PRICE RISK ANALYSIS USING GARCH FAMILY MODELS: EVIDENCE FROM INDIAN NATIONAL STOCK EXCHANGE FUTURE MARKET

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Abstract

The prediction of time-varying volatility plays an important role in financial data. In the paper, a comprehensive analysis of the mean return and conditional variance of NSE index is performed to use GARCH, EGARCH and TGARCH models with Normal innovation and Student's t innovation. Conducting a bootstrap simulation study which shows the Model Confidence Set (MCS) captures the superior models across a range of significance levels. The experimental results show that, under various loss functions, the GARCH using Student's t innovation model is the best model for volatility predictions of NSE among the six models.

Keywords: time-varying volatility, NSE index, bootstrap simulation, GARCH-type models.

1. Introduction

Forecasting market risk is a widely studied subject that has captured the interest of scholars due to its highly non-linearity and volatility. Thus, several approaches use these data for model testing. Generalized auto regressive conditional heteroscedasticity (GARCH) models are one of the most used models to study volatility. Although response to these models is generally good, they are unable to successfully capture extreme changes in the complete time series. Due to this shortcoming, one of the focuses of research has been to work on alternatives that can better approximate the non-linear part of the series mainly using GARCH, such as Markov Switching GARCH. This algorithm has been used extensively in stock markets and for market risk, because it is able to theoretically approximate any non-linear function with minimal error. In practice, the use of MSGARCH allows us to improve forecasting systems as well as forecasts from econometric models with excellent results.

Multi-period volatility forecasts feature prominently in asset pricing, folio allocation, risk-management, and most other areas of finance where horizon measures of risk are necessary. Such forecasts can be constructed quite different ways. The first approach is to estimate a horizonspecific of the volatility, such as a weekly or monthly GARCH, that can then form direct predictions of volatility over the next week, month, etc. approach is to estimate a daily model and then iterate forward the daily to obtain weekly or monthly predictions. The forecasting literature refers first approach as "direct" and the second as "iterated". A third method mixed-data sampling (MIDAS) approach introduced by [1]. A MIDAS model uses, for example, daily returns to produce directly multi-period volatility forecasts and can as a middle ground between the direct and the iterated approaches. Volatility literature (see [2]) has mostly focused regressions-based models. It is the purpose of this paper to introduce ideas similar to MIDAS models in GARCH-type models. The advantages of this are that one focuses directly on multi-period forecasts, as in the direct while one preserves the use of high-frequency.

We propose a unifying framework, based on a generic GARCH-that addresses the issue of volatility forecasting involving forecast a different frequency than the information set. Hence, we propose GARCH models that can handle volatility forecasts over the next days and use past daily data, or tomorrow's expected volatility using intra-daily returns. We call the class of models High Frequency Based Projection-Driven GARCH models as the GARCH dynamics by what we call HYBRID processes. HYBRID-GARCH models - nature - relate to many topics discussed in the extensive literature forecasting. These topics include - but are not limited to - iterated forecasting, temporal aggregation, weak versus semi-strong GARCH, and various estimation procedures.

Exchange rates are a relevant topic of study because they serve as indicators of economic competition between nations and also because commercial relationships between countries are regulated by the value of competing currencies. In the past, the value of an exchange rate was set by the economic authorities of each nation based on monetary policy. However, since 1971, the world economy has changed and currently many countries follow a regime where parities are determined based on the supply and demand of the foreign exchange market, making the exchange rate market more volatile and less predictable. Since then, forecasting the variation in exchange rates has been a matter of interest for the decision-making bodies of government entities, banks, insurers, investors or people who trade with parities. Studying these changes poses several challenges such as determining which variables are relevant for a given currency or which method is superior to another for forecasting. In this sense, the use of time series models to model economic variables has been broad such as Autoregressive Moving Average (ARMA) and its derivatives, which include Vector Autoregressive model (VAR), the Vector Error Correction model (VECM), Cointegration model and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. The remaining paper is organized as follows: In section 2, we present the econometric models. In section 3, we summarize the descriptive statistics of NSE sector. In section 4, we present the estimation procedures. In section 5, we describe the MCS test based on the bootstrap simulation. In section 6, concludes.

ARCH (Autoregressive Conditional Heteroskedasticity) and Generalized ARCH (GARCH) models have emerged as the most proeminent tools for estimating volatility, because they are adequate to capture the random movement of the financial data series. Many researchers have studied over time the performance of GARCH models on explaining volatility of mature stock markets, but only a few have tested GARCH models using daily data from Central and Eastern European stock markets (see, for example, [3],[4],[5],[6],[7],[8]). The focus of our paper is on forecasting stock market volatility in Romania, a market which has not been thoroughly investigated.

Several studies results have confirmed that asymmetric GARCH-models fit better stock markets returns volatility for emerging CEE countries. Lupu [9] found that an EGARCH (Exponential GARCH) model is suitable for the logarithmic returns of the Romanian composite index BET-C covering the period 03/01/2002- 17/11/2005. Furthermore [10] employed different asymmetric GARCH-family models (EGARCH, PGARCH, and TGARCH) using U.S. and Romanian daily stock return data corresponding to the period 2002-2010. They found that volatility estimates given by the EGARCH model exhibit generally lower forecast error and are therefore more accurate than the estimates given by PGARCH and TGARCH models. [11] examines the presence of volatility at the Karachi Stock Exchange(KSE) through the use of Autoregressive Conditional Heteroskedasticity(ARCH)and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models introduced by Engle (1982), Bollerslev (1986) and Nelson (1991). The empirical result confirms the presence of high volatility at Karachi Stock Exchange throughout the study period. The volatility was found in clustering and stochastic manner. The results of GARCH analysis show a random-walk behavior so market can be termed as very uncertain and very risky for short-term and medium-term investors.

2. Material and Methods

The volatility of a stock price can be used as an indicator of the uncertainty of stock returns. In a financial market, volatility is measured in terms of standard deviation σ or σ2 compute variance from a set of observations as follow:

$$\sigma^2 = \frac{1}{n-1}\sum_{t=1}^{n}(y_t - \bar{y})^2 \tag{1}$$

here $\bar{y}$ and y_t are the mean return and return, respectively. Return is defined to be the total gain or loss from an investment over a given period of time. In this paper, we compute the daily closing prices are as

$$y_t = 10\log\left(p_t / p_{t-1}\right) \tag{2}$$

where p_t is stock closed price at time t. Then prices are converted into logarithmic returns, y_t denotes p_t the continuously compounded daily returns of the underlying assets at time t. We assume that the conditional mean equation of stock return is constructed as the constant term plus residuals error

$$y_t = \mu + \varepsilon_t, \varepsilon_t = \sigma_t z_t \tag{3}$$

where $\{z_t\}$ is a sequence of independent identically distributed random variables with zero mean and unit variance, is the conditional variance of derived from mean equation, it is also known as current day's variance or volatility. Larger implies higher volatility and higher risk.

2.1. Parametric models GARCH (1, 1) is written as following is

The standard variance model for financial data is GARCH. GARCH assumes a Gaussian observation model and a linear transition function for the variance: the time-varying variance σ_t^2 is linearly dependent on p previous variance values and q previous squared time series values, so

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{q}\alpha_j + \varepsilon_{t-j}^2 + \sum_{i=1}^{p}\beta_i\sigma_{t-i}^2 \tag{4}$$

Hence, GARCH (1,1) is defined as $\sigma_t^2 = \alpha_0 + \sum_{j=1}^{q}\alpha_j + \varepsilon_{t-j}^2 + \sum_{i=1}^{p}\beta_i\sigma_{t-i}^2$ Where $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta < 1$. First, $\omega > 0$ means that volatility cannot have a zero or negative mean. Second, the positive parameters α, β show that the conditional variance forecasts will increase if there is a large fluctuation in returns, the model thus capturing the stylized feature of volatility clustering. Finally, $\alpha + \beta < 1$ indicates the persistence of shocks to volatility will eventually fade away, which depicts another stylized characteristic of volatility, mean reversion.

2.2. Exponential-GARCH (EGARCH) (1,1) is defined as following is

A more flexible and often cited GARCH extension is Exponential GARCH (EGARCH) (Hamilton, 1994). The default EGARCH (p, q) model in Econometrics is of the form: $\varepsilon_t = \sigma_t z_t$ with Normal innovation or Student's t innovation distributions and

$$\log(\sigma_t^2) = \omega + \beta\log(\sigma_{t-1}^2) + \alpha[\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - E(\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}})] + r(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}) \tag{5}$$

where the parameter α captures the volatility clustering effect and the r measures the leverage effect. The conditional variance is in logarithmic form, which implies that the model has the following features: first, σ_t^2 will always be positive regardless of the sign of the parameters, therefore no constraints of non-negativity are needed. Second, the asymmetrical effect is not quadratic but exponential, if $r < 0$ it indicates a leverage effect. EGARCH model allows good news and bad news to have a different impact on volatility because the level of is included $\frac{\varepsilon_{t-1}}{\sigma_{t-1}}$ with a coefficient r.

2.3. MCS test based on bootstrap simulation:

When obtaining the predicted values, we can compare it with the real Proxy variables of a volatility deviation size. However, the loss of function which is used to measure the prediction error is no consensus. The paper uses two loss functions: mean square error and mean absolute deviation to measure the forecasting error. It is not easy to choose the best model which is always the best under all loss function or all data samples. Since, [12] offers some resolution of this quandary, the metric for assessing the forecasts of volatility models is the Bootstrap method of superior predictive ability (SPA) test. But use SPA test, we must need to choose the basic model, it is very vital to choose it which can affect the result. In order to overcome the defects of SPA test, the paper use the MCS test which is a modified version of SPA test.

2.4. MCS test procedure

We define a set of models which are denoted by $M_0 = \{1,\ldots,m\}$, the models are indexed by $i = 1, \ldots, m$, and model is forecasts of σ_t^2 is denoted by $h_{i,t}^2$, We rank models according to their expected loss using one of two loss functions: MSE, $L(h_{i,t}^2, \sigma_t^2) = (h_{i,t}^2 - \sigma_t^2)^2$ and $L(h_{i,t}^2, \sigma_t^2) = \left|h_{i,t}^2 - \sigma_t^2\right|$. The loss differential between models i and j, is given by $d_{ij,t} = L(h_{i,t}^2, \sigma_t^2) - L(h_{j,t}^2 - \sigma_t^2)\ i, j = 1,\ldots,m, t = 1,\ldots,n$ The MCS is determined after sequentially trimming the set of candidate models, M_0. At each step, the hypothesis

$$H_0 : E(d_{ij,t}) = 0, \text{for all } i, j\hat{I}M\grave{I}M_0 \tag{6}$$

The hypothesis, H_0, is a test for (Equal Predictive Ability) EPA over the models in M and if H_0 is rejected, the worst performing model is eliminated from M. The trimming ends when the first non-rejection occurs. The set of surviving models is the model confidence set $\widehat{M}_\alpha^*$, By holding the significance level,α, fixed at each step of the MCS procedure, we construct a $(1 - \alpha)$-confidence set, $\widehat{M}_\alpha^*$,for the best models in M_0. However, the trimming model which is mentioned in the sequential inspection have a drawback. At each step in the test, we need to test the predictive power of any two prediction models and calculate a test statistic. To overcome this drawback, our tests for EPA employ the rang statistic, T_R, and the semi-quadratic statistic, T_{sq} given by

$$T_R = \max_{i j \hat{I} M} \frac{\left|\sum_{i<j}(\bar{d}_{ij})\right|}{\sqrt{\hat{v}(\bar{d}_{ij})}} \qquad T_{sq} = \frac{\sum_{i<j}(\bar{d}_{ij})^2}{\mathrm{var}(\bar{d}_{ij})} \tag{7}$$

Where the sum is taken over the models in M, and $\hat{v}(\bar{d}_{ij})$ is an estimate of $v(\bar{d}_{ij})$.Both of the test statistic value is larger, it means rejecting the EPA hypothesis. In fact, their distribution is very complicated, and the covariance structure depends on the predictive value of each prediction model. So, the paper uses a bootstrap simulation study to find the p-value of the two statistics.

3. Data and Experimental Results

The whole sample consists of 2537 daily data spanning from 4 Jan. 2010 to 16 Mar. 2023, we select subsample of size 2000, dated from 4 Jan. 2010 to 24 Feb. 2023, as the training set for the parameter's estimation for models and the remaining sample of size 537 daily data, from 25 Feb. 2022 to 16 Mar. 2023 is used as the test set or for out of sample forecasting.

Table 1: *Summary statistics of NSE*

		Descriptive statistics	
Sample	2575	Mean	9.844
Std. dev	8.693	Skewness	0.237
Kurtosis	-1.44	JB	243.87

Then we need to calculate logarithmic returns $y_t = 10\log(p_t / p_{t-1})$. Table 1 summarizes the descriptive statistics of NSE index throughout the whole period. Table 1 remarks that these facts suggest a highly competitive and volatile mark. The Skewness is 0.2371853 > 0, the positive skewness indicates that there is a high probability of gain in the market. The value of the Kurtosis is -1.444026 > 3, it suggests that the market is volatile with high probability of extreme events occurrences. The JB.test is 243.87 which shows that the returns deviate from normal distribution significantly and exhibit leptokurtic. Hence the distribution of the index is not the normal distribution, and it has the feature of asymmetric, zero mean and left side.

Table 1: *Unit Root Test of NSE*

Test	Critical value	P.value	alternative hypothesis
ADF test	-13.512	0.01	Stationary
KPSS test	0.065	0.01	Stationary
PP test	-3225.2	0.01	Stationary

The table 2, reports the unit root tests of the NSE. The Augmented Dickey and Fuller-ADF test for the null of non – stationary. Critical value -13.512.KPSS indicates the Kwiatkowski, Phillips, Schmidt and shin test for the null of stationary. Critical value: 0.065915. PP. test indicates the Phillips-Perron test for the null of non- hypothesis. Critical value -3225.2 it means that the series yt is stationary time series.

3.1. Detecting ARCH effects of NSE returns

From the Fig. 1, we can see that the returns appear to fluctuate around a constant level but exhibit volatility clustering. Large changes in the returns tend to cluster together, and small changes tend to cluster together. So, the preliminary judgment shows that the series exhibits the conditional heteroscedasticity. Now the paper use ARCH-LM to detect whether NSE returns have ARCH effects.

According to the heteroskedasticity test ARCH, the value of F-statistic is 0.004301 and the probability 0.0002< 0.05, R-squared = 9.444 and Adjusted R-squared = 0.9443, the probability is 0.0002 < 0.05, and the number of lags is 1, the test of the residuals for ARCH(1) rejects the null hypothesis of no conditional heteroskedasticity, so it is clear that NSE returns have ARCH effects. Then we can use GARCH-type models to forecast the volatility.

4. Estimation result

Apply the return series to the GARCH and EGARCH models with Normal innovation and Student's innovation, and then we get their parameters. The estimation results and diagnosis are shown in Table 2 and Table 3.

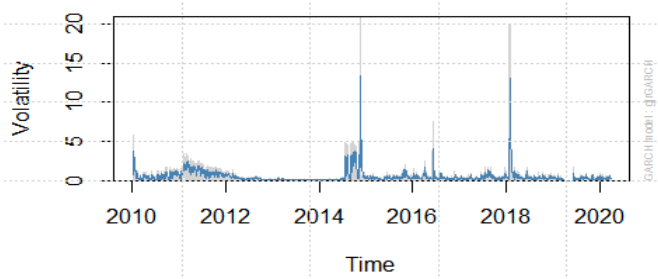

Figure 1. *Daily return and volatility of NSE*

Table 2. *Estimation results by GARCH models*

Statistics	GARCH-T				GARCH-N			
	Parameter	St. Error	t-value	Pr(>|t|)	parameter	St. Error	t-value	Pr(>|t)
mu	-0.002	0.002	-1.104	0.020*	0.023	0.0313	-1.104	0.047*
Omega	0.0008	0.000	4.636	0.000***	0.004	0.025	4.636	0.025*
alpha1	0.114	0.025	4.431	0.000***	0.214	0.356	4.431	0.049*
beta1	0.871	0.24	3.596	0.000***	0.375	0.242	3.039	0.000***
beta2	0.000	0.215	0.6367	0.034*	0.003	0.215	0.580	0.040*
Information Criteria:	Log Likelihood : 476.0085				Log Likelihood : 485.2743			
Akaile	-0.840				-0.740			
Bayes	-0.790				-0.696			

Note. *,**, and *** denotes level of significant at 10%, 5% and 1%, respectively.

Table 3. *Estimation results by EGARCH models*

Statistics	EGARCH-T				EGARCH-N			
	Parameter	St. Error	t-value	Pr(>|t|)	Parameter	St. Error	t-value	Pr(>|t|)
mu	0.036	0.038	0.966	0.333	0.836	0.166	5.027	0.000
ar1	0.982	0.003	2.482	0.000	0.999	0.000	1.512	0.000
ma1	-0.746	0.038	-2 .379	0.000	-0.820	0.019	-4.319	0.000
Omega	-0.027	0.004	-5.884	0.000	-0.012	0.001	-9.364	0.000
alpha1	0.002	0.023	0.101	0.919	0.106	0.021	4.980	0.000
alpha2	0.060	0.031	1.942	0.052	-0.030	0.015	-1.951	0.050
beta1	0.998	0.001	7.618	0.000	0.997	0.000	1.4290	0.000
gamma1	0.493	0.069	7.136	0.000	0.342	0.050	6.799	0.000
gamma2	-0.231	0.074	-3.096	0.001	-0.272	0.049	-5.539	0.000

Information Criteria:	Log Likelihood : 476.0085	Log Likelihood : 485.2743
Akaike	-0.840	-0.740
Bayes	-0.790	-0.696

Note. *,**, and *** denotes level of significant at 10%, 5% and 1%, respectively.

Among the parametric models, with Normal innovation and Student's t innovation, In GARCH-N, the value of LL is 419.724, AIC is -1.989 and BIC -1.063, each parameter is significant. In EGARCH-N, the value of LL is 485.2743, AIC is −0.740 and BIC is −0.863, each parameter is significant. In GARCH-T, the value of LL is 403.819, AIC is -0.929 and BIC is -0.863, each parameter is significant. In EGARCH-T, the value of μ, α1 is not significant. In EGARCH-N, the value of α1 also are not significantly. Hence according to highest value of Log Likelihood (LL) and smallest value of AIC and BIC. Hence the series best fit is EGARCH-N.

4.1. The MCS test results

The Table 4 shows the MCS test results by using bootstrap simulation at 1000 times. Figures in the table represent MCS test p-value. When greater, *p*-value indicates that they more reject the null hypothesis. The paper sets a basis p-value which is p = 0.1. If p-value is less than 0.1, then the volatility forecasting model is poor. So, the model will be removed in the MCS inspection process. Conversely, it survives in MCS.

Table 4. *MCS test results of realized volatility models*

Model	MSE		MAD	
	T_R	T_{sq}	T_R	T_{sq}
GARCH-N	0.161	0.185	0.136	0.042
GARCH-T	0.211	0.231	0.191	0.152
EGARCH-N	0.035	0.052	0.064	0.042
EGARCH-T	0.021	0.035	0.033	0.053

So, the model will be removed in the MCS inspection process. Conversely, it survives in MCS. According to the table, when the loss function is the MSE, the p-values of in the GARCH-N and GARCH-T models are more than 0.1. But other 2 models are less than 0.01. It means that EGARCH-N, EGARCH-T, volatility forecasting models will be removed in the MCS inspection process. Considering the loss function for MAD, we find that only the p-value of GARCH-T model is more than 0.1. Hence using the loss function of MSE and MAD, we find that the value of T_R T_{sq} in the GARCH-T model are more than 0.1. Therefore, GARCH-T model is the best one.

V. Conclusion

The study uses NSE prices to predict daily volatility changes in the stock market. First, we use descriptive statistics to show that the index series has the feature of asymmetric zero mean and left side, it is not the normal distributed. Second, we consider Dickey-Fuller Unit Root Tests to find the series is stationary time series. And then using ARCH-Lagrange multiplier to detect NSE returns have ARCH effects. In this study, NSE index volatility models are estimated with Normal innovation and Student's t innovation distributions to find the effect of distribution selection on forecasting performance of the models. According to highest value of Log likelihood (LL) and smallest value of AIC and BIC, the result suggests that the GARCH model with Student's t innovation enables more accurate forecasting than EGARCH.

The paper use MCS test to find the best model. Under the evaluation criteria of loss functions

MSE and MAD, the empirical results show that GARCH-T model is the best model for forecasting volatility. Although the prediction's results represent that GATCH-T model is not so good, it can be used as an assistant tool in financial applications. The study has also multiple significantly: first, the stock index futures in favor of investors to make rational investment decisions in advance. Second, it helps to improve risk management of institutional and individual investors. Finally, there is conducive to the development of relevant policies and regulatory authorities to improve supervision.

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