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A PENTAGONAL FUZZY-BASED SOLUTION OF MULTIPLE OBJECTIVE LPP

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Abstract

In this paper, the researchers compare proposed approach and Excel solver. In this proposed technique, the researchers converted fuzzy multiple objective linear programming problems (FMOLPP) into multiple objective linear programming problems (MOLPP) with the help of Defuzzified mean of maxima method. Before that, the researchers changed the pentagonal fuzzy numerical valuation to a triangular fuzzy value by using the proposed theorem. Further, the crisp value of MOLPP is solved using standard simplex algorithms. Then the outcomes of the optimal solutions are compared with both the results.

Keywords: pentagonal fuzzy number, triangular fuzzy number, MOLPP, Defuzzified mean of maxima

1. Introduction

In a 2014 study [1], the Pentagonal Fuzzy Number (PFN) and its mathematical operations were described in this article. These operations were used to resolve a few cases. The problem with five points of approximation could be resolved with a pentagonal fuzzy number. In the 2017 study [2], this paper aims to describe the fundamental idea of a PFN. The researchers examine canonical pentagonal fuzzy numbers (CPFN) utilizing internal arithmetic operations and α-cut procedures. Internal arithmetic features had been studied with the idea of PFN. The CPFN and associated arithmetic operations were given special consideration. This research provides a PFN modification. PFNs of many types are created. Here, a specific kind of PFN's arithmetic operation was discussed. This article also discusses the distinction between two pentagonal-valued functions. The abovementioned numbers were used to demonstrate pentagonal fuzzy results for the fuzzy equation [3]. The 2018 concepts [4], portfolio optimization technique builds on the standard Markowitz mean-variance model and uses PFN to represent returns. The valuation and variance of fuzzy numbers were defined using the alpha-level set approach. The proposed model performs more effectively than the conventional mean-variance method. In this study [5], modeling techniques over multiple PFNs were presented. These similarity criteria depend on geometric distance, l_p-metric distance, graded mean integration form, and the perimeter of a PFN. The ideas of symmetric PFNs and quadratic PFNs, as well as their geometrical examples, were introduced in this study [6]. In addition, define the fundamental operations of arithmetic, such as the addition and subtraction of two symmetric PFNs. This work suggests a straightforward method for solving the fuzzy transportation problem in the context of a fuzzy environment, where the rates of conveyance, the availability of resources at the providers, and the consumption at the targets were all represented as PFNs. With the use of a strong ranking method and innovative fuzzy arithmetic on PFNs, the fuzzy transportation problem was solved without having to transform it to its equivalent crisp formulation [7].

The focus of 2019 work [8] discusses the fuzzy optimum solution to fully fuzzy linear programming problems (FFLPP) using PFNs. It had been suggested that new methods using multiple ranking functions would be used to solve FFLPP with PFNs. In this study [9], various methods of interval-valued pentagonal fuzzy numbers (IVPFN) connected to various membership functions (MF) were investigated, taking into account the prevalence of various interval-valued fuzzy numbers in specialized studies. Additionally, the concept of MF was substantially generalized to nonlinear membership functions for observing the asymmetries and symmetries of pentagonal fuzzy structures. By using the parameters as PFNs, the produced intellects were applied to a game challenge, leading to a new approach for simulating actual issues and a better knowledge of the parameters' uncertainties during the testing process. In the 2020 study, using a ranking function and comparing the results with completely fuzzy LPP, a technique was suggested for solving fuzzy LPP utilizing PFNs. It involves observing that which produces optimum results [10]. The 2021 study, implementing the Leasing Strategy, aims to lower the machine's rental price. A strong ranking approach and fuzzy arithmetic pentagonal fuzzy numbers were utilized to solve the fuzzy flow-shop scheduling problem without translating the processing time into its equivalent crisp results [11]. In this research design [12], fuzzy results were used to represent the pertinent samples of imprecise rainfall data that were gathered in southern and northern India. Moreover, fuzzy numbers were removed by explicit classification using the PFN pivot points more extensively than the PFN. After the fuzzy phase, a pertinent statistical technique was applied to assess the hypotheses, allowing for better decision-making. In a 2022 study, three steps make up the suggested study technique. With the mean approach of α-cut, the coefficients are first defuzzified. In the second step, a crisp multi-objective quadratic fractional programming model (MOQFP) was built to create a non-fractional system based on an iterative parametric technique. Then, for the final step, the Σ-constraint approach was used to turn this multi-objective, non-fractional model into one with a single objective [13].

The remaining research is organized as follows: 2. Preliminary and Theorem, 3. Proposed algorithm, 4. Numerical example, and 5. Conclusions.

2. Preliminary and Theorem

2.1. Fuzzy set

Let Q be a non-empty set. A fuzzy set P in Q is identified by its membership function $\mu_{\tilde{P}}(y) : Q \rightarrow [0,1]$ and $\mu_{\tilde{P}}(y)$ is described as the degree to which an element is such a member y in fuzzy set P for each $y \in Q$. Then a fuzzy set P in Q is a collection of ordered pairs [14].

$$\tilde{P} = \{(y, \mu_{\tilde{P}}(y))/y \in Q\}$$

Theorem 1. In this theorem, we convert the pentagonal fuzzy number into a triangular fuzzy number ($\sum_{i=1}^{5} \tilde{Q}_i$ are convert into $\sum_{i=1}^{3} \tilde{Q}_i$). By finding the results of pentagonal gradient points? And then obtain the three-tuples fuzzy process transformed as a Defuzzified mean of the maxima of the crisp process.

Proof. Case 1: Let us take the pentagonal fuzzy number

$$\begin{aligned} \tilde{S} &= \sum_{i=1}^{5} \tilde{Q}_i \\ &= (\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3, \tilde{Q}_4, \tilde{Q}_5; \tilde{\omega}_1, \tilde{\omega}_2) \end{aligned}$$

Since, $(\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3, \tilde{Q}_4, \tilde{Q}_5; \tilde{\omega}_1, \tilde{\omega}_2)$ are the pentagonal fuzzy numbers and also we known that $(\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3, \tilde{Q}_4, \tilde{Q}_5)$ are real numbers.
In this case the pentagonal fuzzy number of $\tilde{\omega}_1$ and $\tilde{\omega}_2$ is the graded point of $\tilde{Q}_2$ and $\tilde{Q}_4$ [15].
So, the value of $\tilde{\omega}_1 = \tilde{\omega}_2 = 0$ means the graded points of $\tilde{Q}_2 = \tilde{Q}_4 = 0$
Thus, the values of pentagonal fuzzy number transformed as triangular fuzzy numbers.
So, we can write as, $\tilde{S} = (\tilde{Q}_1, \tilde{Q}_3, \tilde{Q}_5)$ (or) $\tilde{S} = (\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3)$.
As a result, we can obtain three tuples of fuzzy number value, which we refer to as triangular fuzzy number.
Hence, the triangular fuzzy number is $\tilde{S} = \sum_{i=1}^{3} \tilde{Q}_i$
Case 2: Let us take three-tuples triangular fuzzy number

$$\tilde{S} = \sum_{i=1}^{3} \tilde{Q}_i = (\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3)$$

Now, we are converting the three-tuples fuzzy process to the Defuzzified mean of the maxima (MoM) crisp process.
The Defuzzified process of $MoM = \frac{\sum_{\tilde{Q}_s \in D} \tilde{Q}_s}{|D|}$, Where $|D|$ is the number of counted values of $\tilde{Q}_s$.
Since triangular fuzzy numbers have three tuples only, we submit the three tuples in the MoM process.
Thus, the value of $MoM = \frac{\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3}{3} = R_s$, Here we take three-tuples, so the value of $|D| = 3$ and where R_s is the crisp value.
Therefore, the numerical result has been changed from a fuzzy to a crisp process as per the above format. ■

3. Proposed algorithm

- We take that first pentagonal numerical example; convert it into triangular values, and then Defuzzified as per 1 of the mean of maxima.
- A three-tuples of fuzzy MOLPP utilized in the form of as usual simplex algorithm and the transformed into Defuzzified mean of maxima MOLPP crisp values.
- Here, we used the usual simplex algorithm to solve the objective function and constraints.
- Discover the valuation of each of the particular objective functions that are to be maximized or minimized.
- Test the feasibility of the solution in step 3. If it is feasible, then go to step 6. Otherwise, use the dual simplex method to remove infeasibility.
- To give a name to the optimum valuation of the objective functions as r_w.
- Step 4 should be repeated with $w = 1, 2, 3, 4$ and 5. We are only considering five objective functions in this work.
- Find an optimum solution to each LP problem obtained in step 7.
- Then, LP objective problems with an efficient, optimal solution will receive it. Otherwise, we have to follow the same procedure.

4. Numerical example

$$\text{Max } \tilde{w}_1 = (1, 0, 2, 0, 3)\tilde{x}_1 + (7, 0, 8, 0, 9)\tilde{x}_2$$
$$\text{Max } \tilde{w}_2 = (10, 10, 11, 11, 12)\tilde{x}_1 + (12, 13, 13, 14, 14)\tilde{x}_2$$
$$\text{Max } \tilde{w}_3 = (20, 0, 21, 22, 22)\tilde{x}_1 + (16, 16, 17, 0, 18)x_2\tilde{x}_2$$
$$\text{Min } \tilde{w}_4 = (6, 6, 7, 7, 8)\tilde{x}_1 + (2, 2, 3, 4, 4)\tilde{x}_2$$
$$\text{Min } \tilde{w}_5 = (25, 0, 26, 0, 27)\tilde{x}_1 + (13, 0, 14, 0, 15)\tilde{x}_2$$

S.to

$$
\begin{gathered}
(2,0,3,0,4)\tilde{x}_1 + (11,0,12,0,13)\tilde{x}_2 \leq (4,0,5,0,6) \\
(1,1,2,2,3)\tilde{x}_1 + (6,7,7,8,8)\tilde{x}_2 \leq (9,9,10,11,11) \\
(5,0,6,6,7)\tilde{x}_1 + (3,3,4,0,5)\tilde{x}_2 \leq (14,14,15,16,16) \\
\tilde{x}_2 1, \tilde{x}_2 \geq \tilde{0}
\end{gathered} \tag{1}
$$

By 1 using as per Theorem1 (Case 1) to convert pentagonal fuzzy numerical values into triangular fuzzy values.

$$
\begin{aligned}
\text{Max } \tilde{w}_1 &= (1,2,3)\tilde{x}_1 + (7,8,9)\tilde{x}_2 \\
\text{Max } \tilde{w}_2 &= (10,11,12)\tilde{x}_1 + (12,13,14)\tilde{x}_2 \\
\text{Max } \tilde{w}_3 &= (20,21,22)\tilde{x}_1 + (16,17,18)\tilde{x}_2 \\
\text{Min } \tilde{w}_4 &= (6,7,8)x_1\tilde{x}_1 + (2,3,4)\tilde{x}_2 \\
\text{Min } \tilde{w}_5 &= (25,26,27)\tilde{x}_1 + (13,14,15)\tilde{x}_2
\end{aligned}
$$

S.to

$$
\begin{gathered}
(2,3,4)\tilde{x}_1 + (11,12,13)\tilde{x}_2 \leq (4,5,6) \\
(1,2,3)\tilde{x}_1 + (6,7,8)\tilde{x}_2 \leq (9,10,11) \\
(5,6,7)\tilde{x}_1 + (3,4,5)\tilde{x}_2 \leq (14,15,16) \\
\tilde{x}_1, \tilde{x}_2 \geq \tilde{0}
\end{gathered} \tag{2}
$$

By 2 using as per Theorem1 (Case 2) to convert three-tuples of fuzzy numerical values into crisp values.

$$
\begin{aligned}
\text{Max } w_1 &= 2x_1 + 8x_2 \\
\text{Max } w_2 &= 11x_1 + 13x_2 \\
\text{Max } w_3 &= 21x_1 + 17x_2 \\
\text{Min } w_4 &= 7x_1 + 3x_2 \\
\text{Min } w_5 &= 26x_1 + 14x_2
\end{aligned}
$$

S.to

$$
\begin{gathered}
3x_1 + 12x_2 \leq 5 \\
2x_1 + 7x_2 \leq 10 \\
6x_1 + 4x_2 \leq 15 \\
x_1, x_2 \geq 0
\end{gathered} \tag{3}
$$

As per the algorithm used, the usual simplex techniques and Excel solver approaches.
First objective function:

$$\text{Max } w_1 = 2x_1 + 8x_2$$

S.to

$$3x_1 + 12x_2 + x_3 = 5$$

$$2x_1 + 7x_2 + x_4 = 10$$
$$6x_1 + 4x_2 + x_5 = 15$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Max $w_1 = 3.3336$

Table 1: *Excel solver*

Decision variables	x_1	x_2	Objective value (Max w_1)		
Value	0	0.416667	3.333333333		
Coefficients	2	8			
			LHS		RHS
Constraints 1	3	12	5	$\leq$	5
Constraints 2	2	7	3.3333333333	$\leq$	10
Constraints 3	6	4	107	$\leq$	15

Second objective function:

$$\text{Max } w_2 = 11x_1 + 13x_2$$

S.to

$$3x_1 + 12x_2 + x_3 = 5$$
$$2x_1 + 7x_2 + x_4 = 10$$
$$6x_1 + 4x_2 + x_5 = 15$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Max $w_2 = 18.337$

Table 2: *Excel solver*

Decision variables	x_1	x_2	Objective value (Max w_2)		
Value	1.666667	0	18.33333333		
Coefficients	11	13			
			LHS		RHS
Constraints 1	3	12	5	$\leq$	5
Constraints 2	2	7	3.333333333	$\leq$	10
Constraints 3	6	4	10	$\leq$	15

Third objective function:

$$\text{Max } w_3 = 21x_1 + 17x_2$$

S.to

$$3x_1 + 12x_2 + x_3 = 5$$
$$2x_1 + 7x_2 + x_4 = 10$$
$$6x_1 + 4x_2 + x_5 = 15$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

RT&A, No 4 (76)
Volume 18, December 2023

Max $w_3 = 35.007$

Table 3: *Excel solver*

Decision variables	x_1	x_2	Objective value (Max w_3)		
Value	1.666667	0	35		
Coefficients	21	17			
			LHS		RHS
Constraints 1	3	12	5	$\leq$	5
Constraints 2	2	7	3.333333333	$\leq$	10
Constraints 3	6	4	10	$\leq$	15

Fourthd objective function:

$$\text{Min } w_4 = 7x_1 + 3x_2$$

S.to

$$3x_1 + 12x_2 + x_3 = 5$$
$$2x_1 + 7x_2 + x_4 = 10$$
$$6x_1 + 4x_2 + x_5 = 15$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Min $w_4 = -11.667$

Table 4: *Excel solver*

Decision variables	x_1	x_2	Objective value (Min w_4)		
Value	1.666667	0	-11.66666667		
Coefficients	-7	-3			
			LHS		RHS
Constraints 1	3	12	5	$\leq$	5
Constraints 2	2	7	3.333333333	$\leq$	10
Constraints 3	6	4	10	$\leq$	15

Fifth objective function:

$$\text{Min } w_5 = 26x_1 + 14x_2$$

S.to

$$3x_1 + 12x_2 + x_3 = 5$$
$$2x_1 + 7x_2 + x_4 = 10$$
$$6x_1 + 4x_2 + x_5 = 15$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Min $w_5 = -43.342$

Table 5: *Excel solver*

Decision variables	x_1	x_2	Objective value (Min w_5)		
Value	1.666667	0	-43.33333333		
Coefficients	-26	-14			
			LHS		RHS
Constraints 1	3	12	5	$\leq$	5
Constraints 2	2	7	3.333333333	$\leq$	10
Constraints 3	6	4	10	$\leq$	15

Table 6: *Final table*

Comparison of w		
S.no	Excel solver	Proposed technique
1	3.3333	3.3336
2	18.333	18.337
3	35	35.007
4	-11.667	-11.667
5	-43.333	-43.342

5. Conclusion

In this work, the researchers first take a pentagonal triangular fuzzy number and then convert it into a triangular fuzzy number with the use of Theorem 2.1. Furthermore, the triangular or tri-tuple fuzzy valuations are changed into Defuzzified crisp values. The algorithm employs the mean maxima method of crisp values to solve the standard linear programming simplex method. As a comparison result of our proposed LPP simplex method to Excel's solver simplex method, our proposed results are more optimal than Excel's solver.

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