}, \lambda_{\eta_{Q_2}}, \lambda_{\mu_{Q_2}}, \mu_{Q_2})$$

$$\begin{aligned} \alpha_1 Q_1 \oplus \alpha_2 Q_1 &= \alpha_1\{(-\eta_{Q_1}), (-\lambda_{\eta_{Q_1}}), (1 - (1 - 0.5\lambda^2_{\mu Q_1}))^{\frac{1}{2}}, (1 - (1 - (\mu^2_{Q_1}))^{\frac{1}{2}}\} \\ &\oplus \alpha_2\{(-\eta_{Q_1}), (-\lambda_{\eta_{Q_1}}), (1 - (1 - 0.5\lambda^2_{\mu Q_1}))^{\frac{1}{2}}, (1 - (1 - (\mu^2_{Q_1}))^{\frac{1}{2}}\} \\ &= \{-(-\eta_{Q_1})^{\alpha_1}(-\eta_{Q_1})^{\alpha_2}, -(-\lambda_{\eta_{Q_1}})^{\alpha_1}(-\lambda_{\eta_{Q_1}})^{\alpha_2}, \\ &([1 - (1 - 0.5\lambda^2_{\mu Q_1})^{\alpha_1}] + [1 - (1 - 0.5\lambda^2_{\mu Q_2})^{\alpha_2}] \\ &- [1 - (1 - 0.5\lambda^2_{\mu Q_1})^{\alpha_1}][1 - (1 - 0.5\lambda^2_{\mu Q_2})^{\alpha_2}])^{\frac{1}{2}}, \\ &([1 - (1 - \mu^2_{Q_1})^{\alpha_1}] + [1 - (1 - \mu^2_{Q_2})^{\alpha_2}] \\ &- [1 - (1 - \mu^2_{Q_1})^{\alpha_1})[1 - (1 - \mu^2_{Q_2})^{\alpha_2}])^{\frac{1}{2}}\} \\ &= \{-\prod_{k=1}^{2}(-\eta_{Q_k})^{\alpha_k}, -\prod_{k=1}^{2}(-\lambda_{\eta_{Q_k}})^{\alpha_k}, (1 - (1 - 0.5\lambda^2_{\mu_{Q_1}})^{\alpha_1})(1 - 0.5\lambda^2_{\mu_{Q_2}})^{\alpha_2})^{\frac{1}{2}}, \\ &(1 - (1 - \mu^2_{Q_1})^{\alpha_1}(1 - \mu^2_{Q_2})^{\alpha_2})^{\frac{1}{2}}\} \\ \alpha_1 Q_1 \oplus \alpha_2 Q_2 &= \{-\prod_{k=1}^{2}(-\eta_{Q_k})^{\alpha_k}, -\prod_{k=1}^{2}(-\lambda_{\eta_{Q_k}})^{\alpha_k}, (1 - \prod_{k=1}^{2}(1 - 0.5\lambda^2_{\mu_{Q_k}})^{\alpha_k})^{\frac{1}{2}}, \\ &(1 - \prod_{k=1}^{2}(1 - \mu^2_{Q_k})^{\alpha_k})^{\frac{1}{2}}\} \end{aligned}$$

For $n = s$ assume Equation 1 is true. Thus, the result follows:

$$\begin{aligned} QFWA(Q_1, Q_2, \ldots, Q_s) &= (-\prod_{k=1}^{s}(-\eta_{Q_k})^{\alpha_k}, -\prod_{k=1}^{s}(-\lambda_{\eta_{Q_k}})^{\alpha_k}, \\ &(1 - \prod_{k=1}^{s}(1 - 0.5\lambda^2_{\mu_{Q_k}})^{\alpha_k})^{\frac{1}{2}}, (1 - \prod_{k=1}^{s}(1 - \mu^2_{Q_k})^{\alpha_k})^{\frac{1}{2}}) \end{aligned}$$

For $n = s+1$,

$$\begin{aligned} QFWA(Q_1, Q_2, \dots, Q_{s+1}) &= \oplus_{k=1}^{s}(\alpha_k Q_k) \oplus (\alpha_{s+1} Q_{s+1}) \\ &= (-\prod_{k=1}^{s}(-\eta_{Q_k})^{\alpha_k}, -\prod_{k=1}^{s}(-\lambda_{\eta_{Q_k}})^{\alpha_k}, \\ &(1-\prod_{k=1}^{s}(1-0.5\lambda_{\mu_{Q_k}}^2)^{\alpha_k})^{\frac{1}{2}}, (1-\prod_{k=1}^{s}(1-\mu_{Q_k}^2)^{\alpha_k})^{\frac{1}{2}})) \\ &\oplus ((-\eta_{Q_{s+1}})^{\alpha_{s+1}}, (-\lambda_{\eta_{Q_{s+1}}})^{\alpha_{s+1}}, \\ &(1-(1-0.5\lambda_{\mu_{Q_{s+1}}}^2)^{\alpha_{s+1}})^{\frac{1}{2}}, (1-(1-\mu_{Q_{s+1}}^2)^{\alpha_{s+1}})^{\frac{1}{2}}) \end{aligned}$$

$$\begin{aligned} QFWA(Q_1, \dots, Q_{s+1}) &= (-\prod_{k=1}^{s}(-\eta_{Q_k})^{\alpha_k} \oplus (-\eta_{Q_{s+1}})^{\alpha_{s+1}}, \\ &-\prod_{k=1}^{s}(-\lambda_{\eta_{Q_k}})^{\alpha_k} \oplus (-\lambda_{\eta_{Q_{s+1}}})^{\alpha_{s+1}}, \\ &(1-\prod_{k=1}^{s}(1-0.5\lambda_{\mu_{Q_k}}^2)^{\alpha_k})^{\frac{1}{2}} \oplus (1-(1-0.5\lambda_{\mu_{Q_{s+1}}}^2)^{\alpha_{s+1}})^{\frac{1}{2}}, \\ &(1-\prod_{k=1}^{s}(1-\mu_{Q_k}^2)^{\alpha_k})^{\frac{1}{2}} \oplus (1-(1-\mu_{Q_{s+1}}^2)^{\alpha_{s+1}})^{\frac{1}{2}}) \\ &= \{(-\prod_{k=1}^{s+1}(-\eta_{Q_k})^{\alpha_k}, -\prod_{k=1}^{s+1}(-\lambda_{\eta_{Q_k}})^{\alpha_k}, \\ &(1-\prod_{k=1}^{s}(1-0.5\lambda_{\mu Q_k}^2)^{\alpha_k})^{\frac{1}{2}} + (1-(1-0.5\lambda_{\mu Q_{s+1}}^2)^{\alpha_{s+1}})^{\frac{1}{2}} \\ &-(1-\prod_{k=1}^{s}(1-0.5\lambda_{\mu Q_k}^2)^{\alpha_k})^{\frac{1}{2}}(1-(1-0.5\lambda_{\mu Q_{s+1}}^2)^{\alpha_{s+1}}])^{\frac{1}{2}}, \\ &(1-\prod_{k=1}^{s}(1-\mu_{Q_k}^2)^{\alpha_k})^{\frac{1}{2}} + (1-(1-\mu_{Q_{s+1}}^2)^{\alpha_{s+1}})^{\frac{1}{2}} \\ &-(1-\prod_{k=1}^{s}(1-\mu_{Q_k}^2)^{\alpha_k})^{\frac{1}{2}}(1-(1-\mu_{Q_{s+1}}^2)^{\alpha_{s+1}})^{\frac{1}{2}}\} \end{aligned}$$

$$\begin{aligned} QFWA(Q_1, Q_2, \dots, Q_s) &= (-\prod_{k=1}^{s+1}(-\eta_{Q_k})^{\alpha_k}, -\prod_{k=1}^{s+1}(-\lambda_{\eta_{Q_k}})^{\alpha_k}, \\ &(1-\prod_{k=1}^{s+1}(1-0.5\lambda_{\mu_{Q_k}}^2)^{\alpha_k})^{\frac{1}{2}}, (1-\prod_{k=1}^{s+1}(1-\mu_{Q_k}^2)^{\alpha_k})^{\frac{1}{2}}) \end{aligned}$$

Hence 1 is true for $n = s+1$. By, the method of mathematical induction we conclude 1 is true for any $n > 0$. ■

Example 2: Let $Q_k = (\eta_k, \lambda_{\eta_k}, \lambda_{\mu_k}, \mu_k)$, $k = 1,2,3,4$ be any four Quadrasophic fuzzy set.Consider $Q_1 = (-0.7, -0.3, 0.3, 0.5)$, $Q_2 = (-0.5, -0.2, 025, 0.7)$, Q_3=$(-0.8, -0.4, 0.3, 0.7)$, and $Q_4 = (-0.6, -0.4, 0.5, 0.9)$. We assume the weight vector is $\alpha = (0.2, 0.3, 0.4, 0.1)$. Then

$$\begin{aligned} QFWA(Q_1, Q_2, Q_3, Q_4) &= \oplus_{k=1}^{4}(\alpha_k Q_k) \\ &= 0.2(-0.7, -0.3, 0.3, 0.5) + 0.3(-0.5, -0.2, 025, 0.7) \\ &+ 0.4(-0.8, -0.4, 0.3, 0.7) + 0.1(-0.6, -0.4, 0.5, 0.9) \\ &= (-0.657, -0.307, 0.221, 0.708) \end{aligned}$$

Theorem 4 (Idem-potency Property)**.** If $Q_k = (\eta_k, \lambda_{\eta_k}, \lambda_{\mu_k}, \mu_k)$, k=$1, 2, 3, \dots, n$ be the set of QFS and $Q_k = Q$ $\forall k$. Then $QFWA(Q_1, Q_2, Q_3, \dots, Q_n) = Q$.

Proof. Consider

$$
\begin{aligned}
QFWA(Q_1, Q_2, Q_3, \ldots, Q_n) &= \oplus_{k=1}^{n} (\alpha_k Q_k) \\
QFWA(Q_1, Q_2, \ldots, Q_n) &= (-\prod_{k=1}^{n} (-\eta_{Q_k})^{\alpha_k}, -\prod_{k=1}^{n} (-\lambda_{\eta_{Q_k}})^{\alpha_k}, \\
&\quad (1 - \prod_{k=1}^{n} (1 - 0.5\lambda^2_{\mu_{Q_k}})^{\alpha_k})^{\frac{1}{2}}, (1 - \prod_{k=1}^{n} (1 - \mu^2_{Q_k})^{\alpha_k})^{\frac{1}{2}}) \\
&\quad (since\, Q_k = Q) \\
&= ((-\eta_Q), (-\lambda_{\eta_{Q_k}}), (1 - (1 - 0.5\lambda^2_{\mu_Q}))^{\frac{1}{2}}, \\
&\quad (1 - (1 - \mu^2_Q))^{\frac{1}{2}}) \\
QFWA(Q_1, Q_2, Q_3, \ldots, Q_n) &= Q.
\end{aligned}
$$

■

Theorem 5 (Monotonicity)**.** Let Q_k and $Q_k^{'}$ be the pair of sets in QFS. If $Q_k < Q_k^{'}$ $\forall k$, then $QFWA(Q_1, Q_2, Q_3, \ldots, Q_n) = QFWA(Q_1^{'}, Q_2^{'}, Q_3^{'}, \ldots, Q_n^{'})$.

Proof. If $Q_k, Q_k^{'} \in QFS$ and $Q_k < Q_k^{'}$ $\forall k$, then

$$
\begin{gathered}
(-\prod_{k=1}^{n} (-\eta_{Q_k})^{\alpha_k}, -\prod_{k=1}^{n} (-\lambda_{\eta_{Q_k}})^{\alpha_k}, (1 - \prod_{k=1}^{n} (1 - 0.5\lambda^2_{\mu_{Q_k}})^{\alpha_k})^{\frac{1}{2}}, \\
(1 - \prod_{k=1}^{n} (1 - \mu^2_{Q_k})^{\alpha_k})^{\frac{1}{2}}) \leq (-\prod_{k=1}^{n} (-\eta^{'}_{Q_k})^{\alpha_k}, -\prod_{k=1}^{n} (-\lambda^{'}_{\eta_{Q_k}})^{\alpha_k}, \\
(1 - \prod_{k=1}^{n} (1 - (0.5\lambda^{'}_{\mu_{Q_k}})^2)^{\alpha_k})^{\frac{1}{2}}, (1 - \prod_{k=1}^{n} (1 - (\mu^{'}_{Q_k})^2)^{\alpha_k})^{\frac{1}{2}})
\end{gathered}
$$

$$
QFWA(Q_1, Q_2, Q_3, \ldots, Q_n) = QFWA(Q_1^{'}, Q_2^{'}, Q_3^{'}, \ldots, Q_n^{'}).
$$

■

Example 3: Consider $Q_1 = (-0.7, -0.3, 0.4, 0.8)$ and $Q_2 = (-0.6, -0.2, 0.5, 0.7)$ be two sets in QFS. The weight vector is $\alpha = (0.5, 0.5)$.

Solution: Consider (s) for smaller and (l) for larger value of membership.
We know that,

$$
\begin{gathered}
\prod_{k=1}^{2} (-\eta_{Q_{k_{(s)}}})^{\alpha_k} < \prod_{k=1}^{2} (-\eta_{Q_k})^{\alpha_k} < \prod_{k=1}^{2} (-\eta_{Q_{k_{(l)}}})^{\alpha_k} \\
= (-0.7)^{0.5}(-0.7)^{0.5} < (-0.7)^{0.5}(-0.6)^{0.5} < (-0.6)^{0.5}(-0.6)^{0.5} \\
= -0.7006 < -0.6481 < -0.600.
\end{gathered}
$$

Also,

$$
\begin{gathered}
\prod_{k=1}^{2} (-\lambda_{\eta_{Q_{k_{(s)}}}})^{\alpha_k} < \prod_{k=1}^{2} (-\lambda_{\eta_{Q_k}})^{\alpha_k} < \prod_{k=1}^{2} (-\lambda_{\eta_{Q_{k_{(l)}}}})^{\alpha_k} \\
= -0.299 < -0.244 < -0.199
\end{gathered}
$$

Also,

$$
(1 - \prod_{k=1}^{2} (1 - 0.5\lambda^2_{\mu_{Q_{k_{(s)}}}})^{\alpha_k})^{\frac{1}{2}} < (1 - \prod_{k=1}^{2} (1 - 0.5\lambda^2_{\mu_{Q_k}})^{\alpha_k})^{\frac{1}{2}} < (1 - \prod_{k=1}^{2} (1 - 0.5\lambda^2_{\mu_{Q_{k_{(l)}}}})^{\alpha_k})^{\frac{1}{2}}
$$

$$
= 0.282 < 0.320 < 0.353
$$

Also,

$$(1-\prod_{k=1}^{2}(1-\mu^2_{Q_{k_{(s)}}})^{\alpha_k})^{\frac{1}{2}} < (1-\prod_{k=1}^{2}(1-\mu^2_{Q_k})^{\alpha_k})^{\frac{1}{2}} < (1-\prod_{k=1}^{2}(1-\mu^2_{Q_{k_{(l)}}})^{\alpha_k})^{\frac{1}{2}}$$

$$= 0.700 < 0.756 < 0.800$$

$$\{-\prod_{k=1}^{2}(-\eta_{Q_{k_{(s)}}})^{\alpha_k}, -\prod_{k=1}^{2}(-\lambda_{\eta_{Q_{k_{(s)}}}})^{\alpha_k}, (1-\prod_{k=1}^{2}(1-0.5\lambda^2_{\mu_{Q_{k_{(s)}}}})^{\alpha_k})^{\frac{1}{2}}, (1-\prod_{k=1}^{2}(1-\mu^2_{Q_{k_{(s)}}})^{\alpha_k})^{\frac{1}{2}}\}$$
$$< \{-\prod_{k=1}^{2}(-\eta_{Q_k})^{\alpha_k}), -\prod_{k=1}^{2}(-\lambda_{\eta_{Q_k}})^{\alpha_k}, (1-\prod_{k=1}^{2}(1-0.5\lambda^2_{\mu_{Q_k}})^{\alpha_k})^{\frac{1}{2}}, (1-\prod_{k=1}^{2}(1-\mu^2_{Q_k})^{\alpha_k})^{\frac{1}{2}}\}$$
$$< \{-\prod_{k=1}^{2}(-\eta_{Q_{k_{(l)}}})^{\alpha_k}, -\prod_{k=1}^{2}(-\lambda_{\eta_{Q_{k_{(l)}}}})^{\alpha_k}, (1-\prod_{k=1}^{2}(1-0.5\lambda^2_{\mu_{Q_{k_{(l)}}}})^{\alpha_k})^{\frac{1}{2}}, (1-\prod_{k=1}^{2}(1-\mu^2_{Q_{k_{(l)}}})^{\alpha_k})^{\frac{1}{2}}\}$$

And thus, the set of smaller membership grade is lesser than actual membership grade which is lesser than the larger membership grade.

Table 1: *Comparative study between QFS and Bipolar environment*

Type of operator	Environment	Results
Dombi Weighted [5]	Bipolar Fuzzy environment	$Q_4 > Q_1 > Q_2 > Q_5 > Q_3$ $0.706 > 0.694 > 0.661 > 0.600 > 0.583$
Proposed method	Quadrasophic Fuzzy Set	$Q_4 > Q_1 > Q_2 > Q_5 > Q_3$ $0.134 > 0.124 > 0.121 > 0.057 > 0.017$

Table 2: *Comparative study between QFS and Bipolar Pythagorean environment*

Type of operator	Environment	Results
Weighted Average[6]	Bipolar Pythagorean environment	$Q_1 > Q_2 > Q_3 > Q_4$ $0.383 > 0.189 > -0.031 > -0.102$
Proposed method	Quadrasophic Fuzzy set	$Q_1 > Q_2 > Q_4 > Q_3$ $0.256 > 0.084 > -0.045 > -0.052$

Table 3: *Types, ratio and duration of fertilizer*

Fertilizer types	Composition in ratio	Duration
f_1	120:40:40	Short
f_2	150:60:50	Medium
f_3	150:80:50	Long

Theorem 6 (Bounded property)**.** Consider $Q_k = (\eta_k, \lambda_{\eta_k}, \lambda_{\mu_k}, \mu_k)$, k=1, 2, 3, ..., n be the set of QFS. If
$Q_{(s)} = min(Q_1, Q_2, Q_3, \ldots, Q_n) = (\eta_{k_{(s)}}, \lambda_{\eta_{k_{(s)}}}, \lambda_{\mu_{k_{(s)}}}, \mu_{k_{(s)}})$ and
$Q_{(l)} = max(Q_1, Q_2, Q_3, \ldots, Q_n) = (\eta_{k_{(l)}}, \lambda_{\eta_{k_{(l)}}}, \lambda_{\mu_{k_{(l)}}}, \mu_{k_{(l)}}))$.
Then $Q_{(s)} < QFWA(Q_1, Q_2, Q_3, \ldots, Q_n) < Q_{(l)}$.

Proof. By the example 3, the proof is obvious.

$$(-\prod_{k=1}^{n}(-\eta_{Q_{k_{(s)}}})^{\alpha_k}, -\prod_{k=1}^{n}(-\lambda_{\eta_{Q_{k_{(s)}}}})^{\alpha_k},$$
$$(1-\prod_{k=1}^{n}(1-0.5\lambda^2_{\mu_{Q_{k_{(s)}}}})^{\alpha_k})^{\frac{1}{2}}, (1-\prod_{k=1}^{n}(1-\mu^2_{Q_{k_{(s)}}})^{\alpha_k})^{\frac{1}{2}})$$
$$< (-\prod_{k=1}^{n}(-\eta_{Q_k})^{\alpha_k}, -\prod_{k=1}^{n}(-\lambda_{\eta_{Q_k}})^{\alpha_k}, (1-\prod_{k=1}^{n}(1-0.5\lambda^2_{\mu_{Q_k}})^{\alpha_k})^{\frac{1}{2}}, (1-\prod_{k=1}^{n}(1-\mu^2_{Q_k})^{\alpha_k})^{\frac{1}{2}})$$
$$< (-\prod_{k=1}^{n}(-\eta_{Q_{k_{(l)}}})^{\alpha_k}, -\prod_{k=1}^{n}(-\lambda_{\eta_{Q_{k_{(l)}}}})^{\alpha_k},$$
$$(1-\prod_{k=1}^{n}(1-0.5\lambda^2_{\mu_{Q_{k_{(l)}}}})^{\alpha_k})^{\frac{1}{2}}, (1-\prod_{k=1}^{n}(1-\mu^2_{Q_{k_{(l)}}})^{\alpha_k})^{\frac{1}{2}})$$
$$\implies Q_{(s)} < QFWA(Q_1, Q_2, Q_3, \ldots, Q_n) < Q_{(l)}.$$ ■

Table 4: *Decision Matrix of Quadrasophic Fuzzy Set*

Fertilizer	s_1	s_2	s_3
f_1	(-0.8,-0.4,0.3,0.6)	(-0.7,-0.3,0.5,0.8)	(-0.6,-0.2,0.3,0.7)
f_2	(-0.7,-0.3,0.2,0.4)	(-0.5,-0.3,0.5,0.7)	(-0.4,-0.2,0.4,0.8)
f_3	(-0.4,-0.2,0.4,0.8)	(-0.7,-0.3,0.4,0.8)	(-0.6,-0.3,0.3,0.6)

5. Comparative study

In this segment, to corroborate our method, a comparison study is performed to prove that the proposed method generates better and more accurate results than the other existing methodologies. The Table 1 and 2 gives the study of comparison.

Table 5: *Normalized decision matrix of QFS*

Fertilizer	s_1	s_2	s_3
f_1	(-0.2,-0.1,0.2,0.4)	(-0.7,-0.3,0.5,0.8)	(-0.6,-0.2,0.3,0.7)
f_2	(-0.3,-0.2,0.3,0.6)	(-0.5,-0.3,0.5,0.7)	(-0.4,-0.2,0.4,0.8)
f_3	(-0.6,-0.3,0.1,0.2)	(-0.7,-0.3,0.4,0.8)	(-0.6,-0.3,0.3,0.6)

Table 6: *QFWA value of QFS*

Fertilizer	QFWA value
f_1	(-0.4519,-0.183,0.25,0.683)
f_2	(-0.3923,-0.225,0.28,0.724)
f_3	(-0.628,-0.299,0.2096,0.6259)

6. Application of Quadrasophic Fuzzy Set in agricultural field

The QFS has a special function that allows the difficulties to be solved perfectly. In this segment, we design a technique to deal with the decision-making assessment based on the QFWA

Table 7: *Decision Matrix of Quadrasophic Fuzzy Set*

	M_1	M_2	M_3
B_1	(-0.7,-0.4,0.2,0.6)	(-0.6,-0.3,0.4,0.7)	(-0.5,-0.3,0.1,0.3)
B_2	(-0.5,-0.2,0.1,0.3)	(-0.3,-0.2,0.4,0.8)	(-0.2,-0.1,0.3,0.7)
B_3	(-0.3,-0.1,0.4,0.7)	(-0.6,-0.3,0.2,0.7)	(-0.5,-0.3,0.2,0.5)

Table 8: *Score Value of Quadrasophic Fuzzy Set*

	M_1	M_2	M_3
B_1	-0.1	0.06	-0.13
B_2	-0.1	0.23	0.23
B_3	0.23	0	-0.03

operator and score function of QFS.

Crop yields are significantly influenced by fertilizers. Several types of fertilizers are applied to the soil to improve cultivation. The two primary types of fertilizer used in agriculture are organic and inorganic materials. For various purposes during the cultivation process, three primary plant nutrients: nitrogen, potassium, and phosphorus are used in different ratios. Suppose that the farmer wants to get better yields from the cultivation in medium term. The following Table 3 lists the type, ratio, and length of time employed in the cultivation for better results. Let $F = \{f_1, f_2, f_3\}$ be the set of alternative fertilizers used on agricultural land.

Membership grade of QFS in Agriculture :
$\eta-$ refers to the side effects of using fertilizer
$\lambda_\eta-$refers to the rate of soil pollution
$\lambda_\mu-$ refers to the rate of soil potential
$\mu-$ refers to the rate of fertilizer absorption level
Let $s = \{s_1, s_2, s_3\}$ be the group of criteria satisfied by the fertilizers used in the cultivation land. where, s_1- to upgrade the soil fertility, s_2- to promote the cultivation and s_3- healthy crop.
Step 1: The Table 4 provides the QFS decision matrix.
Step 2: The normalized QFS decision matrix is presented in the Table 5 by considering s_1 as the cost factor.
Step 3: Assume that $\alpha = (0.3, 0.3, 0.4)$ is the weight vector of each criteria.
Step 4: Use the QFWA operator to find the aggregated value. The Table 6 presents the QFWA value of a normalized QFS decision matrix.
Step 5: Apply the formula $sv(Q) = \frac{\mu(x)+\lambda_\mu(x)+\eta(x)+\lambda_\eta(x)}{3}$ to find the score value of QFSDM.

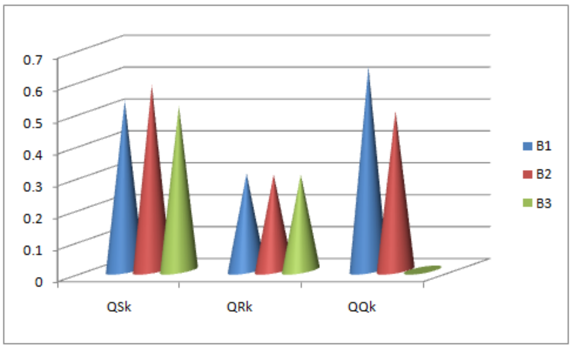

Figure 1: *Values of QS_k, QR_k, and QQ_k*

Table 9: *QFS positive and negative ideal solutions*

	$s_l^{(*)}$	$s_l^{(-)}$
M_1	(-0.3,-0.1,0.4,0.7)	(-0.7,-0.4,0.1,0.3)
M_2	(-0.3,-0.2,0.4,0.8)	(-0.6,-0.3,0.2,0.7)
M_3	(-0.2,-0.1,0.3,0.7)	(-0.5,-0.3,0.1,0.3)

Table 10: *Value of QS_k, QR_k, QQ_k*

	QS_k	QR_k	QQ_k
B_1	0.5324	0.3028	0.6378
B_2	0.5802	0.3000	0.5
B_3	0.51421	0.3000	0
Ranking	B_3, B_1, B_2	$B_3 = B_2, B_1$	B_3, B_2, B_1

$sv(f_1) = 0.0993$, $sv(f_2) = 0.1289$, and $sv(f_3) = -0.0306$.
Step 6: Rank the score value: $sv(f_2) > sv(f_1) > sv(f_3)$.
The fertilizer f_2 is therefore the ideal option for the best production in a medium period of time. Evidently, a medium-duration cultivation technique uses the nutrient ratio 150:60:50. The application gives sufficient justification for QFS to be able to determine the most appropriate decision.

7. A novel MCDM QF-VIKOR technique

A VIKOR technique focuses on ranking options and identifying compromises that are closest to the ideal answer. To define and identify the optimal solution of the Quadrasophic Fuzzy Set, we employ the QF-VIKOR technique in this segment.Consider a businessman who wants to boost profits from their investment in the share marketing sector. Let $M = \{M_1, M_2, M_3\}$ be the set of investment criteria that leads to the alternative businesses B_1, B_2, and B_3.
Step 1: The QFS decision-making matrix is shown in Table 7.
Step 2: The QFS Score matrix is shown in Table 8.
Step 3: The Table 9 shows the values of the QFS ideal solutions, both positive and negative.
Step 4: Assume that the weight vector for each criterion is $\alpha = \{0.3, 0.4, 0.3\}$
Step 5: The values of QS_k, QR_k, QQ_k are provided in the Table 10 using the distance metric 3.1. Rank the alternatives as well. where, [9] $QS_k = \sum_{l=1}^{n} \alpha_l \frac{d(s_l^{(*)}, s_{kl})}{d(s_l^{(*)}, s_l^{(-)})}$, $QR_k = max_l\, \alpha_l \frac{d(s_l^{(*)}, s_{kl})}{d(s_l^{(*)}, s_l^{(-)})}$
$QQ_k = \frac{\beta(QS_k - QS^{(*)})}{(QS^{(-)} - QS^{(*)})} + \frac{(1-Q\beta)(QR_k - QR^{(*)})}{(QR^{(-)} - QR^{(*)})}$ and $QS^{(*)} = minQS_l$, $QS^{(-)} = maxQS_l$, $QR^{(*)} = minQR_l$, and $QR^{(*)} = maxQR_l$, $Q\beta \in [0, 1]$.
Step 6: Figure 1 shows that B_3 is the minimum value. We evaluate how effectively the compromise solution of QQ_k accepts B_3 and B_2. $QQ(B_2) - QQ(B_3) \geq \frac{1}{n-1} \implies 0.5 \geq \frac{1}{3-1} = 0.5 \geq 0.5$ Similarly for QS_k and QR_k.

Hence, the greatest alternative is B_3, whereas B_2 and B_3 are the compromise solutions. Therefore, the ideal firm to invest in for increased profit is B_3.

8. Conclusion

The definitions, characteristics, and some Quadrasophic Fuzzy set operations are defined in this artifact. Theorems and results are also demonstrated using pertinent examples and remarks. To validate our technique, comparative research was conducted in several fuzzy environments. The use of Quadrasophic Fuzzy set in the field of agriculture to identify appropriate fertilizer has been explored. Further,the QF-VIKOR approach is introduced with its new decision-making application. The use of Quadrasophic Fuzzy set is grounded in the notion that it also works in artificial intelligence, neural-networks, and medicine. Other aggregation operators and their uses in various fields will be examined in further work. Due to its unique parameter classification, Quadrasophic Fuzzy set will also be functional in many domains, including corporate management and psychology.

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