A TWO NON IDENTICAL UNITS COLD STANDBY SYSTEM WITH CORRELATED FAILURE TIME AND REPAIR MACHINE FAILURE

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Abstract

The paper deals with a system composed of two-non identical units (unit-1 and unit-2). Initially, unit-1 is operative and unit-2 is kept in cold standby. The cold standby unit can't fail in its standby mode. Each unit of the system has two possible modes: Normal (N) and total failure (F). When the unit-1 fails the cold standby (unit-2) becomes operative instantaneously with the help of a perfect and instantaneous switching device. A single repairman is always available with the system to repair a failed unit and failed RM. Unit-1 gets priority in operation and repair over unit-2. However, the RM gets priority in repair over any of the units. The RM machine is good initially and can't fail unless it becomes operative. The system failure occurs when both the units are in total failure mode. The joint distribution of failure and repair times for each unit is taken bivariate exponential distribution. Each repaired unit works as good as new. Using regenerative point technique, various important measures of system effectiveness have been obtained.

Keywords: Transition probabilities, mean sojourn time, bi-vairate exponential distribution, reliability, MTSF, availability, expected busy period of repairman, net expected profit.

1. Introduction

Two units standby system models have been investigated by a large number of authors including A. Kumar, D. Pawar and S.C. Malik [11], P. Chaudhary, A. Sharma and R.Gupta [3], P. Chaudhary and A. Sharma [2], N.Kumar and N. Nandal [12], P. Gupta and P. Vinodiya [9], R. Gupta and P.Bhardwaj [5], R. Gupta and A. Tyagi [8], N. Kumar, S.C. Malik and N. Nandal [10], P. Chaudhary and S. Masih [1], P. Chaudhary and L. Tyagi[4] by using the concepts of warm standby with common cause failure and human error, correlated failure and repairs, two types of repairmen, two priority units warm standby with preparation for repair, two unit priority standby with repair, two unit cold standby with two operating modes.

In the analysis of above system models it has been assumed that a failed unit is always repairable manually and after repair the unit becomes as good as new. There are many situations where a repair machine (RM) is needed to repair a failed unit and the RM may also fail during the repair of a failed unit .In this case the RM is first taken up for repair and the failed unit waits for getting repair.

For example, in case of nuclear reactors, marine equipment etc. the robots are used for the repair of such type of systems. It is evident that a robot being a Machine, may fail while performing its intended task. In this case the repairman will repair the RM first and then begins the repair of the failed unit.

Keeping above fact in view, the present chapter deals with the analysis of a two non-identical units cold standby system model with constant failure and general repair rates assuming that the first unit gets priority in operation and repair both. The RM may also fail during the repair of a unit. The failure rate of RM is taken as constant and its repair rate as general.

The objective of the present paper is to provide the analysis of a two non-identical unit standby system with correlated failure time and repair machine failure. The joint distributions of failure and repair times of each unit are taken to be bivariate exponential distribution with p.d.f. of the type-

$$f_i(x,y) = \alpha_i\beta_i(1-r_i)e^{-\alpha_i x-\beta_i y} I_0(2\sqrt{\alpha_i\beta_i r_i x y})$$

$$; i = 1,2;\ x, y, \alpha_i, \beta_i > 0; 0 \le r_i < 1$$

$$\text{Where,}\quad I_0 = \sum_{k=0}^{\infty} \frac{(z/2)^{2k}}{(k!)^2}$$

is the modified Bessel function of type-I and order zero. Gupta et al. [6] and Gupta and Shivakar [7] have analyzed some of two unit redundant system models by taking the joint distribution of failure and repair as bivariate exponential having the above form of pdf.

Using regenerative point technique, the following measures of system effectiveness are obtained-

i. Transition probabilities and mean sojourn times in various states.
ii. Reliability and Mean time to system failure (MTSF).
iii. Point-wise and steady-state availabilities of the system as well as expected up time of the system during time interval (0, t).
iv. Expected busy period of repairman in the repair of unit-1 and unit-2 during time interval (0, t).
v. Net expected profit earned by the system in time interval (0, t).

2. System description and assumptions

1. The system consists of two non-identical units (unit-1 and unit-2). Initially, unit-1 is operative and unit-2 is kept in cold standby. The cold standby unit can't fail in its standby mode.
2. Each unit of the system has two possible modes: Normal (N) and total failure (F).
3. When the unit-1 fails the cold standby (unit-2) becomes operative instantaneously with the help of a perfect and instantaneous switching device.
4. A single repairman is always available with the system to repair a failed unit and failed RM.
5. Unit-1 gets priority in operation and repair over unit-2. However, the RM gets priority in repair over any of the units.
6. The RM machine is good initially and can't fail unless it becomes operative.
7. The system failure occurs when both the units are in total failure mode.

8. The joint distribution of failure and repair times for each unit is taken bivariate exponential with density function given by ,

$$f_i(x,y) = \alpha_i\beta_i(1-r_i)e^{-\alpha_i x-\beta_i y}I_0(2\sqrt{\alpha_i\beta_i r_i xy})$$

$$;\ i=1,2;\ x,y,\alpha_i,\beta_i > 0; 0 \le r_i < 1$$

Where, $I_0 = \sum_{k=0}^{\infty}\frac{(z/2)^{2k}}{(k!)^2}$

9. Each repaired unit works as good as new.

3. Notations and states of the system

3.1. Notations:

E	:	Set of regenerative states.
X_i, Y_i	:	Random variable denoting the failure and repair time for unit-1and unit-2 respectively ;(i=1, 2)
$f_i(x,y)$	:	Joint probability density function of (X_i, Y_i) ;(i=1, 2) $=\alpha_i\beta_i(1-r_i)e^{-\alpha_i x-\beta_i y}I_0(2\sqrt{\alpha_i\beta_i r_i xy})dx \quad ; x,y,\alpha_i,\beta_i > 0;\ \ 0\le r_i<1$ Where, $I_0(2\sqrt{\alpha_i\beta_i r_i xy}) = \sum_{k=0}^{\infty}\frac{(\alpha_i\beta_i r_i xy)^k}{(k!)^2}$
$g_i(x)$	:	Marginal p.d.f. of $X_i = x$; (i=1, 2) $=\alpha_i(1-r_i)e^{-\alpha_i(1-r_i)x}$
$k_i(y\|X_i = x)$	:	Conditional p.d.f. of Y_i given $X_i = x$; (i=1, 2) $=\beta_i e^{-(\beta_i y+\alpha_i r_i x)}\sum_{j=0}^{\infty}\frac{(\alpha_i\beta_i r_i xy)^j}{(j!)^2}$
$g_{ij}(\bullet), g_{ij}^{(k)}(\bullet)$	:	P.d.f. of transition time from state S_i to S_j and S_i to S_j via S_k.
$p_{ij}(\bullet), p_{ij}^{(k)}(\bullet)$	:	Steady-state transition probabilities from state S_i to S_j and S_i to S_j via S_k .
$p_{ij\|x}(\bullet), p_{ij\|x}^{(k)}(\bullet)$	:	Steady-state transition probabilities from state S_i to S_j and S_i to S_j via S_k when it is known that the unit has worked for time x before its failure.
*	:	Symbol for Laplace Transform i.e. $g_{ij}^*(s) = \int e^{-st}q_{ij}(t)dt$
~	:	Symbol for Laplace Stieltjes Transform i.e. $\widetilde{Q}_{ij}(s) = \int e^{-st}dQ_{ij}(t)t$
©	:	Symbol for ordinary convolution i.e. $A(t)©B(t)=\int_0^t A(u)B(t-u)du$
θ	:	waiting time of unit-1.

3.2. Symbols for the states of the system

N_{1o}, N_{2o}	:	Unit-1and Unit-2 is in N-mode and operative.
N_{2s}	:	Unit-2 is in N-mode and kept into cold standby.
F_{1r}, F_{2r}	:	Unit-1 and unit-2 is in F-mode and under repair.

F_{1w}, F_{2w}	:	Unit -1 and Unit -2 is in F-mode and waiting for repair.
RM_g	:	Repair machine is good.
RM_o	:	Repair machine is operative.
RM_r	:	Repair machine is failed and under repair.

Considering the above symbols in view of the assumptions stated earlier, we have the following states of the system:

Up States

$$S_0 \equiv \begin{pmatrix} N_{1o}, N_{2s} \\ RM_g \end{pmatrix}$$

$$S_1 \equiv \begin{pmatrix} F_{1r}, N_{2o} \\ RM_o \end{pmatrix}$$

$$S_2 \equiv \begin{pmatrix} F_{1w}, N_{2o} \\ RM_r \end{pmatrix}$$

$$S_5 \equiv \begin{pmatrix} N_{1o}, F_{2w} \\ RM_r \end{pmatrix}$$

$$S_6 \equiv \begin{pmatrix} N_{1o}, F_{2w} \\ RM_r \end{pmatrix}$$

Failed States

$$S_3 \equiv \begin{pmatrix} F_{1r}, F_{2w} \\ RM_o \end{pmatrix}$$

$$S_4 \equiv \begin{pmatrix} F_{1w}, F_{2w} \\ RM_r \end{pmatrix}$$

The transition diagram of the system model along with the transition rate or transition time c.d.f. is shown in Fig.1. The epochs of the transition into state S_4 from S_2 and S_6 from S_5 are non-regenerative.

TRANSITION DIAGRAM

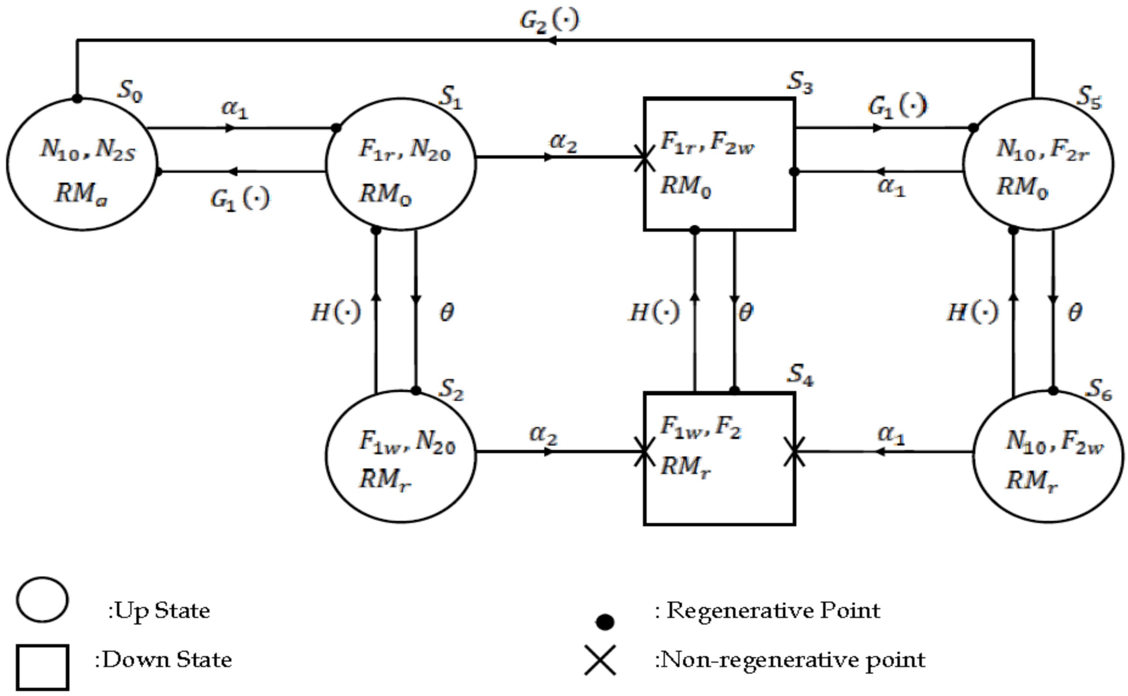

Figure 1: *Correlation Model*

†The limits of integration are 0 to ∞ whenever they are not mentioned.

4. Transition Probabilities and Mean Sojourn Times

Let $X(t)$ be the state of the system at epoch t, then $\{X(t); t \geq 0\}$ constitutes a continuous parametric Markov-Chain with state space $E = \{S_0 to S_6\}$ .The various measures of system effectiveness are obtained in terms of steady-state transition probabilities and mean sojourn times in various states.

$$p_{01} = 1, \qquad p_{21} = \frac{1}{(\theta + \alpha_2(1-r_2))}$$

$$p_{43} = \int dG(t) = 1, \qquad p_{65} = \frac{1}{(\theta + \alpha_1(1-r_1))}$$

$$\begin{aligned}
p_{10|x} &= \int \alpha_2(1-r_2)e^{-\alpha_2(1-r_2)t}e^{-\theta t}dK_1(t \mid x) \\
&= \widetilde{K}_1[\theta + \alpha_2(1-r_2) \mid x] \\
&= k_1^*[\theta + \alpha_2(1-r_2) \mid x],
\end{aligned}$$

$$\begin{aligned}
p_{12|x} &= \theta \int e^{-[\theta+\alpha_2(1-r_2)]t} d\overline{K_1}(t \mid x) \\
&= \theta \int e^{-[\theta+\alpha_2(1-r_2)]t} dt\left[1 - K_1(t \mid x)\right] \\
&= \left(\frac{\theta}{\theta + \alpha_2(1-r_2)}\right)\left(1 - k_1^*[\theta + \alpha_2(1-r_2) \mid x]\right)
\end{aligned}$$

$$\begin{aligned}
p_{13|x} &= \int \alpha_2(1-r_2)e^{-\alpha_2(1-r_2)t}e^{-\theta t}dt\overline{K_1}(t \mid x) \\
&= \alpha_2(1-r_2)\int e^{-[\theta+\alpha_2(1-r_2)]t}dt\left[1 - K_1(t \mid x)\right] \\
&= \left(\frac{\alpha_2(1-r_2)}{\theta + \alpha_2(1-r_2)}\right)\left(1 - k_1^*[\theta + \alpha_2(1-r_2) \mid x]\right)
\end{aligned}$$

$$\begin{aligned}
p_{34|x} &= \int \theta e^{-\theta t}dt\overline{K_1}(t \mid x) \\
&= \int \theta e^{-\theta t}dt\left[1 - K_1(t \mid x)\right] \\
&= 1 - k_1^*(\theta \mid x),
\end{aligned}$$

$$\begin{aligned}
p_{35|x} &= \int e^{-\theta t}dK_1(t \mid x) \\
&= \widetilde{K}_1[\theta \mid x] \\
&= k_1^*(\theta \mid x)
\end{aligned}$$

$$\begin{aligned}
p_{50|x} &= \int e^{-\theta t}dK_2(t \mid x) \\
&= \widetilde{K}_2\left[\theta + \alpha_1(1-r_1) \mid x\right] \\
&= k_2^*(\theta + \alpha_1(1-r_1) \mid x)
\end{aligned}$$

$$\begin{aligned}
p_{53|x} &= \int \alpha_1(1-r_1)e^{-\alpha_1(1-r_1)t}e^{-\theta t}dt\overline{K_2}(t \mid x) \\
&= \alpha_1(1-r_1)\int e^{-[\theta+\alpha_1(1-r_1)]t}dt\left[1 - K_2(t \mid x)\right] \\
&= \left(\frac{\alpha_1(1-r_1)}{\theta + \alpha_1(1-r_1)}\right)\left(1 - k_2^*[\theta + \alpha_1(1-r_1) \mid x]\right)
\end{aligned}$$

$$\begin{aligned}
p_{56|x} &= \theta \int e^{-[\theta+\alpha_1(1-r_1)]t}dt\left[1 - K_2(t \mid x)\right] \\
&= \left(\frac{\theta}{\theta + \alpha_1(1-r_1)}\right)\left(1 - k_2^*[\theta + \alpha_1(1-r_1) \mid x]\right)
\end{aligned}$$

$$p_{23}^{(4)} = \int \left(1 - e^{-\alpha_2(1-r_2)t}\right) dG(t)$$

$$= 1 - \frac{1}{\theta + \alpha_2(1-r_2)},$$

$$p_{63}^{(4)} = \int \left(1 - e^{-\alpha_1(1-r_1)t}\right) dG(t)$$

$$= 1 - \frac{1}{\theta + \alpha_1(1-r_1)}$$

It can be easily verified that

$$p_{01} = p_{45|x} = 1, \qquad p_{12} + p_{13} = 1, \qquad p_{34} = 1$$

$$p_{20|x} + p_{25|x}^{(4)} = 1, \qquad p_{50|x} + p_{52|x}^{(6)} = 1 \qquad (1\text{-}5)$$

From the conditional steady state transition probabilities, the unconditional steady state transition probabilities can be obtained by using the result-

$$p_{ij} = \int p_{ij|x}\, g(x)\, dx$$

Thus,

$$p_{10} = \frac{\beta_1(1-r_1)}{[\alpha_2(1-r_2) + \beta_1(1-r_1) + \theta]}$$

$$p_{12} = \frac{\theta}{\theta + \alpha_2(1-r_2)}\left[1 - \frac{\beta_1(1-r_1)}{[\alpha_2(1-r_2) + \beta_1(1-r_1) + \theta]}\right]$$

$$p_{13} = \frac{\alpha_2(1-r_2)}{\theta + \alpha_2(1-r_2)}\left[1 - \frac{\beta_1(1-r_1)}{[\alpha_2(1-r_2) + \beta_1(1-r_1) + \theta]}\right]$$

$$p_{34} = \left[1 - \frac{\beta_1(1-r_1)}{[\beta_1(1-r_1) + \theta]}\right],$$

$$p_{35} = \frac{\beta_1(1-r_1)}{[\beta_1(1-r_1) + \theta]}$$

$$p_{50} = \frac{\beta_2(1-r_2)}{\left[\theta + \beta_2(1-r_2) + \alpha_1(1-r_1)\right]}$$

$$p_{53} = \frac{\alpha_1(1-r_1)}{\theta + \alpha_1(1-r_1)}\left[1 - \frac{\beta_2(1-r_2)}{[\alpha_1(1-r_1) + \beta_2(1-r_2) + \theta]}\right]$$

$$p_{56} = \frac{\theta}{\theta + \alpha_1(1-r_1)}\left[1 - \frac{\beta_2(1-r_2)}{[\alpha_1(1-r_1) + \beta_2(1-r_2) + \theta]}\right]$$

Thus, we have

$$p_{01} = p_{43} = 1, \qquad p_{10} + p_{12} + p_{13} = 1 \qquad p_{34} + p_{35} = 1$$

$$p_{50} + p_{53} + p_{56} = 1 \qquad p_{21} + p_{23}^{(4)} = 1, \qquad p_{65} + p_{63}^{(4)} = 1 \qquad (6\text{-}11)$$

5. Mean Sojourn Time

The mean sojourn time ψ_i in state S_i is defined as the expected time taken by the system in state S_i before transiting into any other state. If random variable U_i denotes the sojourn time in state S_i then,

$$\psi_i = \int P[U_i > t]\, dt$$

The mean sojourn times in various states are as follows-

$$\psi_0 = \frac{1}{\alpha_1(1-r_1)}, \qquad \psi_1 = \frac{1}{\theta+\alpha_2(1-r_2)+\beta_1(1-r_1)}$$

$$\psi_2 = \frac{1}{\theta+\alpha_2(1-r_2)}, \qquad \psi_3 = \frac{1}{\theta+\beta_1(1-r_1)}$$

$$\psi_4 = \int \overline{G}(t)\, dt = 1, \qquad \psi_5 = \frac{1}{\theta+\alpha_1(1-r_1)+\beta_2(1-r_2)}$$

$$\psi_6 = \frac{1}{\theta+\alpha_1(1-r_1)} \tag{12-18}$$

6. Analysis of Characteristics

6.1. Reliability and MTSF

Let $R_i(t)$ be the probability that the system operates during (0, t) given that at t=0 system starts from $S_i \in E$. To obtain it we assume the failed states S_3 and S_4 as absorbing. By simple probabilistic arguments, the value of $R_0(t)$ in terms of its Laplace Transform (L.T.) is given by

$$R_0^*(s) = \frac{\left(1-q_{56}^* q_{65}^*\right)\left[\left(1-q_{12}^* q_{21}^*\right)Z_0^* + q_{01}^* Z_1^* + q_{01}^* q_{12}^* Z_2^*\right]}{\left(1-q_{56}^* q_{65}^*\right)\left(1-q_{12}^* q_{21}^* - q_{01}^* q_{10}^*\right)} \tag{1}$$

We have omitted the argument's from $q_{ij}^*(s)$ and $Z_i^*(s)$ for brevity. $Z_i^*(s)$; i = 0, 1, 2, 5, 6 are the L. T. of

$$Z_0(t) = e^{-\alpha_1(1-r_1)t}$$

$$Z_1(t) = \int e^{-(\theta+\alpha_2(1-r_2))t}\,\overline{K_1}(t \mid x)\ dt, \qquad Z_2(t) = \int e^{-\alpha_2(1-r_2)t}\,\overline{G}(t)\ dt$$

$$Z_5(t) = e^{-(\theta+\alpha_1(1-r_1))t}, \qquad Z_6(t) = \int e^{-\alpha_1(1-r_1)t}\,\overline{G}(t)\ dt \tag{2-6}$$

Taking the Inverse Laplace Transform of (1), we can get the reliability of the system when system initially starts from state S_0 .

The MTSF is given by,

$$E(T_0) = \int R_0(t) = \lim_{s\to 0} R_0^*(s) = \frac{(1-p_{56}p_{65})\left[(1-p_{12}p_{21})\psi_0 + \psi_1 + p_{12}\psi_2\right]}{(1-p_{56}p_{65})\left[1-p_{12}p_{21}-p_{10}\right]} \tag{7}$$

6.2. Availability Analysis

Let $A_i(t)$ be the probability that the system is up at epoch t, when initially it starts operation from state $S_i \in E$. Using the regenerative point technique and the tools of Laplace transform, one can obtain the value of $A_0(t)$ in terms of its Laplace transforms i.e. $A_0^*(s)$ given as follows-

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)} \tag{8}$$

Where,

$$N_1(s) = \left(1-q_{25}^{(4)*} q_{52}^{(6)*}\right)\left[Z_0^* + q_{01}^* Z_1^*\right] + Z_2^* q_{01}^* \left[q_{12}^* + q_{13}^* q_{34}^* q_{45}^* q_{52}^{(6)*}\right] + Z_5^* q_{01}^* \left[q_{12}^* q_{25}^{(4)*} + q_{13}^* q_{34}^* q_{45}^*\right]$$

and

$$D_1(s) = 1 - q_{25}^{(4)*} q_{52}^{(6)*} - q_{01}^* q_{12}^* q_{20}^* - q_{01}^* q_{13}^* q_{20}^* q_{34}^* q_{45}^* q_{52}^{(6)*} - q_{01}^* q_{12}^* q_{50}^* q_{25}^{(4)*} - q_{01}^* q_{13}^* q_{34}^* q_{45}^* q_{50}^* \tag{9}$$

Where, $Z_i(t)$, i=0,1,2,5,6 are same as given in section 6.1.

The steady-state availability of the system is given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s\ A_0^*(s) \tag{10}$$

We observe that

$$D_1(0) = 0$$

Therefore, by using L. Hospital's rule the steady state availability is given by

$$A_0 = \lim_{s\to 0} \frac{N_1(s)}{D_1'(s)} = \frac{N_1}{D_1'} \tag{11}$$

Where,

$$N_1(0) = \left(1 - p_{25}^{(4)} p_{52}^{(6)}\right)\left[\psi_0 + \psi_1\right] + \psi_2\left[1 - p_{13}p_{50}\right] + \psi_5\left[1 - p_{12}p_{20}\right]$$

and

$$D_1' = \left(\psi_0 + \psi_1\right)\left(1 - p_{25}^{(4)} p_{52}^{(6)}\right) + \psi_3 p_{13}\left(1 - p_{25}^{(4)} p_{52}^{(6)}\right) + n_1\left(1 - p_{13}p_{20}p_{52}^{(6)}\right) + n_2\left(1 - p_{12}p_{20}\right) \tag{12}$$

The expected up time of the system in interval (0, t) is given by

$$\mu_{up}(t) = \int_0^t A_0(u)\,du$$

So that, $\mu_{up}^*(s) = \dfrac{A_0^*(s)}{s}$ (13)

6.3. Busy Period Analysis

Let $B_i^1(t)$ and $B_i^2(t)$ be the respective probabilities that the repairman is busy in the repair of unit-1 failed due to first repair with priority of unit-1 and unit-2 failed due to second repair at epoch t, when initially the system starts operation from state $S_i \in E$. Using the regenerative point technique and the tools of L. T., one can obtain the values of above two probabilities in terms of their L. T. i.e. $B_i^{1*}(s)$ and $B_i^{2*}(s)$ as follows-

$$B_i^{1*}(s) = \frac{N_2(s)}{D_1(s)}, \qquad B_i^{2*}(s) = \frac{N_3(s)}{D_1(s)} \tag{14-15}$$

Where,

$$N_2(s) = Z_1^* q_{01}^*\left[\left(1 - q_{56}^* q_{65}^*\right)\left(1 - q_{34}^* q_{43}^*\right) + q_{35}^* q_{43}^*\left(q_{56}^* q_{63}^{(4)*} + q_{53}^*\right)\right] + Z_3^* q_{01}^*\left(1 - q_{56}^* q_{65}^*\right)\left(q_{12}^* q_{23}^{(4)*} + q_{13}^*\right) \tag{16}$$

and

$$N_3(s) = -q_{01}^* q_{35}^*\left(q_{13}^* + q_{12}^* q_{23}^{(4)*}\right) Z_5^*$$

and $D_1(s)$ is same as defined by the expression (9) of section 6.2.

The steady state results for the above two probabilities are given by-

$$B_0^1 = \lim_{s\to 0} s\ B_0^{1*}(s) = N_2 \setminus D_1' \quad \text{and} \quad B_0^2 = \lim_{s\to 0} s\ B_0^{2*}(s) = N_3 \setminus D_1' \tag{17-18}$$

Where,

$$N_2(0) = \psi_1\left[(1 - p_{56}p_{65})(1 - p_{34}) + p_{35}\left(p_{56}p_{63}^{(4)} + p_{53}\right)\right] + \psi_3\left(1 - p_{56}p_{65}\right)\left(p_{12}p_{23}^{(4)} + p_{13}\right) \tag{19}$$

$$N_4(0) = -p_{35}\left(p_{13} + p_{12}p_{23}^{(4)}\right)\psi_5 \tag{20}$$

and D_1' is same as given in the expression (12) of section 6.2.

The expected busy period in repair of unit-1 failed due to first repair with priority of unit-1 and unit-2 failed due to second repair during time interval (0, t) are respectively given by-

$$\mu_b^1(t) = \int_0^t B_0^1(u)\,du, \qquad \text{and} \qquad \mu_b^2(t) = \int_0^t B_0^2(u)\,du$$

So that,

$$\mu_b^{1*}(s) = \frac{B_0^{1*}(s)}{s} \qquad \text{and} \qquad \mu_b^{2*}(s) = \frac{B_0^{2*}(s)}{s} \tag{21-22}$$

7. Profit Function Analysis

The net expected total cost incurred in time interval (0, t) is given by

P (t) = Expected total revenue in (0, t) - Expected cost of repair in (0, t)

$$= K_0\mu_{up}(t) - K_1\mu_b^1(t) - K_2\mu_b^2(t) \tag{23}$$

Where, K_0 is the revenue per- unit up time by the system during its operation. K_1 and K_2 are the amounts paid to the repairman per-unit of time when the system is busy in repair of unit-1 failed due first repair with priority of unit-1 and unit-2 failed due to second repair respectively.

The expected total profit incurred in unit interval of time is $P = K_0A_0 - K_1B_0^1 - K_2B_0^2$

8. Particular Case

Let, $\overline{G}(t) = e^{-\lambda t}$

In view of above, the changed values of transition probabilities and mean sojourn times.

$$p_{21} = \frac{1}{\lambda + \alpha_2(1-r_2)}, \qquad p_{65} = \frac{1}{\lambda + \alpha_1(1-r_1)}$$

$$p_{23}^{(4)} = 1 - \frac{1}{\lambda + \alpha_2(1-r_2)}, \qquad p_{63}^{(4)} = 1 - \frac{1}{\lambda + \alpha_1(1-r_1)}$$

$$\psi_2 = \frac{1}{\lambda + \alpha_2(1-r_2)}, \qquad \psi_6 = \frac{1}{\lambda + \alpha_1(1-r_1)}$$

9. Graphical Study of Behaviour and Conclusions

For a more clear view of the behaviour of system characteristics with respect to the various parameters involved, we plot curves for MTSF and profit function in Fig. 2 and Fig. 3 w.r.t. $\alpha_{1=}$for three different values of correlation coefficient β_1=0.15, 0.45, 0.85 and two different values of repair parameter r_2 =0.8, 0.9 while the other parameters are θ =0.9,=$\beta_{2=}$=0.4, r_1 =0.7, $\alpha_{2=}$=0.8, λ=0.9. It is clearly observed from Fig. 2 that MTSF increases uniformly as the value of $\alpha_{1=}$and $r_2$increase and it decrease with the increase in $\alpha_{1=}$. Further, to achieve MTSF at least 80 units we conclude for smooth curves that the values of $\alpha_{1=}$must be less than 0.1, 0.13 and 0.4 respectively for β_1=0.15, 0.45, 0.85 when r_2 =0.8. Whereas from dotted curves we conclude that the values of $\alpha_{1=}$must be less than 0.1, 0.11 and 0.22 for β_1=0.15, 0.45, 0.85 when r_2 =0.9.

Similarly, Fig.3 reveals the variations in profit (P) with respect to $\boldsymbol{\alpha_1}$for three different values of β_1=0.15, 0.45, 0.75 and two different values of r_2 =0.01, 0.03, when the values of other parameters θ =0.8,=$\beta_{2=}$=0.98, r_1 =0.3, $\alpha_{2=}$=0.6, λ=0.8, K_0=95, K_1=250 and K_2=225. Here also the same trends in respect of α_1 , β_1 and r_2 are observed in case of MTSF. Moreover, we conclude from the smooth curves that the system is profitable only if α_1 is less than 0.1, 0.19 and 0.59 respectively for β_1 =0.15, 0.45, 0.75 when r_2 =0.01. From dotted curves, we conclude that the system is profitable only if α_1 is less than 0.1, 0.25 and 0.32 respectively for β_1 =0.15, 0.45, 0.75 when r_2 =0.03.

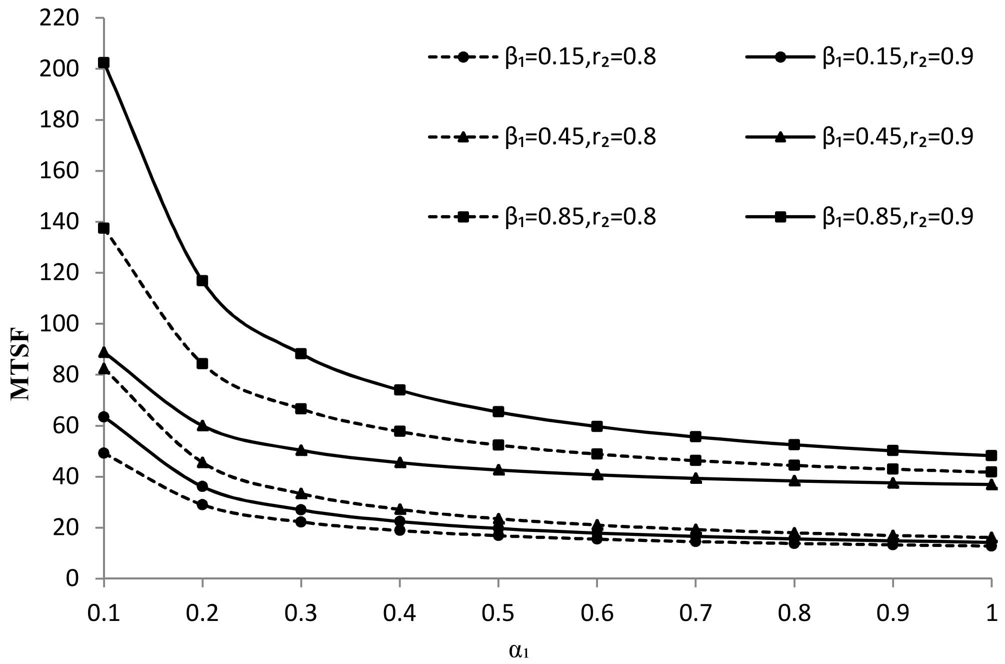

Figure 2: *Behaviour of MTSF with respect to α_1, β_1 and r_2*

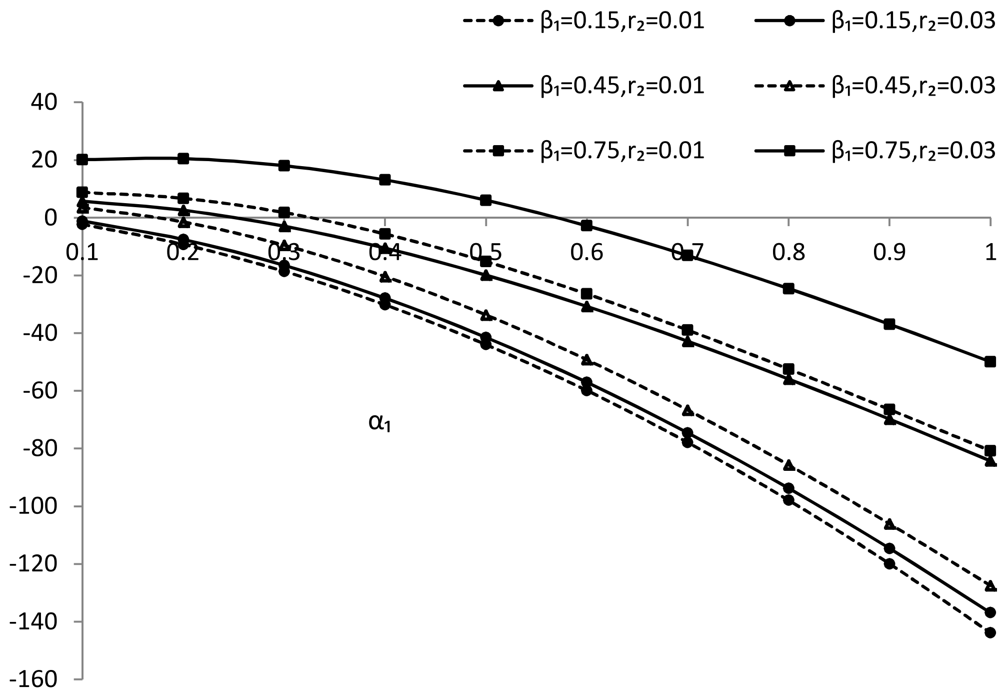

Figure 3: *Behaviour of PROFIT (P) with respect to α_2, β_1 and r_2*

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